

UNIT 7

Data Analysis and Probability

Spencer, Jenny, and Simrit are working on a project on nutrition. They look at the nutrition facts label on a box of breakfast cereal.

- Which type of graph would best display the nutrition facts?
- How could you use a graph to make the cereal appear more nutritious than it really is?
- Who might want to make the cereal appear more nutritious than it is?

What You'll Learn

- Critique ways in which data are presented.
- Solve problems that involve the probability of independent events.

Why It's Important

Graphs are pictures of data. Knowing the best way to present data can help you communicate your ideas. In the media, you hear and read statements about the probability of everyday events. To make sense of these statements, you need to understand probability.





Key Words

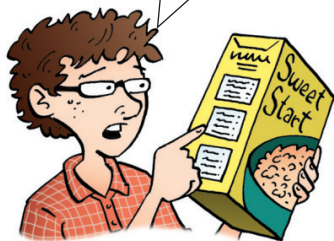
- circle graph
- line graph
- bar graph
- double bar graph
- discrete data
- pictograph
- outcome
- event
- independent events
- probability of an event

Nutrition Facts/Valeur nutritive

Serving 1 cup (55 g) / Portion de 1 tasse (55 g)

Amount per serving	Cereal	With ½ cup 2% milk Avec ½ tasse de lait 2%
Calories / Calories	185	260
	% Daily Value / % valeur quotidienne	
Fat / Lipides 1 g		
Saturated / saturés 0 g		
+ Trans / trans 0 g	0%	8%
Cholesterol / Cholesterol 0 mg	0%	3%
Sodium / Sodium 345 mg	14%	17%
Potassium / Potassium 315 mg	9%	15%
Carbohydrate / Glucides 45 g	15%	17%
Fibre / Fibres 7 g	25%	25%
Sugars / Sucres 16 g		
Starch / Amidon 22 g		
Protein / Protéines 4 g		

"How could this information be displayed differently?"



"I know. We could draw a circle graph or a bar graph. Which one do you think would be better?"



"I don't know. I just want to know my chances of winning this game on the back of the cereal box."



7.1

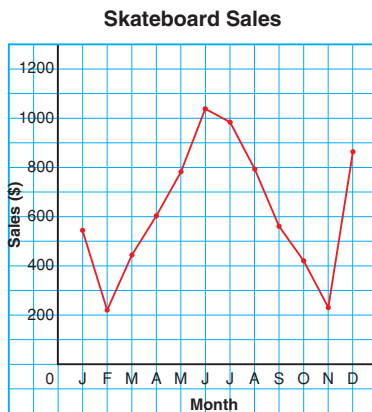
Choosing an Appropriate Graph

Focus Identify the advantages and disadvantages of different types of graphs.

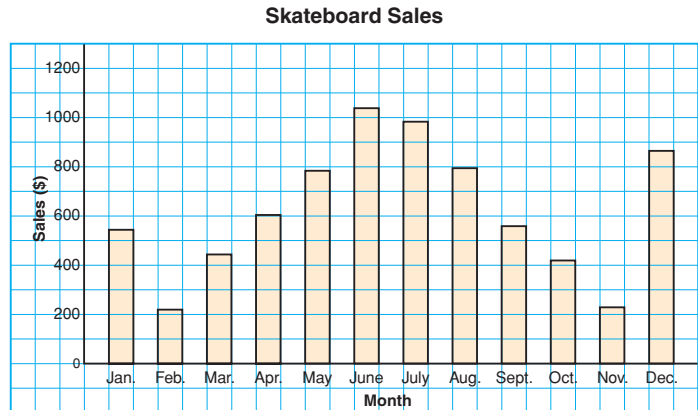
Investigate

Work with a partner.
All these graphs represent
the same data.

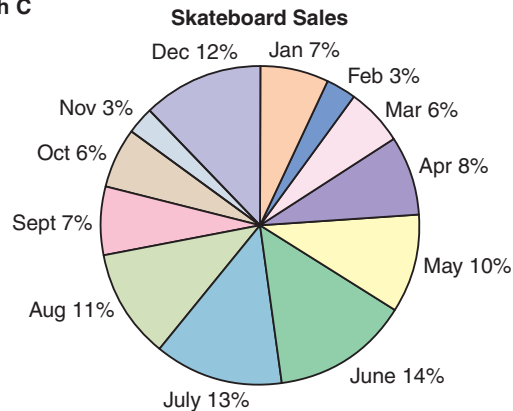
Graph B



Graph A



Graph C



What do you know from each graph?

Which graph is most helpful in answering each question below?

- Which two months had the greatest skateboard sales?
- What is the range in skateboard sales?
- What percent of total skateboard sales occurred in May?

Reflect & Share

Compare your answers with those of another pair of classmates.
What are the advantages and disadvantages of each type of graph?
What can you find out from the bar graph or line graph that you cannot find out from the circle graph?

Connect

Each type of graph has its strengths and limitations.

Line Graphs

A **line graph** displays data that change over time.

On a line graph, line segments join adjacent data points.

A line graph:

- is easy to draw and to read
- can have a zigzag symbol on the vertical axis when the data start at a large number
- is best used to show data gathered over time
- can be used to estimate values between data points and beyond data points.
(This should be done carefully as the trend may not continue.)

Double Bar Graphs

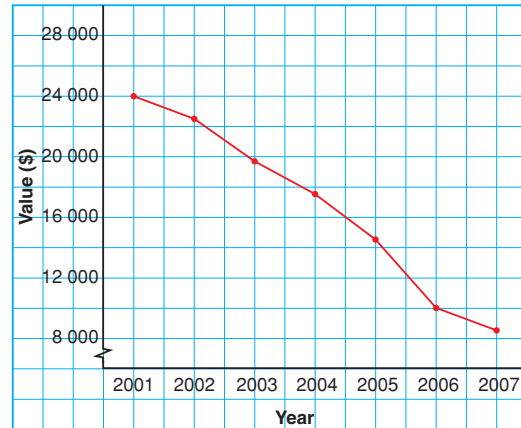
A **double bar graph** displays two sets of data that can be counted.

The lengths of the bars are used to represent and to compare data.

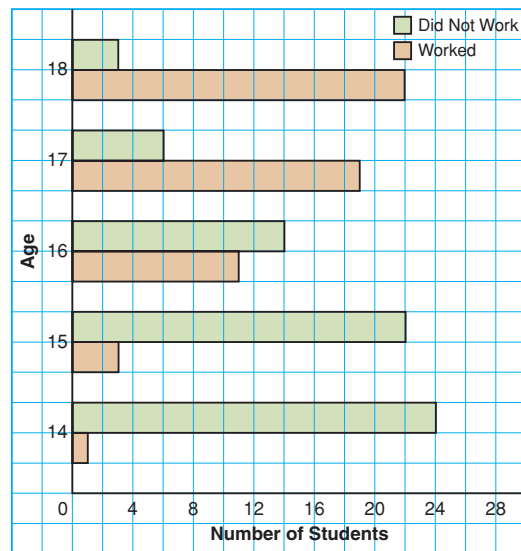
A double bar graph:

- is easy to draw and to read
- can be used to directly compare two sets of data of data
- can only be used to show discrete data
- may be difficult to read accurately depending on the scale used

Value of Car



Students with Part-Time Jobs, Ages 14–18



Discrete data can be counted.

Example 1

Three students surveyed Grade 8 students in their school.

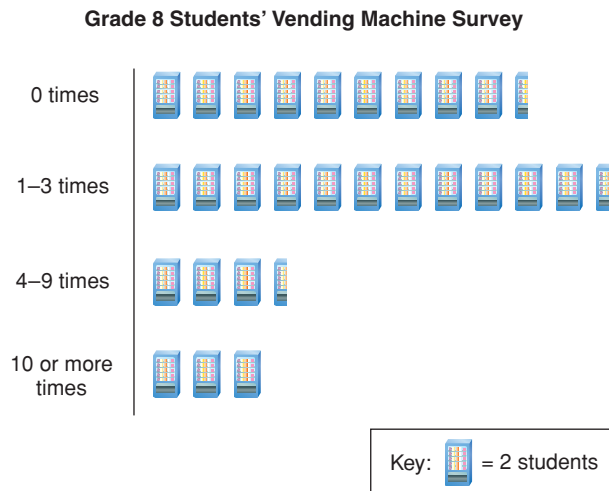
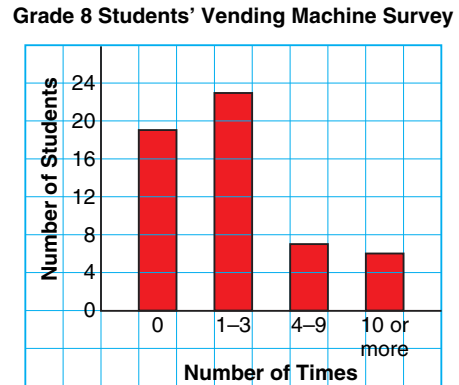
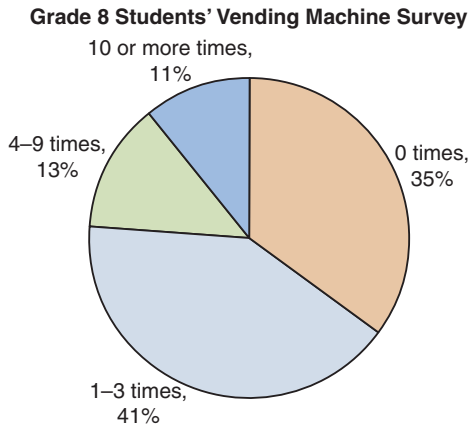
They asked: "How many times did you use a vending machine last week:

0 times, 1–3 times, 4–9 times, or 10 or more times?"

Amrit displayed the results on a circle graph.

Fred used a bar graph. Stella used a pictograph.

- What are the strengths and limitations of each graph?
- Which graph is appropriate? Justify your answer.



A Solution

a)

Type of Graph	Strengths	Limitations
Circle Graph	<ul style="list-style-type: none">• Shows parts of the whole.• Each response is shown as a percent of the number of students who responded.• The sizes of the sectors can be used to compare responses.	<ul style="list-style-type: none">• The graph does not show the number of students who chose each response.• The total number of students cannot be calculated.• Difficult to draw accurately
Bar Graph	<ul style="list-style-type: none">• The heights of the bars can be used to compare responses.• The scale on the vertical axis is: 1 grid square represents 4 students. The scale can be used to calculate the total number of students who responded.• Easy to draw	<ul style="list-style-type: none">• Some people may find this graph difficult to read accurately because none of the bars end on a grid line.• The graph does not show the percent of the students who chose each response.
Pictograph	<ul style="list-style-type: none">• The lengths of the rows of symbols give immediate comparison of responses.• The graph is visually appealing.• The key is 1 symbol equals 2 students. The key can be used to calculate the total number of students who responded. One-half of a symbol represents 1 student.	<ul style="list-style-type: none">• There are a lot of symbols on the graph. For example, there are $11\frac{1}{2}$ symbols for 1–3 times. This makes the graph more difficult to read.• It might be difficult to draw so many vending machine symbols all of the same size.• The graph does not show the percent of the students who chose each response.

- b) To decide which graph is appropriate, we need to know what the students want to display.

For example, if they want to display the fraction of Grade 8 students who did not use a vending machine, then the circle graph is appropriate. The size of that sector can be compared to the whole graph.

If the students want to display the number of students who did not use a vending machine, then the bar graph or pictograph is appropriate. The height of the bar or the key can be used to find the number of students.

Example 2

This table shows the favourite types of video games of the Grade 8 students at L'école Orléans.

Type	Number of Students
Action	15
Role Playing	10
Arcade	4
Strategy	7
Simulation	11
Other	3

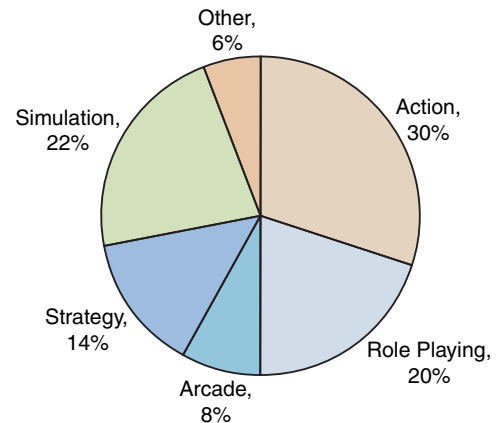
- Graph these data. Justify your choice of graph.
- What are the advantages and disadvantages of the graph you drew?

A Solution

- The circle graph can display these data. The total number of students is the whole. Each sector represents the percent of students who chose each type of game.

