Product Property of Radicals

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

Example $\sqrt{18}$ $\sqrt[3]{24}$

$\sqrt{9}\sqrt{2}$ $\sqrt{4}\sqrt{6}$

$3\sqrt{2}$ $2\sqrt{6}$

To find the Root of a Product

1. Factor the radicand into as many "squares" as possible
2. Use the product property to isolate the perfect "squares"
3. Simplify

Guided Practice:

1A. $\sqrt{12c^6d^3}$ $2c^3d\sqrt{3d}$ B. $\sqrt[3]{27y^{12}z^7}$ $3y^4z^2\sqrt[3]{z}$

Quotient Property of Radicals

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Radicals must be eliminated from the denominator of expressions. To do this, the denominator must be RATIONALIZED. This means to multiply the numerator and denominator by a quantity so that the radicand in the denominator has an exact root.

Example: $\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \rightarrow \frac{2\sqrt{3}}{\sqrt{9}} = \frac{2\sqrt{3}}{3}$

OR $(\sqrt{3})^2 = 3$

Guided Practice (answer on next page)

2A. $\dfrac{\sqrt{a^9}}{\sqrt{b^5}}$

2B. $\sqrt[5]{\dfrac{3}{4y}}$

Answer

2A.
$$\frac{\sqrt{a^9}}{\sqrt{b^5}} = \frac{\sqrt{a^8}\sqrt{a^1}}{\sqrt{b^4}\sqrt{b}} = \frac{a^4\sqrt{a}\sqrt{b}}{b^2\sqrt{b}\sqrt{b}} = \frac{a^4\sqrt{ab}}{b^3}$$

$$\underbrace{\sqrt{b}\sqrt{b}}_{(\sqrt{b})^2} = b$$

2B.
$$\sqrt[5]{\frac{3}{4y}} = \frac{\sqrt[5]{3}}{\sqrt[5]{2^2 y}} \cdot \frac{\sqrt[5]{2^3 y^4}}{\sqrt[5]{2^3 y^4}} = \frac{\sqrt[5]{3 \cdot 2^3 y^4}}{\sqrt[5]{2^5 y^5}} = \frac{\sqrt[5]{24y^4}}{2y}$$

A radical expression is in SIMPLIFIED FORM when the following conditions are met:

1. The index n is as small as possible
2. The radicand contains no factors that are nth powers of an integer or polynomial
3. The radicand contains no fractions
4. No radicals appear in a denominator

Operations with Radicals

Multiply and Divide

use the product and quotient properties

Example 3

GuidedPractice

Simplify.

3A. $6\sqrt{8c^3d^5} \cdot 4\sqrt{2cd^3}$ **$96c^2d^4$**

3B. $2\sqrt[4]{8x^3y^2} \cdot 3\sqrt[4]{2x^5y^2}$ **$12x^2|y|$**

$24\sqrt{16c^4d^8}$

Operations with Radicals

Add and Subtract

in order to add and subtract, radicals must be "like" radicals - ***both index and radicand are identical***

Example 4

GuidedPractice

4A. $4\sqrt{8} + 3\sqrt{50}$ **$23\sqrt{2}$**

4B. $5\sqrt{12} + 2\sqrt{27} - \sqrt{128}$ **$16\sqrt{3} - 8\sqrt{2}$**

Radicals in binomials can be multiplied using the FOIL method:

Example 5

GuidedPractice

Simplify.

5A. $(6\sqrt{3} - 5)(2\sqrt{5} + 4\sqrt{2})$ $12\sqrt{15} - 10\sqrt{5} + 24\sqrt{6} - 20\sqrt{2}$

5B. $(7\sqrt{2} - 3\sqrt{3})(7\sqrt{2} + 3\sqrt{3})$ 71

Binomials in the form $a\sqrt{b}+c\sqrt{d}$ and $a\sqrt{b}-c\sqrt{d}$ are called ***CONJUGATES*** of each other.
You can use conjugates to rationalize denominators containing radical binomials.
Multiply the following pairs of conjugates:

$(2+\sqrt{3})(2-\sqrt{3})$

$4-2\sqrt{3}+2\sqrt{3}-\sqrt{9}$

$4-3$

1

So the product of conjugates is always a ___________________ number.

Example 6

GuidedPractice

6A.

multiply by the conjugate $\sqrt{6}+5$

$$\left(\frac{8}{\sqrt{6}-5}\right)\frac{(\sqrt{6}+5)}{(\sqrt{6}+5)}$$

$$\frac{8\sqrt{6}+40}{6+5\sqrt{6}-5\sqrt{6}-25}$$

$$\frac{8\sqrt{6}+40}{-19}$$

6B.

$$\frac{3-\sqrt{2}}{10+\sqrt{6}}$$

Operations with Radicals

To Multiply Radicals

1. Factor the radicands if possible
2. Use the product property to isolate the perfect "squares"
3. Multiply the factors outside the $\sqrt{\ }$ and the factors left inside the $\sqrt{\ }$

Example: $-2\sqrt{15} \cdot 4\sqrt{21}$

Example: $4\sqrt{3x} \cdot \sqrt{2x^3}$

Example: $\sqrt{2ab^2} \cdot \sqrt{6a^3b^2}$

$5\sqrt[3]{100a^2} \bullet \sqrt[3]{10a}$

To Add Radicals

1. Factor the radicands if possible
2. Use the product property to isolate the perfect "squares"
3. Combine LIKE radicals

like radicals: same index and same radicand

$\sqrt{3}$ $\sqrt[3]{3}$ $\sqrt[4]{5}$ $\sqrt[4]{10}$

$\sqrt{x}$ $\sqrt{x}$ $2\sqrt[3]{5a}$ $3\sqrt[3]{5a}$

SIMPLIFY radicals before looking for like terms

Example: Simplify

$$3\sqrt{45} - 5\sqrt{80} + 4\sqrt{20}$$

Example: Simplify

$$\sqrt[3]{128} + 5\sqrt[3]{16}$$

More Multiply Radicals

Now that you can add and subtract radicals, you can multiply polynomials with radicals

Example: Simplify

$$(2+\sqrt{3})(5-\sqrt{7})$$

Example: Simplify

$$(2\sqrt{3}+3\sqrt{5})(3-\sqrt{3})$$

Using a Conjugate to Rationalize Denominators

If a polynomial with a radical is in the denominator, you must use a CONGUGATE to rationalize

Conjugate is a binomial switching the operations
example: if the denominator is $(2+\sqrt{3})$

the conjugate is $(2-\sqrt{3})$

Using the conjugate eliminates the √ in the denominator !!

Example: Simplify $\dfrac{2+\sqrt{3}}{4-\sqrt{3}}$

Example: Simplify $\dfrac{2+\sqrt{5}}{5-\sqrt{x}}$