

Complex Numbers

The imaginary unit: i

Pure imaginary numbers:

Square roots of negative numbers:

$$\sqrt{-27}$$

$$\sqrt{-18}$$

$$\sqrt{-125}$$

Powers of i

Definitions: $\sqrt{-1} = i$
 $i^2 = -1$

Look for the pattern that is formed when i is raised to the powers 1, 2, 3, 4, and so on.

i^1	$= i$	$= i$	$= i$
i^2	$= -1$	$= -1$	$= -1$
i^3	$= i^2 i$	$= (-1)i$	$= -i$
i^4	$= i^2 i^2$	$=$	$= 1$
i^5	$= i^4 i$	$=$	$=$
i^6	$=$	$=$	$=$
i^7	$=$	$=$	$=$
i^8	$=$	$=$	$=$
i^9	$=$	$=$	$=$
i^{10}	$=$	$=$	$=$
i^{11}	$=$	$=$	$=$
i^{12}	$=$	$=$	$=$

Now simplify:

1. i^{-17}	2. i^{22}	3. i^{38}
4. i^{40}	5. i^{67}	6. i^{90}
7. i^{152}	8. $3i \cdot 6i^3$	9. $2i \cdot (4i^3)^2$

For any positive real number b ,

$$\sqrt{-(b^2)} = \sqrt{b^2} \cdot \sqrt{-1} = bi$$

Example:

$$\begin{aligned}\sqrt{-24} &= \sqrt{24} \cdot \sqrt{-1} \\ &= \sqrt{4 \cdot 6} \cdot i \\ &= 2i\sqrt{6}\end{aligned}$$

Practice

Simplify

10. $\sqrt{-256}$	11. $\sqrt{-80}$	12. $\sqrt{-4} \cdot \sqrt{-9}$
13. $\sqrt{\frac{25}{121}}$	14. $\sqrt{-6} \cdot \sqrt{-12}$	15. $(7-3i)(8+4i)$
16. $(-3\sqrt{-5})^2$	17. $(2\sqrt{-8})(3\sqrt{-2})$	18. $(2i \cdot 3i^2)^2$
19. $(3i)^3 \cdot (2i)^2$	20. $-\sqrt{15} \cdot \sqrt{-15}$	21. $(\sqrt{-13})^2$

Equate complex numbers
set real parts equal to each other
and
set imaginary parts equal to each other

Find the value of x and y that make the
equation true: $2x + yi = 14 - 3i$

Square Root Property

Solve: $x^2 + 36 = 0$

Solve $5y^2 + 20 = 0$

Complex Number $a + bi$,
where a and b are real numbers,
 i is the imaginary unit,
 a is called the real part, and
 bi is called the imaginary part

Add and Subtract Complex Numbers

$$(3 + 5i) + (2 - 4i)$$

Multiply Complex Numbers

In an AC circuit, the voltage E , current I and impedance Z are related by the formula $E = I(Z)$

Find the voltage in a circuit with current $1 + 4j$ amps and impedance $3 - 6j$ ohms.

Divide Complex Numbers