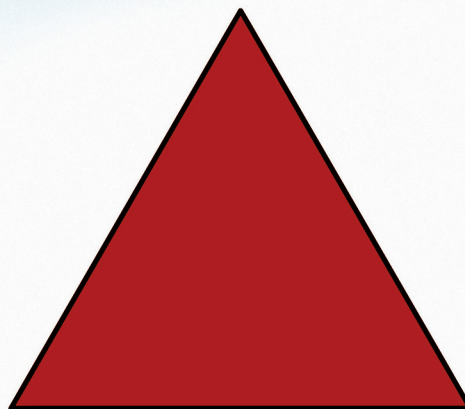


# Operations with Fractions

Look at the shapes at the right. How is each shape made from the previous shape? The equilateral triangle in *Step 1* has side length 1 unit. What is the perimeter of each shape?

What patterns do you see? Suppose the patterns continue. What will the perimeter of the shape in *Step 4* be?

For any shape after *Step 1*, how is its perimeter related to the perimeter of the preceding shape?



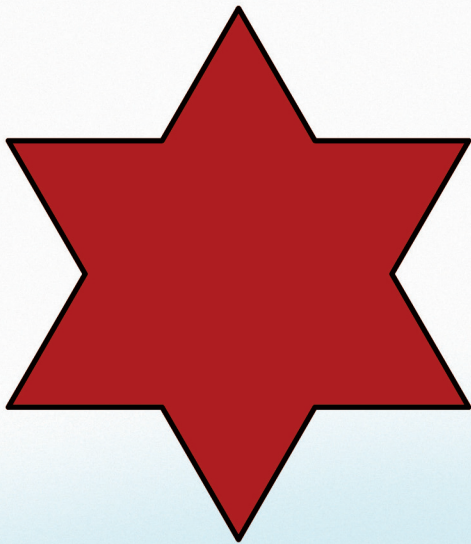
Step 1

## What You'll Learn

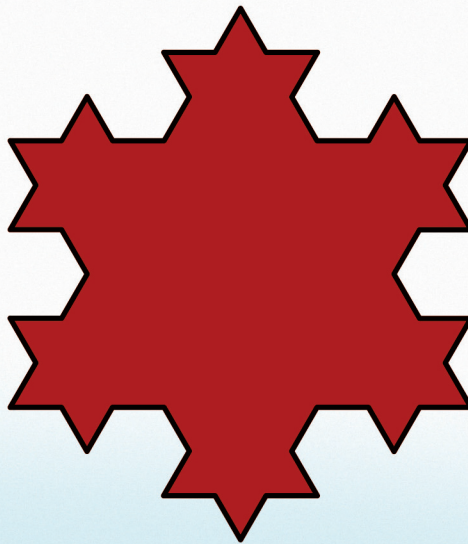
- Estimate the products and quotients of fractions and mixed numbers.
- Multiply a fraction by a whole number and by a fraction.
- Divide a fraction by a whole number and by a fraction.
- Multiply and divide mixed numbers.
- Use the order of operations with fractions.

## Why It's Important

You use fractions when you shop, measure, and work with percents and decimals. You also use fractions in sports, cooking, and business.



Step 2



Step 3

## Key Words

- product
- factor
- proper fraction
- equivalent fraction
- reciprocal
- mixed number
- improper fraction
- quotient
- divisor
- dividend

# 3.1

## Using Models to Multiply Fractions and Whole Numbers

**Focus** Use models to multiply a fraction by a whole number.

How many ways can you find this sum?

$$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$$

### Investigate

Work with a partner.

Use any models to help.

Each of 4 students needs  $\frac{5}{6}$  of a bag of oranges to make a pitcher of freshly squeezed orange juice.

Each bag of oranges contains 12 oranges.

How many bags of oranges are used?

### Reflect & Share

Compare your strategy for solving the problem with that of another pair of classmates.

Did you use addition to solve the problem?

If so, which addition expression did you use?

How could you represent the problem with a multiplication expression?



## Connect

We can find a meaning for:  $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{4}{5}$

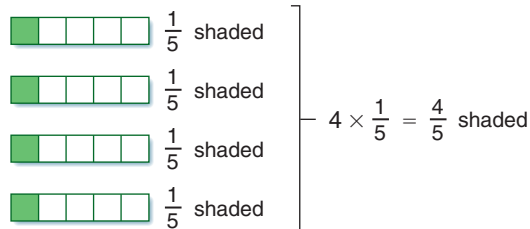
- Repeated addition can be written as multiplication.

$\frac{1}{5}$  is added 4 times.

$$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = 4 \times \frac{1}{5} = \frac{4}{5}$$

$$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{1}{5} \times 4 = \frac{4}{5}$$

We can show this as a picture.



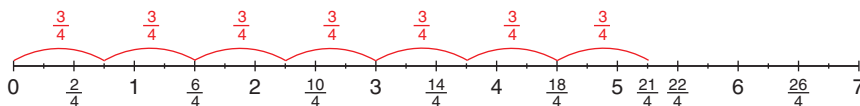
Similarly:  $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{21}{4}$

But,  $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = 7 \times \frac{3}{4} = \frac{21}{4}$

So,  $7 \times \frac{3}{4} = \frac{21}{4}$

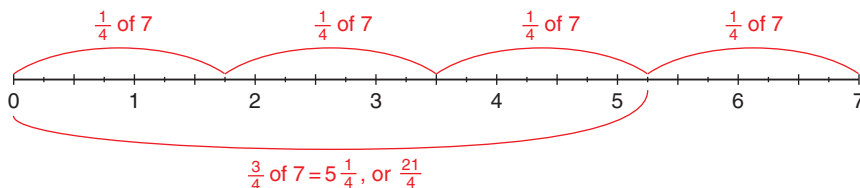
Also,  $\frac{3}{4} \times 7 = \frac{21}{4}$

- We can use a number line divided into fourths to show that  $7 \times \frac{3}{4} = \frac{21}{4}$ .



We can use a different number line to show that  $\frac{3}{4} \times 7 = \frac{21}{4}$ .

For  $\frac{3}{4} \times 7$ , we can think:  $\frac{3}{4}$  of 7



- Another way to multiply  $7 \times \frac{3}{4}$ :

Sketch a rectangle with base 7 units and height 1 unit.

Divide the height into fourths.

Shade the rectangle with base 7 and height  $\frac{3}{4}$ .

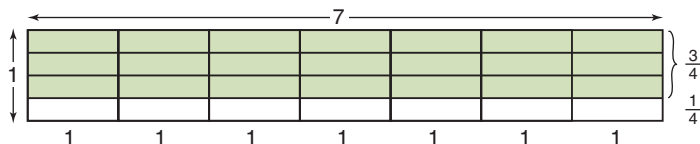
The area of the shaded rectangle is: base  $\times$  height =  $7 \times \frac{3}{4}$

Each small rectangle has area:  $1 \times \frac{1}{4} = \frac{1}{4}$

So, the area of the shaded rectangle is:  $21 \times \frac{1}{4} = \frac{21}{4}$

Then,  $7 \times \frac{3}{4} = \frac{21}{4}$

**This is a multiplication equation.**



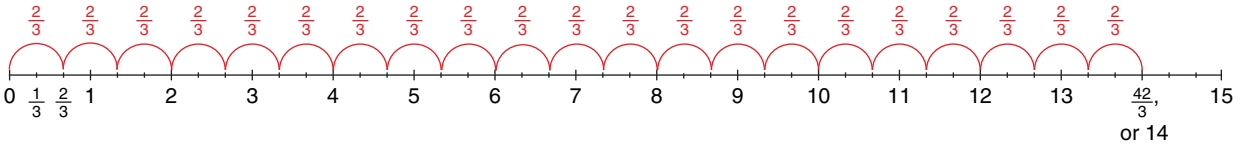
### Example 1

New flooring has been installed in two-thirds of the classrooms in the school.  
There are 21 classrooms in the school.  
How many classrooms have new flooring?

#### A Solution

Multiply:  $21 \times \frac{2}{3}$

Use a number line divided into thirds.



So,  $21 \times \frac{2}{3} = \frac{42}{3}$ , or 14

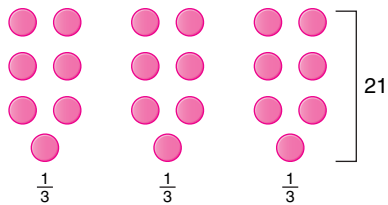
Fourteen classrooms have new flooring.

#### Example 1 Another Solution

Find:  $\frac{2}{3}$  of 21

Use counters.

Model 21 with counters.



Find thirds by dividing the counters into 3 equal groups.

Each group contains 7 counters.

$\frac{1}{3}$  of 21 = 7

So,  $\frac{2}{3}$  of 21 = 14

Fourteen classrooms have new flooring.

### Example 2

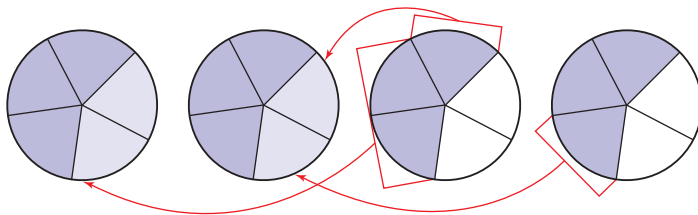
An office building with four floors has rented out  $\frac{3}{5}$  of each floor.  
How many floors of the building have been rented?

### ▶ A Solution

Multiply:  $4 \times \frac{3}{5}$

$$4 \times \frac{3}{5} = \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5}$$

Model the expression  $\frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5}$  with fraction circles.



Put the fifths together to make wholes.

2 wholes and two fifths equal  $2\frac{2}{5}$ .

$$\text{So, } 4 \times \frac{3}{5} = 2\frac{2}{5}$$

$2\frac{2}{5}$  floors of the office building have been rented.

### ▶ Example 2 Another Solution

Multiply:  $4 \times \frac{3}{5}$

Sketch a rectangle with base 4 units and height 1 unit.

Divide the height into fifths.

Shade the rectangle with base 4 and height  $\frac{3}{5}$ .

The area of the shaded rectangle is:

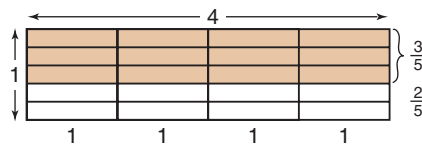
$$\text{base} \times \text{height} = 4 \times \frac{3}{5}$$

Each small rectangle has area:  $1 \times \frac{1}{5} = \frac{1}{5}$

So, the shaded area is:  $12 \times \frac{1}{5} = \frac{12}{5}$ , or  $2\frac{2}{5}$

$$\text{So, } 4 \times \frac{3}{5} = 2\frac{2}{5}$$

$2\frac{2}{5}$  floors of the office building have been rented.



## Discuss the ideas

1. Why can a product be written as repeated addition?
2. When might you not want to use repeated addition to find a product?
3. How could you use a rectangle model to solve the problem in *Example 1*?
4. How could you use a number line to solve the problem in *Example 2*?

## Practice

### Check

5. Write each statement as a multiplication statement in two ways.

- a)  $\frac{5}{9}$  of 45      b)  $\frac{3}{8}$  of 32  
c)  $\frac{1}{12}$  of 36      d)  $\frac{4}{5}$  of 25

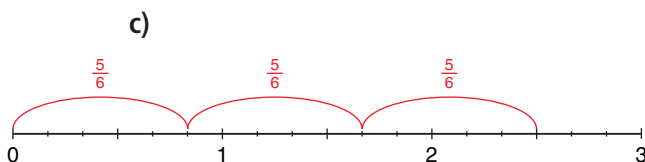
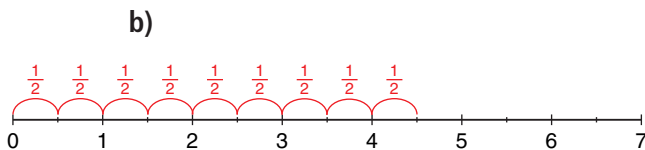
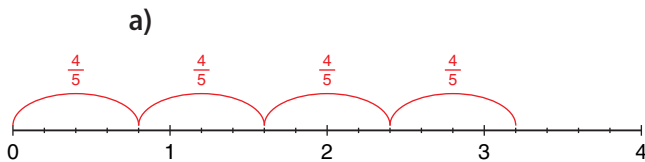
6. Write each repeated addition as a multiplication statement in two ways.

- a)  $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$   
b)  $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$   
c)  $\frac{3}{10} + \frac{3}{10} + \frac{3}{10} + \frac{3}{10}$

7. Use fraction circles to find:  $\frac{2}{3} \times 6$

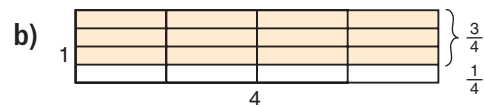
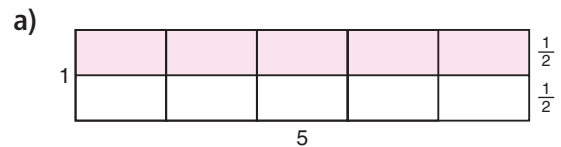
- a) Write the multiplication as repeated addition.  
b) Use fraction circles to find the sum.  
c) Sketch the fraction circles.  
d) Write the multiplication equation the fraction circles represent.

8. Write the two multiplication equations each number line represents.

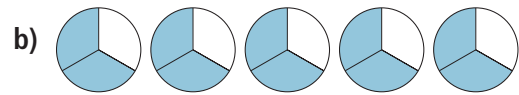
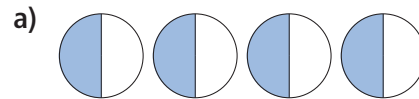


### Apply

9. For each diagram below, state the product the shaded area represents.



10. Write the two multiplication statements each set of fraction circles represents. Then find each product.



11. Use fraction circles to find each product. Sketch the fraction circles. Write a multiplication equation each time.

- a)  $5 \times \frac{1}{8}$     b)  $\frac{2}{5} \times 3$     c)  $4 \times \frac{5}{12}$

12. Use counters to help you find each product.

- a)  $\frac{1}{2} \times 24$     b)  $\frac{1}{3} \times 24$     c)  $\frac{1}{4} \times 24$   
d)  $\frac{1}{6} \times 24$     e)  $\frac{1}{8} \times 24$     f)  $\frac{1}{12} \times 24$

13. Use the results in question 12 to find each product.

- a)  $\frac{2}{2} \times 24$     b)  $\frac{2}{3} \times 24$     c)  $\frac{3}{4} \times 24$   
d)  $\frac{5}{6} \times 24$     e)  $\frac{3}{8} \times 24$     f)  $\frac{5}{12} \times 24$

- 14.** Multiply. Draw a picture or number line to show each product.

a)  $3 \times \frac{4}{7}$       b)  $\frac{2}{15} \times 10$   
c)  $4 \times \frac{9}{4}$       d)  $\frac{2}{5} \times 7$

- 15.** Draw and shade rectangles to find each product.

a)  $\frac{1}{3} \times 12$       b)  $\frac{1}{5} \times 15$   
c)  $\frac{3}{5} \times 15$       d)  $\frac{3}{8} \times 16$

- 16.** Multiply.

a)  $3 \times \frac{4}{5}$     b)  $5 \times \frac{7}{9}$     c)  $\frac{5}{3} \times 6$   
d)  $\frac{1}{2} \times 5$     e)  $12 \times \frac{7}{8}$     f)  $\frac{2}{4} \times 9$

- 17.** It takes  $\frac{2}{3}$  h to pick all the apples on one tree at Springwater Farms. There are 24 trees. How long will it take to pick all the apples? Show your work.

**18. Assessment Focus**

- a) Describe a situation that could be represented by  $5 \times \frac{3}{8}$ .  
b) Draw a picture to show  $5 \times \frac{3}{8}$ .  
c) What meaning can you give to  $\frac{3}{8} \times 5$ ?

Draw a picture to show your thinking.

- 19.** Parri used the expression  $\frac{5}{8} \times 16$  to solve a word problem.

What might the word problem be?  
Solve the problem.

- 20.** Naruko went to the West Edmonton Mall. She took \$28 with her. She spent  $\frac{4}{7}$  of her money on rides. How much money did Naruko spend on rides? Use a model to show your answer.



**21. Take It Further**

- a) Use models. Multiply.  
i)  $2 \times \frac{1}{2}$       ii)  $3 \times \frac{1}{3}$   
iii)  $4 \times \frac{1}{4}$       iv)  $5 \times \frac{1}{5}$   
b) Look at your answers to part a.  
What do you notice?  
How can you explain your findings?  
c) Write two different multiplication statements with the same product as in part a.

- 22. Take It Further** Jacques takes  $\frac{3}{4}$  h to fill one shelf at the supermarket. Henri can fill the shelves in two-thirds Jacques' time. There are 15 shelves. Henri and Jacques work together. How long will it take to fill the shelves? Justify your answer.

## Reflect

Explain how your knowledge of adding fractions helped you in this lesson. Include an example.

# 3.2

## Using Models to Multiply Fractions

**Focus** Use models to multiply fractions.

Describe a picture that shows  $38 \times 25$ .

### Investigate

Work with a partner.

Model this problem with concrete materials.

One-quarter of a cherry pie was left over after dinner.  
Graham ate one-half of the leftover pie for lunch the next day.  
What fraction of the whole pie did he have for lunch?  
What if Graham had eaten only one-quarter of the leftover pie.  
What fraction of the whole pie would he have eaten?



