UNIT 8 Geometry

First Nations artists use their artwork to preserve their heritage.
Haida artist Don Yeomans is one of the foremost Northwest Coast artists.
Look at this print called *The Benefit*, created by Don Yeomans.
Describe any translations, reflections, or rotations you see.

What other art have you seen that demonstrates transformations?

What You'll Learn

- Draw and recognize different views of objects made from rectangular prisms.
- Identify shapes that will tessellate.
- Create tessellations using transformations.
- Identify tessellations in the environment.

Why It's Important

- We learn about the environment by looking at objects from different views. We can combine these views to get a better understanding of these objects.
- Tessellations are found in the environment, in architecture, and in art.

The Benefit

Gunarh and the Whale

Key Words

- isometric
- isometric drawing
- axis of rotation
- plane
- tessellate
- tessellation
- composite shape
- conservation of area

8.1 Sketching Views of Objects

Focus Draw the front, top, and side views of objects from models and drawings.

Which rectangular prisms do you see in this picture?

Choose a rectangular prism.
What does it look like from the top?
From the side?
From the front?

Investigate

Work on your own.
Choose a classroom object that is a rectangular prism,
or is made of more than one rectangular prism.

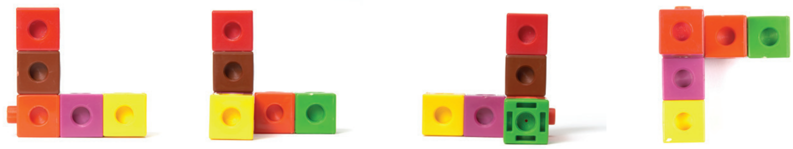
Sketch the object from at least 4 different views.
Label each view.
Use dot paper or grid paper if it helps.

Reflect & Share

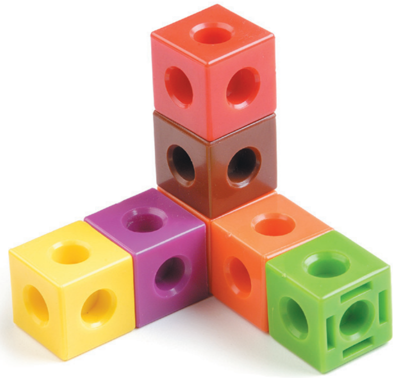
Trade sketches with a classmate.
Try to identify the object your classmate chose.
What strategies did you use to identify it?

Connect

Use linking cubes to make the object at the right.
Rotate the object to match each photo below.



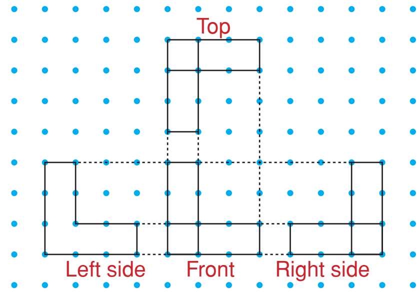
Left side Front Right side Top



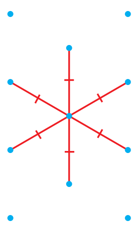
We can use square dot paper to draw each view of the object.
We ignore the holes in each face.

To draw the views of an object:

- Place the top view above the front view, and the side views beside the front view. This way, matching edges are adjacent.
- Use broken lines to show how the views align.
- Show internal line segments only where the depth or thickness of the object changes.

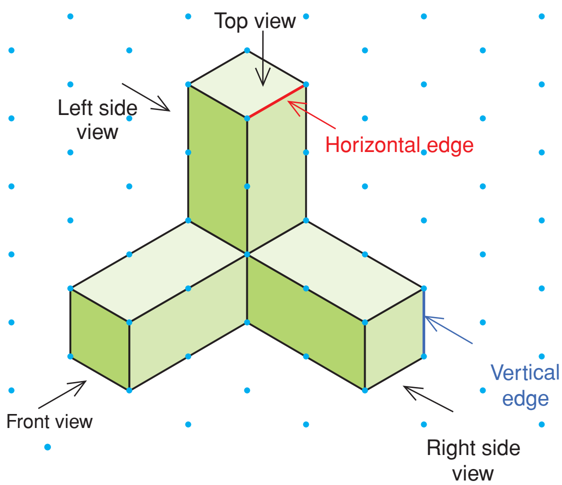


When a photo of an object is not available, the object may be drawn on triangular dot paper.
This is called isometric paper.
Isometric means "equal measure."
The line segments joining 2 adjacent dots in any direction are equal.



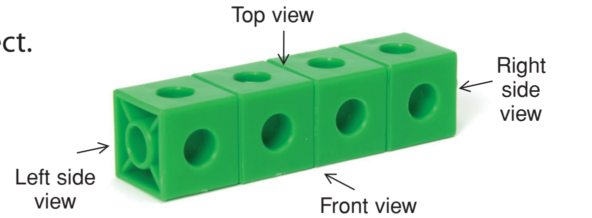
This is called an isometric drawing.

Here is an isometric drawing of the object made from linking cubes at the top of this page.
Each vertical edge on the object is drawn as a vertical line segment.
A horizontal edge is drawn as a line segment going up to the right or down to the left.
The faces are shaded differently to give a three-dimensional look.



Example 1

Here is an object made from linking cubes.
Draw the front, top, and side views of the object.



A Solution

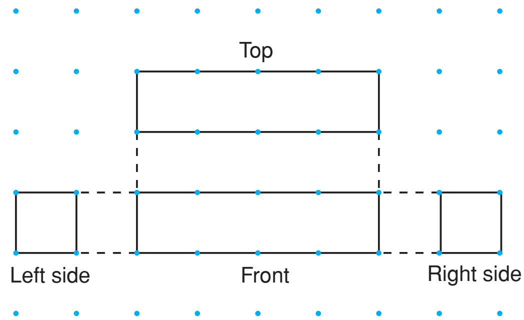
Use linking cubes to make the object.
Use square dot paper.

Draw the front view first.
The front view is a rectangle 4 squares long.

Then draw the top view above the front view.
The top view is a rectangle 4 squares long.
Keep matching edges adjacent.

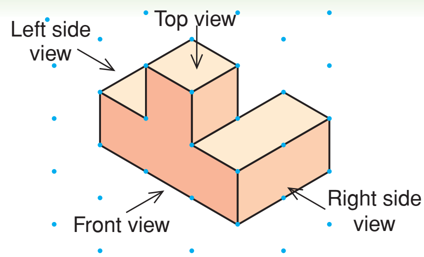
Then draw the side views beside the front view.
Each side view is a square.
Keep matching edges adjacent.

Use broken lines to show how the views align.



Example 2

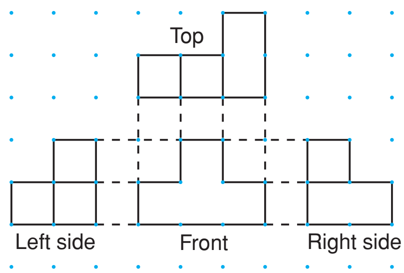
Here is an isometric drawing of an object.
Draw the front, top, and side views of the object.



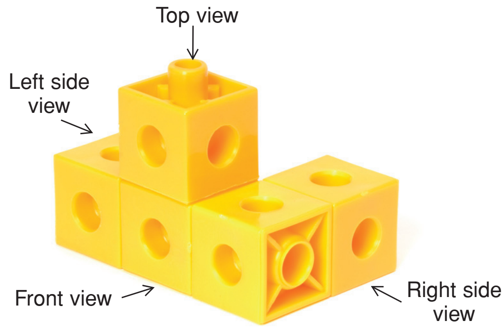
A Solution

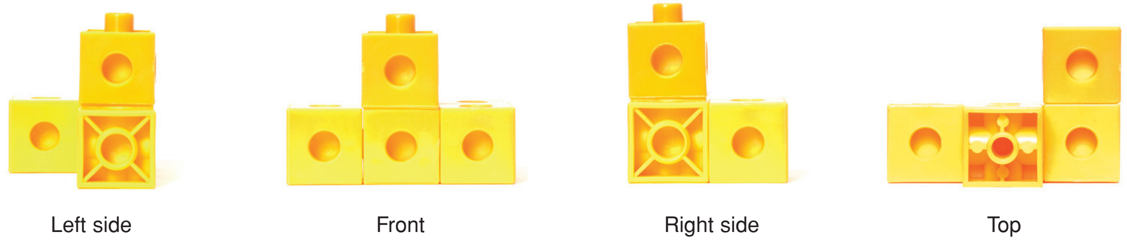
Visualize the object.
Draw the front view first.
Then draw the top view above the front view.
Then draw the side views beside the front view.
Keep matching edges adjacent.

Show internal line segments where the depth of the object changes.



We can build a model of the object in *Example 2*, then rotate the model to compare the views of the object with the actual object.





Discuss the ideas

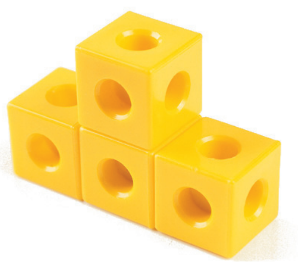
1. In *Example 1*, why are there no internal line segments in any of the views drawn?
2. Do 4 views of an object made from linking cubes always provide enough information to make the object?
3. In what types of jobs or occupations might different views of objects be important?

Practice

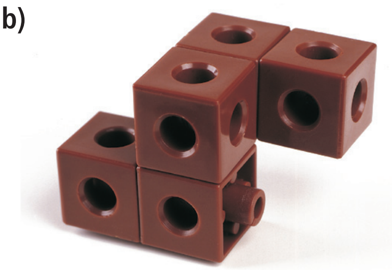
Check

4. Use linking cubes. Make each object. Use square dot paper. Draw the front, top, and side views of each object.

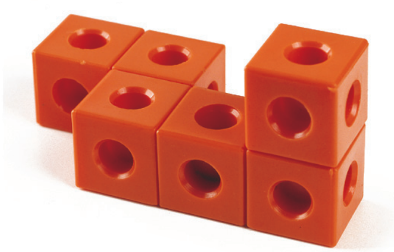
a)



b)



c)



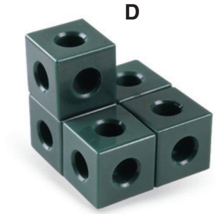
5. Use linking cubes. Make each object A to E. Figures J to Q are views of objects A to E. Match each view (J to Q) to each object (A to E) in as many ways as you can.

A

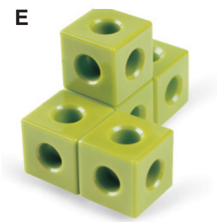
B

C

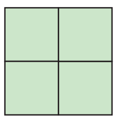
D



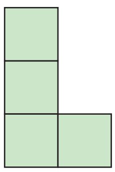
E



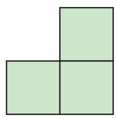
J



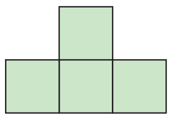
K



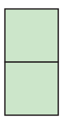
L



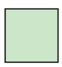
M



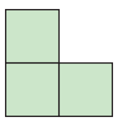
N



P



Q



Apply

6. Sketch the top, front, and side views of this recycling bin. Label each view.

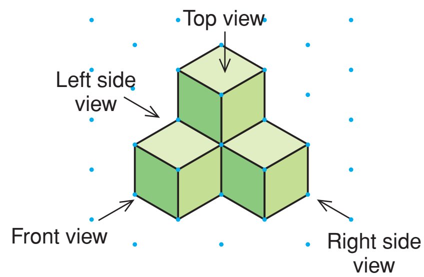
7. Sketch the front, top, and side views of each object in the classroom.

a) filing cabinet

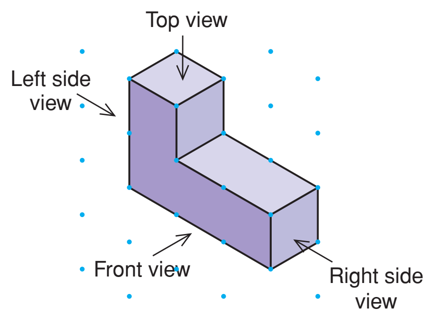
b) whiteboard eraser

c) teacher's desk

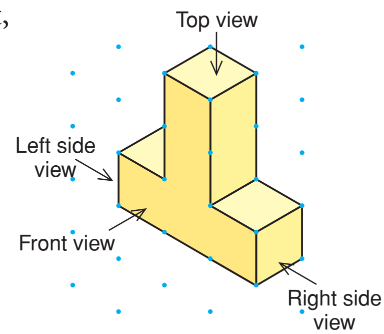
8. Sketch the front, top, and side views of this object drawn on isometric dot paper.



9. Sketch the front, top, and side views of this object.

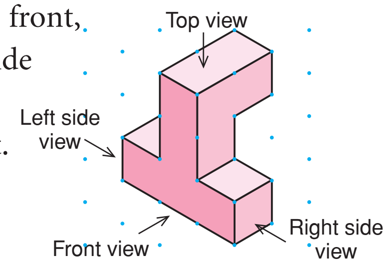


10. Sketch the front, top, and side views of this object.

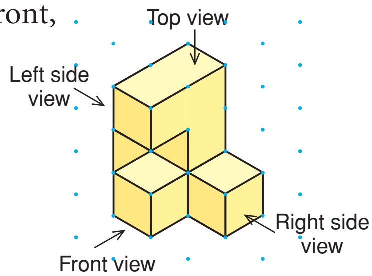


11. Use linking cubes. Make the objects in questions 8, 9, and 10. Rotate the objects to check that the views you drew are correct.

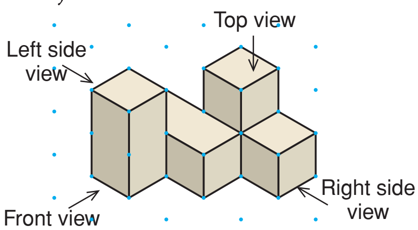
12. Sketch the front, top, and side views of this object.



13. Sketch the front, top, and side views of this object.



14. Sketch the front, top, and side views of this object.



15. Assessment Focus Use 4 linking cubes. Make as many different objects as possible. Draw the front, top, and side views of each object. Label each view. Use your drawings to explain how you know all the objects you made are different.