Type	Number of Students	Fraction	Percent (%)
Action	15	$\frac{15}{50} = \frac{3}{10}$	30
Role Playing	10	$\frac{10}{50} = \frac{1}{5}$	20
Arcade	4	$\frac{4}{50} = \frac{2}{25}$	8
Strategy	7	$\frac{7}{50}$	14
Simulation	11	$\frac{11}{50}$	22
Other	3	$\frac{3}{50}$	6

Favourite Types of Video Games of Grade 8 Students



- Advantages:** The circle graph shows the percent of Grade 8 students who chose each type of video game. The size of each sector can be compared to the whole and to other sectors to make conclusions; such as, action video games were most popular with these Grade 8 students.

Disadvantages: The circle graph does not show the number of students who chose each type of game and the number of students who were surveyed.

In a circle graph, the original data are lost.

A circle graph may be difficult to draw accurately because some calculations may involve approximations.

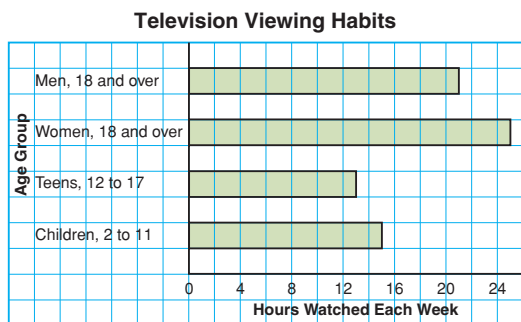
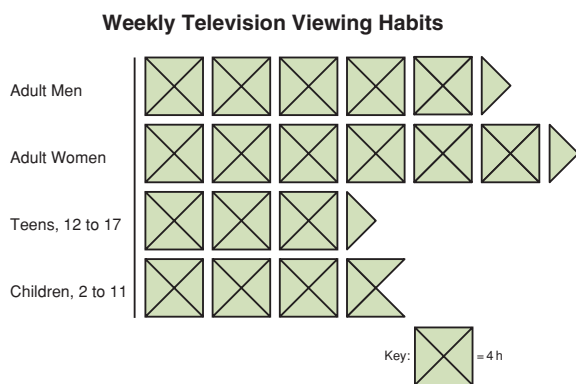
Discuss the ideas

1. Can more than one type of graph be appropriate to display a set of data?
2. In *Example 2*, which other types of graphs could you draw?

Practice

Check

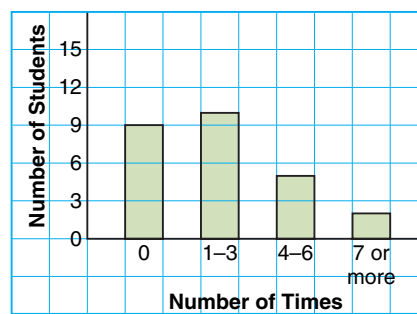
3. Each graph below shows how much time Canadians spend watching TV each week.



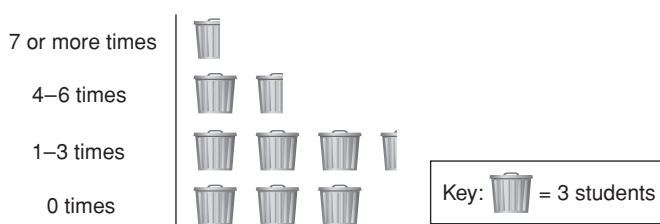
- a) List 3 things you know from the bar graph.
- b) List 3 things you know from the pictograph.
- c) Which graph is more appropriate to display the data? Justify your choice.

4. Each graph below shows the number of times students in a Grade 8 class littered last week.

Number of Times Grade 8 Students Littered



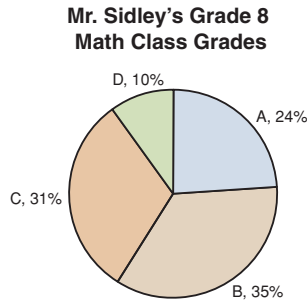
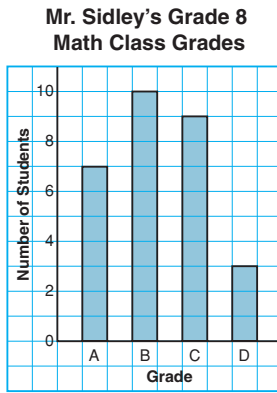
Number of Times Grade 8 Students Littered



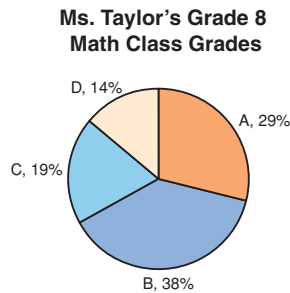
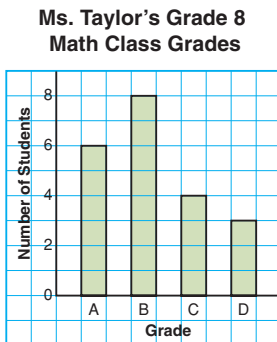
- a) What are the strengths of each graph?
- b) What are the limitations of each graph?
- c) Which graph is more appropriate to display these data? Justify your choice.
- d) Should you use a line graph or a circle graph to display these data? Why or why not?

Apply

5. These graphs show the final grades for Mr. Sidley's Grade 8 math class.



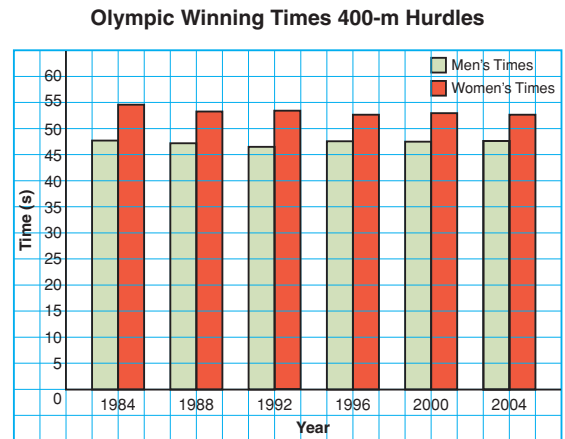
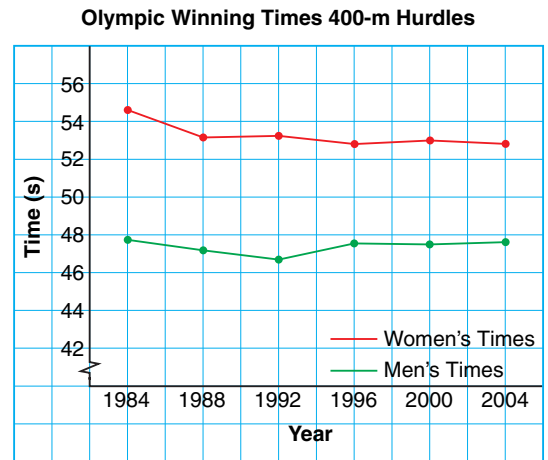
- List 3 things you know from the bar graph.
- List 3 things you know from the circle graph.
- Which graph best shows the number of students who got B as a final grade? Justify your choice.
- These graphs show the final grades for Ms. Taylor's Grade 8 math class.



Which graphs should Mr. Sidley use to show his class has the higher grades? Explain.

- Which class do you think did better? Why do you think so?

6. a) What data does each graph below show?



- What is an advantage of each graph?
- What is a disadvantage of each graph?
- Which graph would you choose in each case? Explain your choice.
 - You want to show how the times changed over time.
 - You want to show the differences between the times for each year.

7. Describe data that could be best represented by each graph below. Explain why you chose each type of data.
- line graph
 - bar graph
 - double bar graph
 - pictograph
 - circle graph

- 8. Assessment Focus** Nina owns a shoe store. These tables show data about the shoe store.

Table A

Sizes of Shoes Sold in May

Size	Number of Pairs Sold
6	60
7	239
8	217
9	156
10	61
11	43
12	36

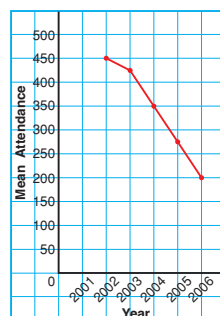
Table B

Yearly Sales

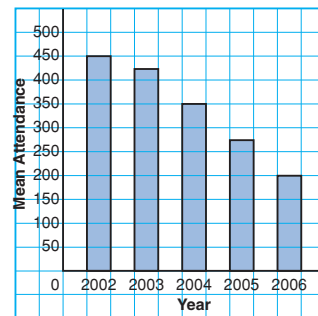
Year	Sales (\$)
2000	579 000
2001	621 000
2002	598 000
2003	634 000
2004	657 000
2005	642 000
2006	675 000

- a) Which data would be best represented with a line graph? Justify your choice.
- b) Which type of graph would be suitable for the other table? Explain.
- 9.** Each graph shows the mean home-game attendance of the school's soccer team over the past 5 years.

Mean Home-Game Attendance of Soccer Team



Mean Home-Game Attendance of Soccer Team



- a) What are the strengths of each graph?
- b) What are the limitations of each graph?
- c) Which type of graph is more appropriate to display these data? Justify your choice.
- d) Could you use a circle graph to display these data? Why or why not?
- 10.** This table shows the number of people employed by the construction industry in Canada from 2002 to 2006.
- a) Graph these data. Justify your choice of graph.
- b) What are the advantages and disadvantages of the graph you drew?

Number of People Employed in the Construction Industry

Year	Number of People (thousands)
2002	865.2
2003	906.0
2004	951.7
2005	1019.5
2006	1069.7

- 11.** This table shows the Canadian Aboriginal population, by province and territory, in 2001.

Canadian Aboriginal Population, 2001	
Province or Territory	Population
Newfoundland and Labrador	18 780
Prince Edward Island	1 345
Nova Scotia	17 015
New Brunswick	16 990
Quebec	79 400
Ontario	188 315
Manitoba	150 040
Saskatchewan	130 185
Alberta	156 220
British Columbia	170 025
Yukon	6 540
Northwest Territories	18 730
Nunavut	22 720

- a) Graph these data. Justify your choice of graph.
 b) What are the advantages and disadvantages of the graph you drew?

- 12. Take It Further** Over a 2-month period, Dinah collected these data about her family.

- i) How many times each week her dad fell asleep while watching television

- ii) How many times her dad cooked dinner and how many times her mom cooked dinner each week
 iii) How many hours a day her brother spent playing video games, doing homework, chatting on-line, doing chores, and eating
 iv) The weekly height of a tomato plant in the garden

Dinah wants to display these data for a school project. Which type of graph would you suggest for each data set? Justify your choices.

- 13. Take It Further** Madan measured the mass of his guinea pig every 5 months, until the pet was 25 months old. The data are shown in the table.

Age (months)	Mass (g)
5	200
10	350
15	480
20	510
25	520

- a) Graph these data. Justify your choice of graph.
 b) What are the advantages and disadvantages of the graph you drew?
 c) Use your graph to predict the mass of the guinea pig at 8 months and at 30 months.

Reflect

Suppose you are given a data set.

How do you choose the most appropriate type of graph to illustrate these data?

Using Spreadsheets to Record and Graph Data



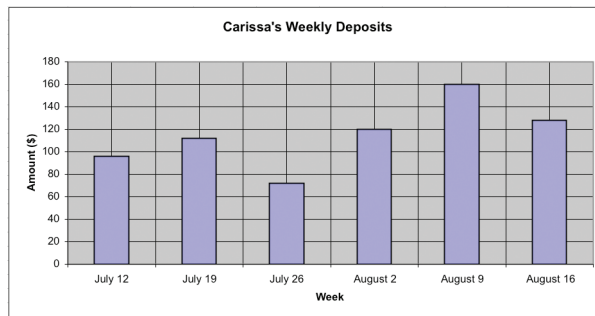
Focus | Display data on graphs using spreadsheets.

Spreadsheets can be used to record and graph data. We can use a spreadsheet to generate different types of graphs for a set of data.

Week	Amount (\$)
July 12	96
July 19	112
July 26	72
Aug. 2	120
Aug. 9	160
Aug. 16	128

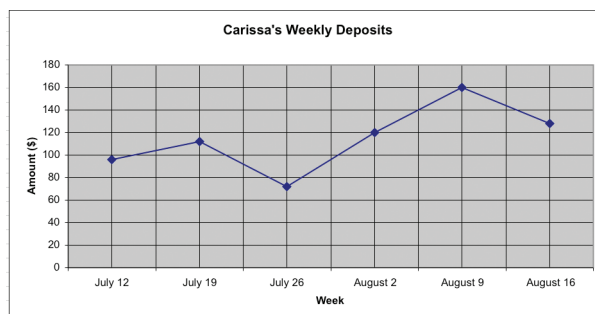
Carissa had a part-time job cutting grass in the summer. She opened a bank account to save all the money she earned. This table shows Carissa's weekly deposits.