### Reflect & Share

How did you solve the problem?  
Compare your solutions and strategies with those of another pair of classmates.  
Was one strategy more efficient than another? Explain.

### Connect

We can use different models to find the product of two fractions.  
The following *Examples* show how we can use Pattern Blocks, counters, and a rectangle model.

### Example 1

Sandi cut  $\frac{2}{3}$  of the grass on a lawn.  
Akiva cut  $\frac{1}{2}$  of the remaining grass.  
What fraction of the lawn did Akiva cut?

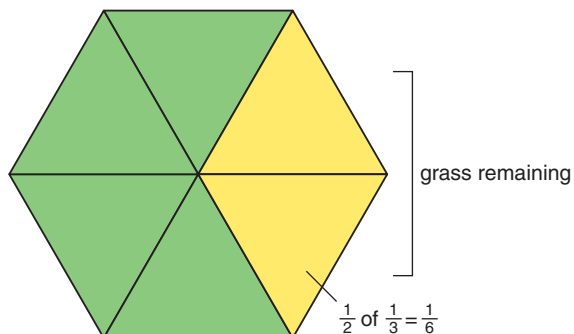
#### A Solution

Use Pattern Blocks.  
Let the yellow hexagon represent the lawn.  
6 green triangles cover the yellow hexagon.  
So, 6 green triangles also represent the lawn.  
4 green triangles represent the grass cut by Sandi.  
2 green triangles represent the remaining grass.

Akiva cut  $\frac{1}{2}$  of the remaining grass.  
One-half of the remaining grass is 1 green triangle.  
One green triangle represents  $\frac{1}{6}$ .

$$\text{So, } \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

Akiva cut  $\frac{1}{6}$  of the lawn.



### Example 2

Multiply:  $\frac{2}{3} \times \frac{6}{8}$

#### A Solution

Use counters.

Think: We want  $\frac{2}{3}$  of  $\frac{6}{8}$  of one whole set of counters.

Model one whole set of eighths with eight counters.  
There are 6 counters in  $\frac{6}{8}$ .

To model thirds, arrange the 6 counters into 3 equal groups.

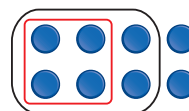
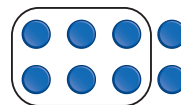
Each group of 2 counters represents  $\frac{1}{3}$ .

So,  $\frac{2}{3}$  of 6 counters is 4 counters.

But, 4 counters are part of a whole set of 8 counters.

So, 4 counters represent  $\frac{4}{8}$ , or  $\frac{1}{2}$  of the original whole set.

$$\text{So, } \frac{2}{3} \times \frac{6}{8} = \frac{1}{2}$$



### Example 3

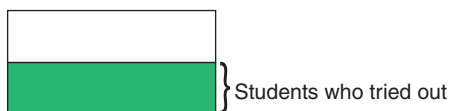
One-half of the Grade 8 students tried out for the school's lacrosse team. Three-quarters of these students were successful. What fraction of the Grade 8 students are on the team?

#### A Solution

Three-quarters of one-half of the Grade 8 students are on the team.

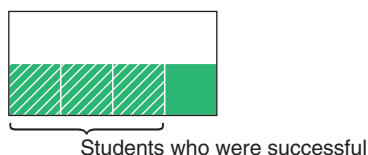
Draw a rectangle.

Show  $\frac{1}{2}$  of the rectangle.



Divide  $\frac{1}{2}$  of the rectangle into quarters.

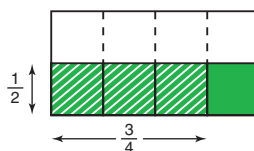
Shade  $\frac{3}{4}$ .



Use broken lines to divide the whole rectangle into equal parts.

There are 8 equal parts.

Three parts are shaded.



$$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$

So,  $\frac{3}{8}$  of the Grade 8 students are on the team.

### Discuss the ideas

1. In *Example 1*, suppose Akiva cut  $\frac{1}{4}$  of the remaining grass. Could we use Pattern Blocks to model the problem? Justify your answer.
2. In *Example 2*, why did we start with 8 counters? Could we have started with a different number of counters? Justify your answer.
3. Look at the area model in *Example 3*. The factors are fractions. How do the numbers of rows and columns in the product relate to the factors? How does the number of parts in the whole relate to the factors?
4. In *Example 3*, could you have divided the rectangle in half a different way? Would you get the same answer? Explain.

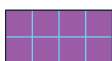
## Practice

### Check

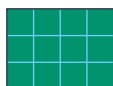
5. Draw a rectangle to multiply  $\frac{3}{5} \times \frac{1}{4}$ .
- Divide the rectangle into fourths vertically. How many fourths will you shade?
  - Divide the rectangle into fifths horizontally. How many fifths will you shade?
  - How many equal parts does the rectangle have?
  - How many of these parts have been shaded twice?
  - What is the product of  $\frac{3}{5} \times \frac{1}{4}$ ?

6. Copy each rectangle onto grid paper. Shade the rectangle to find each product.

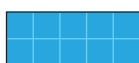
a)  $\frac{1}{2} \times \frac{3}{4}$



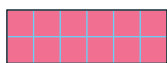
b)  $\frac{3}{4} \times \frac{2}{3}$



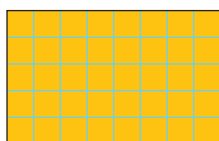
c)  $\frac{2}{5} \times \frac{1}{2}$



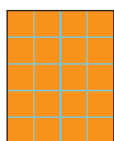
d)  $\frac{5}{6} \times \frac{1}{2}$



e)  $\frac{3}{5} \times \frac{7}{8}$



f)  $\frac{4}{5} \times \frac{3}{4}$



7. Use counters to find each product. Draw a diagram to record your work.

a)  $\frac{3}{4} \times \frac{12}{15}$    b)  $\frac{4}{5} \times \frac{10}{18}$    c)  $\frac{1}{2} \times \frac{4}{12}$

d)  $\frac{1}{4} \times \frac{8}{9}$    e)  $\frac{5}{9} \times \frac{18}{24}$    f)  $\frac{2}{3} \times \frac{15}{20}$

### Apply

8. Find each product.

a)  $\frac{3}{4} \times \frac{5}{8}$    b)  $\frac{4}{9} \times \frac{2}{5}$

c)  $\frac{1}{4} \times \frac{2}{3}$    d)  $\frac{6}{7} \times \frac{2}{3}$

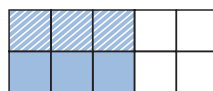
e)  $\frac{2}{3} \times \frac{1}{3}$    f)  $\frac{4}{5} \times \frac{4}{5}$

9. Write 3 multiplication statements using proper fractions. Make sure each statement is different from any statements you have worked with so far. Use a model to illustrate each product.

Recall that, in a proper fraction, the numerator is less than the denominator.

10. Write the multiplication equation represented by each diagram below.

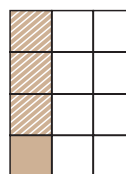
a)



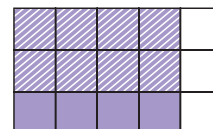
b)



c)



d)



11. Barry used  $\frac{5}{8}$  of the money he had saved to buy a DVD player and 4 DVDs. The cost of the DVD player was  $\frac{2}{5}$  of the amount he spent. What fraction of his savings did Barry spend on the DVD player?

## 12. Assessment Focus

a) Find each product.

i)  $\frac{3}{4} \times \frac{2}{5}$       ii)  $\frac{2}{4} \times \frac{3}{5}$

iii)  $\frac{1}{4} \times \frac{3}{8}$       iv)  $\frac{3}{4} \times \frac{1}{8}$

v)  $\frac{3}{5} \times \frac{4}{6}$       vi)  $\frac{3}{6} \times \frac{4}{5}$

b) What patterns do you see in the answers for part a? Use a model to illustrate the patterns.

c) Write some other products similar to those in part a.

Show your work.

13. Use a model to answer each question.

a) One-third of the students in a class wear glasses. One-half of the students who wear glasses are girls. What fraction of the class is girls who wear glasses?

b) John has  $\frac{2}{3}$  of a tank of gas. He uses  $\frac{3}{4}$  of the gas to get home. What fraction of a tank of gas does John use to get home? What fraction of the tank of gas is left?

c) Justin ate  $\frac{3}{5}$  of a box of raisins. His sister then ate  $\frac{1}{4}$  of the raisins left in the box. What fraction of the box of raisins did Justin's sister eat? What fraction of the box of raisins remained?

14. Gwen used the expression  $\frac{4}{9} \times \frac{1}{5}$  to solve a word problem. What might the word problem be? Solve the problem.

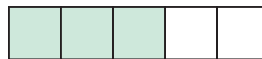
15. **Take It Further** One-eighth of the seats in the movie theatre were empty.  $\frac{2}{7}$  of the seats that were full were filled with teenagers. What fraction of the theatre was filled with teenagers?

16. **Take It Further** Why is  $\frac{5}{8}$  of  $\frac{3}{12}$  equal to  $\frac{3}{8}$  of  $\frac{5}{12}$ ? Use a model to explain your answer.

17. **Take It Further**

a) Look at the diagram.

How does it show  $\frac{3}{5}$  of one whole?



b) Use the same diagram.

Explain how it shows that one whole is  $\frac{5}{3}$  of  $\frac{3}{5}$ .

## Reflect

When you use an area model to multiply two fractions, how do you decide how to draw the rectangle? Include an example in your explanation.

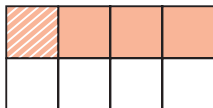
## 3.3

## Multiplying Fractions

**Focus** Develop an algorithm to multiply fractions.

Which multiplication equation does this diagram represent?

How do you know?



## Investigate

Work with a partner.

Use an area model to find each product.

- $\frac{2}{3} \times \frac{4}{5}$
- $\frac{1}{2} \times \frac{3}{8}$
- $\frac{3}{5} \times \frac{4}{7}$
- $\frac{2}{5} \times \frac{3}{8}$

Write the multiplication equations in a table.

Look at the table.

What patterns do you notice?

How can you use patterns to multiply  $\frac{2}{3} \times \frac{4}{5}$ ?

Use your patterns to calculate  $\frac{7}{8} \times \frac{3}{10}$ .

Use an area model to check your product.

**Reflect & Share**

Compare your strategies with those of another pair of classmates.

How does your strategy work?

Does your strategy work with  $\frac{2}{5} \times \frac{3}{4}$ ?

Do you think your strategy will work with all fractions? Explain.

## Connect

Here is an area model to show:  $\frac{4}{7} \times \frac{2}{5} = \frac{8}{35}$

The product of the numerators is:

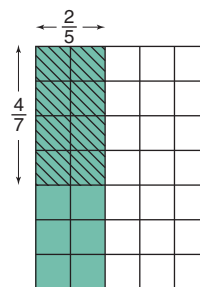
$$4 \times 2 = 8$$

The product of the denominators is:

$$7 \times 5 = 35$$

$$\begin{aligned} \text{That is, } \frac{4}{7} \times \frac{2}{5} &= \frac{4 \times 2}{7 \times 5} \\ &= \frac{8}{35} \end{aligned}$$

Check if there are common factors in the numerator and denominator.



So, to multiply two fractions, multiply the numerators and multiply the denominators.

We can use this method to multiply proper fractions and improper fractions.

### Example 1

Multiply. Estimate to check the product is reasonable.

$$\frac{7}{5} \times \frac{8}{3}$$

#### A Solution

$$\frac{7}{5} \times \frac{8}{3}$$

There are no common factors in the numerators and denominators.

$$\begin{aligned} \text{So, } \frac{7}{5} \times \frac{8}{3} &= \frac{7 \times 8}{5 \times 3} \\ &= \frac{56}{15} \\ &= \frac{45}{15} + \frac{11}{15} \\ &= 3 + \frac{11}{15}, \text{ or } 3\frac{11}{15} \end{aligned}$$

Estimate to check.

$\frac{7}{5}$  is between 1 and 2, but closer to 1.

$\frac{8}{3}$  is between 2 and 3, but closer to 3.

So, the product is about  $1 \times 3 = 3$ .

Since  $3\frac{11}{15}$  is close to 3, the product is reasonable.

Recall that factors are the numbers that are multiplied to get a product; for example, 2 and 5 are factors of 10 because  $2 \times 5 = 10$ .

45 is the multiple of 15 that is closest to 56, and less than 56.



## Example 2

Three-eighths of the animals in a pet store are fish.

Two-fifteenths of the fish are tropical fish.

What fraction of the animals in the pet store are tropical fish?

Use benchmarks to check the solution is reasonable.

### A Solution

Since  $\frac{3}{8}$  of the animals are fish and  $\frac{2}{15}$  of the fish are tropical fish,

then the fraction of animals that are tropical fish is  $\frac{2}{15}$  of  $\frac{3}{8}$ , or  $\frac{2}{15} \times \frac{3}{8}$ .

$$\frac{2}{15} \times \frac{3}{8} = \frac{2 \times 3}{15 \times 8} \quad \text{Multiply the numerators and multiply the denominators.}$$

$$= \frac{6}{120} \quad \text{Simplify. Divide by the common factor, 6.}$$

$$= \frac{6 \div 6}{120 \div 6}$$

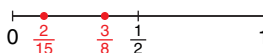
$$= \frac{1}{20}$$

Estimate to check.

$\frac{2}{15}$  is close to 0.

$\frac{3}{8}$  is about  $\frac{1}{2}$ .

So,  $\frac{2}{15} \times \frac{3}{8}$  is close to 0.



Since  $\frac{1}{20}$  is close to 0, the product is reasonable.

One-twentieth of the animals in the pet store are tropical fish.

### Example 2 Another Solution

Here is another way to calculate.

$$\frac{2}{15} \times \frac{3}{8} = \frac{2 \times 3}{15 \times 8}$$

Notice that the numerator and denominator have common factors 2 and 3.

To simplify first, divide the numerator and denominator by these factors.

$$\frac{2}{15} \times \frac{3}{8} = \frac{\cancel{2}^1 \times \cancel{3}^1}{\cancel{15}_5 \times \cancel{8}_4}$$

$$= \frac{1 \times 1}{5 \times 4}$$

$$= \frac{1}{20}$$

$$\begin{array}{l} 2 \div 2 = 1 \quad 3 \div 3 = 1 \\ 15 \div 3 = 5 \quad 8 \div 2 = 4 \end{array}$$

Dividing a fraction by a common factor of the numerator and denominator produces an equivalent fraction.

One-twentieth of the animals in the pet store are tropical fish.

▶ **Example 2**  
**Another Solution**

$$\frac{2}{15} \times \frac{3}{8} = \frac{2 \times 3}{15 \times 8}$$

The numerator and denominator have common factors 2 and 3.  
Write the denominator to show the common factors.

$$\begin{aligned}\frac{2}{15} \times \frac{3}{8} &= \frac{2 \times 3}{3 \times 5 \times 2 \times 4} \\ &= \frac{2}{2} \times \frac{3}{3} \times \frac{1}{5 \times 4} \\ &= 1 \times 1 \times \frac{1}{20} \\ &= \frac{1}{20}\end{aligned}$$

Rewrite making fractions that equal 1.

When multiplying by 1, the value of the fraction does not change.

One-twentieth of the animals in the pet store are tropical fish.

**Discuss**  
**the ideas**

1. Why is it important to estimate to check the product?
2. Look at the different solutions to *Example 2*. Why is it often helpful to simplify the fractions before multiplying?
3. How do you recognize when fractions can be simplified before you multiply them?

**Practice**

**Check**

4. Find the common factors of each pair of numbers.  
a) 4, 12    b) 14, 21    c) 8, 16  
d) 6, 9    e) 10, 15    f) 18, 24
5. Multiply:  $\frac{5}{6} \times \frac{3}{20}$   
a) Multiply. Simplify first.  
b) Use benchmarks to estimate the product.  
c) Is the product reasonable? How do you know?

6. In a First Nations school, five-eighths of the Grade 8 students play the drums. Of these students, three-tenths also play the native flute. What fraction of the Grade 8 students play both the drums and the native flute? Estimate to check the solution is reasonable.



## Apply

7. Multiply. Simplify before multiplying.

Use benchmarks to estimate to check the product is reasonable.

a)  $\frac{3}{4} \times \frac{8}{5}$    b)  $\frac{1}{3} \times \frac{9}{10}$    c)  $\frac{7}{5} \times \frac{15}{21}$   
d)  $\frac{5}{9} \times \frac{3}{5}$    e)  $\frac{2}{9} \times \frac{15}{4}$    f)  $\frac{7}{3} \times \frac{9}{14}$

8. Multiply. Use benchmarks to estimate to check the product is reasonable.

a)  $\frac{3}{5} \times \frac{2}{3}$    b)  $\frac{1}{2} \times \frac{5}{10}$    c)  $\frac{1}{6} \times \frac{1}{4}$   
d)  $\frac{13}{8} \times \frac{3}{2}$    e)  $\frac{5}{4} \times \frac{11}{10}$    f)  $\frac{7}{3} \times \frac{7}{8}$

Which of these questions could have been solved using mental math?

Justify your choice.

9. Solve each problem. Estimate to check the solution is reasonable.

a) Josten took  $\frac{3}{8}$  of his savings on a shopping trip. He used  $\frac{1}{4}$  of the money to buy a new coat.

What fraction of his savings did Josten spend on the coat?

b) Gervais ate  $\frac{1}{3}$  of a baguette with his dinner. Chantel ate  $\frac{1}{4}$  of the leftover baguette as an evening snack.