16. Use linking cubes. Make the letter H. Sketch the front, top, and side views of your model.

17. Take It Further The front, back, top, and side views of a cube are the same. Use the objects in the classroom.

a) Are there any other prisms with all views the same? Explain.

b) Which object has 4 views the same?

c) Which object has only 3 views the same?

d) Which object has no views the same? If you cannot name an object for parts b to d, use linking cubes to make an object.

18. Take It Further Sketch two possible views of each object.

a)

b)

c)

Reflect

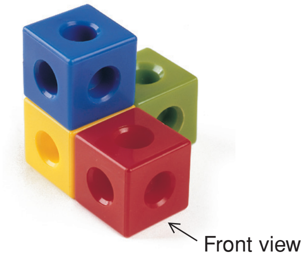
Choose an object made from linking cubes.
What could be a problem with representing only one view?
How many views do you need to draw so someone else can identify the object?
Explain. Sketch the views you describe.

Using a Computer to Draw Views of Objects

Focus Use technology to sketch views of objects.

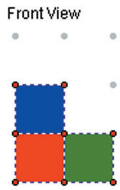
Geometry software can be used to draw different views of objects.
Use available geometry software.

Make this object with linking cubes.
Use the software to create views
of this object.



Open a new sketch.
Check that the distance units
are centimetres.

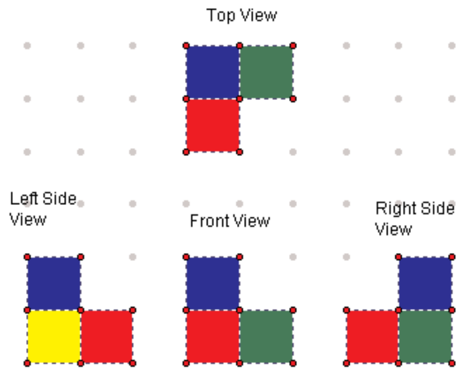
To make a "dot paper" screen, display a coordinate grid.
Change the grid lines to dotted lines.
Then hide the axes.



Use broken line segments to draw the front view.
Label the view.

Use the software to colour the view to match the object.

Draw and label the top and side views.
The views you drew should look similar to these.



Check

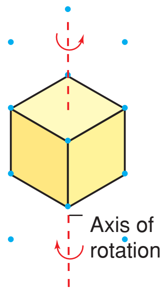
1. Open a new sketch. Draw different views of the objects in Lesson 8.1, *Practice* question 4. Compare the hand-drawn views with the computer-drawn views.

8.2 Drawing Views of Rotated Objects

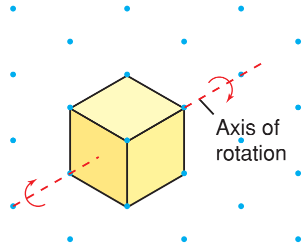
Focus Draw views of objects that result from a given rotation.

An object can be rotated:

horizontally or vertically



When an object is rotated horizontally, the **axis of rotation** is vertical. The object may be rotated clockwise or counterclockwise.



When an object is rotated vertically, the axis of rotation is horizontal. The object may be rotated toward you or away from you.

Investigate

Work with a partner.
You will need 6 linking cubes and square dot paper.

Use the linking cubes to build an object.
Draw the front, top, and side views of the object.

Predict each view when the object is rotated:
- horizontally 90° clockwise
- horizontally 90° counterclockwise
- horizontally 270° clockwise

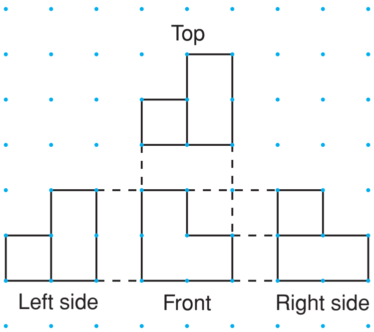
Rotate the object to check your predictions.
Draw the new front, top, and side views after each rotation.

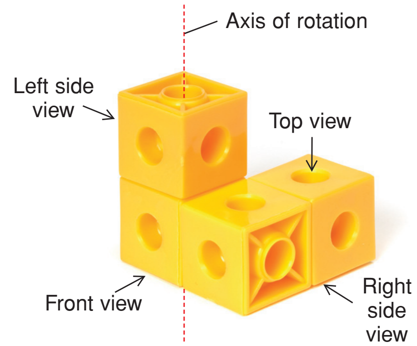
Reflect & Share

Were the views after a 90° clockwise rotation and a 90° counterclockwise rotation the same or different?
If they were the same, what do you notice about the object?
If they were different, talk to another pair of classmates who did get the same views, to see what they found out.

Connect

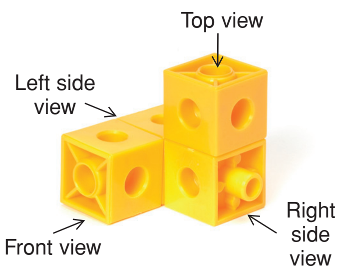
Here are the views of the object at the right.

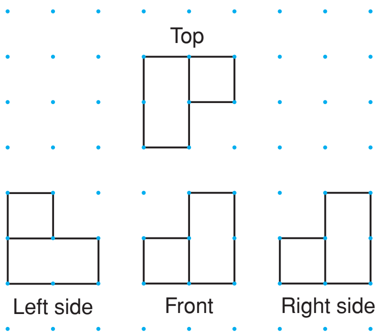




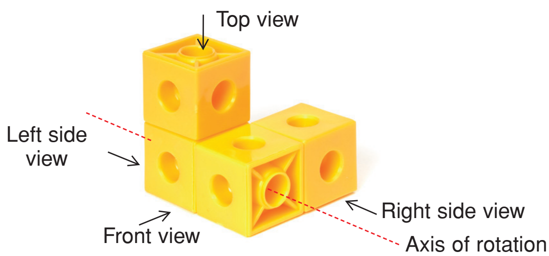
➤ Suppose the object is rotated horizontally 180°, about a vertical axis.
Here are the object and its views after the rotation.

Recall that a rotation of 180° clockwise is the same as a rotation of 180° counterclockwise.

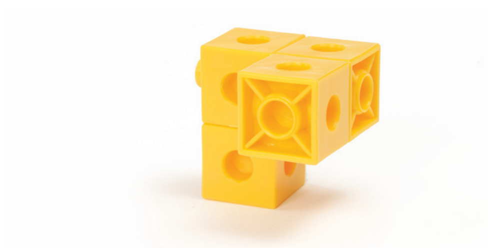


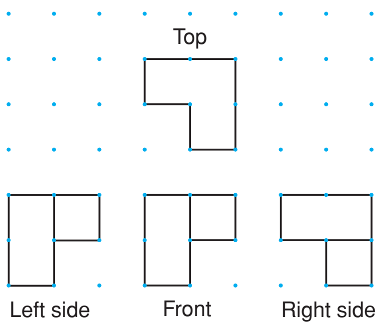


➤ Suppose the object is rotated vertically 180°, about a horizontal axis.



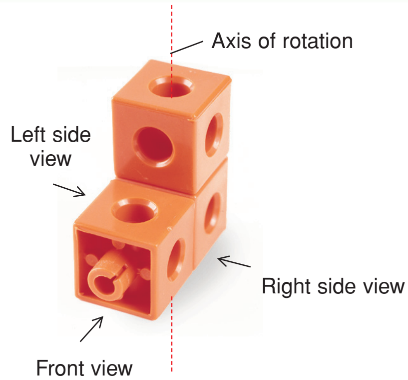
Here are the object and its views after the rotation.





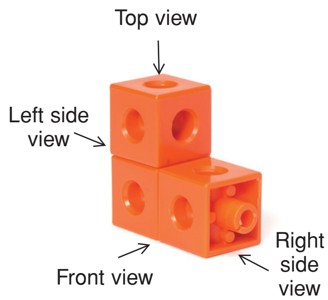
Example 1

Build this object.
It is rotated about the vertical axis shown.
Draw the front, top, and side views after a horizontal rotation of 270° clockwise.

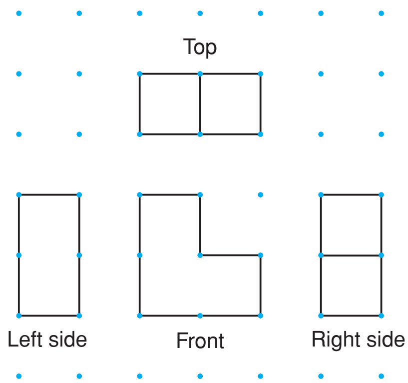


A Solution

After a horizontal rotation of 270° clockwise, the object is in this position.



The views of the object are:



Example 2

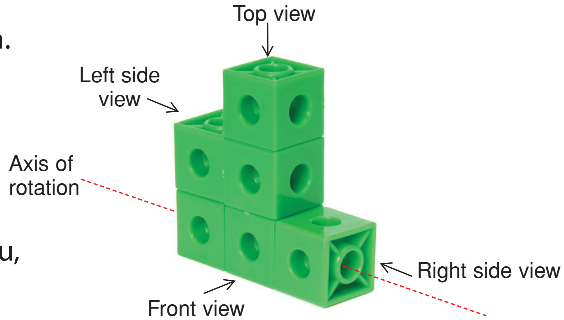
Use the object in *Example 1*.
Draw the front, top, and side views after a horizontal rotation of 90° counterclockwise.

A Solution

A rotation of 90° counterclockwise is the same as a rotation of 270° clockwise.
So, the views will match those in *Example 1*.

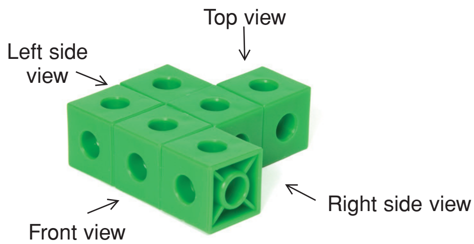
Example 3

Build this object.
It is rotated about the horizontal axis shown.
Draw the front, top, and side views after a vertical rotation of 90° away from you.

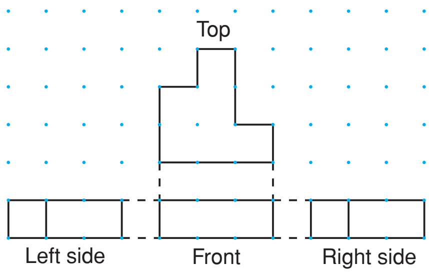


A Solution

After a vertical rotation of 90° away from you, the object is in this position.



The views of the object are:



Discuss the ideas

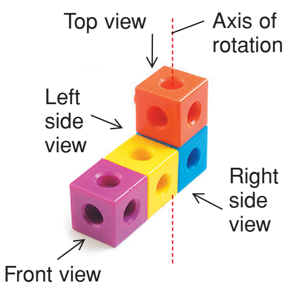
1. An object is rotated vertically 90° toward you. The same object is rotated vertically 270° away from you. Will the views of the object after each rotation be the same? Justify your answer.
2. An object can be rotated vertically or horizontally 90°, 180°, or 270°. The views of the rotated object are always the same. What might the object be?

Practice

Check

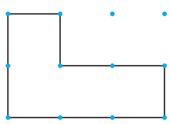
Use linking cubes and square dot paper.

3. This object is rotated horizontally about the axis shown.

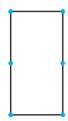


The front view after each rotation is shown below. Describe each rotation.

a)



b)



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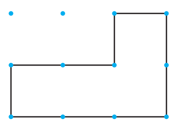
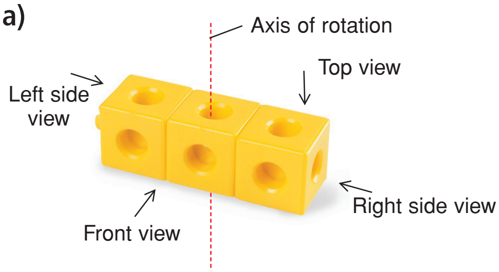
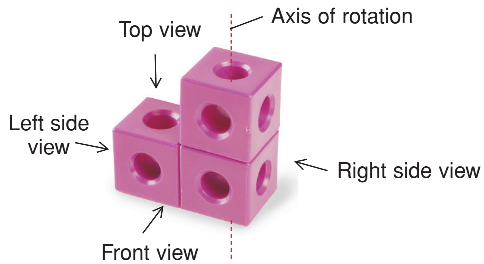


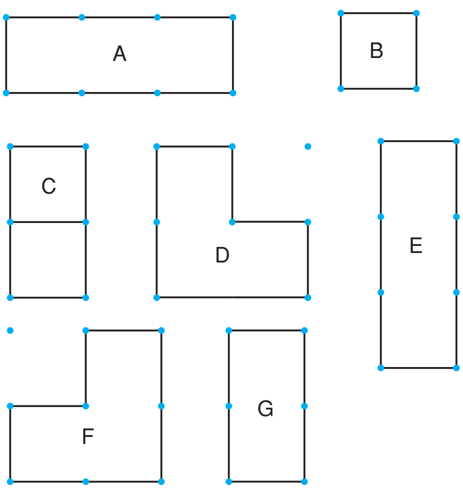
4. Build each object. Rotate each object horizontally 90° clockwise through the axis shown. Match each view (A to G) to the front, top, and side views of each rotated object. A lettered view can be used more than once.

a)



b)

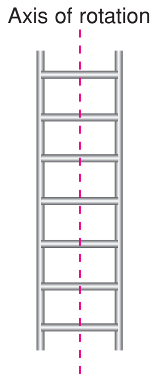




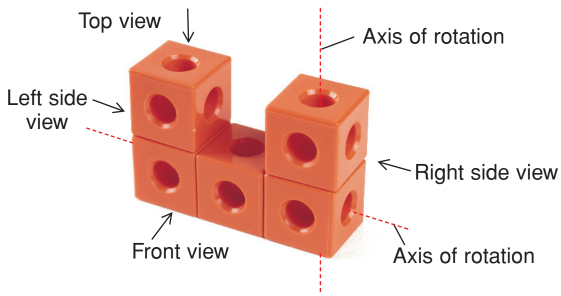
5. Use the objects in question 4. Suppose each object is rotated horizontally 180°. For which object will the views not change? How do you know?