- Enter the data into rows and columns of a spreadsheet. Highlight the data. Include the column heads. Click the graph/chart icon. Select the *Vertical bar graph*. Label the graph and the axes. Your graph may look similar to this:



The bar for August 9 is the tallest. This shows Carissa deposited more that day than any other.

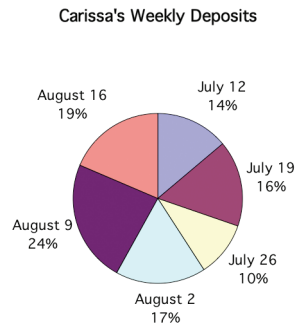
- Use the same data. Highlight the data. Include the column heads. Click the graph/chart icon. Select the *Line graph*. Label the graph and the axes. Your graph may look similar to this:



Generally, the line segments go up to the right. So, Carissa's deposits increased over time.

In most spreadsheet programs, circle graphs are called pie charts.

- Use the same data.
Highlight the data. Do not include the column heads.
Click the graph/chart icon. Select the *Pie chart*.
Label the graph.
Your graph may look similar to this:



From the circle graph, we know that the greatest amount Carissa deposited in a week is 24% of the total amount she deposited.

- What are the strengths of each graph?
- What are the limitations of each graph?
- Which graph is most appropriate to display these data?
- Justify your choice by suggesting what could be found out from each graph.

Check

- The table shows the places where some Canadians met their best friends.

Place	Number of People
School	5700
Work	4100
Club or organization	1400
Religious organization	700
Home/neighbourhood	4300
Through family	1200
Through friend	1100
Other	600

- Use a spreadsheet to draw a bar graph and a circle graph.
- Which graph represents these data better? Justify your answer.

2. a) Use a spreadsheet.
Create one graph to display these data.
- b) Which types of graphs would you not use to represent these data? Explain why you would not use these graphs.

**Average Hours of Television
Viewed per Week by Adolescents Aged 12–17 years,
Fall 2004**

Province	Number of Hours
Newfoundland/Labrador	12.3
Prince Edward Island	12.3
Nova Scotia	13.8
New Brunswick	12.6
Quebec	13.5



3. a) Use a spreadsheet.
Draw two different graphs to display the data in the table.
- b) Which type of graph represents these data better?
Justify your choice.
- c) Which type of graph would you not use to represent these data?
Explain why you would not use that graph.

Average Value of US\$1

Year	Value in C\$
2002	1.570
2003	1.401
2004	1.302
2005	1.212
2006	1.134

7.2

Misrepresenting Data

Focus Examine how data may be misinterpreted.

There are many ways to represent data.

Investigate

Work on your own.

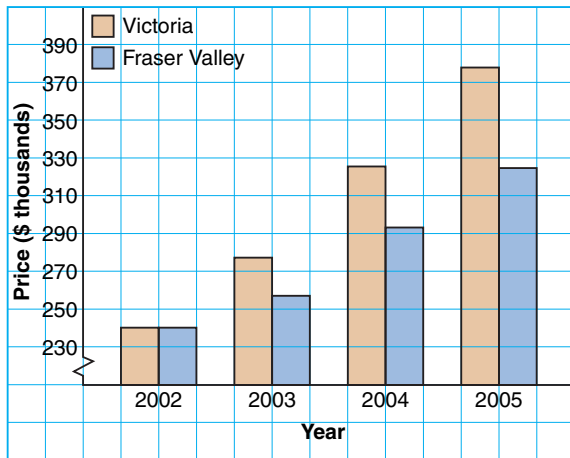
What data do the two graphs below show?

How are the graphs similar? How are they different?

Explain.

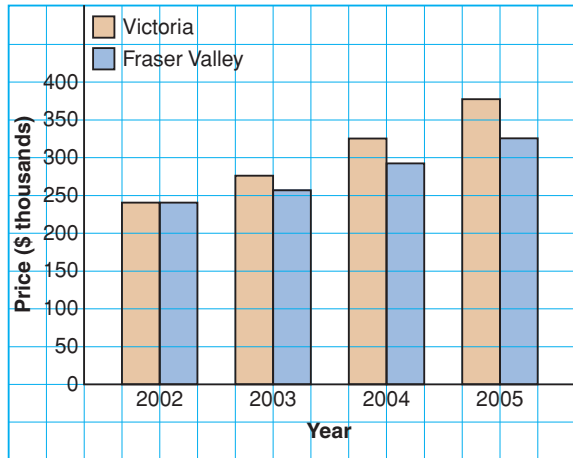
Graph A

Average House Prices



Graph B

Average House Prices



Reflect & Share

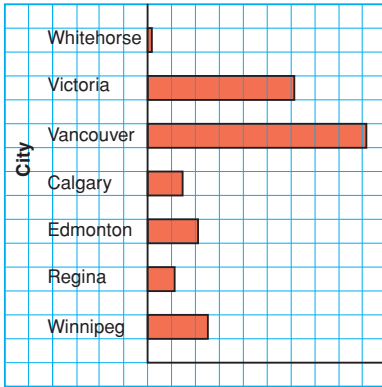
Discuss with a classmate:

- What impression does each graph create?
- Who might want to use each graph?

Connect

Different formats of a graph may lead to misinterpretation of data.

Average Annual Precipitation



This bar graph is misleading.

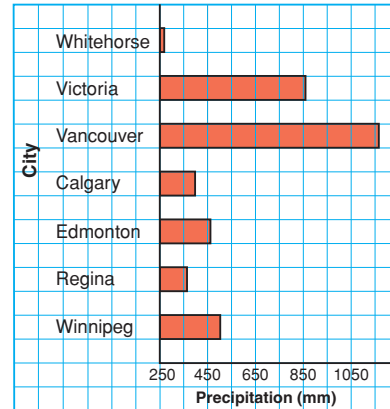
It suggests that Vancouver has more than 30 times as much precipitation as Whitehorse.

This graph has no measurements of the amount of precipitation.

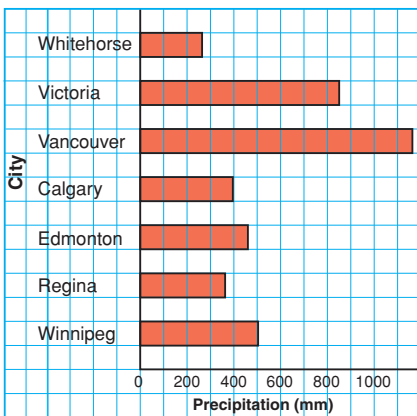
Visually, this graph shows the same information as the first graph. However, the horizontal scale is labelled with the amount of precipitation, and the scale does not start at 0.

The scale shows that Vancouver has about 1170 mm of precipitation and Whitehorse has about 270 mm.

Average Annual Precipitation



Average Annual Precipitation



This graph accurately shows the data.

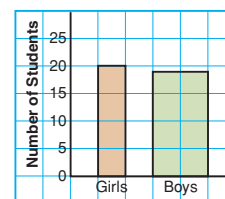
The horizontal scale starts at 0.

The lengths of the bars are shown in the correct ratio. Vancouver has between 4 and 5 times as much precipitation as Whitehorse.

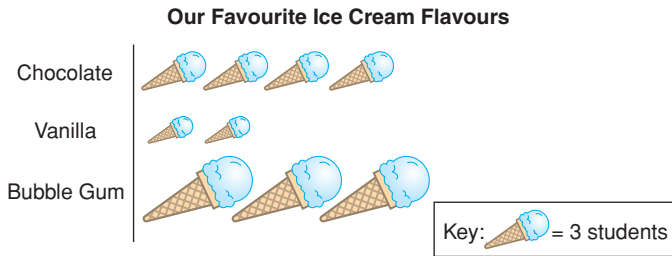
There are many ways in which graphs can be drawn to **misrepresent data**. Graphs like these may be found in the media to create false impressions.

- In this bar graph, the wider bar creates the impression that many more boys than girls scored higher than 80%. In fact, the number of girls who scored higher than 80% is greater than the number of boys.

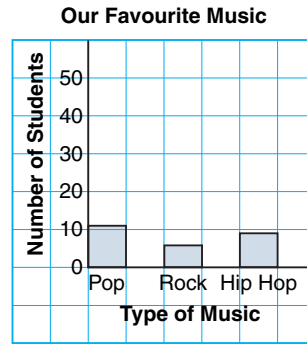
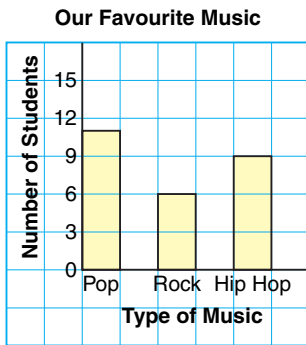
Grade 8 Students Who Scored Higher than 80% on a Math Test



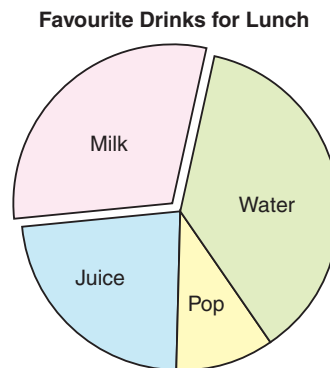
- In this pictograph, the symbols have different sizes.
The three large ice-cream cone symbols give the impression that bubble gum is the favourite flavour.
When the key is used, chocolate is the favourite flavour.



- In the bar graph below left, the scale on the vertical axis is 1 square represents 3 students.
The differences among the heights of the bars are easily seen.
In the bar graph below right, the scale on the vertical axis is 1 square represents 10 students.
This change in scale makes the differences among the heights of the bars less evident.



- A part of a graph may be treated differently to draw attention to it.
A milk company uses this circle graph to draw attention to the milk sector.
The sector for milk is not as large as the sector for water, but the special treatment makes it seem larger.



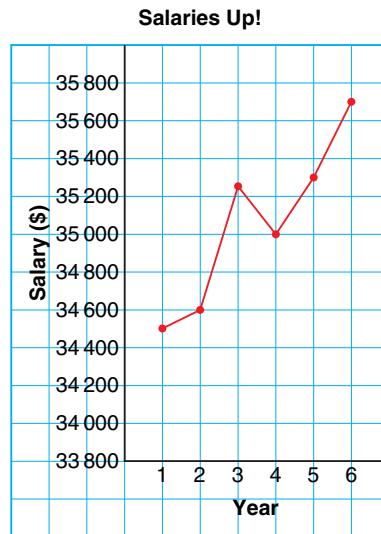
Example 1

From this line graph, Shiva made the conclusion that salaries have almost tripled in 6 years.

- a) Shiva's conclusion is not consistent with the data.

Explain her misinterpretation.

- b) What changes should be made to the graph to accurately show how salaries have changed in 6 years?

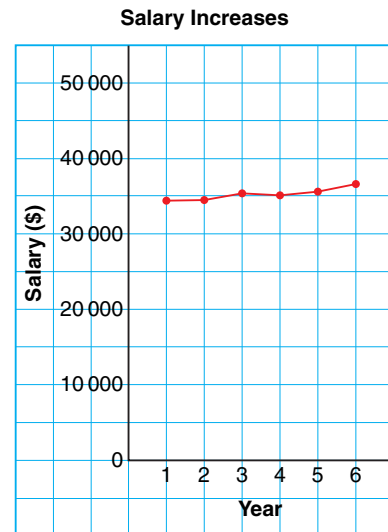


▶ A Solution

- a) The scale on the vertical axis does not start at 0. The upward trend of the graph suggests a rapid rise in salaries. This is not so. The increase in salary is: $\$35\,700 - \$34\,500 = \$1\,200$. So, the mean annual increase is \$200.

- b) To accurately display these data, the scale on the vertical axis should start at \$0. Then use a scale of 1 square to represent \$5,000.

From this graph, it is clear that salaries have increased very little in 6 years.



Example 2

This table shows the number of tonnes of plastic bottles processed by a local recycling company.

Draw a graph to show how the company can display these data to support each statement.

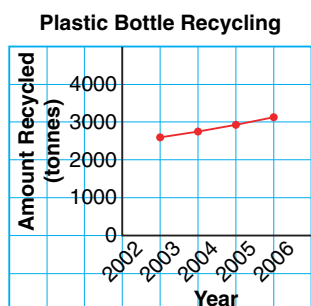
Explain how you created each impression.

