1 Polynomial Functions A **polynomial in one variable** is an expression of the form $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$, where $a_n \neq 0$, a_{n-1}, a_2, a_1, and a_0 are real umbers, and n is a nonnegative integer.

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Polynomial	Expression	Degree	Leading Coefficient
Constant	12	0	12
Linear	$4x - 9$	1	4
Quadratic	$5x^2 - 6x - 9$	2	5
Cubic	$8x^3 + 12x^2 - 3x + 1$	3	8
General	$a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$	n	a_n

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1A. $5x^3 - 4x^2 - 8x + \frac{4}{x}$

1B. $5x^6 - 3x^4 + 12x^3 - 14$

1C. $8x^4 - 2x^3 - x^6 + 3$

If you know an element in the domain of any polynomial function, you can find the corresponding value in the range.

The volume of air in the lungs during a 5-second respiratory cycle can be modeled by $v(t) = -0.037t^3 + 0.152t^2 + 0.173t$, where v is the volume in liters and t is the time in seconds. This model is an example of a polynomial function.

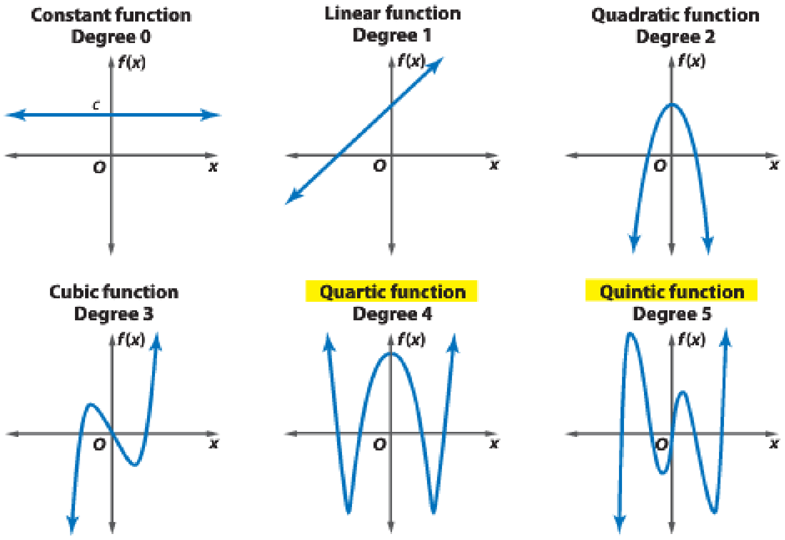
2. Find the volume of air in the lungs 4 seconds into the respiratory cycle.

You can also evaluate functions for variables and algebraic expressions.

Find $g(5a - 2) + 3g(2a)$ if $g(x) = x^2 - 5x + 8$.

Find $h(-4d + 3) - 0.5h(d)$ if $h(x) = 2x^2 + 5x + 3$.

2 Graphs of Polynomial Functions The general shapes of the graphs of several polynomial functions show the *maximum* number of times the graph of each function may intersect the x-axis. This is the same number as the degree of the polynomial.



The domain of any polynomial function is all real numbers. The **end behavior** is the behavior of the graph of $f(x)$ as x approaches positive infinity ($x \to +\infty$) or negative infinity ($x \to -\infty$). The degree and leading coefficient of a polynomial function determine the end behavior of the graph and the range of the function.

StudyTip

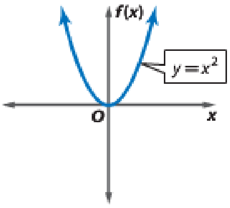
CCSS Sense-Making The leading coefficient and degree are the sole determining factors for the end behavior of a polynomial function. With very large or very small numbers, the rest of the polynomial is insignificant in the appearance of the graph.

KeyConcept End Behavior of a Polynomial Function

Degree: even
Leading Coefficient: positive
End Behavior:

$f(x) \to +\infty$ as $x \to -\infty$

$f(x) \to +\infty$ as $x \to +\infty$

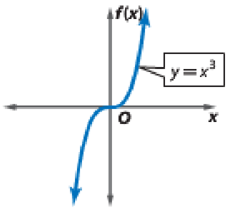


Domain: all real numbers
Range: all real numbers ≥ minimum

Degree: odd
Leading Coefficient: positive
End Behavior:

$f(x) \to -\infty$ as $x \to -\infty$

$f(x) \to +\infty$ as $x \to +\infty$

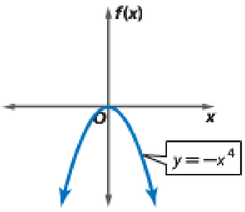


Domain: all real numbers
Range: all real numbers

Degree: even
Leading Coefficient: negative
End Behavior:

$f(x) \to -\infty$ as $x \to -\infty$

$f(x) \to -\infty$ as $x \to +\infty$

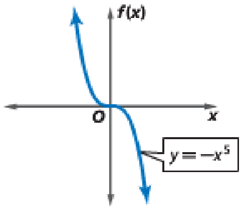


Domain: all real numbers
Range: all real numbers ≤ maximum

Degree: odd
Leading Coefficient: negative
End Behavior:

$f(x) \to +\infty$ as $x \to -\infty$

$f(x) \to -\infty$ as $x \to +\infty$



Domain: all real numbers
Range: all real numbers

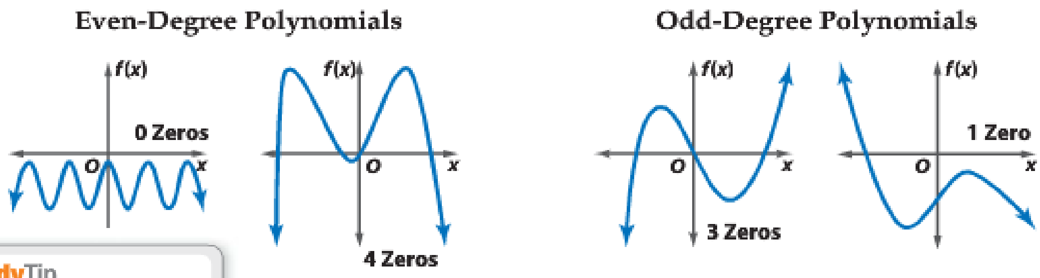
ReviewVocabulary

zero the *x*-coordinate of the point at which a graph intersects the *x*-axis

The number of real zeros of a polynomial function can be determined by examining its graph. Recall that real zeros occur at *x*-intercepts, so the number of times a graph crosses the *x*-axis equals the number of real zeros.

KeyConcept Zeros of Even- and Odd-Degree Functions

Odd-degree functions will always have an odd number of real zeros. Even-degree functions will always have an even number of real zeros or no real zeros at all.



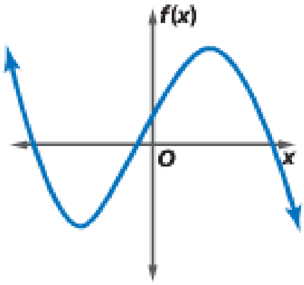
StudyTip

Double roots When a graph is tangent to the *x*-axis, there is a *double root*, which represents two of the same root.

For each graph,

- describe the end behavior,
- determine whether it represents an odd-degree or an even-degree polynomial function, and
- state the number of real zeros.

4A.



4B.