Apply

6. Here is the front view of a ladder. Suppose the ladder is rotated horizontally 90° clockwise about the axis shown. Sketch the new front view.



7. Here is an object made of linking cubes.



a) Predict the front, top, and side views of the object after each horizontal rotation about the vertical axis shown.

b) Build the object. Draw the views after each rotation to verify your predictions.

i) 90° clockwise

ii) 180°

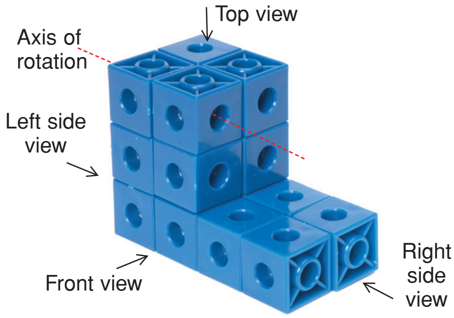
iii) 270° clockwise

8. Draw the front, top, and side views of the object in question 7 after each vertical rotation about the horizontal axis shown.

a) 90° toward you

b) 180°

9. Here is an object made of linking cubes.



a) Predict the front, top, and side views of the object after each vertical rotation about the axis shown.

b) Build the object. Draw the views after each rotation to verify your predictions.

  i) 90° away from you
  ii) 180°
  iii) 270° away from you

10. **Assessment Focus** Use 5 linking cubes. Build an object.

a) Draw the front, top, and side views of the object.

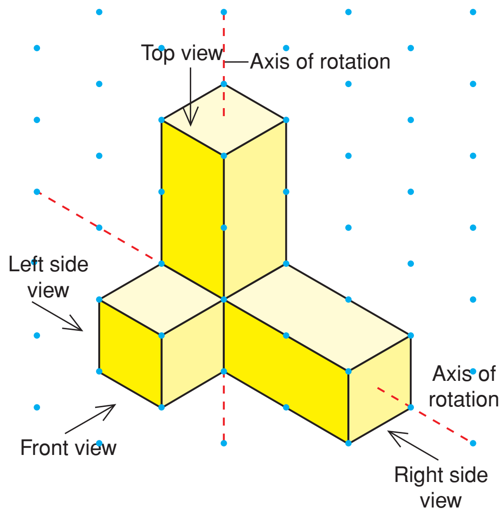
b) Choose a horizontal rotation and a vertical axis. Rotate the object. Draw the new front, top, and side views.

c) Describe a different rotation that will have the same views as the ones you drew in part b.

d) Choose a vertical rotation and a horizontal axis. Rotate the object. Draw the new front, top, and side views.

e) Describe a different rotation that will have the same views as the ones you drew in part d.

11. **Take It Further** Here is an isometric drawing of an object. The object is rotated horizontally 180° about the vertical axis shown. Draw the new front, top, and side views of the object.



12. **Take It Further** Use the object in question 11. Suppose the object is rotated horizontally 90° clockwise. Then the rotated object is rotated vertically 180°. Draw the new front, top, and side views of the object.

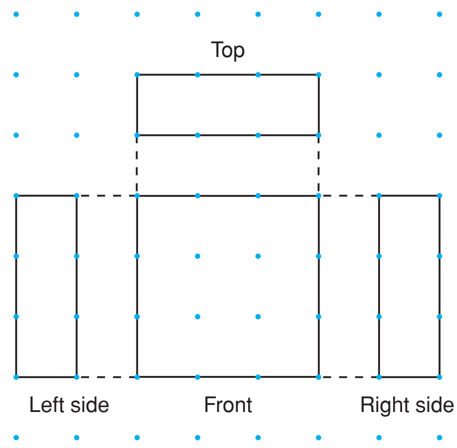
Reflect

For which type of rotation–horizontal or vertical–did you find the views easier to draw? Justify your answer.

8.3 Building Objects from Their Views

Focus Build an object given different views of the object.

Here are the views of an object made from linking cubes.

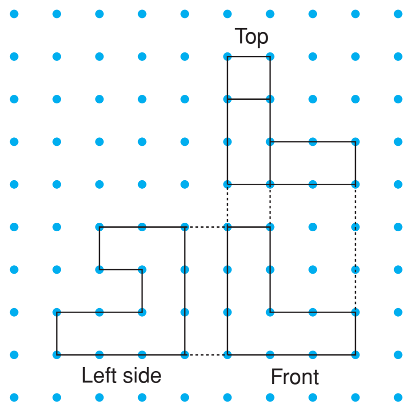


Which view or views can you use to find the height of the object?
The width of the object?
The length of the object?

Investigate

Work in groups of three.

Each student needs 16 linking cubes.
Each student chooses *one* of these views.



Use 8 cubes to build an object that matches the view you chose.
Use the other 8 cubes to build a different object that has the view you chose.

Reflect & Share

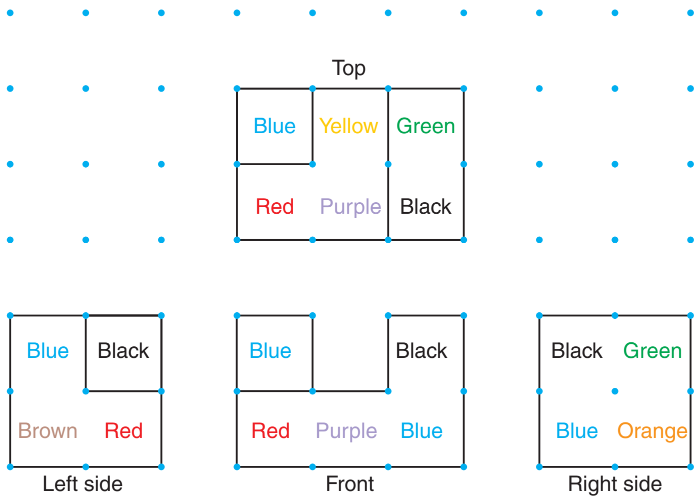
Compare your objects with those of the other members of your group.
Does any object match all 3 views?
If not, work together to build one that does.
What helped you decide the shape of the object?
Are any other views needed to help build the object? Explain.

Connect

Each view of an object provides information about the shape of the object. The front, top, and side views often provide enough information to build the object.
Remember that, when you look at views of an object, internal line segments show changes in depth.

Example 1

Here are the views of an object made with linking cubes.
Colours are labelled to show how the views relate.
Build the object.

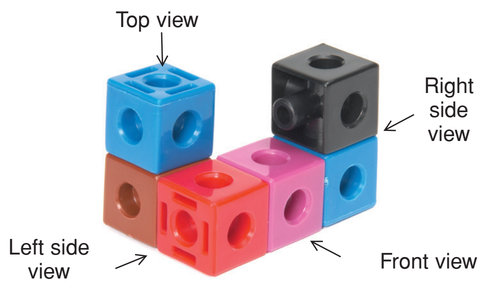


A Solution

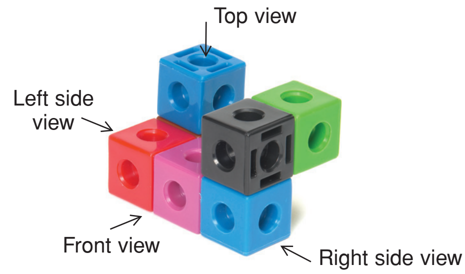
The left side view shows that the object is 2 cubes high and 2 cubes deep.
The black cube is behind or in front of the other cubes.

The front view shows that the object is 3 cubes long and 2 cubes high.
One blue cube is behind the other cubes.
The black cube is in front.

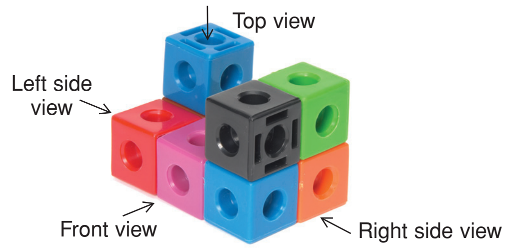
So, in the left side view, the black cube is behind the other cubes.
Use the left side and front views to build part of the object.



The top view shows that the red, purple, and yellow cubes are on the same level; and the blue, green, and black cubes are on a different level. Use the top view to continue building the object.

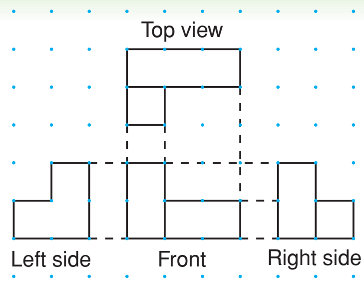


The right side view shows that the bottom right cube is orange.
Insert that cube to complete the object.



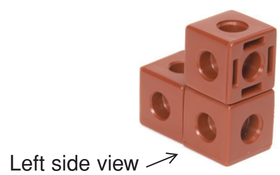
Example 2

Use linking cubes.
Build an object that has these views.

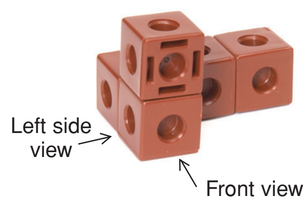


A Solution

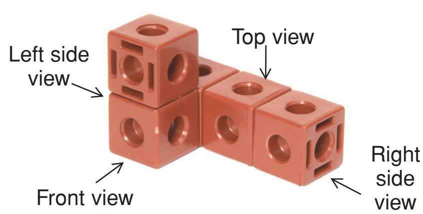
The left side view shows no change in depth.
So, use linking cubes to build the left side first.
The left side is shaped like a backward L.



Rotate the L shape horizontally 90° clockwise, about a vertical axis.
Compare the front views.
There is a change in depth of the object.
So, to match the given front view, add 2 cubes to the back of the object.



Check the top and right side views of the object.
The views match the given views.
So, the object is correct.



Discuss the ideas

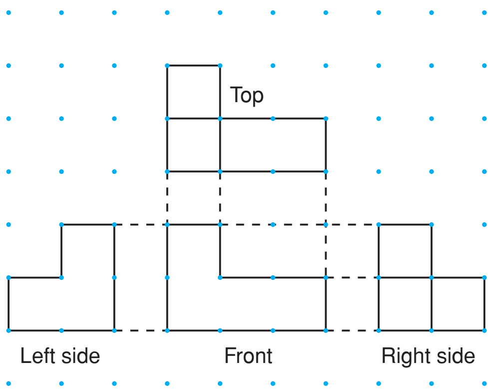
1. Build the object in *Example 1*.
   This time, start with the right side view.
   Which way was easier? Justify your choice.
2. When you build an object, how do you decide which view to start with? Explain.
3. Can you build more than one object for a given set of views? Justify your answer.

Practice

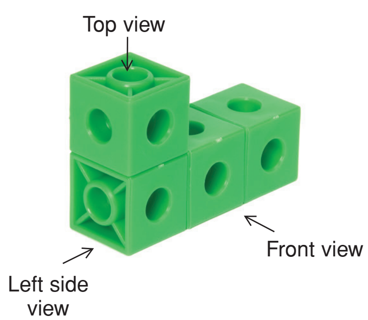
Check

You will need linking cubes.

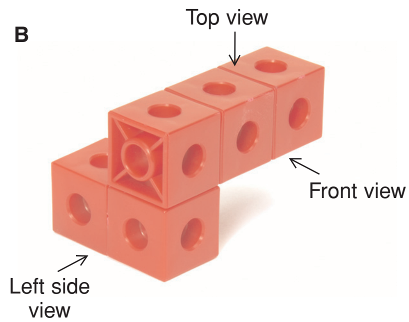
4. Which object has these views? How do you know?



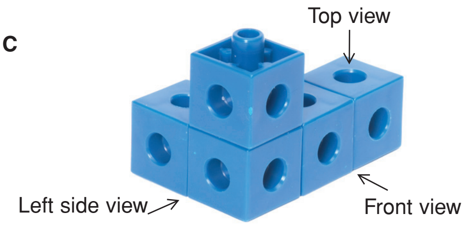
A



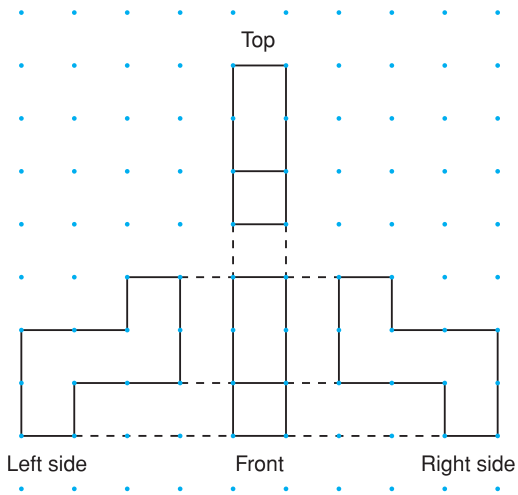
B



C

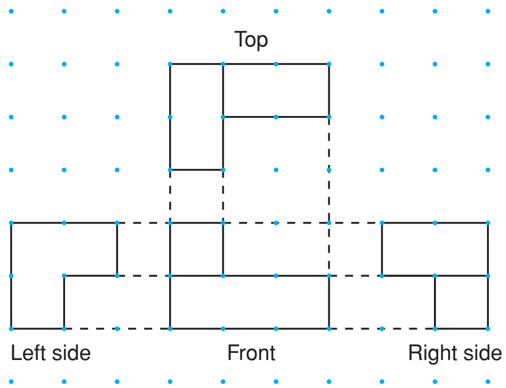


5. Use these views to build an object.

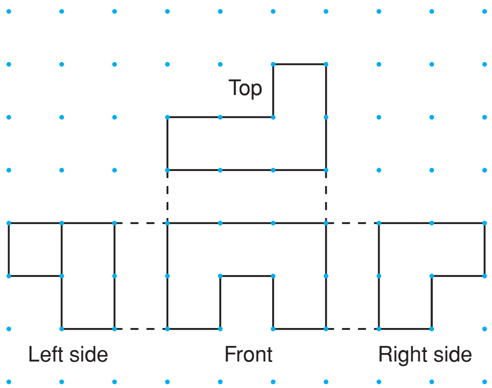


Apply

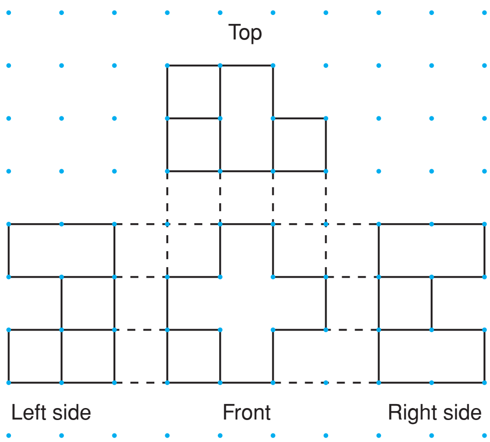
6. Use these views to build an object.