What fraction of the baguette did Chantel eat as a snack?

10. Write a story problem that can be represented by the expression  $\frac{7}{8} \times \frac{1}{2}$ . Solve your problem.

Trade problems with a classmate.

Solve your classmate's problem.

Check to see that your solutions are the same.

11. Eeva spent  $\frac{5}{6}$  of  $\frac{3}{4}$  of her total allowance on a hair crimper. What fraction of her total allowance did Eeva have left?

12. a) Find each product.

i)  $\frac{3}{4} \times \frac{4}{3}$    ii)  $\frac{1}{5} \times \frac{5}{1}$

iii)  $\frac{7}{2} \times \frac{2}{7}$    iv)  $\frac{5}{6} \times \frac{6}{5}$

- b) What do you notice about the products in part a? Write 3 more pairs of fractions that have the same product. What can you say about the product of a fraction and its reciprocal?

$\frac{11}{12}$  and  $\frac{12}{11}$  are reciprocals.

13. **Assessment Focus** In question 12, each product is 1.

a) Write a pair of fractions that have each product.

i) 2   ii) 3   iii) 4   iv) 5

b) Write a pair of fractions that have the product 1. Change only one numerator or denominator each time to write a pair of fractions that have each product.

i) 2   ii) 3   iii) 4   iv) 5

c) How can you write a pair of fractions that have the product 10? Show your work.

14. The sum of two fractions is  $\frac{7}{12}$ . The product of the same two fractions is  $\frac{1}{12}$ . What are the two fractions? Describe the strategy you used.

**15.** Multiply. Estimate to check the product is reasonable.

a)  $\frac{33}{40} \times \frac{15}{55}$

b)  $\frac{26}{39} \times \frac{9}{13}$

c)  $\frac{51}{64} \times \frac{8}{17}$

d)  $\frac{76}{91} \times \frac{7}{19}$

**16. a)** Multiply  $\frac{24}{25} \times \frac{85}{96}$  using each strategy below.

i) Simplify before multiplying.

ii) Multiply first, then simplify.

b) Which strategy in part a did you find easier? Justify your choice.

**17.** The product of 2 fractions is  $\frac{3}{4}$ . What might the fractions be? How many pairs of fractions could have a product of  $\frac{3}{4}$ ? How do you know?

**18. Take It Further** Keydon baked a wild blueberry upside-down cobbler.

Shawnie ate  $\frac{1}{6}$  of the cobbler.

Iris ate  $\frac{1}{5}$  of what was left.

Chan ate  $\frac{1}{4}$  of what was left after that.

Cami ate  $\frac{1}{3}$  of what was left after that.

Demi ate  $\frac{1}{2}$  of what was left after that.

How much of the original cobbler remained?

**19. Take It Further** The product of two fractions is  $\frac{2}{3}$ . One fraction is  $\frac{3}{5}$ . What is the other fraction? How do you know?

**20. Take It Further** Eddie used the expression  $\frac{4}{7} \times \frac{3}{5}$  to solve a word problem. Which of these word problems better fits the expression? How do you know? Solve the problem.

a)  $\frac{4}{7}$  of the Grade 8 students voted to have Spirit Day.  $\frac{3}{5}$  of those students wanted Spirit Day to be on the first day of classes. What fraction of the Grade 8 students wanted Spirit Day to be on the first day of classes?

b)  $\frac{3}{5}$  of the Grade 7 students voted to have a school dance.  $\frac{4}{7}$  of those students wanted the dance to be on the day before Spring Break. What fraction of the Grade 7 students wanted the dance to be on the day before Spring Break?

**21. Take It Further**

Find each square root.

Explain the strategy you used.

a)  $\sqrt{\frac{4}{9}}$

b)  $\sqrt{\frac{16}{25}}$

c)  $\sqrt{\frac{36}{81}}$

d)  $\sqrt{\frac{49}{169}}$

## Reflect

When we multiply 2 whole numbers, the product is always greater than either factor.

Is this always true when we multiply 2 fractions?

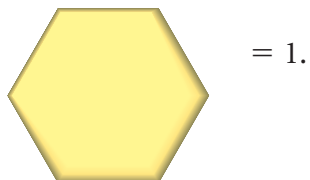
Use examples and diagrams to explain your answer.

# 3.4

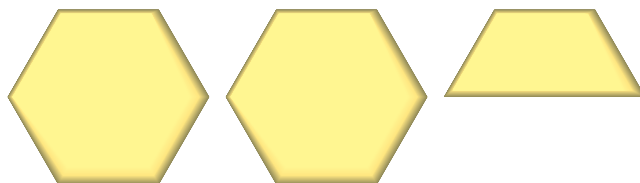
## Multiplying Mixed Numbers

**Focus** Apply knowledge of multiplying fractions to multiply mixed numbers.

Suppose



How can you write the fraction representing



in 2 ways?

### Investigate

Work with a partner.

During the salmon drift, volunteers collect catch information from fisherpeople. Akecheta volunteered for  $3\frac{1}{2}$  h.

Onida volunteered for  $\frac{2}{3}$  of the time that Akecheta volunteered.

For how long did Onida volunteer?

How can you find out?

Show your work.

Use models or diagrams to justify your strategy.

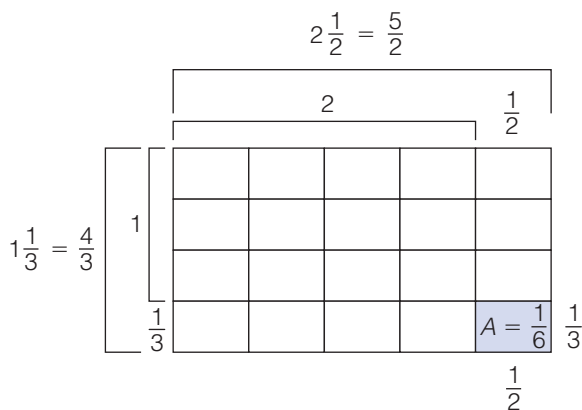


### Reflect & Share

Compare your strategy with that of another pair of classmates. Do you think your strategy will work with all mixed numbers? Test it with  $\frac{3}{4} \times 2\frac{1}{3}$ .

## Connect

Here is an area model to show:  $2\frac{1}{2} \times 1\frac{1}{3}$



Write each mixed number as an improper fraction.

$$2\frac{1}{2} = 2 + \frac{1}{2} = \frac{4}{2} + \frac{1}{2} = \frac{5}{2}$$

$$1\frac{1}{3} = 1 + \frac{1}{3} = \frac{3}{3} + \frac{1}{3} = \frac{4}{3}$$

Each small rectangle has area:  $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

There are  $5 \times 4$ , or 20 small rectangles.

$$\begin{aligned} \text{So, the area is: } 20 \times \frac{1}{6} &= \frac{20}{6} \\ &= \frac{10}{3} \\ &= 3\frac{1}{3} \end{aligned}$$

After we write the mixed numbers as improper fractions, we can multiply the same way we multiplied proper fractions.

$$\begin{aligned} 2\frac{1}{2} \times 1\frac{1}{3} &= \frac{5}{2} \times \frac{4}{3} \\ &= \frac{5 \times 4}{2 \times 3} \\ &= \frac{20}{6} \\ &= \frac{20 \div 2}{6 \div 2} \\ &= \frac{10}{3} \\ &= 3\frac{1}{3} \end{aligned}$$

This is the same product as when we used the area model.

We can also use a rectangle model to multiply two mixed numbers.

### Example 1

Multiply:  $2\frac{1}{2} \times 1\frac{1}{3}$

#### A Solution

Use a rectangle model.

$$2\frac{1}{2} \times 1\frac{1}{3} = (2 \times 1) + (\frac{1}{2} \times 1) + (2 \times \frac{1}{3}) + (\frac{1}{2} \times \frac{1}{3})$$

$$= 2 + \frac{1}{2} + \frac{2}{3} + \frac{1}{6}$$

$$= 2 + \frac{3}{6} + \frac{4}{6} + \frac{1}{6}$$

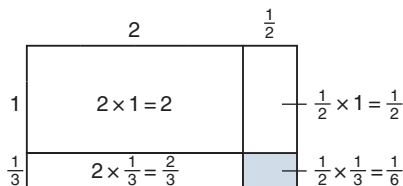
$$= 2 + \frac{8}{6}$$

$$= 2 + \frac{6}{6} + \frac{2}{6}$$

$$= 2 + 1 + \frac{2}{6}$$

$$= 3\frac{2}{6}, \text{ or } 3\frac{1}{3}$$

Add. Use common denominators.



Remember to write the product in simplest form.

### Example 2

Multiply. Estimate to check the product is reasonable.

$$2\frac{1}{4} \times 3\frac{2}{5}$$

#### A Solution

$$2\frac{1}{4} \times 3\frac{2}{5}$$

$$2\frac{1}{4} = 2 + \frac{1}{4}$$

$$= \frac{8}{4} + \frac{1}{4}$$

$$= \frac{9}{4}$$

$$\text{and } 3\frac{2}{5} = 3 + \frac{2}{5}$$

$$= \frac{15}{5} + \frac{2}{5}$$

$$= \frac{17}{5}$$

$$\text{So, } 2\frac{1}{4} \times 3\frac{2}{5} = \frac{9}{4} \times \frac{17}{5}$$

$$= \frac{153}{20}$$

$$= \frac{140}{20} + \frac{13}{20}$$

$$= 7\frac{13}{20}$$

Estimate to check.

$2\frac{1}{4}$  is between 2 and 3, but closer to 2.

$3\frac{2}{5}$  is between 3 and 4, but closer to 3.

So, the product is about  $2 \times 3 = 6$ .

Since  $7\frac{13}{20}$  is close to 6, the product is reasonable.

### Example 3

Multiply. Estimate to check the product is reasonable.

$$3\frac{3}{8} \times 4\frac{2}{3}$$

#### A Solution

$$3\frac{3}{8} \times 4\frac{2}{3}$$

$$3\frac{3}{8} = 3 + \frac{3}{8}$$

$$= \frac{24}{8} + \frac{3}{8}$$

$$= \frac{27}{8}$$

and

$$4\frac{2}{3} = 4 + \frac{2}{3}$$

$$= \frac{12}{3} + \frac{2}{3}$$

$$= \frac{14}{3}$$

$$\text{So, } 3\frac{3}{8} \times 4\frac{2}{3} = \frac{9\cancel{27}}{4\cancel{8}} \times \frac{14\cancel{7}}{\cancel{3}^1}$$

$$= \frac{9 \times 7}{4 \times 1}$$

$$= \frac{63}{4}$$

$$= \frac{60}{4} + \frac{3}{4}$$

$$= 15\frac{3}{4}$$

Divide by common factors.

$$\begin{array}{l} 27 \div 3 = 9 \\ 8 \div 2 = 4 \end{array}$$

$$\begin{array}{l} 14 \div 2 = 7 \\ 3 \div 3 = 1 \end{array}$$

Estimate to check.

$3\frac{3}{8}$  is between 3 and 4, but closer to 3.

$4\frac{2}{3}$  is between 4 and 5, but closer to 5.

So, the product is about  $3 \times 5 = 15$ .

Since  $15\frac{3}{4}$  is close to 15, the product is reasonable.

### Discuss the ideas

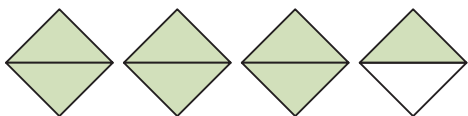
1. What is the difference between a proper fraction and an improper fraction?
2. How is multiplying two mixed numbers like multiplying two fractions?
3. How is the rectangle model useful when you multiply 2 mixed numbers?

## Practice

### Check

4. Write the mixed number and improper fraction represented by each picture.

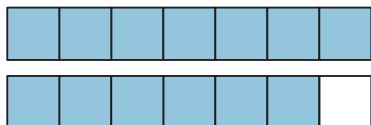
a)



b)



c)



5. Write each mixed number as an improper fraction.

- a)  $2\frac{3}{10}$       b)  $4\frac{1}{8}$       c)  $3\frac{5}{6}$   
 d)  $1\frac{2}{3}$       e)  $3\frac{2}{5}$       f)  $5\frac{1}{2}$   
 g)  $2\frac{4}{7}$       h)  $3\frac{5}{9}$       i)  $6\frac{2}{3}$

6. Write each improper fraction as a mixed number.

- a)  $\frac{11}{3}$       b)  $\frac{15}{4}$       c)  $\frac{21}{5}$   
 d)  $\frac{11}{8}$       e)  $\frac{19}{6}$       f)  $\frac{31}{7}$   
 g)  $\frac{11}{2}$       h)  $\frac{43}{10}$       i)  $\frac{37}{8}$

7. Use estimation. Which number is each product closer to?

- a)  $2\frac{1}{8} \times 3\frac{3}{4}$       6 or 8  
 b)  $3\frac{5}{9} \times 1\frac{5}{6}$       6 or 8  
 c)  $7\frac{3}{8} \times 2\frac{4}{5}$       21 or 24  
 d)  $4\frac{7}{9} \times 3\frac{5}{12}$       15 or 20

8. Multiply:  $3\frac{3}{5} \times 2\frac{2}{9}$
- Estimate the product.
  - Write each mixed number as an improper fraction.
  - Multiply the improper fractions. Simplify first.
  - Is the product reasonable? How do you know?

### Apply

9. Multiply. Estimate to check the product is reasonable.

- a)  $3 \times 2\frac{1}{4}$       b)  $4 \times 2\frac{1}{8}$   
 c)  $1\frac{2}{3} \times 2$       d)  $3\frac{1}{5} \times 3$

10. Use an area model to find each product.

- a)  $1\frac{1}{2} \times 1\frac{1}{3}$       b)  $2\frac{3}{4} \times 2\frac{2}{3}$   
 c)  $1\frac{1}{5} \times 3\frac{1}{3}$       d)  $1\frac{1}{2} \times 2\frac{2}{5}$

11. Use improper fractions to find each product. Estimate to check the product is reasonable.

- a)  $1\frac{7}{8} \times 2\frac{2}{3}$       b)  $4\frac{1}{6} \times 3\frac{2}{5}$   
 c)  $2\frac{3}{7} \times 1\frac{5}{9}$       d)  $3\frac{1}{2} \times 2\frac{2}{7}$   
 e)  $2\frac{1}{4} \times 2\frac{2}{3}$       f)  $1\frac{4}{5} \times 2\frac{1}{3}$

12. Multiply. Estimate to check the product is reasonable.

- a)  $1\frac{3}{4} \times 2\frac{1}{2}$       b)  $3\frac{2}{3} \times 2\frac{1}{5}$   
 c)  $4\frac{3}{8} \times 1\frac{1}{4}$       d)  $3\frac{3}{4} \times 3\frac{3}{4}$   
 e)  $4\frac{3}{10} \times \frac{4}{5}$       f)  $\frac{7}{8} \times 2\frac{3}{5}$

- 13.** A restaurant in Richmond, BC, lists the prices on its menu in fractions of a dollar. Three friends have lunch at the restaurant. Each of 3 friends orders a veggie mushroom cheddar burger for  $11\frac{3}{4}$ , with a glass of water to drink.
- What was the total bill before taxes, in fractions of a dollar?
  - What was the total bill before taxes, in dollars and cents?
- 14.** During the school year, the swim team practises  $2\frac{3}{4}$  h per week. During the summer, the weekly practice time is increased to  $2\frac{1}{3}$  times the school-year practice time. How many hours per week does the team practise during the summer?
- 15.** Write a story problem that can be represented by the expression  $3\frac{1}{2} \times 2\frac{1}{8}$ . Solve your problem. Trade problems with a classmate. Solve your classmate's problem. Check to see that your solutions are the same.
- 16.** In a baseball game, the starting pitcher for the home team pitched  $4\frac{2}{3}$  innings. The starting pitcher for the visiting team pitched  $1\frac{1}{2}$  times as many innings. How many innings did the visiting team's pitcher pitch?
- 17. Assessment Focus** Students baked cookies for a charity bake sale. Elsa baked  $2\frac{1}{2}$  dozen cookies. Layton baked  $2\frac{1}{6}$  times as many cookies as Elsa. Meghan and Josh together baked  $5\frac{1}{3}$  times the number of cookies that Elsa baked.
- Estimate. About how many dozen cookies did Layton bake? About how many dozen cookies did Meghan and Josh bake altogether?
  - Calculate how many dozen cookies Layton baked.
  - Calculate how many dozen cookies Meghan and Josh baked.
  - How many dozen cookies did these 4 students bake altogether?
  - How many cookies did these 4 students bake altogether? Show your work.
- 18. Take It Further** Use estimation. Which expression below has the greatest product? The least product? How do you know?
- $\frac{4}{3} \times \frac{8}{6}$
  - $2\frac{1}{8} \times 1\frac{1}{5}$
  - $1\frac{3}{8} \times \frac{9}{4}$
  - $\frac{7}{2} \times 2\frac{3}{10}$
- 19. Take It Further** Multiply. Estimate to check the product is reasonable.
- $2\frac{4}{9} \times 2\frac{2}{3} \times 2\frac{1}{2}$
  - $3\frac{3}{5} \times 2\frac{3}{4} \times 1\frac{1}{4}$
  - $4\frac{3}{8} \times 1\frac{1}{5} \times 2\frac{1}{4}$

## Reflect

Describe 2 strategies you can use to multiply  $3\frac{1}{2} \times 5\frac{1}{4}$ . Which strategy do you prefer? Why?