- More money is needed to advertise the importance of recycling.
- Recycling is increasing so more staff should be hired.

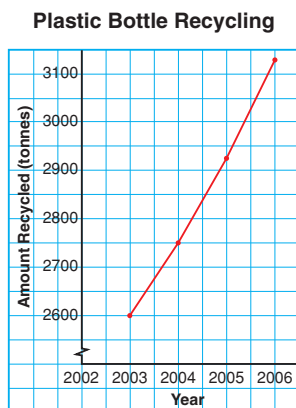
Year	Amount Recycled (t)
2003	2600
2004	2750
2005	2925
2006	3130

A Solution

- The company could draw a line graph that suggests the amount recycled has not changed much over the years. For example, the scale on the vertical axis could be 1 grid square represents 1000 t.



- The company could draw a line graph that suggests the amount recycled is increasing rapidly. For example, the scale on the vertical axis could start at 2600, and each grid square could represent 50 t. This graph exaggerates the increase in recycling.



Discuss the ideas

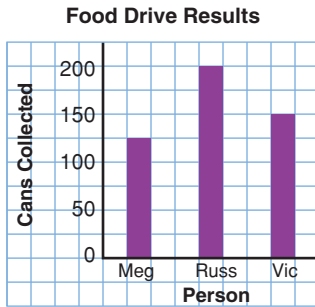
- Look at the graph in *Example 1*. Who might have drawn the graph to create this impression?
- How can the same data set be used to support different views on a topic?

Practice

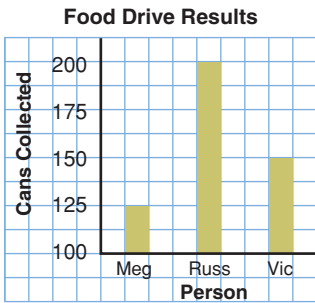
Check

3. These graphs display the same data. Which graph is misleading? Why?

Graph A



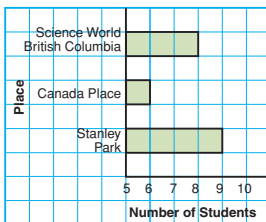
Graph B



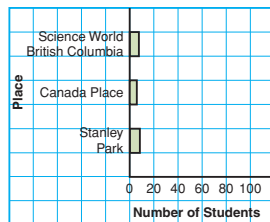
4. A Grade 8 class in Vancouver was surveyed to find out where they would like to go for an end-of-year trip. The results were graphed by two students.

Where We Would Like to Go

Graph A



Graph B



- a) What impression does each graph give?

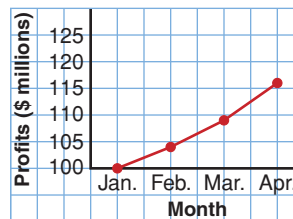
- b) How is each graph misleading?
 c) Where do you think the creator of Graph A would like to go? Why do you think so?
 d) Where do you think the creator of Graph B would like to go? How was the graph drawn to give this impression?
 e) What changes would you make to graph these data accurately?

Apply

5. Graphs A and B display the Read Books Company's profits for a four-month period.

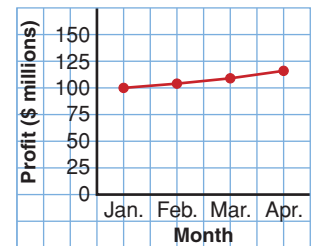
Graph A

Read Books Co. Profits



Graph B

Read Books Co. Profits

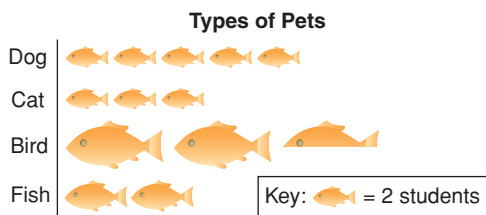


Which conclusions are incorrect?

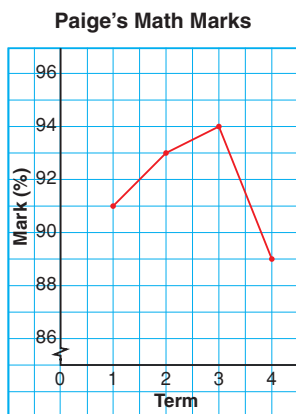
- a) The profits have tripled in 3 months.
 b) The profit in March is double the profit in January.
 c) The profit in March is about \$10 000 000 more than the profit in January.
 d) The profit in April is \$16 000 000 more than the profit in January.

Explain how each incorrect conclusion may have been made.

6. The pictograph shows the number of students in a Grade 8 class who have different types of pets. From this graph, Nick concluded that the most popular pet is a bird.
- Is Nick's conclusion correct? If yes, justify his conclusion. If not, explain his misinterpretation.
 - If your answer to part b was no, what changes would you make to accurately display these data?
 - Which type of pet do you think the creator of this pictograph has? Why do you think so?

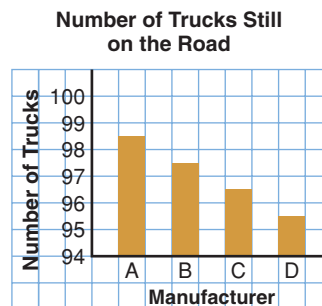


7. This graph appears to show that Paige's math mark dropped greatly in the 4th term.

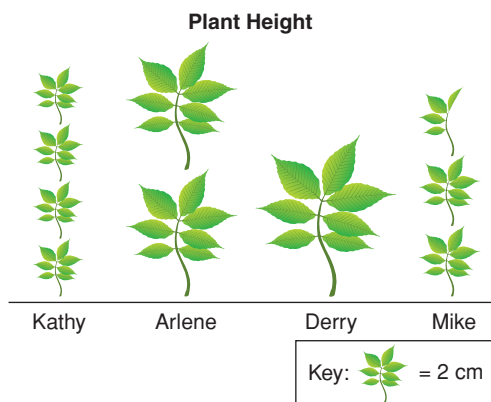


- Do you think Paige should be very concerned about this drop? Why or why not?
- What changes could you make to graph these data more accurately?

8. Manufacturer A uses this graph to advertise that more than 98 out of 100 of its trucks sold in the last 10 years are still on the road.

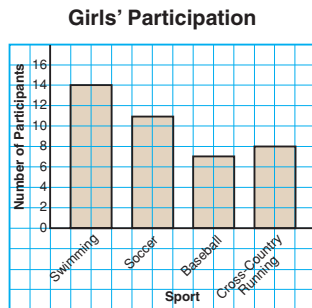
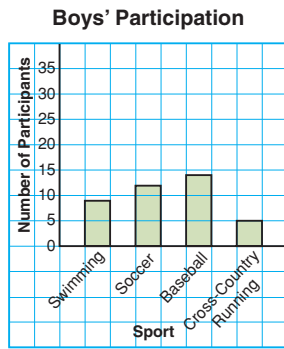


- What impression does this graph give?
 - How many trucks, out of 100, are still on the road for Manufacturer B? C? D?
 - Do you think Manufacturer A's trucks are more dependable than the other manufacturers' trucks? Why or why not?
 - What changes would you make to the graph to accurately display these data?
9. Four students conducted a science experiment to see who could grow the tallest plant over a given period of time. This pictograph shows the results.



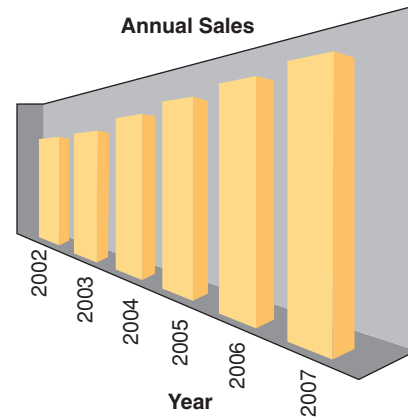
- Whose plant grew the most?
- How does the graph misrepresent these data?
- How could the graph be changed to present the data accurately?
- Do you think Kathy drew this pictograph? Why or why not?

10. Assessment Focus Giada surveyed her classmates to find out which sports they participated in. She drew these graphs.



- What impressions do these graphs give?
- Describe how the graphs create a false impression.
- What features of the graphs make it seem that the girls participate in sports more than the boys?
- How could the graphs be changed to present the data accurately?
- Suggest a different graph that could be used to accurately display these data.

- 11.** This graph was used by a sales manager at the Dust Destroyer Vacuum Cleaner Sales Company. She said that sales had increased greatly in 5 years.



This table shows the actual data.

Annual Sales

Year	Sales (\$1000s)
2002	15 450
2003	15 550
2004	16 000
2005	16 300
2006	16 600
2007	16 800

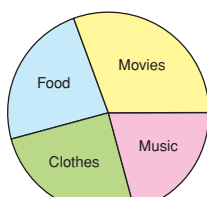
- How is the graph misleading?
 - What changes could you make to the graph to display these data accurately?
- 12.** Use newspapers, magazines, or the Internet. Find a graph that creates a false impression.
- Describe how the graph creates a false impression.
 - Why might the misleading graph be used?
 - How could the graph be changed to present the data accurately?

13. Why do some graphs display data in a misleading way? Describe three ways a graph might be drawn to misrepresent data.

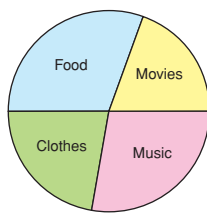
14. The graphs show how two students spend their allowance. From these graphs, a student concluded that Mark spends more money on movies than Tina does.

- From the graphs, can you tell which student spends more money on movies? Why or why not?
- Which type of graph could you use to display these data accurately?

How Students Spend Their Allowance



Mark
Graph A



Tina
Graph B



15. Take It Further Draw a graph to show how the data in the table can be displayed in each way.

Board of Directors' Expenses

Quarter	Amount (\$)
1st	85 000
2nd	104 000
3rd	125 000
4th	155 000

- The directors want the expenses to look low.
 - The shareholders want to show the expenses are too high.
 - Prospective shareholders want to see the data displayed accurately.
- Explain how you created each impression.

16. Take It Further Draw a graph to show how the data in the table below can be displayed in each way.

Submarine Sandwich Sales at
Submarine Sue's

Type	Mean Daily Sales
Pizza	275
Assorted	200
Veggie	260
Meatball	250

- The pizza submarine is by far the most popular sandwich.
- About an equal number of each type of submarine are sold daily.
- Very few assorted submarines are sold. Explain how you created each impression.

Reflect

When you see a graph in the media, what can you do to check whether it represents the data accurately?



Using Spreadsheets to Investigate Formatting

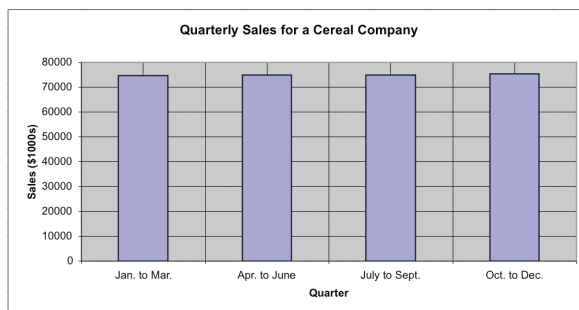
Focus Recognize how formatting may lead to misinterpretation of graphs.

Spreadsheets can be used to record, then graph, data. The format of the graph can affect its visual impact.

This table shows the quarterly sales figures for a cereal company.

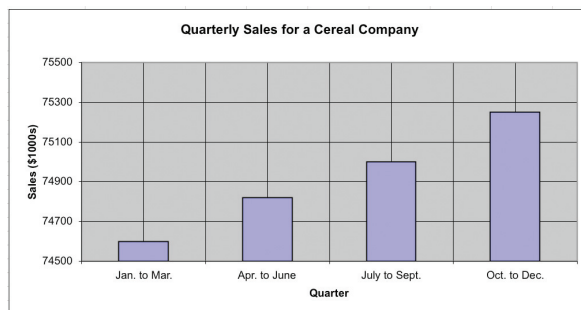
Quarterly Sales	
Quarter	Sales (\$1000s)
Jan. to Mar.	74 600
Apr. to June	74 820
July to Sept.	75 000
Oct. to Dec.	75 250

- Use a spreadsheet. Enter the data into rows and columns. Highlight the data, then create a bar graph. Your graph should look similar to this:



The table shows a slight increase in sales over the year. This is difficult to see on the bar graph.