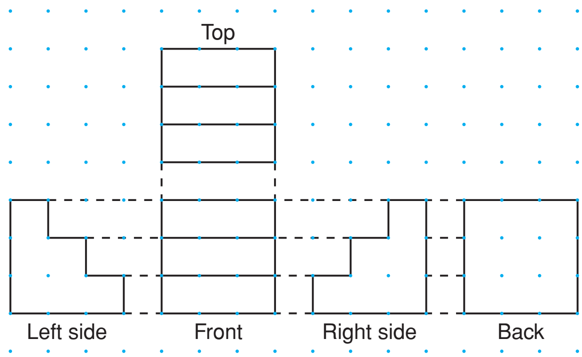
7. Use these views to build an object.



8. Use these views to build an object.



9. **Assessment Focus** For the set of views shown below:



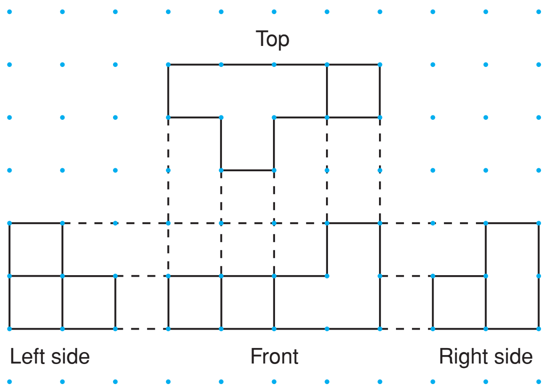
a) Use linking cubes to build the object.

b) Describe the object.

c) Suppose one side view had not been shown. Would you have been able to build the object? Explain.

d) Suppose the back view had not been shown. Could you have built a different object that matches the other views? If your answer is yes, build the object.

10. Use these views to build an object.



11. The front, top, and side views of a cube are the same. Build another object that has the front, top, and side views the same.

12. Use 6 linking cubes to build an object.

a) Draw the front, top, and side views of your object.

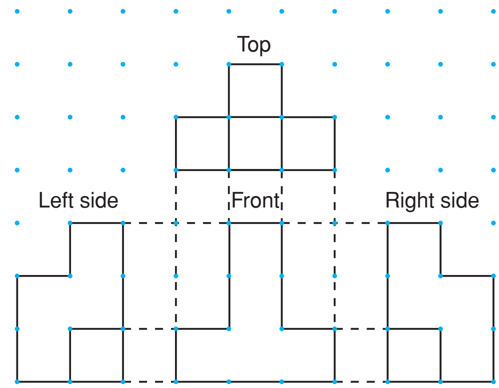
b) Show a classmate 2 views in part a. Have her use the views to build an object. Does her object match your object? Explain.

c) Show your classmate 3 views in part a. Have her use the views to build an object. Does her object match your object? Explain.

d) Show your classmate all the views in part a. Have her use the views to build an object. Does her object match your object? Explain.

e) Given all 4 views, is it possible that your classmate's object still does not match your object? Justify your answer.

13. Take It Further These four views show an object made with linking cubes.

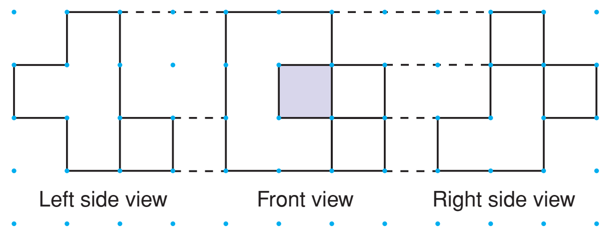


a) Build the object.

b) Assume each linking cube has edge length 2 cm. What are the surface area and volume of the object?

14. Take It Further

a) Use these views to build an object.



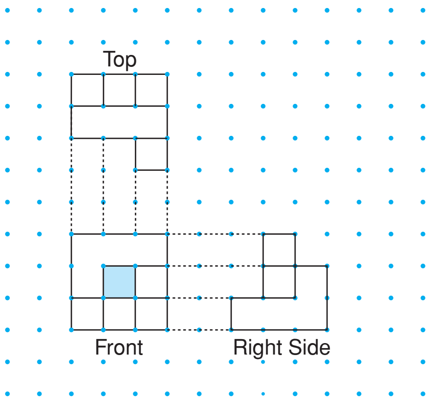
A shaded region has no cubes.

b) Draw the top view of the object.

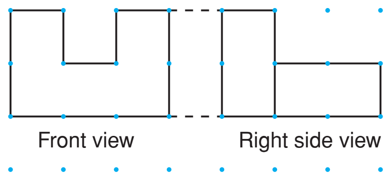
15. **Take It Further**

a) Use these views to build an object. A shaded region has no cubes.

b) Explain the steps you took to build the object.



16. **Take It Further** Use these views to build an object.



a) What is the greatest number of cubes you can use to make the object? How do you know?

b) What is the least number of cubes you can use to make the object? How do you know?

c) How many different possible objects can you make? Explain.

Reflect

Why do architects need to draw the front, top, back, and side views when planning the construction of a building?

Construct two different objects that have the same front, top, and side views.

Math Link

Art

An IMAX® 3-D film gives the illusion of three-dimensional depth. The scenes are filmed from two slightly different angles. One camera lens represents the right eye and the other lens represents the left eye. The specialized IMAX 3-D Projection system allows separate right- and left-eye images to be projected onto the screen alternately at a rate of 48 frames (pictures) per second, which your brain naturally fuses into one 3-D image.

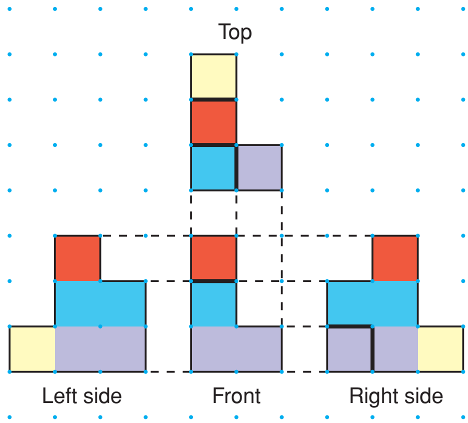
© IMAX Corporation

Using a Computer to Construct Objects from Their Views

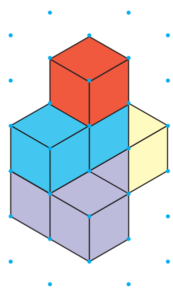
Focus Use a computer to build an object given its views.

A computer can be used to create an object from its views.
Use an interactive isometric drawing tool.

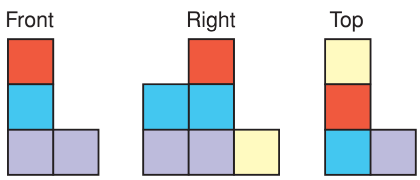
Here are the views of an object.



Should you need help at any time, use the Help or Instructions Menu.



- Use the views to build the object on screen.
  Use the cube tool on the screen to build the object.
  Use the paintbrush tool to colour the cubes as indicated.
- When your object is complete, use the View icon to show the views of your object.



If the views match the views given, your object is correct.
If the views do not match, make changes to your object.
Continue to check the views until they match exactly those given.

Check

1. Use the set of views in *Example 1* of Lesson 8.3, and the interactive isometric drawing tool to build an object.
   Check that the views of your object match the views.

Mid-Unit Review

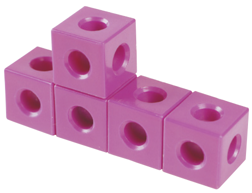
LESSON

8.1

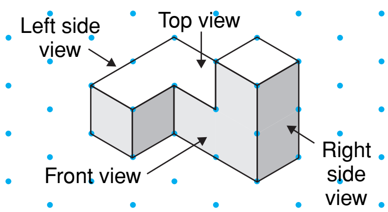
1. Sketch the front, top, and side views of this object. Label each view.

2. Use linking cubes. Make each object. Use square dot paper. Draw the front, top, and side views of each object.

a)



b)

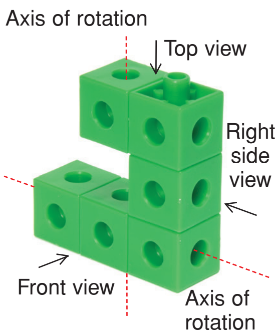


8.2

3. Here is an object made of linking cubes. Draw the front, top, and side views of the object after each rotation.

a) horizontally 90° clockwise

b) vertically 180°



4. Suppose the object in question 3 is rotated horizontally 270° counterclockwise. Predict the new top, front, and side views of the object. Explain how you found your answer.

8.3

5. Suppose you had to build an object with linking cubes. Which would you prefer to build from?

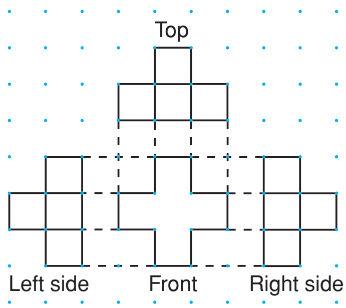
a) a drawing of the object on isometric dot paper

b) the different views of the object on square dot paper

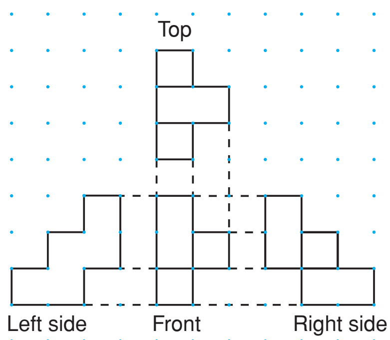
Justify your choice.

6. Use linking cubes. Use each set of views to build an object.

a)



b)



8.4 Identifying Transformations

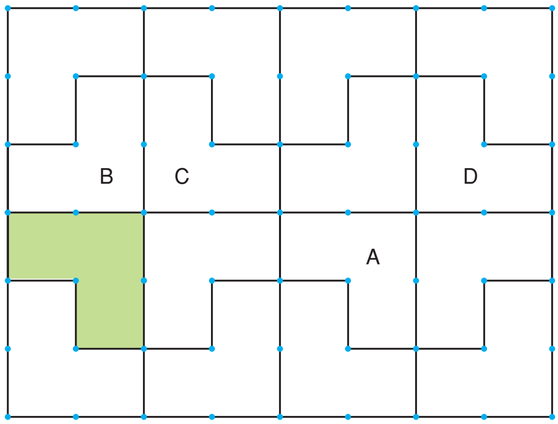
Focus Recognize transformation images.

Look around the classroom.
What transformations do you see?
How did you identify each transformation?

Investigate

Work with a partner.
Your teacher will give you a large copy of this design.

Tia used this design when she laid interlocking paving stones in her driveway.



To create the design, Tia translated, rotated, and reflected the shaded shape.
Each labelled shape is the image after a transformation.
Identify a transformation that produced each image.
Explain how you know.

Reflect & Share

Discuss your strategies for identifying each transformation with another pair of classmates.
How does the image relate to the original shape for each transformation?

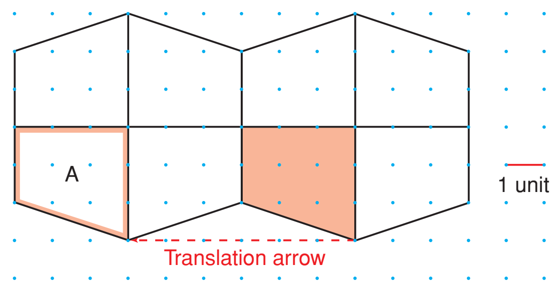
- a reflection
- a translation
- a rotation

Connect

Here is a design that shows 3 different transformations.

Translation

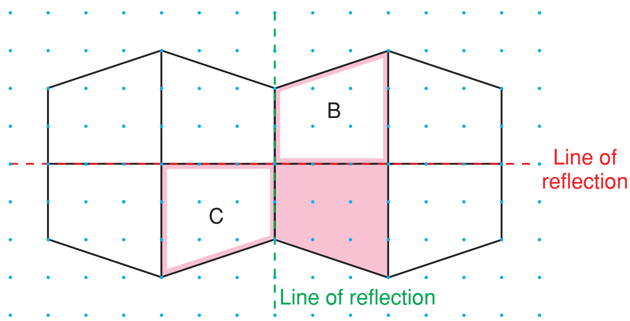
The shaded shape is translated 6 units left. Its translation image is Shape A.
The translation arrow shows the movement in a straight line.
The translation image and the shaded shape are congruent and have the same orientation.



Reflection

The shaded shape is reflected in the red line of reflection.
Its reflection image is Shape B.

The shaded shape is reflected in the green line of reflection.
Its reflection image is Shape C.

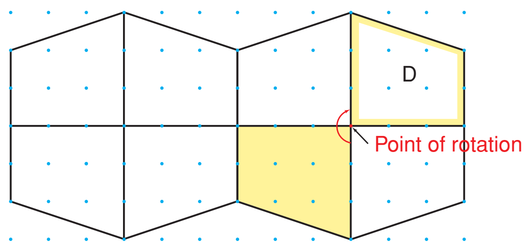


The shaded shape and each reflection image have opposite orientations.
Each reflection image and the shaded shape are congruent.

Rotation

The shaded shape is rotated 180° clockwise about the point of rotation.
The rotation image is Shape D.