# Spinning Fractions



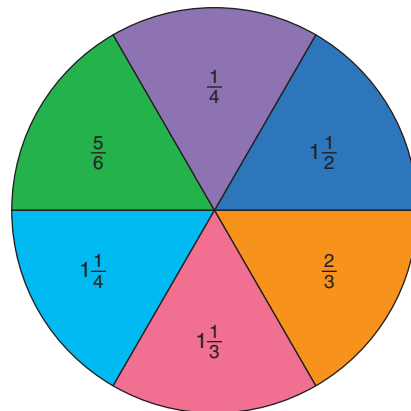
## HOW TO PLAY

Your teacher will give you a copy of the spinner.

Use an open paper clip as the pointer.

Use a sharp pencil to keep the pointer in place.

Record the scores in a chart.



1. Player A spins the pointer twice.  
Player A adds the fractions.  
Player B multiplies the fractions.  
The player with the greater result gets one point.
2. Player B spins the pointer twice.  
Player A adds the fractions.  
Player B multiplies the fractions.  
The player with the greater result gets one point.
3. Players continue to take turns spinning the pointer.  
The first person to get 12 points wins.

## YOU WILL NEED

A copy of the spinner;  
an open paper clip;  
a sharp pencil

## NUMBER OF PLAYERS

2

## GOAL OF THE GAME

To be the first to get  
12 points

## REFLECT

- Do you think this game is fair?  
How many games do you need to play to find out?
  - a) If this game is fair, explain how you know.
  - b) If this game is not fair, how could you make the game a fair game?
- Without playing the game many times, how else could you find out if the game is fair?

# Mid-Unit Review

## LESSON

### 3.1

1. Write each multiplication statement as repeated addition. Draw a picture to show each product.

a)  $4 \times \frac{1}{8}$       b)  $7 \times \frac{3}{5}$

c)  $\frac{5}{6} \times 3$       d)  $\frac{2}{9} \times 6$

2. Multiply. Draw a number line to show each product.

a)  $\frac{1}{4} \times 7$       b)  $8 \times \frac{3}{8}$

c)  $6 \times \frac{7}{10}$       d)  $\frac{5}{12} \times 3$

3. Sasha had 16 tomatoes in his garden. He gave: Samira  $\frac{1}{8}$  of the tomatoes; Amandeep  $\frac{1}{2}$  of the tomatoes; and Amina  $\frac{1}{4}$  of the tomatoes.

- a) How many tomatoes did Sasha give away?  
b) How many tomatoes did Sasha have left?  
c) What fraction of the tomatoes did Sasha have left?

### 3.2

4. Draw a rectangle to find each product.

a)  $\frac{5}{8} \times \frac{1}{2}$       b)  $\frac{2}{3} \times \frac{3}{4}$

c)  $\frac{1}{2} \times \frac{4}{5}$       d)  $\frac{5}{6} \times \frac{3}{10}$

5. Use counters to find each product. Draw a diagram each time.

a)  $\frac{1}{2} \times \frac{4}{9}$       b)  $\frac{2}{3} \times \frac{6}{15}$

c)  $\frac{3}{4} \times \frac{8}{11}$       d)  $\frac{2}{5} \times \frac{10}{12}$

### 3.3

6. Multiply. Use benchmarks to estimate to check each product is reasonable.

a)  $\frac{1}{2} \times \frac{2}{3}$       b)  $\frac{4}{5} \times \frac{1}{4}$

c)  $\frac{3}{4} \times \frac{3}{8}$       d)  $\frac{4}{9} \times \frac{15}{18}$

7. Aiko says that  $\frac{2}{3}$  of her stamp collection are Asian stamps. One-fifth of her Asian stamps are from India. What fraction of Aiko's stamp collection is from India? Estimate to check the solution is reasonable.

### 3.4

8. Use an area model to find each product.

a)  $2\frac{2}{3} \times 1\frac{7}{8}$       b)  $\frac{10}{3} \times \frac{5}{2}$

c)  $4\frac{3}{4} \times \frac{3}{8}$       d)  $1\frac{5}{6} \times 4\frac{1}{2}$

9. Multiply. Estimate to check each product is reasonable.

a)  $2\frac{1}{2} \times 3\frac{1}{4}$

b)  $4\frac{2}{5} \times \frac{1}{4}$

c)  $\frac{7}{3} \times \frac{6}{5}$

d)  $5\frac{1}{2} \times 2\frac{5}{8}$

10. Alek has a  $1\frac{1}{4}$ -h dance lesson every Saturday. The time he spends practising dance during the week is  $3\frac{3}{5}$  times the length of his dance lesson. How long does Alek practise dance each week?

# 3.5

## Dividing Whole Numbers and Fractions

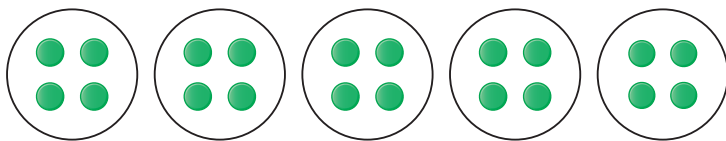
### Focus

Use models to divide proper fractions and whole numbers.

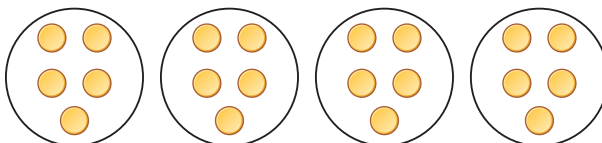
When you first studied division, you learned two ways: sharing and grouping

For example,  $20 \div 5$  can be thought of as:

- Sharing 20 items equally among 5 sets  
There are 4 items in each set.



- Grouping 20 items into sets of 5  
There are 4 sets.



Recall that multiplication and division are inverse operations.

We know:  $20 \div 5 = 4$

What related multiplication facts do you know?

### Investigate

Work with a partner.

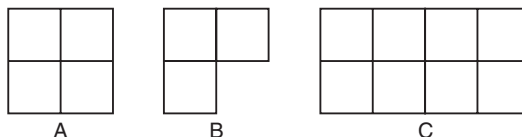
You will need scissors.

Your teacher will give you 2 large copies of this diagram.

Square A models the whole number 1.

Figure B represents  $\frac{3}{4}$  of Square A.

Which number does Rectangle C model?



Use paper cutting.

- Use one copy of the diagram.  
Find:  $2 \div \frac{1}{4}$
- Use the other copy of the diagram.  
Find:  $2 \div \frac{3}{4}$   
Illustrate your answers.

### Reflect & Share

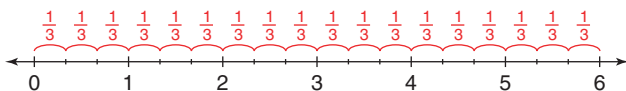
Compare your answers with those of another pair of classmates.

Did you deal with the leftover pieces the same way?

If not, explain your method to your classmates.

## Connect

- We can use a number line to divide a whole number by a fraction. To find how many thirds are in 6, divide 6 into thirds.

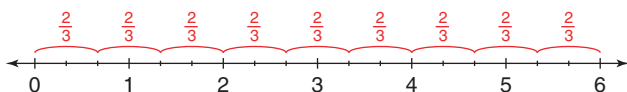


There are 18 thirds in 6.

Write this as a division equation.

$$6 \div \frac{1}{3} = 18$$

- Use the same number line to find how many two-thirds are in 6.



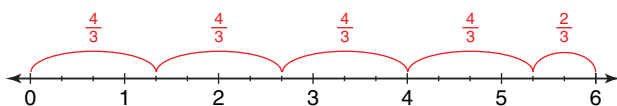
Arrange 18 thirds into groups of two-thirds.

There are 9 groups of two-thirds.

We write:  $6 \div \frac{2}{3} = 9$

- Use the number line again to find how many four-thirds are in 6; that is,  $6 \div \frac{4}{3}$ .

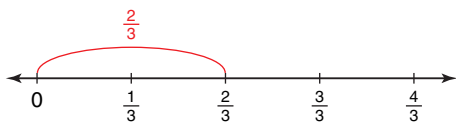
Arrange 18 thirds into groups of four-thirds.



There are 4 groups of 4 thirds.

There are 2 thirds left over.

Think: What fraction of 4 thirds is 2 thirds?



From the number line,  $\frac{2}{3}$  is  $\frac{1}{2}$  of  $\frac{4}{3}$ .

So,  $6 \div \frac{4}{3} = 4\frac{1}{2}$

We can also use a number line to divide a fraction by a whole number. This is illustrated in *Example 1*.

### Example 1

Find each quotient.

- Benny has one-half a litre of milk to pour equally among 3 glasses.  
How much milk should he pour into each glass?
- Chen and Luke equally shared  $\frac{3}{4}$  of a pizza.  
How much of the whole pizza was each person's share?

#### A Solution

- Find:  $\frac{1}{2} \div 3$

Think: Share  $\frac{1}{2}$  into 3 equal parts.

Use a number line. Mark  $\frac{1}{2}$  on the line.

Divide the interval 0 to  $\frac{1}{2}$  into 3 equal parts.



$$\frac{1}{2} = \frac{3}{6}$$

Each part is  $\frac{1}{6}$ .

$$\text{So, } \frac{1}{2} \div 3 = \frac{1}{6}$$

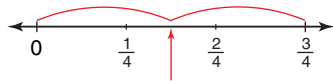
Benny should pour  $\frac{1}{6}$  of a litre of milk into each glass.

- Find:  $\frac{3}{4} \div 2$

Think: Share  $\frac{3}{4}$  into 2 equal parts.

Use a number line. Mark  $\frac{3}{4}$  on the line.

Divide the interval 0 to  $\frac{3}{4}$  into 2 equal parts.



To label this point, divide the fourths into eighths.



$$\frac{3}{4} = \frac{6}{8}$$

Each part is  $\frac{3}{8}$ .

$$\text{So, } \frac{3}{4} \div 2 = \frac{3}{8}$$

Each person's share was  $\frac{3}{8}$  of the pizza.

Sometimes, when we divide fractions and whole numbers, there is a remainder.  
This remainder is written as a fraction of the divisor.

## Example 2

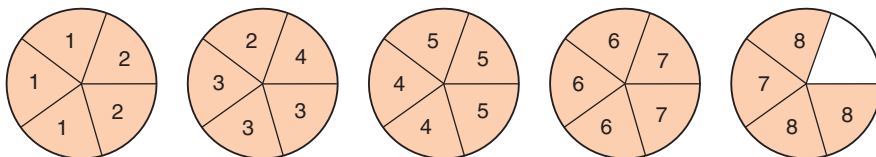
Use a model to divide:  $5 \div \frac{3}{5}$

### A Solution

$$5 \div \frac{3}{5}$$

Think: How many  $\frac{3}{5}$  are in 5 wholes?

Use fraction circles in fifths to model 5.



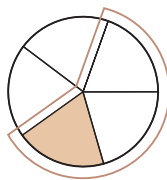
Count groups of three-fifths.

There are 8 groups of three-fifths.

There is 1 fifth left over.

The diagram shows that  $\frac{1}{5}$  is  $\frac{1}{3}$  of  $\frac{3}{5}$ .

$$\text{So, } 5 \div \frac{3}{5} = 8\frac{1}{3}$$



## Discuss the ideas

- In *Example 1a*, the quotient is less than the dividend; that is,  $\frac{1}{6} < \frac{1}{2}$ .  
In *Example 2*, the quotient is greater than the dividend; that is,  $8\frac{1}{3} > 5$ .  
Why do you think this happens?
- How can you use multiplication to check the quotient?

## Practice

### Check

3. Use each picture to find the quotient.  
Write the division equation each time.

a)  $4 \div \frac{1}{3}$



b)  $3 \div \frac{1}{6}$



c)  $4 \div \frac{2}{3}$



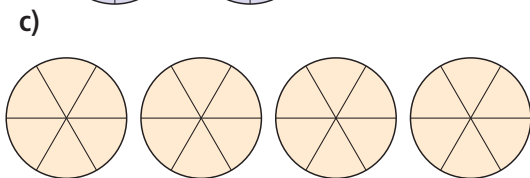
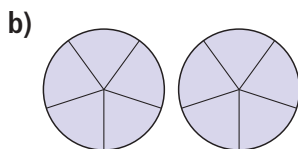
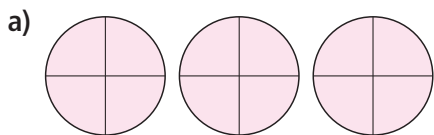
d)  $3 \div \frac{3}{5}$



- 4.** Use fraction circles to find:  $4 \div \frac{5}{6}$
- Use fraction circles to model 4.  
How did you know which fraction circles to use?
  - How many groups of five-sixths are in 4? What is the remainder?
  - What fraction of  $\frac{5}{6}$  does the remainder represent?
  - Write the division equation.
- 5.** Ioana wants to spend  $\frac{4}{5}$  of an hour studying each subject. She has 4 h to study. How many subjects can she study?
- 6.** Use fraction circles.  
Find each quotient.
- $2 \div \frac{1}{2}$
  - $3 \div \frac{1}{3}$
  - $4 \div \frac{1}{4}$
  - $2 \div \frac{1}{6}$
  - $3 \div \frac{1}{2}$
  - $6 \div \frac{3}{4}$

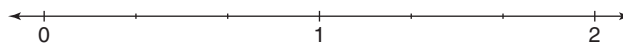
### Apply

- 7.** Which division statement might each picture represent? How many different statements can you write each time?  
Use fraction circles if they help.

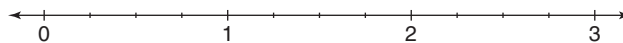


- 8.** Use a number line to find each quotient.

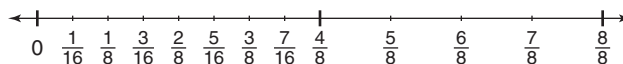
a) i)  $2 \div \frac{1}{3}$       ii)  $2 \div \frac{2}{3}$



b) i)  $3 \div \frac{1}{4}$       ii)  $3 \div \frac{2}{4}$       iii)  $3 \div \frac{3}{4}$



c) i)  $\frac{4}{8} \div 2$       ii)  $\frac{4}{8} \div 4$       iii)  $\frac{4}{8} \div 8$



- 9.** Use a model.  
Find each quotient.

a)  $5 \div \frac{2}{3}$

b)  $4 \div \frac{3}{4}$

c)  $\frac{1}{2} \div 5$

d)  $\frac{5}{8} \div 2$

- 10.** Find each quotient. Use number lines to illustrate the answers.

a) How many  $\frac{1}{4}$ -size sheets can be cut from 5 sheets of paper?

b) How many  $\frac{2}{3}$ -cup servings are in 6 cups of fruit?

c) Janelle feeds her cat  $\frac{4}{5}$  of a tin of cat food each day. Janelle has 12 tins of cat food. How many days' supply of cat food does Janelle have?



**11.** Find each quotient. Use number lines to illustrate the answers.

a) Three-quarters of a whole pizza is shared equally among 5 people.

What fraction of the whole pizza does each person get?

b) One-third of a carton of eggs is used to make a large omelette.

How many large omelettes can be made from 4 cartons of eggs?

c) Brandon planted trees for  $\frac{11}{12}$  h. He planted 5 trees. Assume Brandon took the same amount of time to plant each tree. What fraction of an hour did it take to plant 1 tree?

**12. Assessment Focus** Copy these boxes.

$$\square \div \frac{\square}{\square}$$

a) Write the digits 2, 4, and 6 in the boxes to find as many division expressions as possible.

b) Which expression in part a has the greatest quotient? The least quotient? How do you know?

Show your work.

**13.** Is  $\frac{2}{3} \div 4$  the same as  $4 \div \frac{2}{3}$ ?

Use number lines in your explanation.

**14. Take It Further**

a) Divide:  $8 \div \frac{1}{3}$

b) Divide:  $\frac{1}{8} \div 3$

c) Look at the quotients in parts a and b. What do you notice? How can you explain this?

**15. Take It Further** The numbers  $\frac{9}{2}$  and 3 share this property: their difference is equal to their quotient.

That is,  $\frac{9}{2} - 3 = \frac{3}{2}$  and  $\frac{9}{2} \div 3 = \frac{3}{2}$ .

Find other pairs of numbers with this property. Describe any patterns you see.

## Math Link

### Your World

Usually, a chef measures fractional amounts, such as  $1\frac{1}{2}$  cups.

Occasionally, a chef changes a recipe to serve more or fewer people. To do this, the amounts of ingredients are increased or decreased, often by multiplying or dividing fractions, mixed numbers, or whole numbers.



## Reflect

When you divide a whole number by a proper fraction, is the quotient greater than or less than the whole number? Include an example in your explanation.

# 3.6

## Dividing Fractions

**Focus** Develop algorithms to divide fractions.

You have used grouping to divide a whole number by a fraction:  $4 \div \frac{2}{3} = 6$

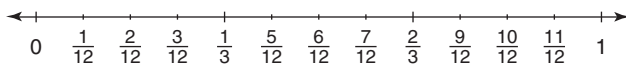
You have used sharing to divide a fraction by a whole number:  $\frac{2}{3} \div 4 = \frac{1}{6}$

You will now investigate dividing a fraction by a fraction.

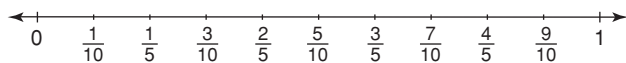
### Investigate

Work with a partner.