- Investigate the effect of changing the scale on the vertical axis. Create a graph that exaggerates the increase in sales. Your graph may look similar to this:



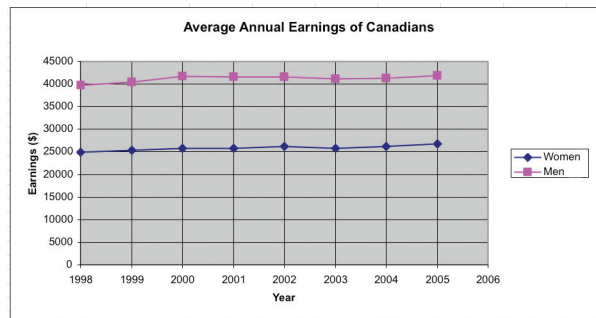
What impression does this graph give? Who might want to use this graph?

These data come from the Statistics Canada Web site.

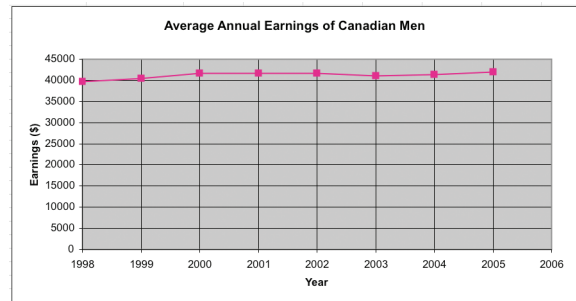
Average Annual Earnings of Canadians

Year	Women (\$)	Men (\$)
1998	24 900	39 700
1999	25 300	40 400
2000	25 800	41 700
2001	25 800	41 600
2002	26 200	41 600
2003	25 800	41 000
2004	26 200	41 300
2005	26 800	41 900

- Use a spreadsheet. Enter the data into rows and columns. Highlight the data, then create a double line graph. Your graph should look similar to the one below.



- Investigate the effect of not graphing both data sets on one grid. Create 2 line graphs to exaggerate the increase in women's average annual earnings compared to men's average annual earnings. Your graphs may look similar to these:



1. This table shows the NHL's top point scorers for the 2005–2006 season.
 - a) Create a graph that Joe Thornton might use to negotiate a new contract. Justify your choice.
 - b) Create a graph that the NY Rangers might use to negotiate a new contract with Jaromir Jagr. Justify your choice.
2. This table shows the numbers of male and female athletes in the Winter Olympic Games 1988–2006.

Top Point Scorers, 2005–2006

Player	Points
Joe Thornton, San Jose	125
Jaromir Jagr, NY Rangers	123
Alexander Ovechkin, Washington	106
Dany Heatley, Ottawa	103
Daniel Alfredsson, Ottawa	103
Sidney Crosby, Pittsburgh	102
Eric Staal, Carolina	100

Number of Athletes in Winter Olympic Games

Year	Location	Men	Women
1988	Calgary, Canada	1122	301
1992	Albertville, France	1313	488
1994	Lillehammer, Norway	1215	522
1998	Nagano, Japan	1389	787
2002	Salt Lake City, USA	1513	886
2006	Torino, Italy	1548	960

- a) Create a graph to accurately display these data. Justify your choice of graph.
 - b) Create two graphs to exaggerate the increase in the number of female athletes.
 - c) Create two graphs to give the impression that there are more female athletes than male athletes.
3. To collect data about students from 8 to 18 years old, Statistics Canada developed the *Census at School* Web site.
 - Your teacher will give you the Web address for *Census at School*. Open the Web site.
 - At the *Home Page*, on the left menu bar, click: *Data and results*
 - Under *Canadian summary results*, click on any topic that interests you from the list under “*Summary tables for 2005/06*.”
 - Create a graph to accurately display these data.
 - Change the formatting of the graph. Explain the effect of your new graph.

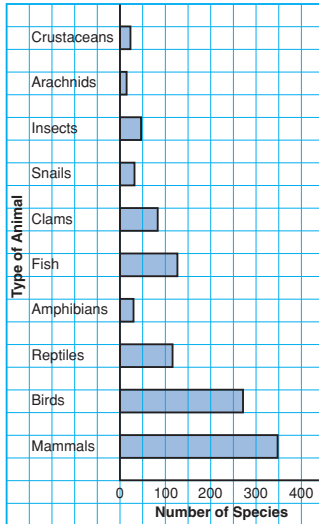
Mid-Unit Review

LESSON

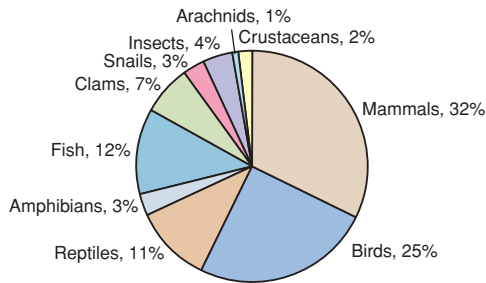
7.1

1. a) What data does each graph show?

Endangered and Threatened Animal Species, 2005



Endangered and Threatened Animal Species, 2005

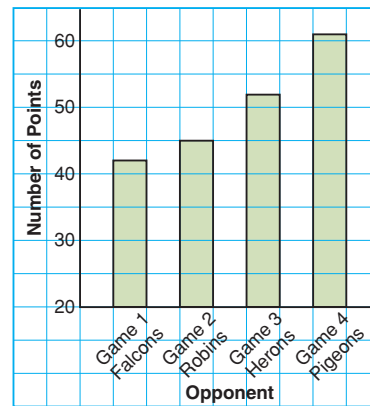


- What are the strengths of each graph?
- What are the limitations of each graph?
- Which graph is more appropriate to display these data? Justify your choice.
- Should a line graph be used to display these data? Why or why not?

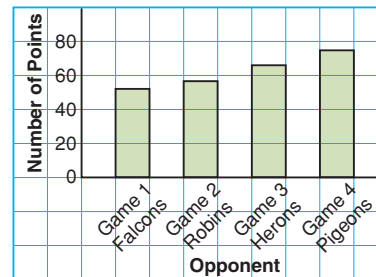
7.2

2. The coach of the Ravens used these graphs to show how much better his team is than the Hawks. He said, “We have scored more points per game and we are improving faster.”

Points Per Game: Ravens



Points Per Game: Hawks



- Are the coach’s claims accurate? Explain.
- What features of the graphs make it seem that the Ravens are the better team?
- How could the graphs be changed to compare the teams accurately?

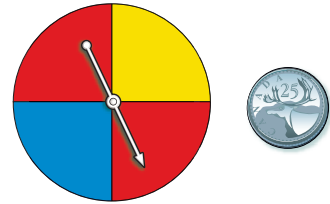
7.3

Probability of Independent Events

Focus

Develop and apply a rule to determine the probability of two independent events.

James played a game.
He spun the pointer on this spinner and tossed the coin.
What is the probability that the pointer lands on red?
Does the spinner result affect the coin toss result?



Investigate

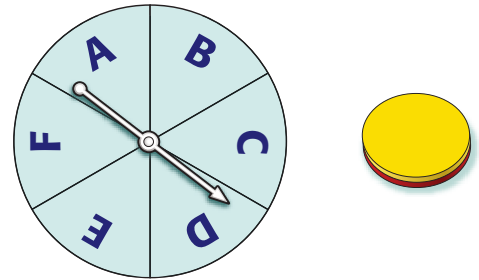
Work with a partner.

- Use a tree diagram.

List the possible outcomes of spinning the pointer on this spinner and tossing the two-coloured counter.

What is the probability of each event?

- landing on F
- tossing red
- landing on F and tossing red



- Each of Kelsey and Sidney has a standard deck of 52 playing cards.

Each student turns over a card, then the students compare suits.

Make a table to list the possible outcomes.

What is the probability of each event?

- Kelsey turns over a spade.
- Sidney turns over a heart.
- Kelsey turns over a spade and Sidney turns over a heart.

- In each situation above, how does the probability of each individual event relate to the probability of the combined events?

Write a rule to find the probability of two independent events.

Use your rule to find the probability of tossing heads on a coin and

drawing a red tile from a bag that contains 2 red tiles and 3 green tiles.

Use a tree diagram to check your probability.

Reflect & Share

Compare your results and rule with those of another pair of classmates.

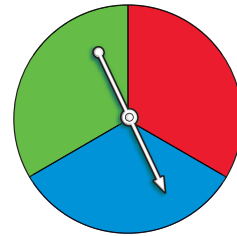
Did you write the same rule? If not, do both rules work?

Does the order in which the events are performed matter? Why or why not?

Connect

Two events are **independent events** when one event does not affect the other event.

The pointer on this spinner is spun twice. Landing on red and landing on blue are examples of two independent events.



Use a table to find the probability of landing on red twice.

There are 9 possible outcomes:

RR, RB, RG, BR, BB, BG, GR, GB, GG

Only one outcome is RR.

So, the probability of landing on red twice is $\frac{1}{9}$.

The probability of landing on red on the first spin is $\frac{1}{3}$.

The probability of landing on red on the second spin is $\frac{1}{3}$.

Note that: $\frac{1}{9} = \frac{1}{3} \times \frac{1}{3}$

probability of landing on red twice = probability of landing on red on the first spin \times probability of landing on red on the second spin

First Spin

	R	B	G
Second Spin			
R	RR	RB	RG
B	BR	BB	BG
G	GR	GB	GG

This illustrates the rule below for two independent events.

Suppose the probability of event A is written as $P(A)$.

The probability of event B is written as $P(B)$.

Then, the probability that both A and B occur is written as $P(A \text{ and } B)$.

If A and B are independent events, then: $P(A \text{ and } B) = P(A) \times P(B)$

Example 1

A coin is tossed and a regular tetrahedron labelled 5, 6, 7, and 8 is rolled.

- Find the probability of tossing heads and rolling an 8.
- Find the probability of tossing heads or tails and rolling an even number.

Use a tree diagram to verify your answers.



A Solution

Since the outcome of tossing the coin does not depend on the outcome of rolling the tetrahedron, the events are independent.

- a) When the coin is tossed, there are 2 possible outcomes.

One outcome is heads.

$$\text{So, } P(\text{heads}) = \frac{1}{2}$$

When the tetrahedron is rolled, there are 4 possible outcomes.

One outcome is an 8.

$$\text{So, } P(8) = \frac{1}{4}$$

$$\begin{aligned} P(\text{heads and } 8) &= P(\text{heads}) \times P(8) \\ &= \frac{1}{2} \times \frac{1}{4} \\ &= \frac{1}{8} \end{aligned}$$

- b) When the coin is tossed, there are 2 possible outcomes.

Two outcomes are heads or tails.

$$\text{So, } P(\text{heads or tails}) = \frac{2}{2} = 1$$

When the tetrahedron is rolled, there are 4 possible outcomes.

Two outcomes are even numbers: 6 and 8

$$\text{So, } P(\text{even number}) = \frac{2}{4} = \frac{1}{2}$$

$$\begin{aligned} P(\text{heads or tails and even number}) &= P(\text{heads or tails}) \times P(\text{even number}) \\ &= 1 \times \frac{1}{2} \\ &= \frac{1}{2} \end{aligned}$$

Use a tree diagram to check your answers.

Toss	Roll	Possible Outcomes
Heads	5	Heads/5
	6	Heads/6
	7	Heads/7
	8	Heads/8
Tails	5	Tails/5
	6	Tails/6
	7	Tails/7
	8	Tails/8

There are 8 possible outcomes.

One outcome is heads/8.

So the probability of tossing heads and rolling 8 is $\frac{1}{8}$.

Four outcomes have heads or tails and

an even number: heads/6, heads/8, tails/6, tails/8

So, the probability of tossing heads or tails and rolling an even number is $\frac{4}{8}$, or $\frac{1}{2}$.

Example 2

The pocket of a golf bag contains 9 white tees, 7 red tees, and 4 blue tees.

The golfer removes 1 tee from her bag without looking, notes the colour, then returns the tee to the pocket.

The process is repeated.

Find the probability of each event.

- Both tees are red.
- The first tee is not red and the second tee is blue.



A Solution

Since the first tee is returned to the pocket, the events are independent.

- There are 20 tees in the pocket.

$$P(\text{red}) = \frac{7}{20}$$

$$\text{So, } P(\text{red and red}) = P(\text{red}) \times P(\text{red})$$

$$= \frac{7}{20} \times \frac{7}{20}$$

$$= \frac{49}{400}$$

- $P(\text{not red}) = P(\text{white or blue})$

$$= \frac{13}{20}$$

$$\text{So, } P(\text{not red, blue}) = P(\text{not red}) \times P(\text{blue})$$

$$= \frac{13}{20} \times \frac{4}{20}$$

$$= \frac{13}{20} \times \frac{1}{5}$$

$$= \frac{13}{100}$$

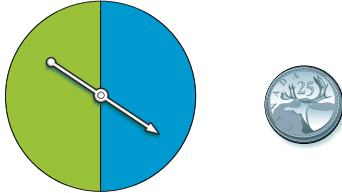
Discuss the ideas

- In a word problem, what are some words that can be used to suggest the events are independent?
- In *Example 1*, how can you find the probability of *not* rolling an 8?