We get the same image if the shaded shape is rotated 180° counterclockwise about the point of rotation.
The rotation image and the shaded shape are congruent and have the same orientation.

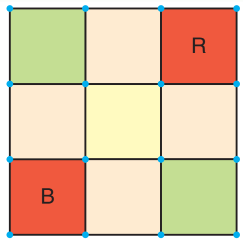


Under any transformation, the original shape and its image are always congruent.

Example 1

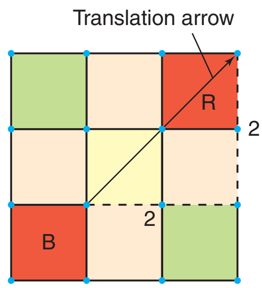
Look at this design of squares.
Describe each transformation.

a) a translation for which Square R is an image of Square B
b) a reflection for which Square R is an image of Square B

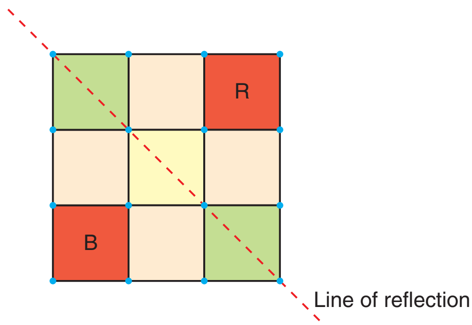


A Solution

a) Square R is the image of Square B after a translation 2 units right and 2 units up. The translation arrow shows the movement.



b) Square R is the image of Square B after a reflection in the slanted line. Use a Mira to verify the image.



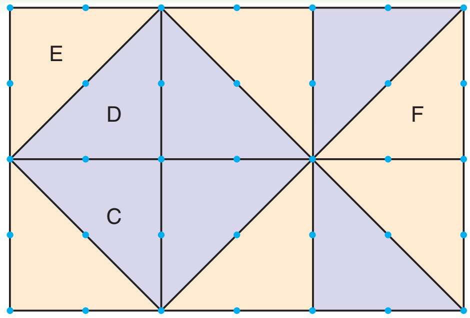
Example 1 shows that an image may be the result of more than one type of transformation.

Example 2

Look at this design of triangles.
Describe each rotation.

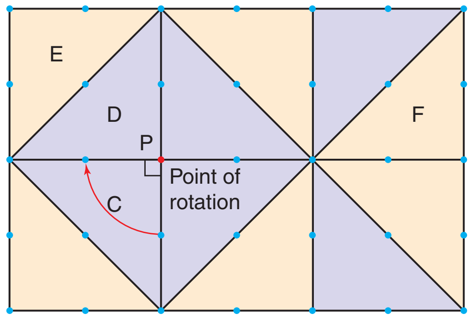
a) a rotation for which Triangle D is an image of Triangle C

b) a rotation for which Triangle F is an image of Triangle E

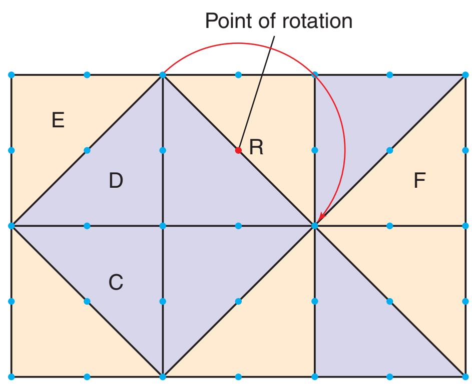


A Solution

a) Triangle D is the image of Triangle C after a rotation of 90° clockwise about P. P is a vertex the two triangles share. The same image is also the result of a rotation of 270° counterclockwise about P.



b) Triangle F is the image of Triangle E after a rotation of 180° about R. The point of rotation, R, is *not* on the shape being rotated.



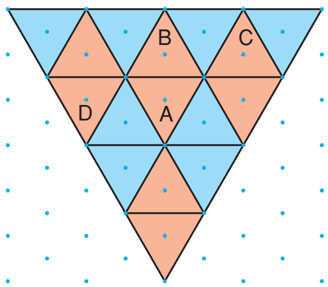
Discuss the ideas

1. In *Connect*, what does it mean when we say that a shape and its reflection image have opposite orientations?
2. In *Example 1b*, how is a point on the image related to a point on the original shape?
   How is the line segment that joins these points related to the line of reflection?
3. In *Example 1*, identify another transformation for which Square R is an image of Square B.
4. Can you change the size of a shape by translating, reflecting, or rotating it? Justify your answer.

Practice

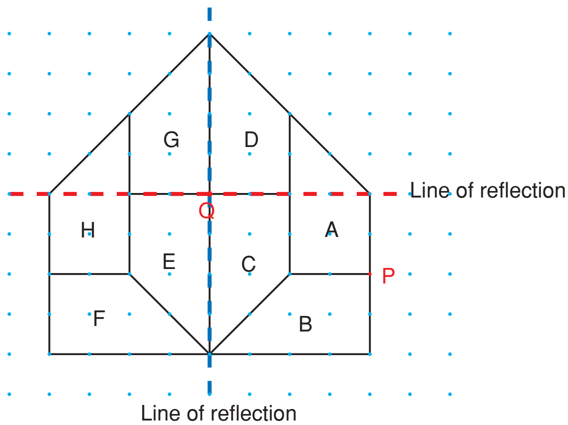
Check

5. In the design below, identify each transformation.



a) Shape B is the image of Shape A.
b) Shape C is the image of Shape A.
c) Shape D is the image of Shape A.
d) Shape C is the image of Shape B.

6. Use this design.

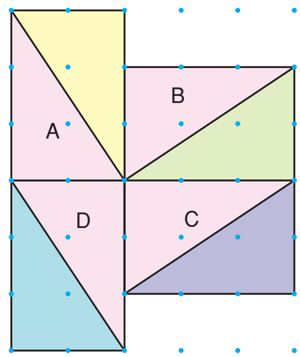


Match each transformation to a transformation image.

a) Rotate Shape A 90° counterclockwise about point P.
b) Reflect Shape C in the red line of reflection.
c) Translate Shape D 2 units right and 2 units down.
d) Rotate Shape G 180° about point Q.
e) Reflect Shape B in the blue line of reflection.

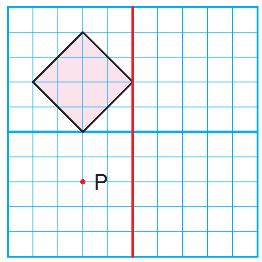
Apply

7. Identify each transformation.



a) Shape A is the image of Shape B.
b) Shape B is the image of Shape C.
c) Shape C is the image of Shape D.
d) Shape D is the image of Shape A.

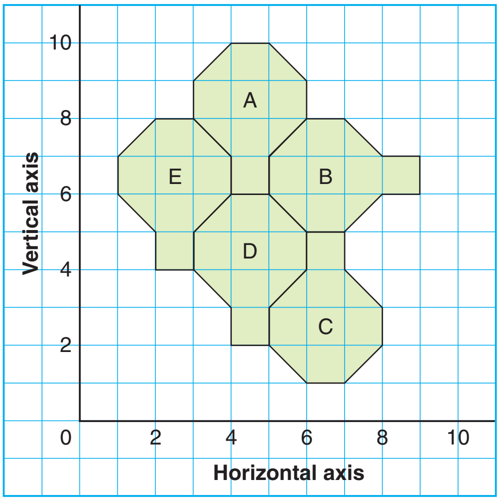
8. On grid paper, copy this square, the red and blue lines, and point P.



Draw the image of the original square after each transformation to create a design.

a) a translation 2 units right
b) a reflection in the red line
c) a rotation of 90° clockwise about P
d) a translation 2 units right and 4 units down
e) a reflection in the blue line

9. **Assessment Focus** How many different ways can each shape be described as a transformation of another shape? Explain.



10. **Take It Further** Use grid paper. In each case, describe the shape you drew.

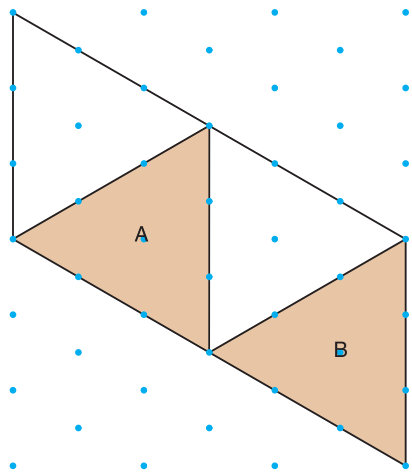
a) Draw a shape for which a translation image is also a reflection image and a rotation image. Draw the translation image.

b) Draw a shape for which a translation image is also a reflection image, but *not* a rotation image. Draw the translation image.

c) Draw a shape for which a translation image is *not* a reflection image *nor* a rotation image. Draw the translation image.

11. **Take It Further** This is a photo of a silk-screen print, *Haida Frog*, by Northwest Coast artist Bill Reid. Describe as many transformations in the print as you can. Include at least one translation, one reflection, and one rotation.

12. **Take It Further** Describe Shape A as a transformation image of Shape B in as many different ways as possible.



Reflect

When you see a shape and its transformation image in a design, how do you identify the transformation? Use diagrams in your explanation.

8.5 Constructing Tessellations

Focus Construct and analyse tessellations.

One of the basic ideas of Geometry is that of a *plane*. A plane is a flat surface. It has the property that a line joining any two points lies completely on its surface. What does this make you visualize?

Investigate

Work with a partner.
You will need tracing paper, plain paper, and a ruler.

One partner draws a triangle on his paper.
The other partner draws a quadrilateral on her paper.

Trace your shape.
Use the tracing to cover the paper with copies of your shape.
Try to do this with no overlaps or gaps.

You can rotate or flip the shape to try to make it fit.

Reflect & Share

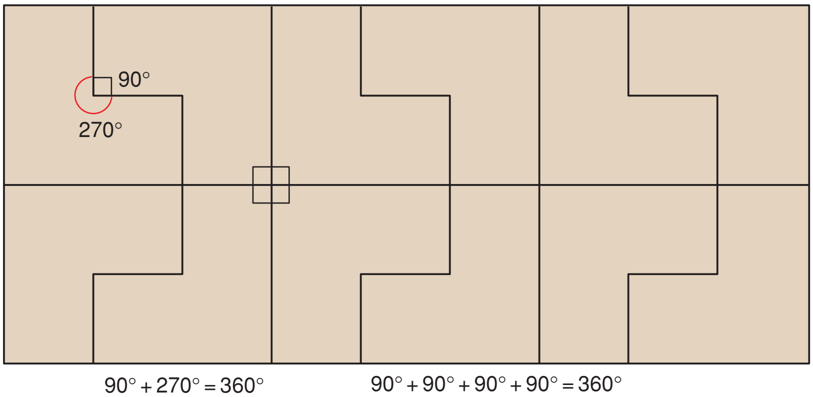
Compare your results with those of other classmates.
Can congruent copies of any triangle cover a plane with no overlaps or gaps? Justify your answer.
Can congruent copies of any quadrilateral cover a plane with no overlaps or gaps? Justify your answer.
What do you notice about the sum of the angles at a point where vertices meet?

Connect

When congruent copies of a shape cover a plane with no overlaps or gaps, we say the shape **tessellates**.
The design created is called a **tessellation**.

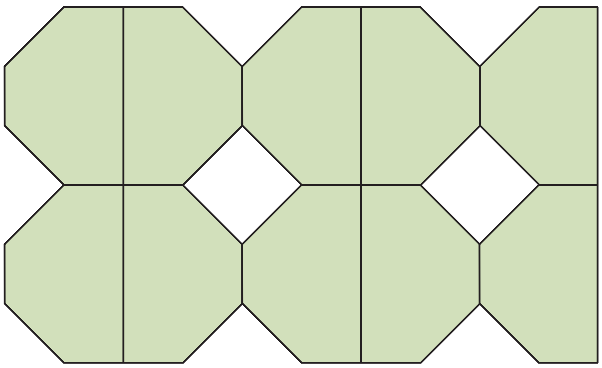
Not all polygons tessellate.

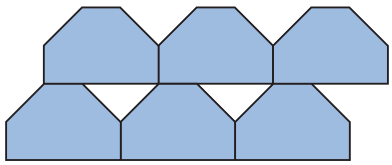
➤ This hexagon *does* tessellate.



➤ This hexagon *does not* tessellate.
Here are two different pictures to illustrate this.

There are gaps among the hexagons.



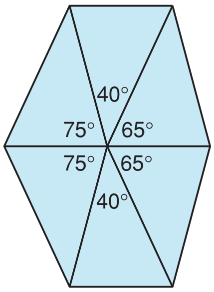


For copies of a polygon to tessellate, the sum of the angles at any point where vertices meet must be 360°. We say the *polygons surround a point*.

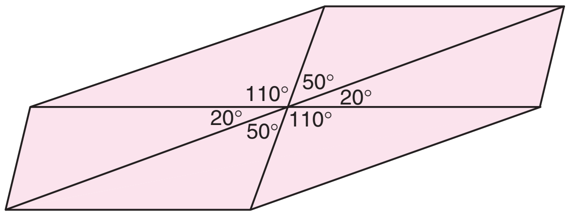
In *Investigate*, you found that triangles and quadrilaterals tessellate.

At any point where vertices meet, the sum of the angle measures is 360°.

Acute triangle



Obtuse triangle

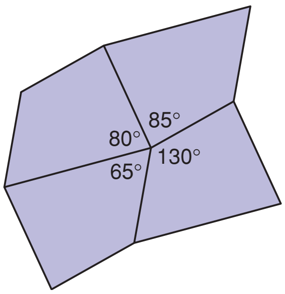


Six congruent triangles surround a point.

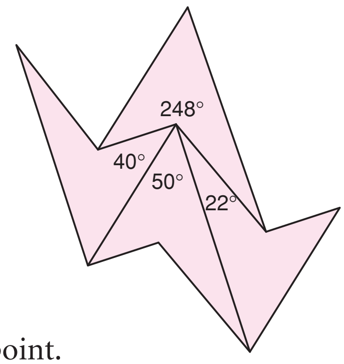
At each point:
$75° + 40° + 65° + 65° + 40° + 75°$
$= 360°$

At each point:
$20° + 50° + 110° + 20° + 50° + 110°$
$= 360°$

Convex quadrilateral



Concave quadrilateral



Four congruent quadrilaterals surround a point.