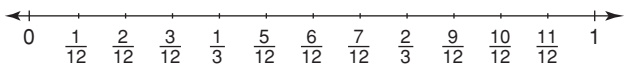
Use this number line. Find:  $\frac{2}{3} \div \frac{1}{4}$



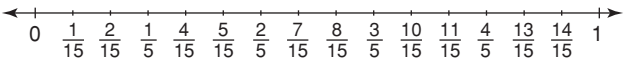
Use this number line. Find:  $\frac{3}{5} \div \frac{1}{2}$



Use this number line. Find:  $\frac{1}{3} \div \frac{1}{4}$



Use this number line. Find:  $\frac{4}{5} \div \frac{2}{3}$



Look at the quotients.

How do the numbers in the numerators and denominators relate to the quotients?

Try to find a strategy to calculate the quotient without using a number line.

Use a different division problem to check your strategy.

### Reflect & Share

Compare your strategy with that of another pair of classmates.

Does your strategy work with their problem? Explain.

Does their strategy work with your problem? Explain.

## Connect

Here are two ways to divide fractions.

- Use common denominators.

To divide:  $\frac{4}{5} \div \frac{1}{10}$

Write each fraction with a common denominator.

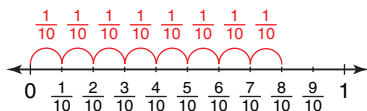
Since 5 is a factor of 10, 10 is a common denominator.

$$\frac{4}{5} = \frac{8}{10}$$

$\times 2$   
 $\times 2$

So,  $\frac{4}{5} \div \frac{1}{10} = \frac{8}{10} \div \frac{1}{10}$

This means: How many 1 tenths are in 8 tenths?



From the number line, this is the same as  $8 \div 1 = 8$ .

So,  $\frac{4}{5} \div \frac{1}{10} = 8$

When the denominators are the same, divide the numerators.

- Use multiplication.

Every whole number can be written as a fraction with denominator 1. Here are some division equations from Lesson 3.5 and their related multiplication equations.

Each multiplication equation uses the reciprocal of the divisor.

This pattern is always true.

Division Equation	Related Multiplication Equation
$5 \div \frac{3}{5} = \frac{25}{3}$ or $\frac{5}{1} \div \frac{3}{5} = \frac{25}{3}$	$\frac{5}{1} \times \frac{5}{3} = \frac{25}{3}$
$4 \div \frac{3}{4} = \frac{16}{3}$ or $\frac{4}{1} \div \frac{3}{4} = \frac{16}{3}$	$\frac{4}{1} \times \frac{4}{3} = \frac{16}{3}$
$6 \div \frac{5}{3} = \frac{18}{5}$ or $\frac{6}{1} \div \frac{5}{3} = \frac{18}{5}$	$\frac{6}{1} \times \frac{3}{5} = \frac{18}{5}$
$\frac{1}{2} \div 3 = \frac{1}{6}$ or $\frac{1}{2} \div \frac{3}{1} = \frac{1}{6}$	$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
$\frac{3}{4} \div 2 = \frac{3}{8}$ or $\frac{3}{4} \div \frac{2}{1} = \frac{3}{8}$	$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$
$\frac{4}{8} \div 4 = \frac{1}{8}$ or $\frac{4}{8} \div \frac{4}{1} = \frac{1}{8}$	$\frac{4}{8} \times \frac{1}{4} = \frac{1}{8}$

We can use the same patterns to divide two fractions.

For example, to find  $\frac{3}{5} \div \frac{1}{4}$ , write the reciprocal of the divisor, then multiply.

$$\begin{aligned} \frac{3}{5} \div \frac{1}{4} &= \frac{3}{5} \times \frac{4}{1} \\ &= \frac{12}{5}, \text{ or } 2\frac{2}{5} \end{aligned}$$

### Example 1

Divide. Estimate to check each quotient is reasonable.

$$\frac{3}{4} \div \frac{5}{6}$$

#### ▶ A Solution

$$\frac{3}{4} \div \frac{5}{6}$$

Use multiplication.

$$\frac{3}{4} \div \frac{5}{6} \text{ can be written as}$$

$$\begin{aligned} \frac{3}{4} \times \frac{6}{5} &= \frac{3 \times \cancel{6}^3}{\cancel{4}_2 \times 5} \text{ Simplify. Divide by common factor 2.} \\ &= \frac{9}{10} \end{aligned}$$

Estimate to check.

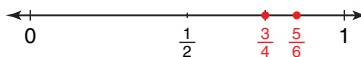
$$\frac{3}{4} \text{ is about } 1. \quad \frac{5}{6} \text{ is about } 1.$$

$$\text{So, } \frac{3}{4} \div \frac{5}{6} \text{ is about } 1 \div 1 = 1.$$

Since  $\frac{5}{6}$  is greater than  $\frac{3}{4}$ ,

$$\frac{3}{4} \div \frac{5}{6} \text{ is less than } 1.$$

Since  $\frac{9}{10}$  is close to 1, and less than 1, the quotient is reasonable.



Dividing by  $\frac{5}{6}$  is the same as multiplying by  $\frac{6}{5}$ .

### Example 2

$$\text{Divide: } \frac{7}{8} \div \frac{1}{4}$$

#### ▶ A Solution

$$\frac{7}{8} \div \frac{1}{4}$$

Use common denominators.

Since 8 is a multiple of 4, 8 is a common denominator.

Multiply the numerator and denominator by 2:  $\frac{1}{4} = \frac{2}{8}$

$$\begin{aligned} \frac{7}{8} \div \frac{1}{4} &= \frac{7}{8} \div \frac{2}{8} \\ &= 7 \div 2 \\ &= \frac{7}{2}, \text{ or } 3\frac{1}{2} \end{aligned}$$

Since the denominators are the same, divide the numerators.

Estimate to check.  $\frac{7}{8}$  is close to 1, but less than 1.

Since there are 4 quarters in one whole,  $\frac{7}{8} \div \frac{1}{4}$  is close to 4, but less than 4.

Since  $3\frac{1}{2}$  is close to 4, and less than 4, the quotient is reasonable.

### Example 3

Use a number line to illustrate the quotient.

$$\frac{3}{5} \div \frac{1}{4}$$

#### A Solution

$$\frac{3}{5} \div \frac{1}{4}$$

Use common denominators.

Write each fraction with a common denominator.

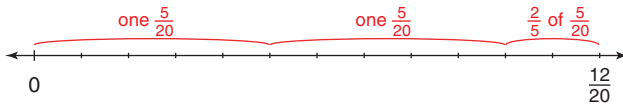
Since 5 and 4 have no common factors, a common denominator is  $5 \times 4 = 20$ .

$$\frac{3}{5} = \frac{12}{20}$$

$$\frac{1}{4} = \frac{5}{20}$$

$$\frac{3}{5} \div \frac{1}{4} = \frac{12}{20} \div \frac{5}{20}$$

This means: How many 5 twentieths are in 12 twentieths?

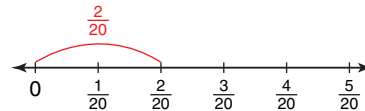


From the number line, there are 2 groups of 5 twentieths with remainder 2 twentieths.

Write the remainder as a fraction of  $\frac{5}{20}$ .

2 twentieths is  $\frac{2}{5}$  of 5 twentieths.

$$\text{So, } \frac{3}{5} \div \frac{1}{4} = 2\frac{2}{5}$$



### Discuss

### the ideas

1. Without dividing, how do you know if the quotient of  $\frac{5}{6} \div \frac{2}{3}$  is less than or greater than 1?
2. Why is it important to estimate to check the quotient?
3. What is a strategy for dividing two fractions?

## Practice

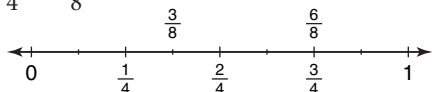
### Check

4. Write the reciprocal of each fraction.

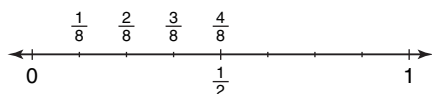
a)  $\frac{5}{9}$       b)  $\frac{3}{7}$       c)  $\frac{7}{8}$       d)  $\frac{14}{15}$

5. Use a copy of each number line to illustrate each quotient.

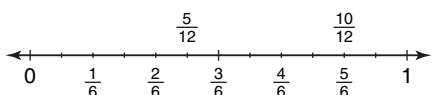
a)  $\frac{3}{4} \div \frac{3}{8}$



b)  $\frac{1}{2} \div \frac{1}{8}$



c)  $\frac{5}{6} \div \frac{5}{12}$



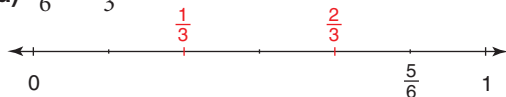
6. Divide:  $\frac{3}{5} \div \frac{9}{10}$

- What is the reciprocal of  $\frac{9}{10}$ ?
- Use multiplication. Simplify first.
- Estimate the quotient.
- Is the quotient reasonable?  
How do you know?

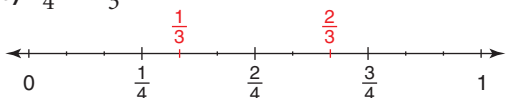
### Apply

7. Use a copy of each number line to illustrate each quotient.

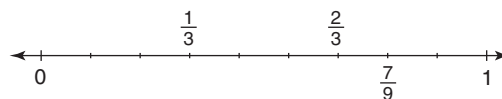
a)  $\frac{5}{6} \div \frac{1}{3}$



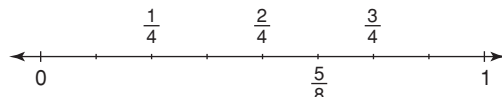
b)  $\frac{3}{4} \div \frac{1}{3}$



c)  $\frac{7}{9} \div \frac{1}{3}$



d)  $\frac{5}{8} \div \frac{1}{4}$



8. Find each quotient.

a)  $\frac{7}{10} \div \frac{3}{10}$       b)  $\frac{5}{9} \div \frac{2}{9}$

c)  $\frac{3}{5} \div \frac{2}{5}$       d)  $\frac{4}{5} \div \frac{2}{5}$

9. Use multiplication to find each quotient.

a)  $\frac{8}{5} \div \frac{3}{4}$       b)  $\frac{9}{10} \div \frac{5}{3}$

c)  $\frac{7}{2} \div \frac{4}{3}$       d)  $\frac{1}{2} \div \frac{7}{6}$

10. Use common denominators to find each quotient.

a)  $\frac{7}{12} \div \frac{1}{4}$       b)  $\frac{3}{5} \div \frac{11}{10}$

c)  $\frac{5}{2} \div \frac{1}{3}$       d)  $\frac{5}{6} \div \frac{9}{8}$

11. Divide. Estimate to check each quotient is reasonable.

a)  $\frac{5}{3} \div \frac{3}{5}$       b)  $\frac{4}{9} \div \frac{4}{9}$       c)  $\frac{1}{6} \div \frac{5}{2}$

12. Suppose you have  $\frac{11}{12}$  of a cake.

How many servings can you make of each size?

a)  $\frac{1}{4}$  of the cake      b)  $\frac{1}{3}$  of the cake

c)  $\frac{1}{6}$  of the cake      d)  $\frac{1}{2}$  of the cake

13. a) Find each quotient.

i)  $\frac{3}{4} \div \frac{5}{8}$       ii)  $\frac{5}{8} \div \frac{3}{4}$

iii)  $\frac{7}{12} \div \frac{2}{5}$       iv)  $\frac{2}{5} \div \frac{7}{12}$

v)  $\frac{5}{3} \div \frac{4}{5}$       vi)  $\frac{4}{5} \div \frac{5}{3}$

b) In part a, what patterns do you see in the division statements and their quotients? Write two more pairs of division statements that follow the same pattern.

14. As a busboy in a restaurant, Amiel takes  $\frac{1}{12}$  h to clear and reset a table. How many tables can Amiel clear in  $\frac{2}{3}$  h? Estimate to check the solution is reasonable.

15. Divide. Estimate to check each quotient is reasonable.

a)  $\frac{27}{28} \div \frac{9}{14}$       b)  $\frac{15}{22} \div \frac{3}{11}$

c)  $\frac{32}{51} \div \frac{8}{17}$       d)  $\frac{57}{69} \div \frac{19}{115}$

16. To conduct a science experiment, each pair of students requires  $\frac{1}{16}$  cup of vinegar. The science teacher has  $\frac{3}{4}$  cup of vinegar. How many pairs of students can conduct the experiment?

### 17. Assessment Focus

a) Copy the boxes below. Write the digits 2, 3, 4, and 5 in the boxes to make as many different division statements as you can.

$$\frac{\square}{\square} \div \frac{\square}{\square}$$

b) Which division statement in part a has the greatest quotient? The least quotient? How do you know? Show your work.

18. Tahoe used the expression  $\frac{7}{8} \div \frac{1}{4}$  to solve a word problem. What might the word problem be? Solve the problem. Estimate to check the solution is reasonable.

19. **Take It Further** Copy each division equation. Replace each  $\square$  with a fraction to make each equation true. Explain the strategy you used.

a)  $\frac{2}{3} \div \square = \frac{8}{9}$       b)  $\frac{3}{11} \div \square = \frac{12}{55}$

c)  $\frac{1}{4} \div \square = \frac{9}{20}$       d)  $\frac{4}{5} \div \square = \frac{28}{45}$

20. **Take It Further** Write as many division statements as you can that have a quotient between  $\frac{1}{2}$  and 1. Explain the strategy you used.

## Reflect

Explain how your knowledge of common denominators can help you divide two fractions. Include an example in your explanation.

# 3.7

## Dividing Mixed Numbers

### Focus

Apply knowledge of dividing fractions to divide mixed numbers.

What strategies can you use to divide  $\frac{5}{6} \div \frac{2}{3}$ ?

### Investigate

Work with a partner.

Jeffrey wants to share some round Belgian waffles with his friends.

Suppose Jeffrey makes each portion  $\frac{3}{4}$  of a waffle.

How many portions will he get from  $4\frac{1}{2}$  waffles?

How can you find out?

Show your work.

Use models or diagrams to justify your strategy.

### Reflect & Share

Compare your strategy with that of another pair of classmates.

Do you think your strategy will work with all mixed numbers?

Test it with  $1\frac{1}{2} \div \frac{5}{8}$ .



## Connect

Here are three ways to divide mixed numbers.

In each method, the mixed numbers are first written as improper fractions.

- Use a number line.

To divide:  $4\frac{2}{5} \div 1\frac{1}{2}$

$$4\frac{2}{5} = \frac{22}{5} \text{ and } 1\frac{1}{2} = \frac{3}{2}$$

Write each fraction with a common denominator.

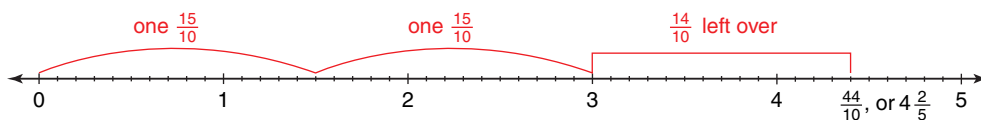
Since 2 and 5 have no common factors, a common denominator is  $2 \times 5 = 10$ .

$$\frac{22}{5} = \frac{44}{10} \text{ and } \frac{3}{2} = \frac{15}{10}$$

$$\text{So, } 4\frac{2}{5} \div 1\frac{1}{2} = \frac{44}{10} \div \frac{15}{10}$$

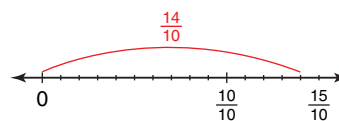
This means: How many 15 tenths are in 44 tenths?

Use a number line divided in tenths.



From the number line, there are 2 groups of 15 tenths, with remainder 14 tenths.

Think: What fraction of 15 tenths is 14 tenths?



From the number line, 14 tenths is  $\frac{14}{15}$  of 15 tenths.

$$\text{So, } 4\frac{2}{5} \div 1\frac{1}{2} = 2\frac{14}{15}$$

- Use common denominators.

Divide:  $4\frac{2}{5} \div 1\frac{1}{2}$

$$4\frac{2}{5} = \frac{22}{5} \text{ and } 1\frac{1}{2} = \frac{3}{2}$$

Write each fraction with a common denominator.

$$\frac{22}{5} = \frac{44}{10} \text{ and } \frac{3}{2} = \frac{15}{10}$$

$$\text{So, } 4\frac{2}{5} \div 1\frac{1}{2} = \frac{44}{10} \div \frac{15}{10}$$

$$= 44 \div 15$$

$$= \frac{44}{15}, \text{ or } 2\frac{14}{15}$$

Since the denominators are the same, divide the numerators.

- Use multiplication.

$$4\frac{2}{5} \div 1\frac{1}{2} = \frac{22}{5} \div \frac{3}{2}$$

Recall that dividing by  $\frac{3}{2}$  is the same as multiplying by  $\frac{2}{3}$ .

$$\begin{aligned} \text{So, } \frac{22}{5} \div \frac{3}{2} &= \frac{22}{5} \times \frac{2}{3} \\ &= \frac{22 \times 2}{5 \times 3} \\ &= \frac{44}{15} \\ &= 2\frac{14}{15} \end{aligned}$$

$\frac{2}{3}$  is the reciprocal of  $\frac{3}{2}$ .