Practice

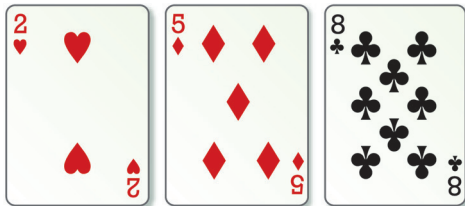
Check

3. A spinner has 2 congruent sectors coloured blue and green. The pointer is spun once, and a coin is tossed.



Find the probability of each event:

- blue and tails
 - blue or green and heads
4. Stanley has two sets of three cards face down on a table. Each set contains: the 2 of hearts, the 5 of diamonds, and the 8 of clubs. He randomly turns over one card from each set.

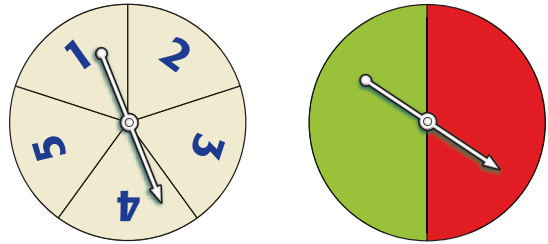


Find the probability of each event:

- Both cards are red.
- The first card is red and the second card is black.
- Both cards are even numbers.
- The sum of the numbers is greater than 8.

Which strategy did you use each time?

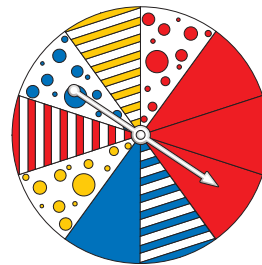
5. Raoul spins the pointer on each spinner. Find the probability of each event.



- green and a 2
 - red and an even number
 - green and a prime number
- Use a tree diagram or a table to verify your answers.

Apply

6. Find the probability of each event:
- The pointer lands on a blue spotted sector, then a solid red sector.
 - The pointer lands on a red sector, then a spotted sector.



- The pointer lands on a striped sector, then a solid blue sector.
 - The pointer lands on a blue or red sector, then a spotted sector.
- b) Use a different strategy to verify your answers in part a.

- 7.** Bart and Bethany play a game. They each roll a regular 6-sided die labelled 1 to 6. Find the probability of each event:
- Each player rolls a 6.
 - Bart rolls a 6 and Bethany rolls a 2.
 - Bart does not roll a 4 and Bethany rolls an even number.
 - Bart rolls an even number and Bethany rolls an odd number.
 - Bart rolls a number greater than 3 and Bethany rolls a number less than 4.
- 8.** An experiment consists of rolling a die labelled 3 to 8 and picking a card at random from a standard deck of playing cards.
- What is the probability of each event?
 - rolling a 6 and picking a spade
 - not rolling a 4 and picking an ace
 - Use a tree diagram to verify your answer to part a, i.
 - What is the probability of picking the ace of spades and rolling a 5? What is the advantage of using the rule instead of a tree diagram?
- 9.** A game at a school carnival involves rolling a regular tetrahedron. Its four faces are coloured red, orange, blue, and green. A player rolls the tetrahedron twice. To win, a player must roll the same colour both times. Marcus has been watching the game. He says he has figured out the probability of a player winning. “The probability of rolling any colour is $\frac{1}{4}$. So, the probability of

rolling the same colour again is $\frac{1}{4}$. Since the events are independent, the probability of rolling the same colour both times is $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$.”

Do you agree with Marcus?

Justify your answer.

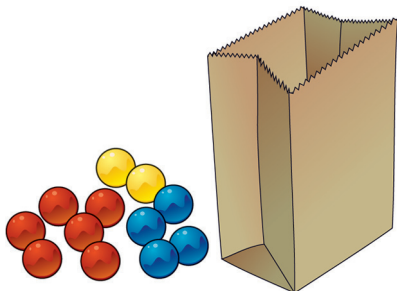
Use a tree diagram to show your thinking.



- 10.** A dresser drawer contains five pairs of socks of these colours: blue, brown, green, white, and black. The socks in each pair are folded together. Pinto reaches into the drawer and takes a pair of socks without looking. He wants a black pair.
- What is the probability that Pinto takes the black pair of socks on his first try?
 - What is the probability that Pinto takes the green pair of socks on his first and second tries?
 - What assumptions do you make?
- 11.** Suppose it is equally likely that a baby be born a boy or a girl.
- What is the probability that, in a family of 2 children, both children will be boys?
 - Verify your answer to part a using a different method.

12. Assessment Focus A bag contains 6 red marbles, 4 blue marbles, and 2 yellow marbles. A student removes 1 marble without looking, records the colour, then returns the marble to the bag. The process is repeated.

- a) What is the probability of each outcome?
- i) a red marble, then a yellow marble
 - ii) 2 blue marbles
 - iii) not a blue marble, then a yellow marble
- b) Suppose the marbles are not returned to the bag. Could you use the rule for two independent events to find each probability in part a? Why or why not?



13. Luke and Salina play the card game “Slam.” Each player has 10 cards numbered 1 to 10. Both players turn over one card at the same time. The player whose card has the greater

value gets one point. If both cards are the same, a tie is declared and no point is given. After each round, the cards are returned to the pile and all the cards are shuffled.

Find the probability of each event:

- a) Luke will get one point when he turns over a 3.
- b) Salina will get one point when she turns over a 10.
- c) Luke and Salina will tie.
- d) Salina will get one point when she turns over a 1.

14. Take It Further Neither Andrew nor David like to set the table for dinner. They each toss a coin to decide who will set the table. If both coins show heads, David sets the table. If both coins show tails, Andrew sets the table. If the coins show a head and a tail, both Andrew and David set the table.

What is the probability David will set the table alone 2 days in a row? Show your work.

15. Take It Further A coin is tossed and a die labelled 1 to 6 is rolled. Write an event that has each probability below.

- a) $\frac{1}{2}$ b) $\frac{1}{6}$ c) $\frac{1}{3}$

Reflect

Which method of finding the probability of 2 independent events do you prefer? Why? When might the rule not be the best method? When might the tree diagram or table not be the best method? Include an example in your explanation.

Doing Your Best on a Test

Have you ever written a test and have not been happy with the results? There are many things you can do to help you do well on a test.



Getting ready

- Find out what will be covered in the test.
 - Give yourself plenty of time to review.
 - Review your notes, the textbook, past quizzes, and tests. Try sample questions often.
 - Think about what worked well for you on previous tests.
 - Practise difficult questions with a friend or family member.
 - Get a good night's sleep.
 - Eat a nutritious breakfast.
 - Bring all the supplies you need to the test.
 - Think about what you are good at.
- Stay calm and confident.

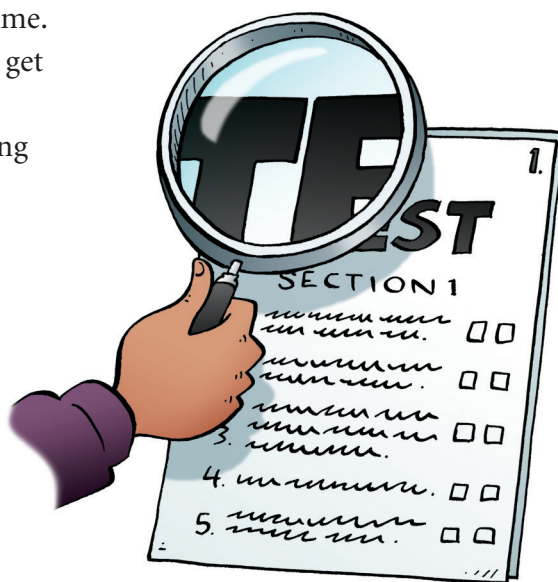


When you first get the test

- Listen carefully to the instructions.
- Find out how much time you will have.
- Look over the whole test before you begin.
- Read the instructions. Ask questions about anything that is unclear.

Answering the questions

- Read each question carefully.
- Start by answering the quick and easy questions.
- Next, answer the questions that you know how to do but take more time.
- Leave for later any questions you get stuck on.
- Remember strategies for answering different kinds of questions.
- Look for key words, such as: compare, describe, determine, explain, and solve.
- Show your work.



When you have finished the questions

Use any time you have left to check your work.

For each question, ask yourself:

- Did I answer the question?
- Did I leave anything out?
- Did I give a complete solution?
- Did I show my work so someone else can follow my thinking?
- Is my answer reasonable? (Does it make sense?)



Empty the Rectangles

HOW TO PLAY

1. Each player draws 6 rectangles on a piece of paper.



YOU WILL NEED

2 dice labelled 1 to 6;
12 counters

NUMBER OF PLAYERS

2

GOAL OF THE GAME

To remove all counters
from all rectangles

Label each rectangle from 0 to 5.

2. Each player places her 6 counters in any or all of the rectangles.
You can place 1 counter in each rectangle, or 2 counters in each of 3 rectangles, or even 6 counters in 1 rectangle.
3. Take turns to roll the dice.
Find the difference of the numbers.
You remove counters from the rectangle labelled with that number.
For example, if you roll a 6 and a 4, then $6 - 4 = 2$;
so, remove all counters from rectangle 2.
4. The winner is the first person to have all rectangles empty.

What strategies can you use to improve your chances of winning this game?

7.4

Solving Problems Involving Independent Events

Focus

Solve a problem that involves finding the probability of independent events.

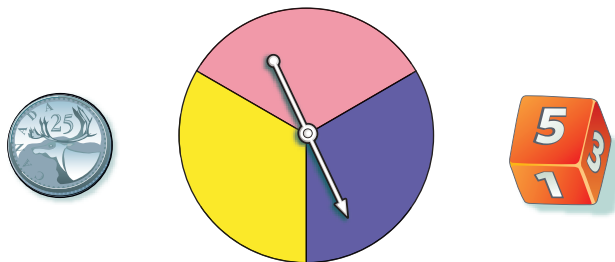
In Lesson 7.3, you learned that when A and B are independent events, the probability of both A and B happening is the product of the probability of event A and the probability of event B.

$$P(A \text{ and } B) = P(A) \times P(B)$$

Investigate

Work with a partner.

- A probability experiment involves tossing a coin, rolling a die labelled 1 to 6, and spinning the pointer on a spinner with 3 congruent sectors coloured pink, purple, and yellow. Use a tree diagram or a table to find the probability of each event.
 - tossing heads, rolling a 2, and landing on purple
 - tossing tails, rolling an even number, and landing on yellow
 - tossing heads, rolling a 1 or 2, and landing on pink



- Predict a rule to find the probability of three independent events. Use your rule to verify the probabilities you found above.

Reflect & Share

Compare your results and your rule with those of another pair of classmates.

Use your rule to find the probability of getting heads on 3 consecutive tosses of a coin.

Connect

The rule for the probability of two independent events can be extended to three or more independent events.

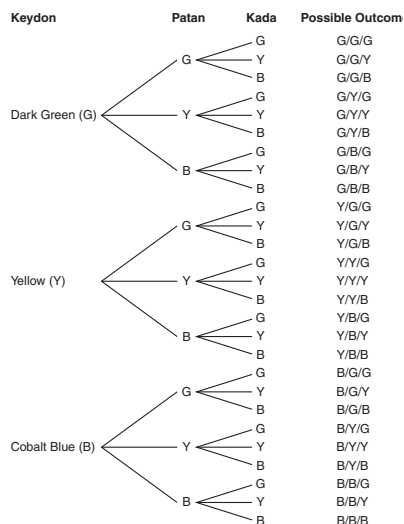
Example 1



The students in a Grade 8 class were making buffalo horn beaded chokers. Students could choose from dark green, yellow, and cobalt blue Crow beads. Each student has the same number of beads of each colour. Keydon, Patan, and Kada take their first beads without looking. Find the probability that Keydon takes a yellow bead, Patan takes a dark green bead, and Kada takes a yellow bead.

A Solution

Since each student has her own set of beads, the events are independent. Use a tree diagram.



There are 27 possible outcomes. One outcome is Y/G/Y.

So the probability that Keydon takes a yellow bead, Patan takes a dark green bead, and Kada takes a yellow bead is $\frac{1}{27}$.

In *Example 1*, the probability that Keydon takes a yellow bead is $P(Y) = \frac{1}{3}$.

The probability that Patan takes a dark green bead is $P(G) = \frac{1}{3}$.