At each point:
$80° + 85° + 130° + 65° = 360°$

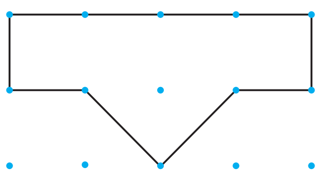
At each point:
$50° + 40° + 22° + 248° = 360°$

It is also possible for combinations of shapes to tessellate.

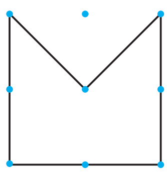
Example 1

Does each shape tessellate?
Justify your answer.

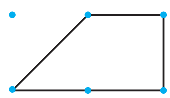
a)



b)



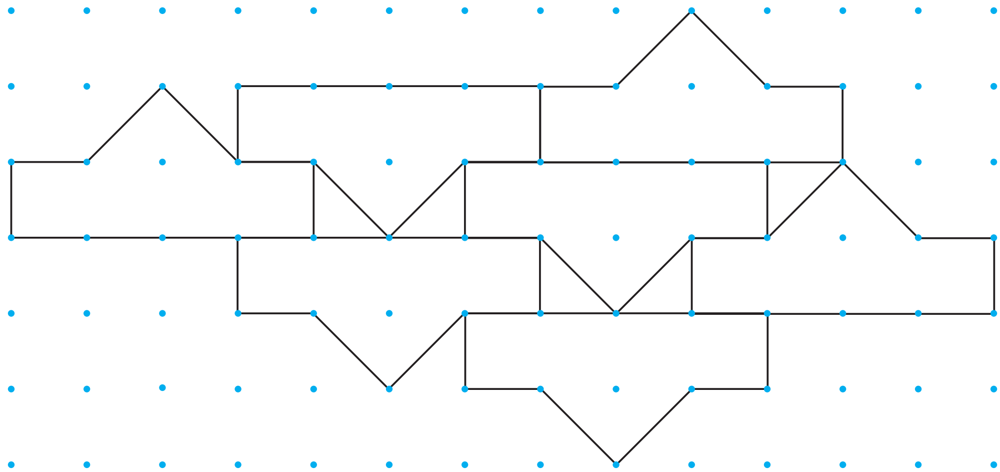
c)



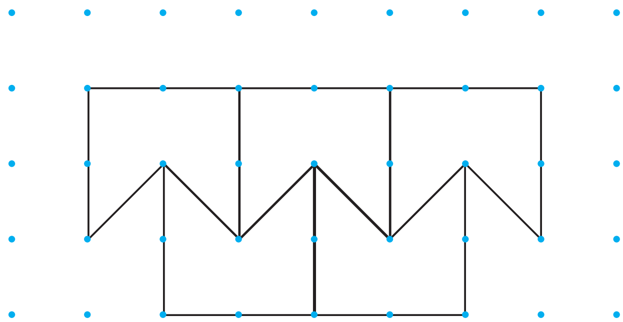
A Solution

Trace each shape.
Try to cover a page with the shape
so there are no gaps or overlaps.

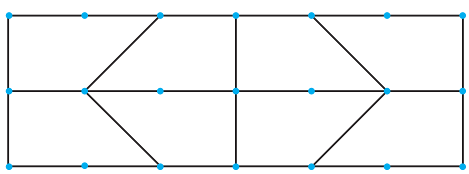
a) The shape does not tessellate.
There are gaps that are triangles.



b) The shape does tessellate.
There are no gaps or overlaps.



c) The shape does tessellate.
There are no gaps or overlaps.

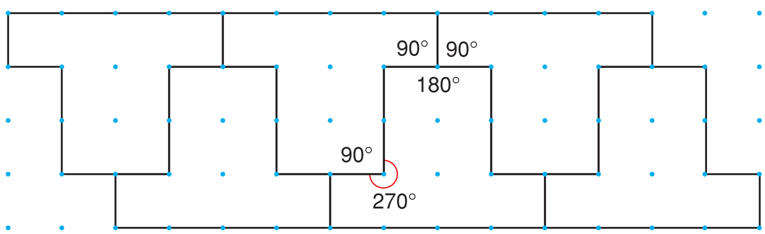


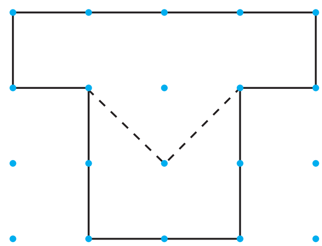
Example 2

Look at each shape in *Example 1* that does not tessellate.
Try to combine it with other shapes in *Example 1* so that the combined shape tessellates.
How many ways can you do this?

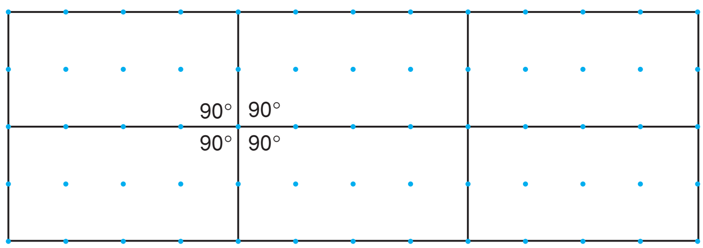
A Solution

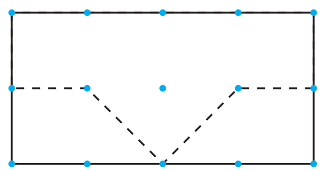
The shapes in parts a and b can be combined to make a T shape.
Tessellate with this new shape.





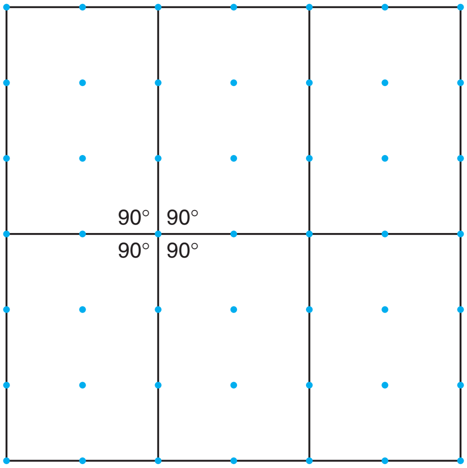
$90° + 90° + 180° = 360°$
$90° + 270° = 360°$
At each point where vertices meet, the sum of the angle measures is 360°.
So, the T shape tessellates.
The shape in part a and 2 of the shapes in part c can be combined to make a rectangle.

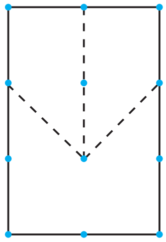




$90° + 90° + 90° + 90° = 360°$
At each point where vertices meet, the sum of the angle measures is 360°.
So, the rectangle tessellates.

The shape in part b and 2 of the shapes in part c can be combined to make a rectangle.





$90° + 90° + 90° + 90° = 360°$
At each point where vertices meet, the sum of the angle measures is 360°.
So, the rectangle tessellates.

Examples 1 and *2* illustrate that a shape that does *not* tessellate may be combined with one or more shapes to make a new shape that tessellates. This new shape is called a **composite shape**.
In the *Practice* questions, you will investigate to find which other shapes and combinations of shapes tessellate.

Discuss the ideas

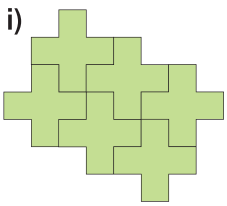
1. What is meant by "a shape tessellates"?
2. How can you tell that a shape does not tessellate?
3. Look around the classroom. Where do you see examples of tessellations? Describe each tessellation you see.
4. How do you know that all triangles tessellate?
5. How do you know that all quadrilaterals tessellate?

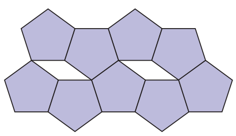
Practice

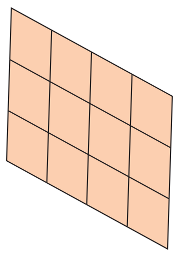
Check

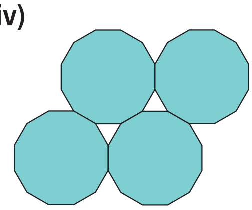
Keep all the tessellations you create for Lesson 8.6.

6. a) Which of these designs are tessellations? Justify your answer.

i)


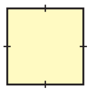
ii)


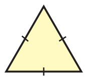
iii)


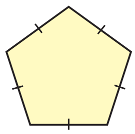
iv)


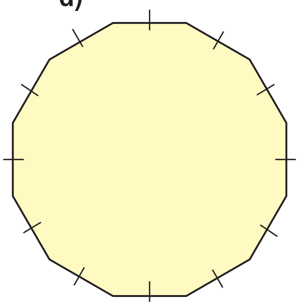
b) Which designs in part a are *not* tessellations? Justify your answer.

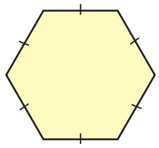
7. Use copies of each polygon. Does the polygon tessellate? If your answer is yes, create the tessellation. If your answer is no, explain how you know the shape does not tessellate.

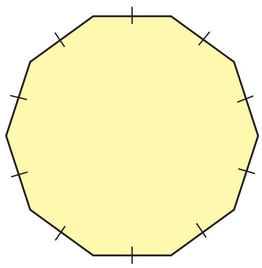
a) 

b) 

c) 

d) 

e) 

f) 

8. a) Use angle measures to justify your answers in question 7.

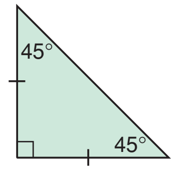
b) Which regular polygons tessellate? How do you know?

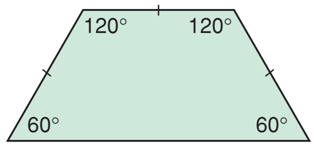
Apply

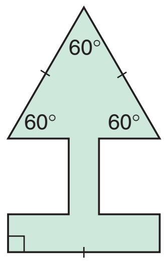
9. Use copies of the polygons in question 7. Find polygons that combine to make a composite shape that tessellates. How many different composite shapes can you find? Show your work.

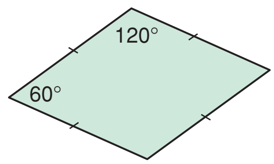
10. Use Pattern Blocks. Does the composite shape of a square and a regular hexagon tessellate? If your answer is yes, show the tessellation. If your answer is no, explain why not.

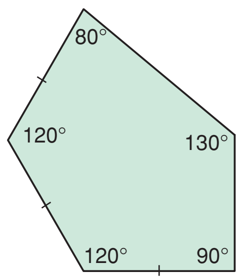
11. Use copies of each polygon. Which of these irregular polygons tessellate? Use angle measures to justify your answer.

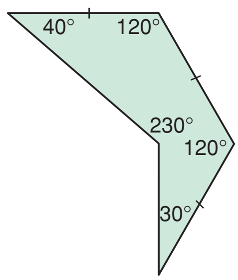
a) 

b) 

c) 

d) 

e) 

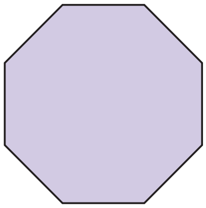
f) 

12. **Assessment Focus**

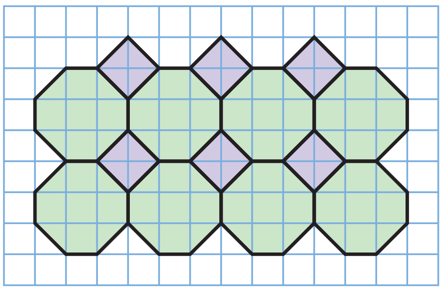
a) Which polygons in question 11 combine to make a composite shape that tessellates? Justify your answer in 2 ways.

b) How many different composite shapes can you find that tessellate? Show your work.

13. Here is a regular octagon. Trace this octagon. Does this octagon tessellate? Justify your answer.



14. Identify the two shapes that combine to make the composite shape in this tessellation. Explain how you know that this composite shape tessellates.



15. Where have you seen tessellations outside the classroom? What shapes were used in each tessellation?

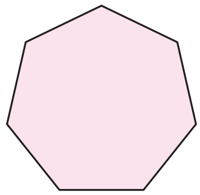
16. **Take It Further** In question 13, you discovered that a regular octagon does not tessellate. Use dot paper. Draw an octagon that tessellates. Explain why it tessellates.

17. **Take It Further** A 7-sided shape is called a heptagon.

a) Does a regular heptagon tessellate? Justify your answer.

b) If your answer to part a is yes, draw the tessellation.

c) If your answer to part a is no, draw a heptagon that tessellates.

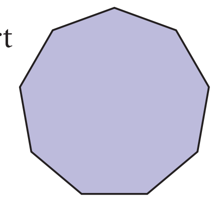


18. **Take It Further** A 9-sided shape is called a nonagon.

a) Does a regular nonagon tessellate? Justify your answer.

b) If your answer to part a is yes, draw the tessellation.

c) If your answer to part a is no, draw a nonagon that tessellates.



Reflect

How can you tell if a polygon or a composite shape tessellates? Use examples in your explanation.

Game

Target Tessellations

Players score points for each tessellation they create.
The first player to score 18 points wins.

YOU WILL NEED

Computer with geometry software, paper and pencil

NUMBER OF PLAYERS

2

GOAL OF THE GAME

To hit the target number exactly

HOW TO PLAY

1. Take turns to use the software to construct a shape that you think will tessellate.
   Use the software to check.

2. If the shape tessellates, the player scores 1 point for each side of the shape.
   For example, a 5-sided shape scores 5 points.
   If the shape does not tessellate, no points are scored.

3. Use the software to construct a different shape that you think will tessellate.
   The shape cannot have the same number of sides as the shape you used in *Step 1*.
   Use the software to check.

4. Play continues with a different shape each time.
   The first player to score 18 points wins.

What strategies did you use to hit the target number exactly?