### Example 1

Divide. Estimate to check the quotient is reasonable.

$$1\frac{7}{8} \div 1\frac{1}{4}$$

#### A Solution

$$1\frac{7}{8} \div 1\frac{1}{4}$$

Change the mixed numbers to improper fractions.

$$1\frac{7}{8} = \frac{15}{8} \quad \text{and} \quad 1\frac{1}{4} = \frac{5}{4}$$

$$\begin{aligned} \text{So, } 1\frac{7}{8} \div 1\frac{1}{4} &= \frac{15}{8} \div \frac{5}{4} \\ &= \frac{15}{8} \times \frac{4}{5} \\ &= \frac{3\cancel{15}^1}{2\cancel{8}_1} \times \frac{\cancel{4}^1}{\cancel{5}_1} \\ &= \frac{3 \times 1}{2 \times 1} \\ &= \frac{3}{2}, \text{ or } 1\frac{1}{2} \end{aligned}$$

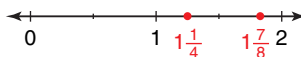
Estimate to check.

$1\frac{7}{8}$  is close to 2.  $1\frac{1}{4}$  is close to 1.

So,  $1\frac{7}{8} \div 1\frac{1}{4}$  is about  $2 \div 1 = 2$ .

Since  $1\frac{7}{8}$  is less than 2, and  $1\frac{1}{4}$  is greater than 1, the quotient will be less than 2.

Since  $1\frac{1}{2}$  is close to 2, the quotient is reasonable.



## Example 2

Brittany has a summer job in a bakery.

One day, she used  $3\frac{3}{4}$  cups of chocolate chips to make chocolate-chip muffins.

A dozen muffins requires  $\frac{3}{4}$  cup chocolate chips.

How many dozen chocolate-chip muffins did Brittany make that day?

### A Solution

$$3\frac{3}{4} \div \frac{3}{4}$$

Change the mixed number to an improper fraction.

$$3\frac{3}{4} = \frac{15}{4}$$

So,  $3\frac{3}{4} \div \frac{3}{4} = \frac{15}{4} \div \frac{3}{4}$  Since the denominators are the same, divide the numerators.

$$= 15 \div 3$$

$$= \frac{15}{3}$$

$$= 5$$

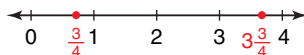
Estimate to check.

$3\frac{3}{4}$  is close to 4.  $\frac{3}{4}$  is close to 1.

So,  $3\frac{3}{4} \div \frac{3}{4}$  is about  $4 \div 1 = 4$ .

Since 5 is close to 4, the solution is reasonable.

Brittany made 5 dozen chocolate-chip muffins.



## Discuss the ideas

1. How is dividing mixed numbers similar to dividing fractions?
2. You have seen 3 methods for dividing mixed numbers. Which method are you likely to use most often? Justify your choice.
3. Why do we often write the quotient as a mixed number when we divide with mixed numbers?

## Practice

### Check

4. Write each mixed number as an improper fraction.

a)  $4\frac{3}{8}$     b)  $3\frac{2}{7}$     c)  $6\frac{1}{6}$     d)  $2\frac{1}{4}$   
 e)  $1\frac{7}{10}$     f)  $7\frac{2}{3}$     g)  $2\frac{5}{9}$     h)  $5\frac{2}{5}$

5. Write each improper fraction as a mixed number.

a)  $\frac{14}{9}$     b)  $\frac{16}{7}$     c)  $\frac{24}{5}$     d)  $\frac{21}{10}$   
 e)  $\frac{15}{6}$     f)  $\frac{23}{7}$     g)  $\frac{17}{3}$     h)  $\frac{25}{12}$

6. Use estimation. Which number is each quotient closer to?

a)  $6\frac{1}{8} \div 2\frac{3}{4}$     2 or 3  
 b)  $7\frac{4}{5} \div 1\frac{3}{4}$     3 or 4  
 c)  $3\frac{1}{8} \div 2\frac{3}{4}$     1 or 2  
 d)  $9\frac{4}{7} \div 2\frac{1}{9}$     4 or 5

7. Divide:  $1\frac{4}{5} \div 2\frac{7}{10}$

- a) Estimate the quotient.  
 b) Write each mixed number as an improper fraction.  
 c) Divide the improper fractions. Simplify first.  
 d) Is the quotient reasonable? How do you know?

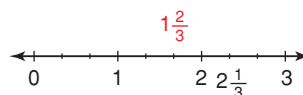
### Apply

8. Use common denominators to find each quotient. Estimate to check the quotient is reasonable.

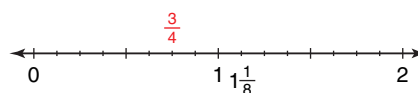
a)  $3\frac{3}{4} \div 1\frac{1}{8}$     b)  $1\frac{1}{6} \div 4\frac{1}{3}$   
 c)  $3\frac{1}{4} \div 3\frac{1}{4}$     d)  $\frac{2}{3} \div 1\frac{1}{9}$

9. Use a copy of each number line to illustrate each quotient.

a)  $2\frac{1}{3} \div 1\frac{2}{3}$



b)  $1\frac{1}{8} \div \frac{3}{4}$



10. Use multiplication to find each quotient. Estimate to check the quotient is reasonable.

a)  $3\frac{2}{3} \div 5\frac{1}{4}$   
 b)  $4\frac{3}{8} \div 1\frac{5}{16}$   
 c)  $1\frac{3}{10} \div 3\frac{3}{5}$   
 d)  $3\frac{2}{3} \div 3\frac{2}{3}$

11. Divide. Estimate to check the quotient is reasonable.

a)  $1\frac{9}{10} \div 2\frac{2}{3}$     b)  $2\frac{3}{4} \div 2\frac{1}{3}$   
 c)  $1\frac{4}{5} \div 3\frac{1}{2}$     d)  $1\frac{3}{8} \div 1\frac{3}{8}$

12. Maxine took  $12\frac{1}{2}$  h to build a model airplane. She worked for  $1\frac{1}{4}$  h each evening. How many evenings did Maxine take to complete the model?

13. Glenn ran  $3\frac{1}{3}$  laps in  $11\frac{2}{3}$  min. Assume Glenn took the same amount of time to complete each lap. How long did Glenn take to run one lap?

**14. Assessment Focus** Amelia has her own landscaping business. She ordered  $10\frac{5}{8}$  loads of topsoil to fill large concrete planters. Each planter holds  $1\frac{1}{2}$  loads of topsoil.

- Estimate the number of planters that Amelia can fill.
- Sketch a number line to illustrate the answer.
- Calculate the number of planters that Amelia can fill.
- What does the fraction part of the answer represent?

**15.** Write a story problem that could be solved using the expression  $4\frac{2}{3} \div \frac{3}{5}$ . Find the quotient to solve the problem. Estimate to check the solution is reasonable.

**16.** Use estimation. Which expression below has the greatest quotient? The least quotient? How do you know?

- $\frac{8}{5} \div \frac{4}{3}$
- $2\frac{3}{4} \div 1\frac{7}{8}$
- $4\frac{8}{9} \div 2\frac{1}{8}$
- $2\frac{1}{10} \div 1\frac{5}{6}$

**17. Take It Further**

- Which of these quotients is a mixed number? How can you tell without dividing?
  - $4\frac{3}{8} \div 3\frac{2}{5}$
  - $3\frac{2}{5} \div 4\frac{3}{8}$
- Find the quotients in part a. What can you say about the order in which you divide mixed numbers?

**18. Take It Further** Which expression below has the greatest value? Give reasons for your answer. How could you find out without calculating each answer?

- |                                      |                                      |
|--------------------------------------|--------------------------------------|
| a) $3\frac{1}{5} \times \frac{1}{2}$ | b) $3\frac{1}{5} \times \frac{2}{3}$ |
| c) $3\frac{1}{5} \div \frac{2}{3}$   | d) $3\frac{1}{5} \div \frac{2}{1}$   |
| e) $3\frac{1}{5} + \frac{2}{3}$      | f) $3\frac{1}{5} + \frac{3}{2}$      |

**19. Take It Further** One way to divide fractions is to use multiplication.

- How could you multiply fractions by using division? Explain.
- Do you think you would want to multiply fractions by using division? Why or why not?

## Reflect

Suppose you divide one mixed number by another.

How can you tell, before you divide, if the quotient will be:

- greater than 1?
- less than 1?
- equal to 1?

Use examples in your explanation.

# 3.8

## Solving Problems with Fractions

### Focus

Identify the operation required to solve a problem involving fractions.

A Grade 8 class is going to a canoe competition.  
 There are 24 students in the class.  
 Each canoe holds 4 people.  
 How many canoes does the class need?  
 How did you know which operation to use?

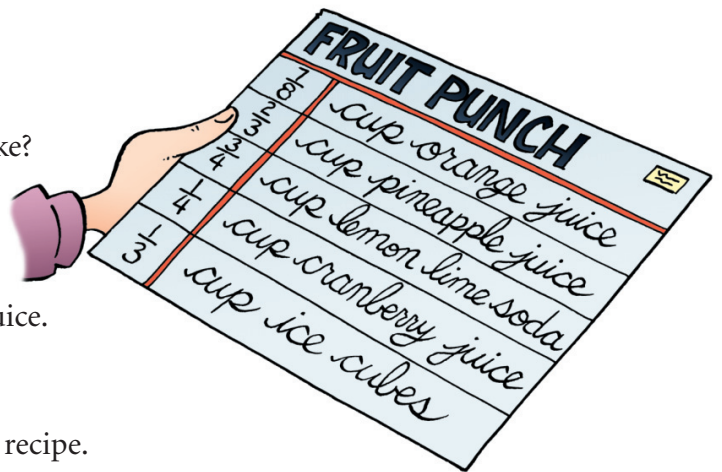


### Investigate

Work with a partner.

Aidan wants to make this fruit punch recipe.  
 Answer each of these questions.  
 Show your work.

- How many cups of punch does the recipe make?
- Suppose Aidan makes the punch, then pours himself  $\frac{3}{4}$  cup of punch.  
 How much punch does he have left?
- Suppose Aidan only has  $\frac{1}{3}$  cup of pineapple juice.  
 How much of each of the other ingredients does he need to keep the flavour the same?
- Suppose Aidan decides to make one-third the recipe.  
 How much soda will he need?



### Reflect & Share

Compare your answers and strategies with those of another pair of classmates.

Did you solve the problems the same way?

How did you decide which operation to use each time?

## Connect

When solving word problems, it is important to identify the operation or operations needed to solve the problem.

To identify the operation:

- Think about the situation.
- Make sense of the problem by explaining it in your own words, drawing a picture, or using a model.
- Think about what is happening in the problem. Sometimes, key words can help you identify the operation to use.

For example, “total” suggests adding,

“less than” suggests subtracting,

“times” suggests multiplying, and

“shared” suggests dividing.

Although key words may help identify an operation, the operation must make sense in the context of the problem.

### Example 1

Kassie worked on her science project for  $\frac{3}{4}$  h on Tuesday and  $\frac{5}{6}$  h on Wednesday.

She spent Thursday finishing her math homework.

- How long did Kassie work on her science project altogether?
- How much longer did Kassie work on the project on Wednesday than on Tuesday?
- Altogether, Kassie spent 2 h on school work over the 3 days.  
How long did Kassie spend on her math homework?

### A Solution

- The word “altogether” suggests addition.

$$\text{Add: } \frac{3}{4} + \frac{5}{6}$$

$$\begin{aligned}\frac{3}{4} + \frac{5}{6} &= \frac{9}{12} + \frac{10}{12} \\ &= \frac{19}{12} \\ &= \frac{12}{12} + \frac{7}{12} \\ &= 1\frac{7}{12}\end{aligned}$$

Kassie worked on her science project for  $1\frac{7}{12}$  h.

Write an equivalent fraction with denominator 12 for each fraction.

$$\frac{3}{4} = \frac{9}{12} \quad \frac{5}{6} = \frac{10}{12}$$

- b) The words “longer ... than” suggest subtraction.

To find how much longer Kassie worked on Wednesday than on Tuesday,

$$\begin{aligned}\text{subtract: } & \frac{5}{6} - \frac{3}{4} \\ \frac{5}{6} - \frac{3}{4} &= \frac{10}{12} - \frac{9}{12} \\ &= \frac{1}{12}\end{aligned}$$

Kassie worked  $\frac{1}{12}$  h longer on Wednesday than on Tuesday.

- c) The word “altogether” suggests addition.

However, to find the time Kassie spent doing math homework, we subtract.

From part a, we know Kassie spent  $1\frac{7}{12}$  h on her science project.

Altogether, Kassie spent 2 h on school work over the 3 days.

So, the time spent on her math homework is:

$$\begin{aligned}2 - 1\frac{7}{12} &= 1\frac{12}{12} - 1\frac{7}{12} \\ &= \frac{5}{12}\end{aligned}$$

Kassie spent  $\frac{5}{12}$  h on her math homework.

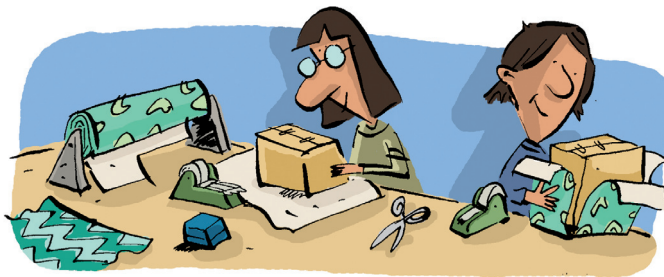
## Example 2

Dakota volunteered at a gift-wrapping booth for a local charity.

He volunteered for  $2\frac{3}{4}$  h and wrapped 11 gift boxes.

His friend Winona volunteered  $1\frac{1}{3}$  times as long.

- a) How long did Dakota spend wrapping each gift box?  
What assumptions do you make?
- b) How many hours did Winona volunteer?



## A Solution

- a) Think what the problem means:

We are given a time for many and we have to find a time for one.

This suggests division.

$$2\frac{3}{4} \div 11$$

Write the mixed number as an improper fraction.

$$2\frac{3}{4} = \frac{11}{4}$$

$$\text{So, } 2\frac{3}{4} \div 11 = \frac{11}{4} \div 11 \quad \text{Write the reciprocal of the divisor, then multiply.}$$

$$= \frac{11}{4} \times \frac{1}{11} \quad \text{Simplify. Divide by the common factor 11.}$$

$$= \frac{\cancel{11} \times 1}{4 \times \cancel{11}}$$

$$= \frac{1}{4}$$

Dakota spent  $\frac{1}{4}$  h wrapping each gift box.

It was assumed that Dakota wrapped for the entire time and that he spent the same amount of time wrapping each box.

- b) The words “times as long” suggest multiplication.

So, Winona volunteered for  $1\frac{1}{3}$  of the  $2\frac{3}{4}$  h that Dakota volunteered.

$$2\frac{3}{4} \times 1\frac{1}{3}$$

Write the mixed numbers as improper fractions.

$$2\frac{3}{4} = \frac{11}{4} \quad \text{and} \quad 1\frac{1}{3} = \frac{4}{3}$$

$$\text{So, } 2\frac{3}{4} \times 1\frac{1}{3} = \frac{11}{4} \times \frac{4}{3}$$

$$= \frac{11 \times \cancel{4}^1}{\cancel{4}^1 \times 3} \quad \text{Simplify. Divide by the common factor 4.}$$

$$= \frac{11}{3}$$

$$= 3\frac{2}{3}$$

Winona volunteered for  $3\frac{2}{3}$  h.

## Discuss the ideas

1. What other words indicate addition, subtraction, multiplication, and division?
2. What other questions could you ask using the data in *Example 2*?  
What operation would you use to answer each question?

## Practice

### Check

3. Which operation would you use to solve each problem? How can you tell?
  - a) Noel used  $\frac{2}{3}$  cup of milk and  $\frac{1}{4}$  cup of oil to make cookies. How much liquid did he use altogether?
  - b) One-third of the cars in the parking lot are silver. There are 165 cars in the lot. How many cars are silver?
  - c) Shania has  $\frac{3}{8}$  cup of yogurt. She needs  $\frac{3}{4}$  cup of yogurt to make a smoothie. How much more yogurt does she need?
  - d) Part of a pizza was shared equally between two friends. Each friend got  $\frac{5}{12}$  of the whole pizza. How much pizza was shared?

Solve each problem. For each problem, explain how you decided which operation or operations to use.

4. Chad mixed  $\frac{2}{3}$  of one can of yellow paint and  $\frac{1}{4}$  of one can of white paint to paint a wall in his bedroom. How much paint did he have altogether?



5. Vivi scored 5 goals in the Saskatoon Sticks lacrosse tournament. This was  $\frac{1}{8}$  of her team's goals. How many goals did Vivi's team score altogether?

### Apply

6. Parent-teacher interviews were held on Thursday. Of those parents who attended,  $\frac{1}{6}$  attended in the morning,  $\frac{1}{3}$  attended in the afternoon, and the rest attended in the evening.
  - a) What fraction of the parents attended in the evening?
  - b) Thirty parents attended the interviews. How many parents attended in the evening?
7. Patti works in a coffee shop. She usually takes  $\frac{3}{4}$  h for lunch. One day the shop was very busy and Patti's manager asked her to shorten her lunch break by  $\frac{1}{6}$  h. What fraction of an hour did Patti take for lunch that day?
8. Katrina's monthly salary is \$2400. She uses  $\frac{2}{5}$  of this money for rent. How much rent does Katrina pay?
9. A snail travelled 48 cm in  $\frac{2}{3}$  h. Suppose the snail moved at a constant speed and made no stops. How far would the snail travel in 1 h?