The probability that Kada takes a yellow bead is $P(Y) = \frac{1}{3}$.

Note that: $\frac{1}{27} = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$

Suppose the probability of Event A is $P(A)$, the probability of Event B is $P(B)$, and the probability of Event C is $P(C)$.

Then, the probability that all A, B, and C occur is $P(A \text{ and } B \text{ and } C)$.

If A, B, and C are independent events,

then $P(A \text{ and } B \text{ and } C) = P(A) \times P(B) \times P(C)$.

Example 2

On a particular day in July, there is a 20% probability of rain in Vancouver, a 65% probability of rain in Calgary, and a 75% probability of rain in Saskatoon. What is the probability that it will rain in all 3 cities on that day?

A Solution

The events are independent.

Write each percent as a decimal, then multiply the decimals.

$$P(\text{Rain in Vancouver}) = 20\%, \text{ or } 0.20$$

$$P(\text{Rain in Calgary}) = 65\%, \text{ or } 0.65$$

$$P(\text{Rain in Saskatoon}) = 75\%, \text{ or } 0.75$$

$$\begin{aligned} \text{So, } P(\text{Rain V and C and S}) &= P(\text{Rain V}) \times P(\text{Rain C}) \times P(\text{Rain S}) \\ &= 0.20 \times 0.65 \times 0.75 \\ &= 0.0975, \text{ or } 9.75\% \end{aligned}$$

Use a calculator.

The probability that it will rain in all 3 cities on that day is 9.75%.

Example 2 Another Solution

Assume the events are independent.

$$P(\text{Rain in Vancouver}) = 20\%, \text{ or } \frac{20}{100} = \frac{1}{5}$$

$$P(\text{Rain in Calgary}) = 65\%, \text{ or } \frac{65}{100} = \frac{13}{20}$$

$$P(\text{Rain in Saskatoon}) = 75\%, \text{ or } \frac{75}{100} = \frac{3}{4}$$

$$\begin{aligned} \text{So, } P(\text{Rain V and C and S}) &= P(\text{Rain V}) \times P(\text{Rain C}) \times P(\text{Rain S}) \\ &= \frac{1}{5} \times \frac{13}{20} \times \frac{3}{4} \\ &= \frac{39}{400} \end{aligned}$$

The probability that it will rain in all 3 cities on that day is $\frac{39}{400}$.

Note that $\frac{39}{400} = 0.0975$.

Discuss

the ideas

1. How can the rule for the probability of 2 independent events be extended to 4 or 5 independent events?
2. In *Example 2*, how can you use the answer to find the probability of it not raining in all 3 cities on that day?
3. Why are the events in *Example 2* considered independent?

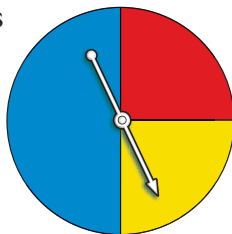
Practice

Check

4. One coin is tossed 3 times. Find the probability of each event:
 - a) 3 heads
 - b) 3 tails
 - c) tails, then heads, then tailsUse a tree diagram to verify your answers.

5. A red die, a blue die, and a green die are rolled. Each die is labelled 1 to 6. Find the probability of each event:
 - a) a 2 on the red die, a 3 on the blue die, and a 4 on the green die
 - b) a 4 on the red die, an even number on the blue die, and a number less than 3 on the green die

6. A spinner has 3 sectors coloured red, blue, and yellow. The pointer on the spinner is spun 3 times. Find the probability of each event:
 - a) red, blue, and yellow
 - b) blue, blue, and not red
 - c) blue, blue, and blue



Apply

7. Stanley's bicycle lock has 4 dials, each with digits from 0 to 9. What is the probability that someone could guess his combination on the first try by randomly selecting a number from 0 to 9 four times?



8. A coffee shop has a contest. When you "lift the lid," you might win a prize. The probability of winning a prize is $\frac{1}{10}$. Suppose your teacher buys one coffee each day. Find the probability of each event:
 - a) Your teacher will win a prize on each of the first 3 days of the contest.
 - b) Your teacher will win a prize on the third day of the contest.
 - c) Your teacher will not win a prize in the first 4 days of the contest.

9. Assessment Focus Nadine, Joshua, and Shirley each have a standard deck of playing cards. Each student randomly draws a card from the deck. Find the probability of each event:

- a) Each student draws a heart.
- b) Nadine draws a spade, Joshua draws a spade, and Shirley draws a red card.
- c) Nadine does not draw a heart, Joshua draws a black card, and Shirley draws an ace. Show your work.

10. Preet writes a multiple-choice test.

The test has 5 questions.

Each question has 4 possible answers.

Preet guesses each answer.

Find the probability of each event:

- a) She answers all 5 questions correctly.
- b) She answers only the first 3 questions correctly.
- c) She answers all the questions incorrectly.

11. Rocco chooses a 3-letter password for his e-mail account. He can use a letter more than once. What is the probability that someone else can access his e-mail by randomly choosing 3 letters?

12. Vanessa has 16 songs on a Classic Rock CD. Six of the songs are by the Beatles, 4 are by the Rolling Stones, 4 are by the Who, and 2 are by the Doors. Vanessa plays the CD. She selects a setting that randomly chooses songs to play. Find the probability of each event:

- a) The first 3 songs played are by the Beatles.
- b) The first 2 songs played are by the Rolling Stones and the next 2 songs are by the Beatles.
- c) The first 2 songs played are by the Doors, and the next song played is either by the Beatles or the Rolling Stones.



13. A bag contains 5 blue marbles and 1 white marble. Susan draws a marble from the bag without looking, then replaces it in the bag. This is done 5 times.

- a) What is the probability that the white marble is drawn 5 times in a row? Express your answer as a percent.
- b) Suppose the white marble is drawn 5 times in a row. What is the probability the white marble will be picked on the next draw? Explain.
- c) Is your answer to part b the same as the probability of drawing the white marble 6 times in a row? Why or why not?

- 14.** Pancho wants to buy his teacher some flowers. The flower shop has 3 vases of cut flowers. One vase contains roses: 1 red, 4 yellow, and 3 white. A second vase contains carnations: 5 pink and 1 red. A third vase contains daisies: 1 yellow and 3 white. Pancho cannot decide so he closes his eyes and picks one flower from each vase. Find the probability of each event.
- Pancho picks a red rose, a pink carnation, and a white daisy.
 - Pancho picks a yellow or white rose, a red carnation, and a yellow daisy.
 - Pancho picks a red rose, a red carnation, and a red daisy.



- 15. Take It Further** In gym class, students take turns shooting a basketball at the net. Each successful shot is worth 1 point. A player is awarded a second shot only if the first shot is successful. The player can score 0, 1, or 2 points in this situation. Suppose a player shoots with 70% accuracy. Find the probability of each event.
- She scores 0 points.
 - She scores 1 point.
 - She scores 2 points.

- 16. Take It Further** A regular 6-sided die is rolled three times.
- What is the probability of rolling three 6s in a row?
 - What is the probability of *not* rolling three 6s in a row?
 - Find the sum of your answers in parts a and b. Explain the result.

Reflect

Give examples of random independent events outside the classroom. Why do you think the events are independent?

Math Link

Your World

The Coquihalla Highway is the only toll road in British Columbia. Those who run toll roads use probability to model the arrival of cars and trucks at the toll booths. This allows decisions to be made about how many toll booths should be open and how many toll booth operators are needed at any time of the day.



Using Technology to Investigate Probability

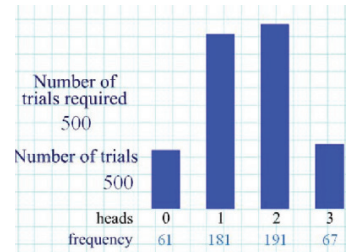


Focus Use virtual manipulatives to investigate the probability of independent events.

You have used manipulatives, such as spinners, dice, and coins. Many Web sites offer virtual manipulatives. These Web sites imitate, or *simulate*, spinning pointers, rolling dice, or tossing coins.

Your teacher will give you a Web address. Use the Web site to simulate tossing 3 coins 500 times.

This is what you might see:



The theoretical probability of tossing 3 tails is:

$$\begin{aligned}P(T \text{ and } T \text{ and } T) &= P(T) \times P(T) \times P(T) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8}, \text{ or } 0.125\end{aligned}$$

Recall:

The probability of tossing tails on the first coin is $\frac{1}{2}$, on the second coin is $\frac{1}{2}$, and on the third coin is $\frac{1}{2}$.

From the graph in this simulation, the experimental probability of tossing 3 tails (or 0 heads) is: $\frac{61}{500} = 0.122$

This experimental probability, 0.122, is very close to the theoretical probability, 0.125.

Combine your experimental results with those of 9 classmates.

Find the experimental probability of tossing 3 tails.

How do the experimental and theoretical probabilities compare?

Check

1. Use a Web site to simulate spinning 2 identical spinners 500 times. Each spinner has 4 congruent sectors coloured red, blue, green, and yellow. Combine your experimental results with those of 9 classmates.
 - a) What is the experimental probability of each event?
 - i) red and yellow
 - ii) blue and green or yellow
 - b) Use the rule for the probability of independent events to find the probability of each event in part a.
 - c) How do your results in parts a and b compare? Explain.

If you have forgotten the difference between experimental and theoretical probability, turn to the *Glossary*.

Your teacher will give you the Web address.

Unit Review

What Do I Need to Know?

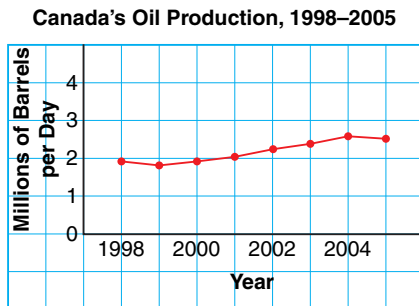
- ✓ Different graphs are used to visually represent data.
- ✓ Graphs can be used to misrepresent data.
For example, this can be done by changing: the scale on the vertical axis; the width of the bars; and the appearance of the graph.
- ✓ Suppose the probability of event A is $P(A)$ and the probability of event B is $P(B)$.
Then, the probability that both A and B occur is $P(A \text{ and } B)$.
When A and B are independent events, then $P(A \text{ and } B) = P(A) \times P(B)$.
- ✓ When A, B, and C are independent events, then $P(A \text{ and } B \text{ and } C) = P(A) \times P(B) \times P(C)$.

What Should I Be Able to Do?

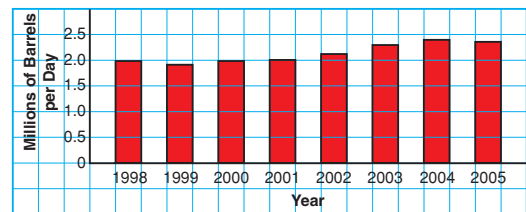
LESSON

7.1

1. Each graph shows Canada's oil production from 1998 to 2005.

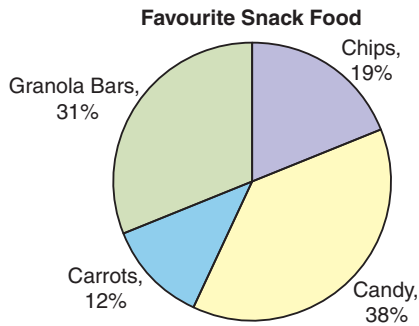


Canada's Oil Production, 1998–2005



- a) List 3 things you know from each graph.
- b) Which graph is more appropriate to display these data? Justify your choice.

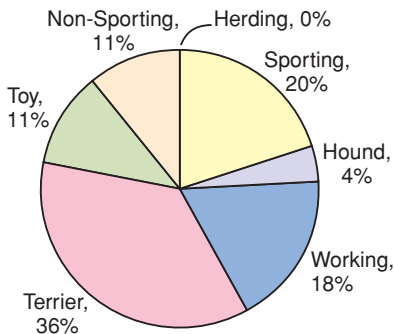
2. Each graph below shows the snacks preferred by Chris' Grade 8 class.



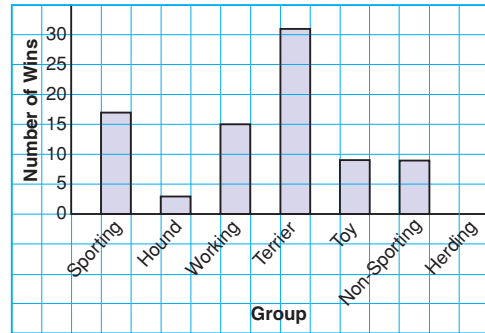
- What are the advantages of each graph?
- Describe a situation when the circle graph is the better graph to display these data.
- Describe a situation when the pictograph is the better graph to display these data.