TAKE IT FURTHER

Play the game again.
This time you choose the target number.
You cannot use shapes that have been used before.

8.6 Identifying Transformations in Tessellations

Focus Create and analyse tessellations using transformations.

In Lesson 8.5, you drew tessellations.
In this lesson, we will use transformations to describe some of these tessellations.

Investigate

Work on your own.
You will need isometric dot paper and Pattern Blocks.
Choose two Pattern Blocks that form a composite shape that tessellates.

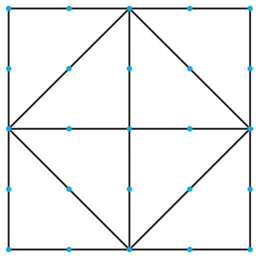
Make a tessellation to cover a plane.
Copy your tessellation onto dot paper.
Label each composite shape in your tessellation.
Explain the tessellation in terms of transformation images.
That is, how do you rotate, translate, or reflect each composite shape to create the tessellation? Write your instructions carefully.

Reflect & Share

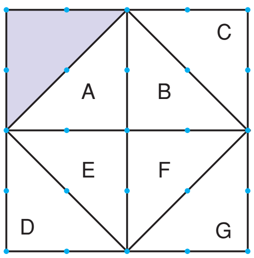
Trade instructions and the composite shape with a classmate.
Create your classmate's tessellation.
Check your tessellation with your classmate's tessellation.
How do they compare?

Connect

We can describe a tessellation in terms of transformations.

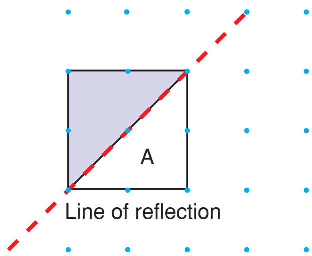


Label the shapes in the tessellation.

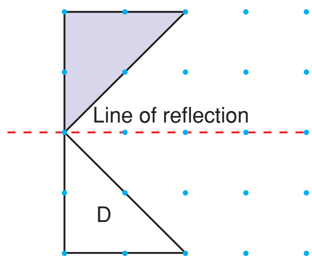


Start with the shaded shape.

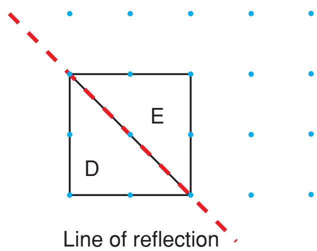
Step 1 To get Shape A, reflect the shaded shape in the line of reflection shown.



Step 2 To get Shape D, reflect the shaded shape in the line of reflection shown.



Step 3 To get Shape E, reflect Shape D in the line of reflection shown.



Repeat similar reflections to get shapes B, C, F, and G.

Under each transformation, the area of the shape does not change. This is known as **conservation of area**. This means that all the triangles in the tessellation have the same area.

Sometimes, a tessellation may be described by more than one type of transformation.

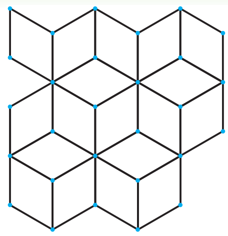
Example 1

In this tessellation, identify:

a) a translation

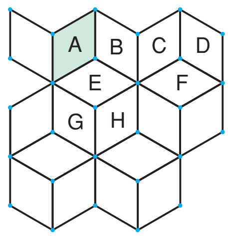
b) a reflection

c) a rotation

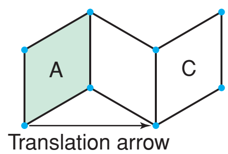


A Solution

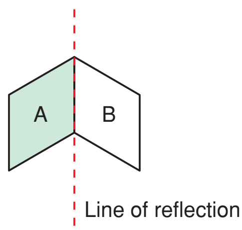
Label and shade Shape A as the original shape.
Label some more shapes.



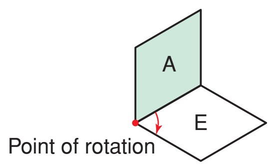
a) Shape C is a translation image of Shape A.
Shape A has been translated right to get Shape C.



b) Shape B is a reflection image of Shape A.
The line of reflection is the side the two shapes share.

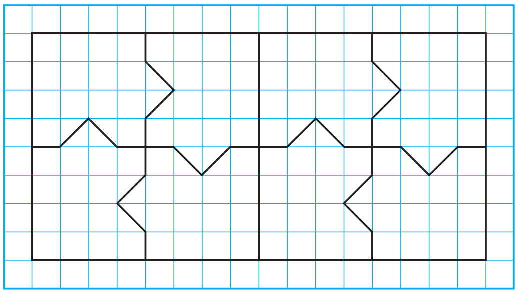


c) Shape E is a rotation image of Shape A.
The point of rotation is a vertex they share.
Shape A is rotated 60° clockwise or 300° counterclockwise to its image, Shape E.



Example 2

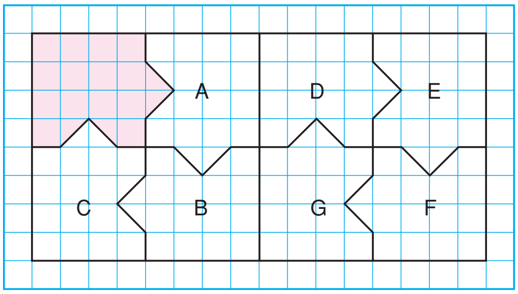
a) Identify a combination of transformations in this tessellation.



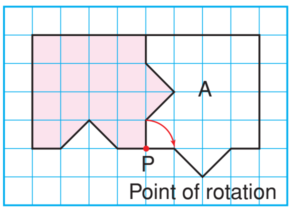
b) How do you know that the area of each shape is conserved?

A Solution

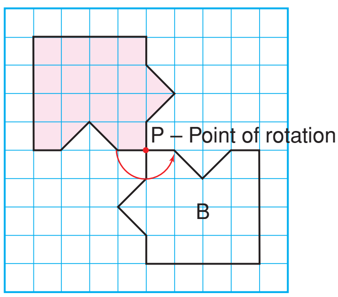
a) Label the shapes in the tessellation.



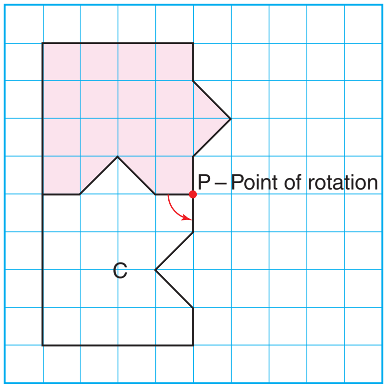
Start with the shaded shape.
To get Shape A, rotate the shaded shape 90° clockwise about P.



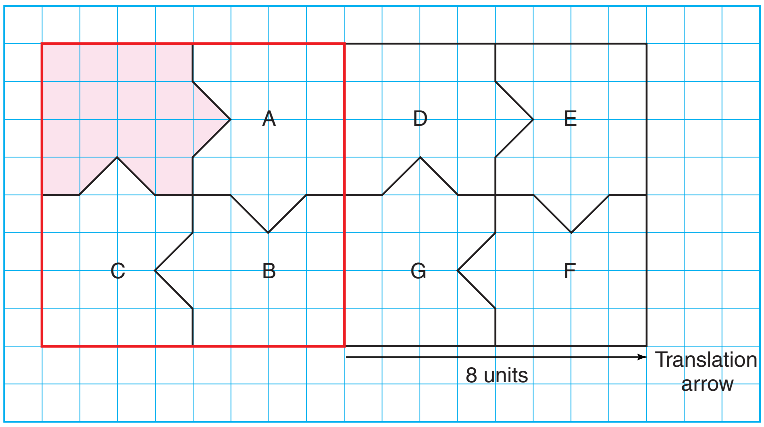
To get Shape B, rotate the shaded shape 180° about P.



To get Shape C, rotate the shaded shape 90° counterclockwise about P.



To get shapes D, E, F, and G, translate the square that contains the shaded shape, and shapes A, B, and C 8 units right.



b) Each image is congruent to the original shape.
Congruent shapes are identical.
So, the area of each image is equal to the area of the original shape.
This means that the area of each shape is conserved.

Discuss the ideas

1. Suppose the area of the shaded shape in *Example 1* is 8 cm². What is the area of Shape B? Shape E? How do you know?

2. In *Example 2*, identify a different combination of transformations in the tessellation.

Practice

Check

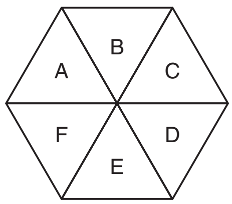
3. In each tessellation, Shape A is the original shape. In each tessellation, identify:

a) a translation

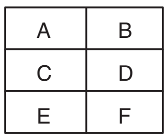
b) a reflection

c) a rotation

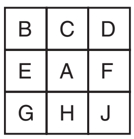
i)



ii)



iii)



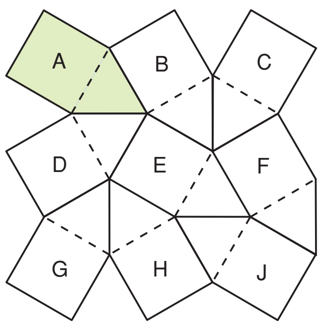
4. In each tessellation, Shape A is the original composite shape. In each tessellation, identify:

a) a translation

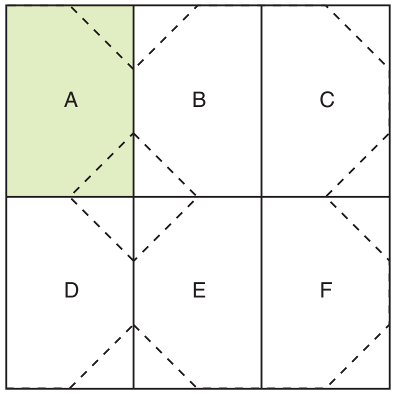
b) a reflection

c) a rotation

i)

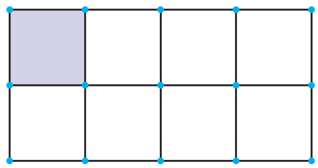


ii)

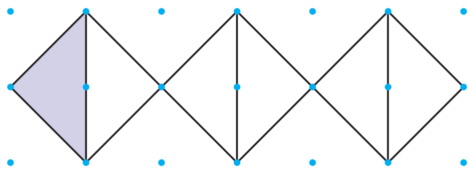


5. Here are three patterns. Describe the transformations that can be used to create each pattern. Start with the shaded shape.

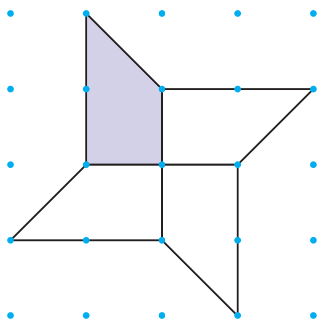
a)



b)

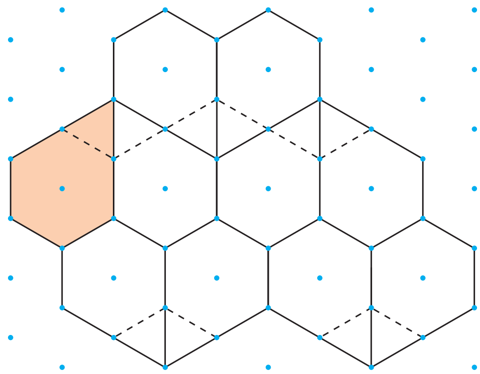


c)



Apply

6. Here is a tessellation one student drew for *Practice* question 9 in Lesson 8.5.

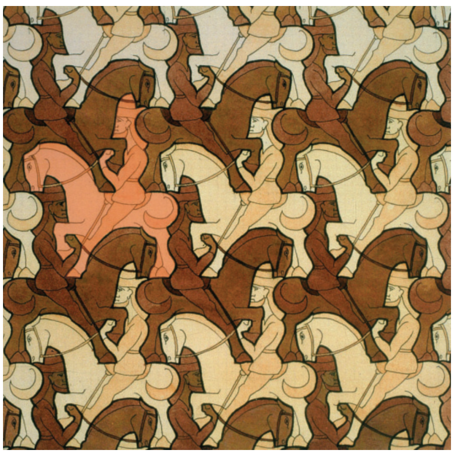


a) Describe the tessellation in terms of translations.

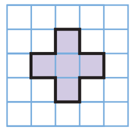
b) Describe the tessellation in terms of reflections.

c) Is area conserved when each shape is transformed? How do you know?

7. Look at the picture called *Knights on Horseback*, by M.C. Escher. Identify different transformations that may have been used to create the tessellation. Start with the red shape.



8. Use this shape and transformations to create a tessellation on grid paper.



Describe the tessellation in terms of transformations and conservation of area.

9. Here is a quilt design. Use a copy of the design.

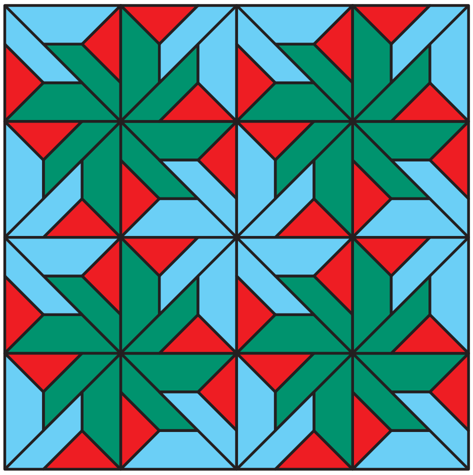
Find as many transformations in the design as you can. Ignore the different patterns on the material. Consider only the shapes.

10. Use transformations to create your own quilt design. Describe the transformations you used.

11. Choose 2 more tessellations you created in Lesson 8.5. Describe each tessellation in terms of transformations and conservation of area.