**10.** Beven has a collection of 72 music CDs. One-sixth of the CDs are dance music,  $\frac{1}{4}$  are hip hop, and  $\frac{3}{8}$  are reggae. The rest of the CDs are rock music. What fraction of her CDs are rock music?

**11. Assessment Focus** A jug contains  $2\frac{1}{2}$  cups of juice. Shavon pours  $\frac{3}{8}$  cup of juice into each of three glasses, then  $\frac{5}{6}$  cup of juice into a fourth glass.

- Estimate the fraction of the apple juice that remains in the jug.
- Calculate the total amount of juice in the 3 glasses that contain the same amount.
- Calculate the total amount of juice in the 4 glasses.
- How much juice remains in the jug after the 4 glasses have been poured?

**12.** The Hendersons went on a driving vacation. They decided to travel  $\frac{1}{3}$  of the distance on the first day. Owing to bad weather, they had to stop for the night having gone only  $\frac{1}{4}$  of the distance to the one-third point. What fraction of the total distance did they travel the first day?

**13.** Howie works at a petting zoo. He fed a piglet  $\frac{1}{5}$  of a bottle of milk, then gave  $\frac{3}{4}$  of what was left to a calf. How much of the bottle of milk did the calf drink?



**14.** Nathan used  $2\frac{5}{6}$  loaves of bread to make sandwiches for a lunch. He made equal numbers of 4 different types of sandwiches. What fraction of a loaf of bread did Nathan use for each type of sandwich?

**15. Take It Further** A steward reported to an airport official before takeoff that three-fifths of the passengers were women, three-eighths were men, and one-twentieth were children. After thinking for a moment, the official seemed puzzled and asked the steward to repeat the fractions. Why do you think the official was puzzled? Explain.

## Reflect

Use the fractions  $\frac{1}{2}$  and  $\frac{7}{8}$ . Write 4 problems. Each problem should require a different operation (addition, subtraction, multiplication and division).

Solve each problem. How did you decide which operation to use?

## 3.9

## Order of Operations with Fractions

## Focus

Use the order of operations to evaluate expressions.

Suppose you had to answer this skill-testing question to win a contest.

What answer would you give?

$$5 + 20 \times 2 - 36 \div 9$$

Explain the strategy you used.

Every fraction can be written as a decimal.

So, we use the same order of operations for fractions as for whole numbers and decimals.



## Investigate

Work with a partner.

Use these fractions:  $\frac{9}{4}$ ,  $\frac{3}{8}$ ,  $\frac{15}{16}$

Use any operations or brackets.

Write an expression that has value 4.

Show your work.

**Reflect  
Share &**

Compare your expression with that of another pair of classmates.

If the expressions are different, check that both expressions have value 4.

What strategies did you use to arrive at your expression?

## Connect

In *Investigate*, you used the order of operations to write an expression with a specific value. To make sure everyone gets the same value for a given expression, we add, subtract, multiply, and divide in this order:

- Do the operations in brackets first.
- Then divide and multiply, in order, from left to right.
- Then add and subtract, in order, from left to right.

### Example 1

Evaluate:  $\frac{5}{16} - \frac{3}{8} \times \frac{2}{3}$

#### A Solution

$$\begin{aligned}\frac{5}{16} - \frac{3}{8} \times \frac{2}{3} \\ &= \frac{5}{16} - \frac{\cancel{3}^1}{4\cancel{8}_2} \times \frac{\cancel{2}^1}{\cancel{3}_1} \\ &= \frac{5}{16} - \frac{1}{4} \\ &= \frac{5}{16} - \frac{4}{16} \\ &= \frac{1}{16}\end{aligned}$$

Multiply. Simplify first.

Use common denominators to subtract.

### Example 2

Evaluate:  $\frac{3}{4} - \frac{2}{3} \div \frac{4}{5} \times (\frac{1}{8} + \frac{1}{4})$

#### A Solution

$$\begin{aligned}\frac{3}{4} - \frac{2}{3} \div \frac{4}{5} \times (\frac{1}{8} + \frac{1}{4}) \\ &= \frac{3}{4} - \frac{2}{3} \div \frac{4}{5} \times (\frac{1}{8} + \frac{2}{8}) \\ &= \frac{3}{4} - \frac{2}{3} \div \frac{4}{5} \times (\frac{3}{8}) \\ &= \frac{3}{4} - \frac{\cancel{2}^1}{3} \times \frac{5}{\cancel{4}_2} \times (\frac{3}{8}) \\ &= \frac{3}{4} - \frac{5}{2} \times \frac{\cancel{3}^1}{8} \\ &= \frac{3}{4} - \frac{5}{16} \\ &= \frac{12}{16} - \frac{5}{16} \\ &= \frac{7}{16}\end{aligned}$$

First do the operation in brackets.

Use common denominators to add.

Divide and multiply from left to right.

To divide by  $\frac{4}{5}$ , multiply by  $\frac{5}{4}$ . Simplify first.

Multiply. Simplify first.

Use common denominators to subtract.

### Discuss

### the ideas

1. A student suggested that brackets should be put around  $\frac{3}{8} \times \frac{2}{3}$  in *Example 1*. What is your response to this suggestion?
2. Why are the brackets necessary in *Example 2*?
3. Do you think most people would get the skill-testing question in the introduction correct? If not, what answer do you think they would give?

## Practice

### Check

4. Which operation do you do first?

a)  $\frac{1}{3} \times (\frac{7}{8} - \frac{3}{4})$

b)  $\frac{7}{8} \div (\frac{1}{3} \times \frac{1}{8})$

c)  $\frac{9}{5} \times (\frac{3}{5} \div \frac{1}{10})$

d)  $(\frac{5}{3} + \frac{7}{12}) \times \frac{4}{9}$

5. Raj and Rena evaluated this expression:

$$\frac{5}{9} + \frac{2}{3} \times \frac{1}{2}$$

Raj got  $\frac{8}{9}$ . Rena got  $\frac{11}{18}$ . Who is correct?

What mistake did the other person make?

6. Evaluate. Which operation is done first?

a)  $\frac{1}{2} \times \frac{3}{5} + \frac{1}{4}$

b)  $\frac{2}{3} + \frac{5}{6} \div \frac{1}{2}$

c)  $\frac{4}{5} \div \frac{7}{10} + \frac{1}{3}$

d)  $\frac{1}{4} \times (\frac{11}{12} - \frac{5}{6})$

e)  $\frac{1}{2} \times (\frac{4}{5} \div \frac{3}{10})$

f)  $(\frac{3}{5} + \frac{7}{15}) \times \frac{5}{6}$

### Apply

7. Evaluate. Show all steps.

a)  $\frac{1}{8} \times \frac{3}{4} \times \frac{7}{5} \div \frac{7}{10}$

b)  $\frac{14}{15} \div \frac{2}{3} \times \frac{5}{8} + \frac{3}{4}$

c)  $\frac{2}{3} - \frac{1}{4} + \frac{1}{2} \div \frac{2}{5}$

d)  $\frac{5}{6} - \frac{1}{5} \times \frac{5}{8} + \frac{2}{3}$

8. Emma thinks that the expressions

$$1\frac{1}{2} \div \frac{1}{4} \times \frac{2}{3} \text{ and } 1\frac{1}{2} \div (\frac{1}{4} \times \frac{2}{3})$$

have the same value. Is Emma correct?

Explain.

9. Evaluate.

a)  $\frac{7}{10} - (\frac{1}{5} + \frac{1}{4}) \times \frac{2}{3}$

b)  $(\frac{1}{4} + \frac{5}{6} - \frac{1}{3}) \times \frac{8}{5}$

c)  $(\frac{6}{5} + \frac{4}{10}) \times (\frac{3}{8} - \frac{1}{16})$

10. Evaluate.

a)  $\frac{5}{2} + \frac{1}{4} \times \frac{4}{5} \div \frac{1}{10} - \frac{1}{2}$

b)  $\frac{4}{9} \times (\frac{2}{3} - \frac{1}{6}) - \frac{1}{8} \times \frac{4}{3}$

11. **Assessment Focus** Robert, Myra, and

Joe evaluated this expression:

$$4 \times (\frac{3}{4} - \frac{1}{2}) + \frac{13}{6} \times \frac{1}{2}$$

Robert's answer was  $5\frac{1}{3}$ , Myra's answer was  $2\frac{1}{12}$ , and Joe's answer was  $4\frac{5}{6}$ .

a) Who had the correct answer?

How do you know?

b) Show and explain how the other two students got their answers.

Where did they go wrong?

12. **Take It Further** Evaluate.

a)  $\frac{14}{15} \div 4\frac{2}{3} \times \frac{5}{8} + 2\frac{3}{4}$

b)  $3\frac{2}{3} - 2\frac{1}{4} + \frac{1}{2} \div 2\frac{2}{5}$

c)  $8\frac{7}{10} - (2\frac{1}{5} + 2\frac{1}{4}) \times \frac{2}{3}$

## Reflect

Write an expression that contains fractions and three operations.

Talk to a partner. Discuss the steps you would follow to evaluate the expression.

## Checking and Reflecting

Have you ever finished your math homework and thought “What was that all about?”

One purpose of homework is to increase your understanding of the math you are learning.

When you have finished your homework, how do you know you have understood everything you are supposed to understand?



It is important to reflect on what you have done and to check your understanding.

Solve this problem:

Chris used  $\frac{2}{3}$  of a roll of stamps.

His sister then used  $\frac{1}{4}$  of the stamps left on the roll.

What fraction of the roll of stamps did Chris' sister use?

What fraction of the roll of stamps is left?



When you have solved the problem, ask yourself these questions.

- What math idea did this problem involve?
- How difficult was this problem for me?
- Did I need help with this problem?
- What could I say to help someone else understand the problem?
- What would make this problem easier to solve?
- What would make this problem more difficult to solve?
- What other math ideas does this problem remind me of?



### Using a Traffic Light Strategy

You can use “traffic light” signals to help you reflect on your understanding. It is a good way to signal what you *know* you can do, what you *think* you can do, and what you need help with.

When you have finished your homework, reflect on all the questions. Label each question with a green dot, a yellow dot, or a red dot.



A red dot means: I do not understand this. I cannot answer the question. I need some help.



A yellow dot means: I am not sure I understand this. I might be able to answer this question with a little help but I am not confident about my answer. I might need to ask a question or two to get started.



A green dot means: I understand this. I can answer this question and am confident about my answer. If someone else is struggling with this question, I can help.

Answer these questions.

When you have finished, use the “Traffic Light” strategy to label each question in your notebook with a green, yellow, or red dot.

1. Evaluate.

a)  $\frac{3}{5} \times 5$

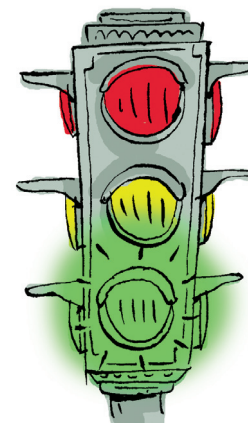
b)  $\frac{3}{7} \times \frac{14}{15}$

c)  $\frac{5}{2} \div \frac{6}{7}$

d)  $\frac{5}{6} - \frac{1}{9} \times \frac{3}{4}$

2. A glass holds  $\frac{2}{3}$  cup of juice. A jug contains 8 cups of juice. How many glasses can be filled from the juice in the jug?

3. A soccer practice lasted  $1\frac{1}{4}$  h. Drills took up  $\frac{2}{3}$  of the time. How much time was spent on drills?



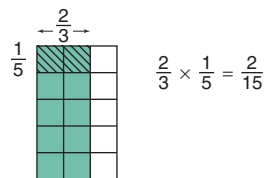
When you have completed the *Unit Review* and *Practice Test*, use the “Traffic Light” strategy to check your understanding. Get help with questions labelled with yellow and red dots.

# Unit Review

## What Do I Need to Know?

- ✓ **To multiply two fractions:**  
Multiply the numerators and multiply the denominators.

$$\frac{2}{3} \times \frac{1}{5} = \frac{2 \times 1}{3 \times 5} = \frac{2}{15}$$



**To multiply two mixed numbers:**

Write each number as an improper fraction, then multiply.

$$\begin{aligned} 1\frac{1}{2} \times 2\frac{5}{6} &= \frac{3}{2} \times \frac{17}{6} \\ &= \frac{17}{4}, \text{ or } 4\frac{1}{4} \end{aligned}$$

Simplify first, when you can.

- ✓ **To divide two fractions:**

Method 1:

Use common denominators.

$$\frac{4}{5} \div \frac{3}{2} = \frac{8}{10} \div \frac{15}{10} = \frac{8}{15}$$

Method 2:

Use multiplication.

$$\frac{4}{5} \div \frac{3}{2} \text{ is the same as } \frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$$

**To divide two mixed numbers:**

Write each number as an improper fraction, then divide.

$$\begin{aligned} 3\frac{1}{2} \div 1\frac{2}{3} &= \frac{7}{2} \div \frac{5}{3} \\ &= \frac{7}{2} \times \frac{3}{5} \\ &= \frac{21}{10}, \text{ or } 2\frac{1}{10} \end{aligned}$$

- ✓ **To identify the operation:**

- Think about the situation.
- Make sense of the problem.
- Think about what is happening in the problem. Use key words to help.

- ✓ **The order of operations with whole numbers and decimals applies to fractions.**

- Do the operations in brackets first.
- Then divide and multiply, in order, from left to right.
- Then add and subtract, in order, from left to right.

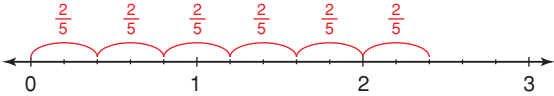
## What Should I Be Able to Do?

### LESSON

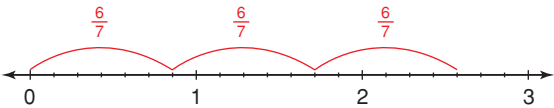
#### 3.1

1. Write the multiplication equation each number line represents.

a)



b)



2. Multiply. Draw a picture or a number line to show each product.

a)  $\frac{1}{3} \times 3$     b)  $7 \times \frac{1}{2}$     c)  $8 \times \frac{2}{5}$

3. Solve each problem.

- a) There are 30 students in a class. Three-fifths of the students are girls. How many girls are in the class?
- b) Six glasses are  $\frac{2}{3}$  full. How many full glasses could be made?
- c) There are 75 cars in the parking lot of a car dealership. Two-thirds of the cars are new. How many of the cars are new?
- d) One serving is  $\frac{1}{12}$  of a cake. How many cakes are needed for 18 servings?

#### 3.2

4. Draw an area model to find each product.

a)  $\frac{2}{3} \times \frac{3}{8}$     b)  $\frac{4}{5} \times \frac{3}{10}$   
 c)  $\frac{7}{10} \times \frac{3}{4}$     d)  $\frac{3}{7} \times \frac{1}{3}$

5. Fasil donated  $\frac{3}{5}$  of  $\frac{1}{4}$  of his allowance to a charity. What fraction of his allowance did Fasil donate?

#### 3.3

6. Multiply. Use benchmarks to estimate to check each product is reasonable.

a)  $\frac{1}{2} \times \frac{3}{10}$     b)  $\frac{3}{5} \times \frac{1}{8}$   
 c)  $\frac{7}{8} \times \frac{2}{5}$     d)  $\frac{3}{11} \times \frac{44}{63}$

7. Twenty Grade 8 students are going on a school trip. They pre-order sandwiches. Three-quarters of the students order a turkey sandwich, while  $\frac{1}{4}$  of the students order a roasted vegetable sandwich. Of the  $\frac{3}{4}$  who want turkey,  $\frac{2}{5}$  do not want mayonnaise. What fraction of the students do not want mayonnaise?

8. Write a story problem that could be solved using the expression  $\frac{5}{7} \times \frac{3}{8}$ . Find the product to solve the problem. Estimate to check the solution is reasonable.

#### 3.4

9. Write each mixed number as an improper fraction.

a)  $7\frac{1}{2}$     b)  $2\frac{7}{8}$     c)  $10\frac{7}{10}$

10. Use an area model to find each product.

a)  $1\frac{1}{2} \times 2\frac{1}{3}$     b)  $\frac{19}{3} \times \frac{6}{5}$   
 c)  $3\frac{1}{5} \times \frac{1}{4}$     d)  $2\frac{1}{4} \times 3\frac{1}{3}$

**LESSON**

**11.** Multiply. Estimate to check the product is reasonable.

a)  $1\frac{2}{3} \times 1\frac{9}{10}$

b)  $4\frac{1}{2} \times \frac{5}{8}$

c)  $\frac{9}{5} \times \frac{14}{8}$

d)  $1\frac{3}{10} \times 6\frac{2}{3}$

**12.** Jonathan works for a landscape maintenance company. It took Jonathan  $1\frac{3}{4}$  h to mow Mr. Persaud's lawn. The lawn he will mow next is  $2\frac{1}{3}$  times as large as Mr. Persaud's lawn. How long will it take Jonathan to mow the next lawn? What assumptions do you make?

**3.5**

**13.** Find each quotient. Use number lines to illustrate the answers.

a) One-half of a cake is shared equally among 5 people. What fraction of the whole cake does each person get?

b) Nakkita's dog eats  $\frac{3}{4}$  of a can of dog food each day. Nakkita has 9 cans of dog food. How many days' supply of dog food does Nakkita have?