3. Each graph shows the best in show awards, by group, at the Westminster Dog Show, from 1924 to 2007.

Westminster Dog Show, Best in Show by Group, 1924–2007



Westminster Dog Show, Best in Show by Group, 1924–2007

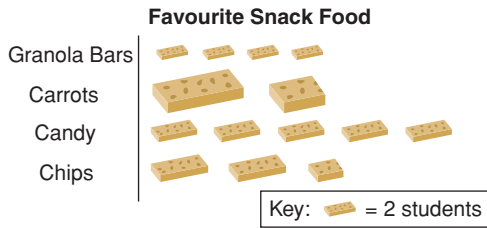


- What can you tell from the bar graph that you cannot tell from the circle graph?
 - Which graph is more appropriate to display these data? Justify your choice.
 - Could you use a line graph to display these data? Why or why not?
 - Could you use a pictograph to display these data? Why or why not?
4. Paola owns a pizza parlour. This table shows the number of toppings on the pizzas she sold during the Grey Cup game.

Number of Toppings	Number of Pizzas Sold
2	26
3	48
4	60
5	50
6	16

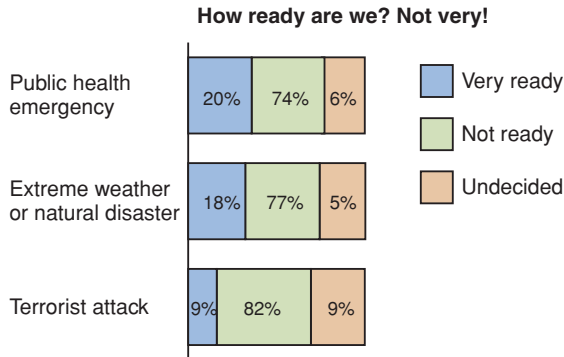
- Graph these data. Justify your choice of graph.
- What are the advantages and disadvantages of the graph you drew?

5. Stacy thought the pictograph in question 2 made it look like the students in her class were not making healthy choices. She redrew the graph.



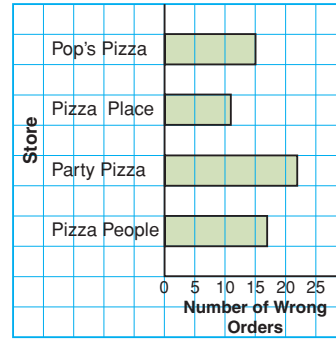
- Did Stacy use the data from question 2? How do you know?
- From the pictograph, which snack appears to be the favourite?
- Why is this pictograph misleading?

6. How is this graph misleading? Explain.

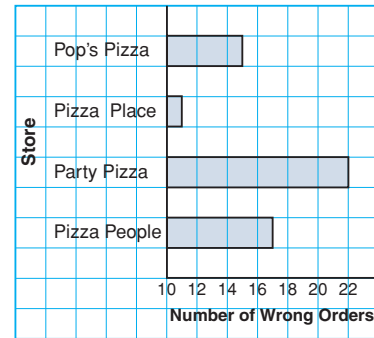


7. These graphs show the same data. Which graph is misleading? Explain why it is misleading.

Wrong Pizza Orders

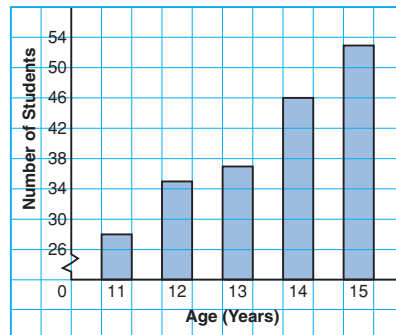


Wrong Pizza Orders



8. From this graph, Cleo concluded that the number of 15-year-old students with cell phones is about 5 times the number of 11-year-old students with cell phones. Is Cleo's conclusion correct? If yes, justify her conclusion. If not, explain how the graph may have led to the incorrect conclusion.

Cell Phones in Cleo's School



9. Draw a graph to show how these data can be displayed in each way.

Bicycle Bonanza's Profit

Year	Profit (\$)
2002	350 000
2003	400 000
2004	560 000
2005	610 000
2006	660 000

- a) The employees want to show a large increase in profit each year with the hope of getting a raise.
 b) The president wants to show a very small increase in profit each year to avoid having to pay his employees more money.

Explain how you created each impression.

7.3

10. A drawer contains one red shirt, two blue shirts, and one yellow shirt. Another drawer contains 2 pairs of brown pants and one pair of blue pants. You choose one item from each drawer without looking.
 a) What is the probability of choosing a red shirt and a pair of brown pants?
 b) Use a tree diagram to verify your answer.
11. A die has 2 faces labelled D, 2 faces labelled E, and 2 faces labelled F. The die is rolled twice. Find the probability of each event:
 a) a D on the first roll
 b) an E on the first roll

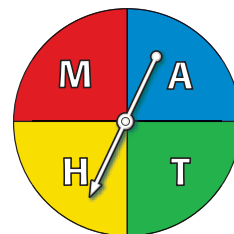
- c) rolling a D followed by an E
 d) rolling an E followed by a D
 e) rolling 2 As

12. On her way to the market, Akita passes through 3 sets of traffic lights. For each set of lights, the probability that the light is green is $\frac{1}{3}$. Find the probability of each event:
 a) The first set of lights is green.
 b) All the lights are green.
 c) None of the lights is green.

7.4

13. Three cards are drawn from a standard deck of 52 playing cards. After each card is drawn, the card is replaced and the deck is shuffled. Find the probability of each event:
 a) Three aces are drawn.
 b) The ace of spades is drawn 3 times.
 c) The first card is an ace, the second card is not a heart, and the third card is a face card.
14. One coin is tossed 10 times. Find the probability of tossing heads 10 times in a row.

15. The pointer on this spinner is spun 3 times. Find the probability of each event:



- a) M, M, M
 b) M, A, T
 c) A, A, H
 d) not A, not a consonant, A

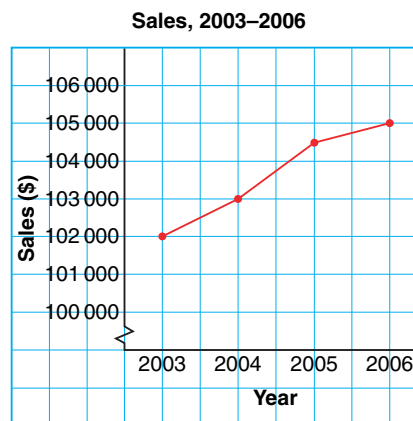
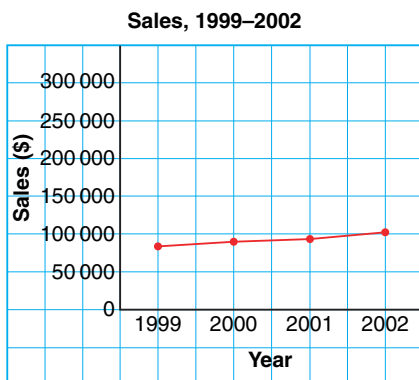
Practice Test

1. This table shows the student population of H. J. Cambie Secondary School in 2007.

Grade	Number of Students
8	125
9	155
10	162
11	200
12	185

- a) What are the limitations of graphing these data with:
 - i) a bar graph?
 - ii) a circle graph?
- b) Would a pictograph be a good choice? Would a line graph be a good choice? Why or why not?
- c) Which graph is most appropriate to display these data? Justify your choice.

2. The new manager of a company created these two graphs to show shareholders how much better the company is doing since she took over in 2003.



- a) What impressions do these graphs create?
- b) Describe how the graphs create a false impression. What features of the graphs cause this impression?
- c) How could the graphs be changed to present the data accurately?
- d) Is the company doing better since the new manager was hired? Explain.

- 3.** A bag contains four 2007 pennies, two 2005 pennies, and six 2001 pennies. A student removes 1 penny without looking, records the year, then returns the penny to the bag. The process is repeated. Find the probability of each event:
- two 2001 pennies
 - a 2007 penny, then a 2005 penny
 - a 2001 penny, then a 2005 penny
 - not a 2001 penny, then a 2007 penny

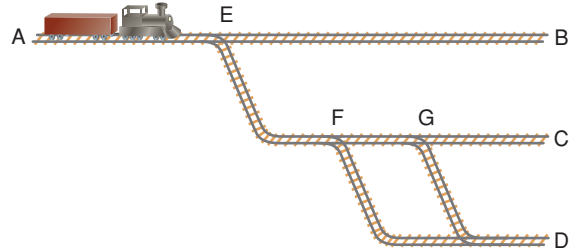
- 4.** This diagram shows part of a model railroad track. At each of the junctions E, F, and G, the probability of the train going straight on is $\frac{3}{4}$ and the probability of it turning right is $\frac{1}{4}$. At any other junction,

the train always goes straight on.

The train starts at A.

Find the probability of each event:

- The train reaches B.
- The train reaches C.
- The train reaches D.



- 5.** This table shows the sales, in thousands of dollars, of 2 employees at Electronics Warehouse over a 6-month period.
- Which graph do you think would be most appropriate to display Jamar's sales?
 - Which graph would be most appropriate to compare Jamar's and Laura's sales?
 - Both employees want to be named salesperson of the year.

Electronics Warehouse Sales Figures

Month	Jamar (\$1000s)	Laura (\$1000s)
Jan.	117	124
Feb.	118	125
Mar.	119	124
Apr.	121	126
May	124	127
June	127	128

- Graph these data to make Jamar's sales figures look much greater than Laura's.
 - Graph these data to make Laura's sales figures appear to be increasing much faster than Jamar's.
- Describe how you created each impression.

You have created your own cereal.
To promote your cereal, you will design two advertisements.
You will also develop a game for the back of your cereal box.

Part A

Creating Your Cereal

Create a name, logo, and slogan for your cereal.
Decide to whom you will market your cereal.
Create two data sets for your cereal that may include:

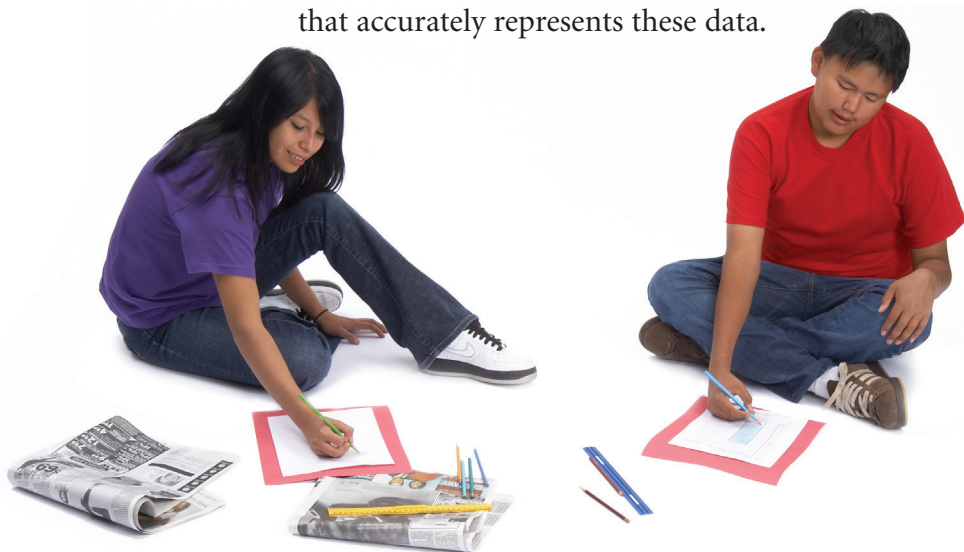
- nutritional information
- sales figures
- market research data

Part B

Promoting Your Cereal

To promote your cereal, create two advertisements for a local newspaper.

- Use one of the data sets in Part A.
- Create an advertisement that includes a graph that could be misinterpreted.
- Use the other data set from Part A.
- Create another advertisement that includes a graph that accurately represents these data.



Part C

Creating a Game

To make your cereal packaging more appealing, create a game for the back of the box.

Your game must involve two or more independent events.

Your game can include spinners, cards, or anything else that can be cut out from the box.

Be sure to include the rules of the game.

Everything needed to play the game should be on the back of the box.

Determine the probability of winning the game.

Check List

Your work should show:

- ✓ data sets and two graphs
- ✓ a game for your box and its rules
- ✓ how you calculated the probability
- ✓ the correct use of math language



Reflect on Your Learning

Why do you think it is important to be able to represent a set of data in different ways? What are the advantages and disadvantages of representing a set of data in different ways?