**14.** Find each quotient.

a)  $3 \div \frac{4}{5}$

b)  $4 \div \frac{5}{6}$

c)  $\frac{3}{10} \div 2$

d)  $2\frac{5}{8} \div 3$

**15.** A glass holds  $\frac{3}{4}$  cup of milk. A jug contains 12 cups of milk. How many glasses can be filled from the milk in the jug?

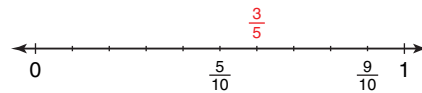
**16.** Kayla uses  $\frac{2}{3}$  of a scoop of detergent to do one load of laundry. Kayla has 9 scoops of detergent. How many loads of laundry can Kayla do?

**17.** When you divide a fraction by a whole number, is the quotient greater than or less than 1? Include examples in your explanation.

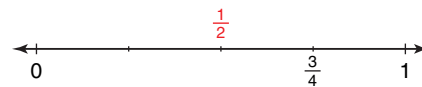
**3.6**

**18.** Use a copy of each number line to illustrate each quotient.

a)  $\frac{9}{10} \div \frac{3}{5}$



b)  $\frac{3}{4} \div \frac{1}{2}$



**19.** Divide. Estimate to check each quotient is reasonable.

a)  $\frac{3}{4} \div \frac{3}{8}$       b)  $\frac{1}{4} \div \frac{7}{8}$

c)  $\frac{5}{12} \div \frac{1}{3}$       d)  $\frac{1}{2} \div \frac{3}{5}$

**20.** Midori lives on a farm. Midori has  $\frac{7}{8}$  of a tank of gas. Each trip to town and back uses  $\frac{1}{6}$  of a tank of gas. How many trips to town and back can Midori make until she needs more gas? Estimate to check the solution is reasonable.

**21.** When you divide a proper fraction by its reciprocal, is the quotient less than 1, greater than 1, or equal to 1? Use examples in your explanation.

**LESSON**

**3.7**

- 22.** Write each mixed number as an improper fraction.
- a)  $3\frac{7}{11}$                       b)  $5\frac{1}{6}$   
 c)  $4\frac{8}{9}$                          d)  $2\frac{5}{12}$
- 23.** Divide. Estimate to check the quotient is reasonable.
- a)  $1\frac{3}{4} \div 2\frac{1}{8}$   
 b)  $3\frac{5}{6} \div 2\frac{1}{5}$   
 c)  $3\frac{1}{2} \div 1\frac{3}{8}$   
 d)  $2\frac{1}{5} \div 4\frac{2}{5}$
- 24.** A recipe for cereal squares calls for  $1\frac{1}{4}$  bags of regular marshmallows. The recipe makes a cookie sheet of squares. Marcus has  $\frac{3}{4}$  of a bag of marshmallows. He buys 5 more bags. How many sheets of cereal squares can Marcus make?

**3.8**

- 25.** A cookie recipe calls for  $\frac{3}{4}$  cup of rolled oats. Norma has  $\frac{5}{8}$  cup of rolled oats. How much more rolled oats does she need to make the cookies? How did you decide which operation to use?
- 26.** In a lottery for a local charity, 1000 tickets are sold. Of these tickets,  $\frac{1}{1000}$  will win \$1000,  $\frac{1}{500}$  will win \$50,  $\frac{1}{200}$  will win \$25,  $\frac{1}{100}$  will win \$10, and  $\frac{1}{10}$  will win \$5. How many tickets will not win a prize? How did you decide which operations to use?

- 27.** There are 30 students in a Grade 8 class. One-third of the students take a school bus,  $\frac{1}{5}$  take public transportation,  $\frac{1}{6}$  are driven by family, and the rest walk to school.
- a) What fraction of the students in the class walk to school?  
 b) How many of the students in the class walk to school? How did you decide which operations to use?

**3.9**

- 28.** Evaluate. State which operation you do first.
- a)  $\frac{1}{5} + \frac{2}{3} \times \frac{3}{5}$     b)  $\frac{4}{5} \div (\frac{2}{3} - \frac{3}{10})$   
 c)  $\frac{7}{3} + \frac{1}{6} \times \frac{2}{5}$     d)  $\frac{7}{8} \div \frac{5}{6} \times \frac{4}{7}$
- 29.** Evaluate.
- a)  $\frac{2}{3} + \frac{1}{4} - \frac{1}{6}$     b)  $\frac{3}{2} \times (\frac{4}{3} - \frac{1}{6})$   
 c)  $\frac{9}{8} \div (\frac{3}{4} + \frac{3}{2})$     d)  $\frac{2}{3} \times (\frac{1}{8} + \frac{5}{6} - \frac{3}{4})$
- 30.** Carlton evaluated this expression:  
 $2\frac{4}{5} \div (\frac{2}{3} + \frac{1}{12})$   
 His work is shown below.  
 Where did Carlton go wrong?  
 What is the correct answer?
- $$\begin{aligned} 2\frac{4}{5} \div (\frac{2}{3} + \frac{1}{12}) &= 2\frac{4}{5} \div (\frac{8}{12} + \frac{1}{12}) \\ &= 2\frac{4}{5} \div (\frac{9}{12}) \\ &= \frac{14}{5} \div \frac{9}{12} \\ &= \frac{14}{5} \times \frac{9}{12} \\ &= \frac{14}{5} \times \frac{9^3}{12^2} \\ &= \frac{21}{10} \\ &= 2\frac{1}{10} \end{aligned}$$

# Practice Test

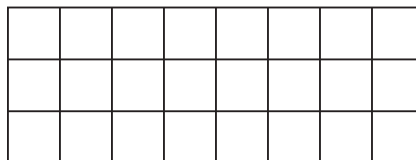
1. Use a number line to divide:  $5 \div \frac{5}{6}$



2. Copy this rectangle.

Shade the rectangle to find the product:

$$\frac{5}{8} \times \frac{2}{3}$$



3. Find each product and quotient.

a)  $\frac{1}{4} \times 28$

b)  $\frac{3}{8} \times \frac{1}{2}$

c)  $\frac{5}{6} \div 2$

d)  $\frac{1}{5} \div \frac{2}{3}$

4. Find each product and quotient.

Estimate to check each solution is reasonable.

a)  $\frac{5}{8} \times 3\frac{1}{4}$

b)  $3\frac{1}{3} \times 2\frac{1}{10}$

c)  $2\frac{1}{4} \div \frac{7}{8}$

d)  $1\frac{2}{5} \div 1\frac{1}{2}$

5. Three-fifths of the Grade 8 class are in the band.

- a) On Tuesday, only  $\frac{1}{3}$  of these students went to band practice.

What fraction of the class went to band practice on Tuesday?

- b) How many students might be in the class?  
How do you know?



6. Predict without calculating. Which statement below has the greatest value?

How do you know?

a)  $\frac{7}{3} \times \frac{3}{4}$

b)  $\frac{7}{3} - \frac{3}{4}$

c)  $\frac{7}{3} \div \frac{3}{4}$

d)  $\frac{7}{3} + \frac{3}{4}$

7. Multiply a fraction by its reciprocal. What is the product?

Use an example and a model to explain.

8. Evaluate.

a)  $\frac{1}{2} - \frac{3}{5} \times \frac{1}{6}$

b)  $\frac{1}{4} \div \frac{1}{8} + (\frac{1}{2} - \frac{3}{8})$

9. A dog groomer takes  $\frac{5}{6}$  h to groom a poodle.
- Estimate the number of poodles the groomer can groom in  $4\frac{1}{2}$  h.
  - Sketch a number line to illustrate the answer.
  - Calculate the number of poodles the groomer can groom in  $4\frac{1}{2}$  h.
  - What assumptions do you make?



10. Solve each problem.

Explain how you decided which operation or operations to use.

- a) Haden is making a milkshake.

He has  $\frac{1}{3}$  cup of milk in a glass.

How much milk must Haden add so there will be  $\frac{4}{5}$  cups of milk in the glass?

- b) Lacy, Lamar, and Patti own a dog-walking business.

Lacy owns  $\frac{5}{12}$  of the business and Lamar owns  $\frac{1}{3}$  of the business.

How much of the business does Patti own?

- c) The science teacher has  $2\frac{1}{4}$  cups of baking soda and  $1\frac{1}{3}$  cups of salt.

To conduct an experiment, each student needs  $\frac{1}{8}$  cup of baking soda and  $\frac{1}{12}$  cup of salt. There are 18 students in the class.

- i) Does the teacher have enough baking soda for each student?

If not, how much more baking soda does the teacher need?

- ii) Does the teacher have enough salt for each student?

If not, how much more salt does the teacher need?

11. Which of these statements is always true?

Use examples to support your answer.

- Division always results in a quotient that is less than the dividend.
- Division always results in a quotient that is greater than the dividend.
- Division sometimes results in a quotient that is less than the dividend and sometimes in a quotient that is greater than the dividend.

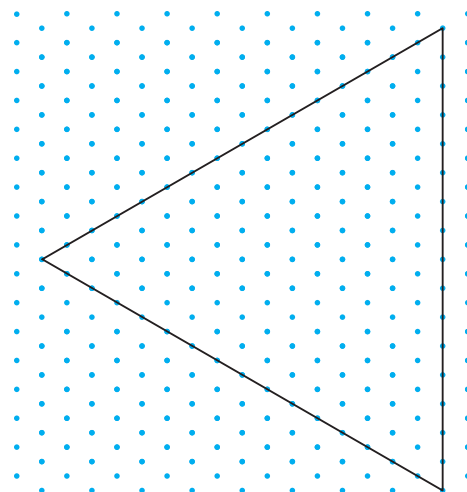
A *fractal* is a shape that can be subdivided into parts, each of which is a reduced-size copy of the original. Many natural objects – blood vessels, lungs, coastlines, galaxy clusters – can be modelled by fractals.

The Sierpinski Triangle is a famous fractal.

Use triangular dot paper.  
Draw an equilateral triangle with side length 16 units.

### Step 1

Find the midpoint of each side.  
Connect each midpoint to form a triangle inside the original triangle.  
Shade the new triangle.  
Label this diagram Figure 1.



Suppose the area of the original triangle is 1 square unit.  
Find the total area *not* shaded in Figure 1.  
Record your results in a copy of the table below.

Area of Sierpinski Triangle						
Figure	Original	1	2	3	4	5
Area Not Shaded (square units)	1					

### Step 2

Use Figure 1. Repeat *Step 1* with each triangle that is not shaded.  
Label this diagram Figure 2.  
Repeat this process 3 more times to get Figures 3, 4, and 5.

### Step 3

Describe any patterns you see in the table you completed in *Step 1*.

Write a pattern rule for the areas not shaded.

Suppose the pattern continues.

Use your rule to find the areas not shaded in Figure 6 and Figure 7.

How did you use multiplication of fractions to help?

### Step 4

Suppose the side length of the original triangle is 1 unit.

Record the perimeter of this triangle in a copy of the table below.

Find the total perimeter of the triangles not shaded in Figure 1.

Record your results in the table.

Explain how you found the perimeter.

Use the figures drawn in *Step 2* to complete the table.

Perimeter of Sierpinski Triangle						
Figure	Original	1	2	3	4	5
Perimeter of Triangles Not Shaded (units)						

Describe any patterns you see in the table.

Write a pattern rule for the perimeters of the triangles not shaded.

Use your rule to find the perimeters of the triangles not shaded in Figure 6 and Figure 7.

How did you use multiplication of fractions to help?

### Check List

Your work should show:

- ✓ all diagrams, tables, and calculations in detail
- ✓ clear explanations of the patterns you found
- ✓ how you used multiplication of fractions to find area and perimeter

## Reflect on Your Learning

What do you now know about fractions that you did not know before this unit? Use examples in your explanation.

## UNIT

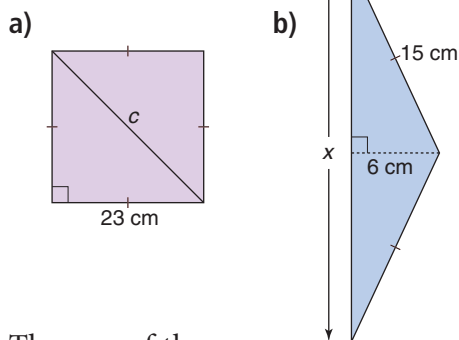
1

- 1.** a) List the factors of each number in ascending order.  
 i) 84            ii) 441  
 iii) 236        iv) 900  
 b) Which numbers in part a are square numbers? How can you tell?

- 2.** Estimate each square root to 1 decimal place. Show your work.

a)  $\sqrt{52}$             b)  $\sqrt{63}$   
 c)  $\sqrt{90}$            d)  $\sqrt{76}$

- 3.** Find each length indicated. Sketch and label the triangle first. Give your answers to one decimal place where needed.



- 4.** The area of the square on each side of a triangle is given. Is the triangle a right triangle? How do you know?  
 a)  $16 \text{ cm}^2$ ,  $8 \text{ cm}^2$ ,  $30 \text{ cm}^2$   
 b)  $16 \text{ cm}^2$ ,  $8 \text{ cm}^2$ ,  $24 \text{ cm}^2$

- 5.** Identify the sets of numbers that are Pythagorean triples. How did you decide?  
 a) 2, 5, 6            b) 6, 10, 8  
 c) 12, 9, 7         d) 18, 30, 24

- 6.** The dimensions of a rectangle are 3 cm by 4 cm. What is the length of a diagonal? Explain your reasoning.

- 7.** Jacobi wants to install an underground sprinkler system in her backyard. The backyard is rectangular with side lengths 17 m and 26 m. The water pipe will run diagonally across the yard. About how many metres of water pipe does Jacobi need?

2

- 8.** Evaluate.  
 a)  $(-9) \times (+8)$     b)  $(+14) \times (+8)$   
 c)  $(-18) \times (-1)$    d)  $(+21) \times (-6)$

- 9.** The Brandon Birdies junior golf team has 4 golfers. Each golfer is  $-5$  on her round for the day. What is the team score for the day?  
 a) Write this problem as a multiplication expression using integers.  
 b) Solve the problem.

- 10.** Write a word problem that could be solved using the expression  $(+8) \times (-5)$ . Solve the problem.

- 11.** Divide.  
 a)  $(-77) \div (+7)$     b)  $(+63) \div (+3)$   
 c)  $(-30) \div (-1)$    d)  $\frac{-24}{+6}$   
 e)  $\frac{+51}{-3}$                 f)  $\frac{0}{-8}$



**12.** Divide.

- a)  $(+84) \div (-12)$     b)  $(-75) \div (-15)$   
 c)  $(-78) \div (+13)$     d)  $(+98) \div (+14)$

**13.** Four friends had lunch at a restaurant. They share the bill equally. The bill is \$52. How much does each friend owe?

- a) Write this problem as a division expression using integers.  
 b) Solve the problem.

**14.** For each number below, find two integers for which that number is:

- i) the sum  
 ii) the difference  
 iii) the product  
 iv) the quotient  
 a)  $-8$     b)  $-2$     c)  $-12$     d)  $-3$

**15.** Evaluate.

- a)  $(-3)[5 - (-3)]$   
 b)  $[8 \div (-4)] - 10(3)$   
 c)  $\frac{(-6) + 12 \div (-4) \times (-5)}{(-18) \div 6}$

**3**

**16.** Write each improper fraction as a mixed number.

- a)  $\frac{7}{3}$     b)  $\frac{15}{2}$     c)  $\frac{21}{8}$     d)  $\frac{19}{5}$

**17.** Multiply. Estimate to check each product is reasonable.

- a)  $\frac{7}{9} \times 36$     b)  $\frac{5}{6} \times 21$   
 c)  $\frac{5}{6} \times \frac{5}{6}$     d)  $\frac{9}{4} \times \frac{8}{3}$   
 e)  $1\frac{1}{2} \times 2\frac{3}{4}$     f)  $3\frac{1}{5} \times 4\frac{1}{8}$

**18.** Divide. Estimate to check each quotient is reasonable.

- a)  $9 \div \frac{3}{5}$     b)  $\frac{5}{6} \div 3$   
 c)  $\frac{5}{3} \div \frac{3}{4}$     d)  $\frac{5}{3} \div \frac{4}{3}$   
 e)  $1\frac{2}{3} \div 1\frac{1}{3}$     f)  $2\frac{1}{4} \div 3\frac{1}{2}$

**19.** A recipe for one bowl of punch calls for  $1\frac{1}{8}$  bottles of sparkling water.

For a party, Aaron wants to make  $5\frac{1}{2}$  bowls of punch.

How many bottles of sparkling water does Aaron need to make the punch?

**20.** Evaluate.

- a)  $\frac{3}{20} \div \frac{2}{5} \div \frac{1}{2} \times \frac{3}{4}$   
 b)  $\frac{7}{8} - \frac{7}{12} + 2$   
 c)  $\frac{5}{9} \div (\frac{1}{2} \times \frac{7}{9})$   
 d)  $4 \div \frac{2}{3} - 3\frac{1}{4} + \frac{7}{12}$

**21.** Solve each problem. Explain how you decided which operation to use.

- a) Justine usually runs  $7\frac{1}{2}$  laps of the track before breakfast. This morning, she had to be at school early so she only ran  $3\frac{1}{4}$  laps. By how many laps did Justine cut her run short?
- b) Lyle slept for  $6\frac{1}{2}$  h on Monday night. On Tuesday night, Lyle slept  $1\frac{1}{4}$  times as long as he did on Monday night. How many hours of sleep did Lyle get on Tuesday night?