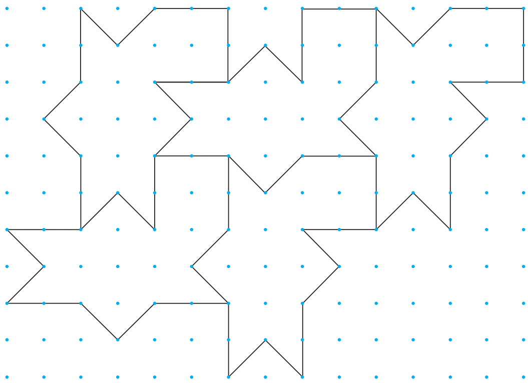
12. **Assessment Focus** Look at the tessellations you created in Lesson 8.5. Choose a tessellation that can be described in more than one way. Copy the tessellation onto dot paper. Label the shapes. Colour the tessellation. Describe the tessellation in terms of transformations and conservation of area. Describe the tessellation in as many ways as you can.

13. **Take It Further** Here is a flooring pattern. Use a copy of this pattern. Use transformations to describe the patterns in one square.



14. **Take It Further** The Alhambra is a walled city and fortress in Granada, Spain. It was built in the 14th century.

Here is part of one of its many tiling patterns. The pattern is a tessellation. Copy this tessellation onto dot paper.



a) Identify the composite shape that tessellates.

b) Continue the tessellation to cover the page.

c) Use transformations to describe the tessellation.

Reflect

When you use transformations to describe a tessellation, how do you decide which transformations to use? Include a tessellation in your explanation.

Using a Computer to Create Tessellations

Technology

Focus Use technology to create and analyse tessellations.

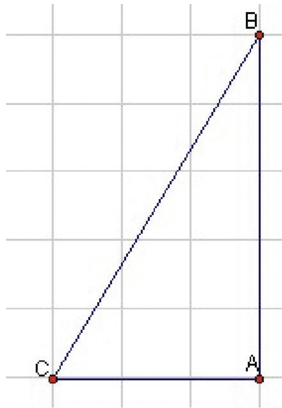
Geometry software can be used to create tessellations.
Use available geometry software.

Should you need help at any time, use the software's Help Menu.

Open a new sketch. Check that the distance units are centimetres. To display a "grid paper" screen, display a coordinate grid. Then hide the axes.

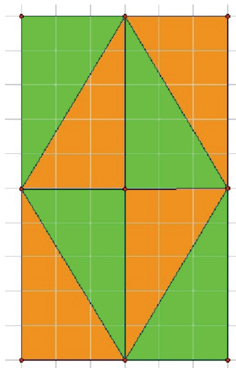
To create a tessellation:

Construct a triangle ABC.



Select the triangle.
Use the software to translate, reflect, or rotate the triangle.

Continue to transform the triangle or an image triangle to create a tessellation.
Colour your design to make it attractive.
Print your design.



Check

Construct two different shapes.
Use these shapes to make a composite shape.
Use any or all of the transformations you know to create a tessellation that covers the screen.
Colour your tessellation.
Describe your tessellation in terms of transformations.
What can you say about the area of each composite shape in your tessellation?

Explaining Your Answer

How many times have you been asked to "explain your answer" when you answer a math question?

The explanation for your answer shows your thinking.
It also helps you to review the solution when you study for a test.

Part of explaining your answer to a question is showing how you know your answer is correct.

Here is a problem.

What is the ones digit when 9 is multiplied by itself 500 times?

Compare these two solutions.

Solution 1

When 9 is multiplied by itself 500 times means...
9 x 9 x 9 x 9... x 9 (There are 500 9's.)
I know 9 x 9 = 81
9 x 9 x 9 = 729
9 x 9 x 9 x 9 = 6561 (Use a calculator.)
9 x 9 x 9 x 9 x 9 = 59049
9 x 9 x 9 x 9 x 9 x 9 = 531441
Every time 9 is multiplied an even number of times, the ones digit is 1.
Since 500 times is an even number of times, the ones digit will be 1.
The ones digit when 9 is multiplied by itself 500 times is 1.

Solution 2

Answer: 1

How does the solution on the right explain the answer?

Strategies for Success

Here are some things you can do to show how you know your answer is correct.

- Show all the steps in a logical order so that someone else can follow your thinking.
- Show all calculations.
- When a question involves one of the four operations, use estimation to check.
- Use a different strategy. For example: if the question involves subtraction, use addition to check.
- Verify the solution. For example: when solving an equation, substitute the solution into the original equation to check.
- Use thinking words and phrases such as:
  - because
  - so that means . . .
  - as a result
  - if you . . . then . . .
- Include labelled sketches, diagrams, or tables to help explain your answer.

Practice

Answer these questions.
Write a complete solution that explains your answer.

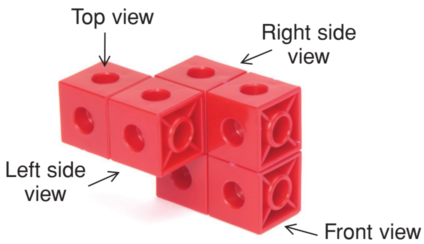
1. Hori is buying carpet for his living room. The living room is rectangular, with dimensions 4 m by 5 m. The regular price of the carpet is $9.99 per square metre. How much will Hori save when he buys the carpet on sale for 20% off?

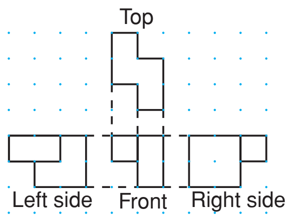
2. Suppose you are in charge of setting up the cafeteria for a graduation dinner. One hundred twenty-two people will attend.
The tables seat either 8 or 10 people. You do not want empty seats.
How many of each size table will you need to make sure everyone has a seat?
List all possible combinations.

Unit Review

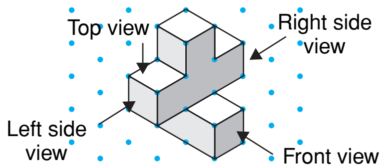
What Do I Need to Know?

☑ Each view of an object provides information about the shape of the object.

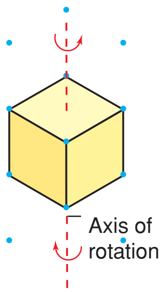




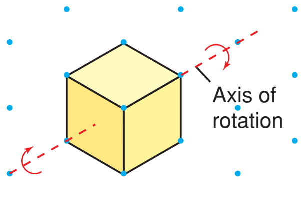
☑ An isometric drawing shows the three dimensions of an object.



☑ An object may be rotated: horizontally about a vertical axis

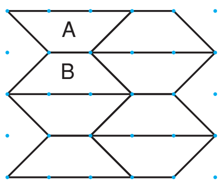


or vertically about a horizontal axis

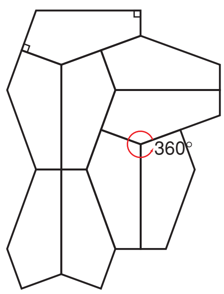


☑ Under a transformation, the area of a shape does not change. This is called *conservation of area*.

☑ When congruent copies of a shape cover a plane with no overlaps or gaps, we say the shape *tessellates*. The design created is called a *tessellation*. The tessellation can be described using transformations.



For a polygon or a composite polygon to tessellate, the sum of the angle measures where vertices meet must be 360°. The only regular polygons that tessellate are triangles, squares, and hexagons.

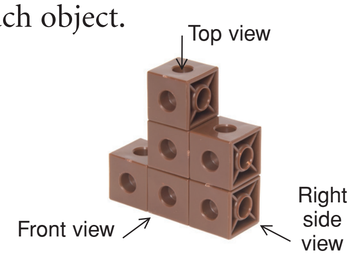


What Should I Be Able to Do?

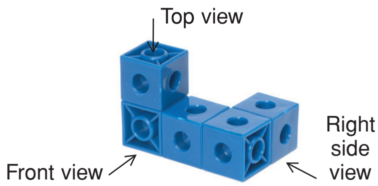
LESSON
8.1

1. Sketch the front, top, and side views of each object.

a)



b)



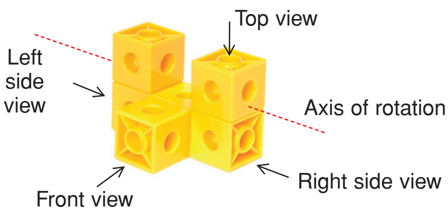
2. Use 3 linking cubes. Use square dot paper.

a) Build an object. Draw the front, top, and side views of the object.

b) How many different objects can you make with 3 linking cubes? Make each object. Draw its front, top, and side views.

8.2

3. Here is an object made from linking cubes.

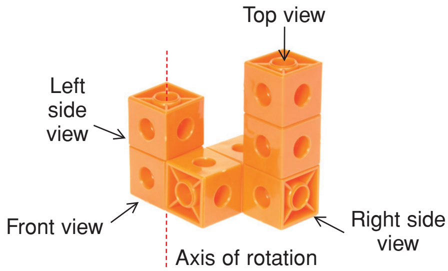


The object is rotated 180° vertically toward you.

a) Predict the front, top, and side views after the rotation. Sketch your predictions.

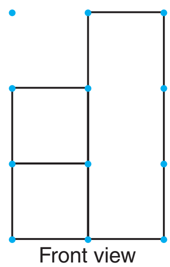
b) Build the object. Rotate the object to check your predictions. Draw each view after the rotation.

4. This object is rotated horizontally.

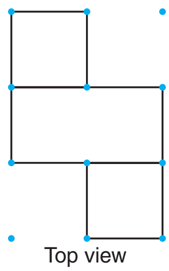


A new view is shown. Describe the rotation that produced each view.

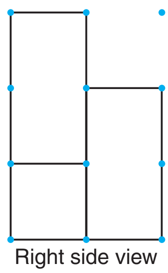
a) front view



b) top view



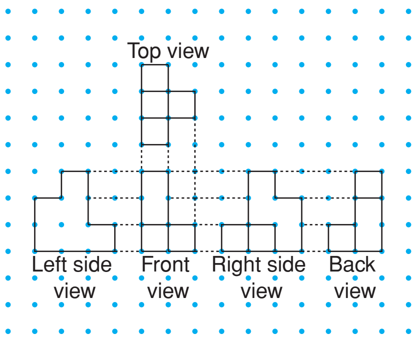
c) right side view



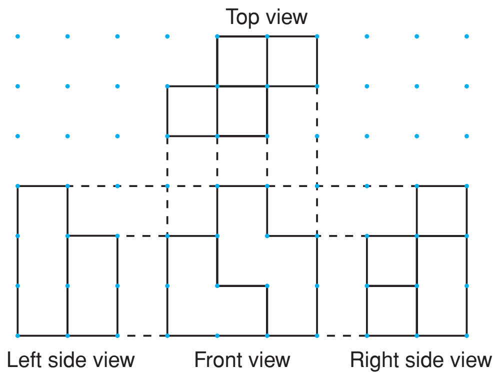
LESSON

8.3

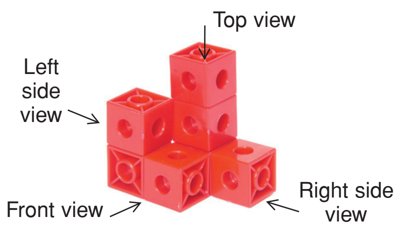
5. Use linking cubes. Use this set of views to build an object. How can you check that your object is correct?



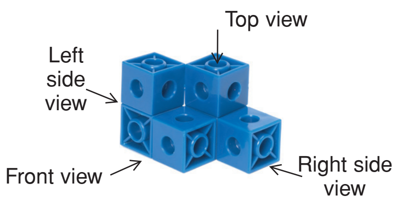
6. Which object below has these views? How do you know?



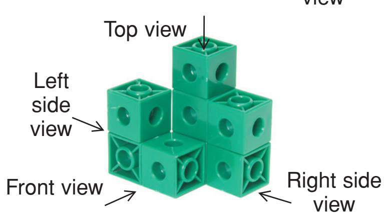
a)



b)



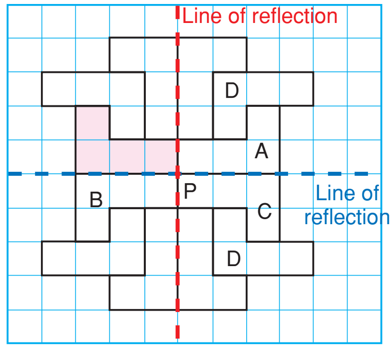
c)



7. Two objects in question 6 did not have views that matched those given. Draw the top, front, and side views of these objects.

8.4

8. Use a copy of this tessellation.



Match each transformation of the shaded shape to a transformation image.

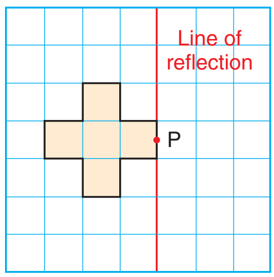
a) a translation 3 units down and 4 units right

b) a rotation of 180° about point P

c) a reflection in the red line

d) a reflection in the blue line

9. On grid paper, copy this shape, the red line, and point P.



Draw the image of the shape after each transformation. What do you notice?

a) a rotation of 180° about P

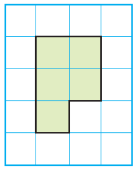
b) a reflection in the red line

c) a translation 3 units right

LESSON

8.5

10. Copy this shape on grid paper.



a) How many ways can you use the shape to tessellate? Show each way.

b) Use transformations to describe each tessellation in part a.

11. Is it possible to create a tessellation using each polygon below? If it is possible, use dot paper to draw the polygon and show the tessellation.
If it is not possible, explain why not.

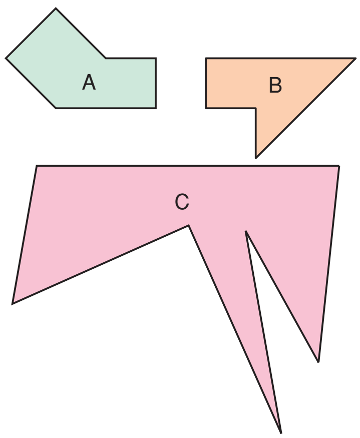
a) a regular octagon

b) a hexagon

c) a parallelogram

12. Use grid paper. Find a concave polygon that tessellates. Explain how it tessellates.

13. Use these shapes.



a) Try to tessellate with each shape. Identify the shapes that tessellate.

b) For each shape that does not tessellate:

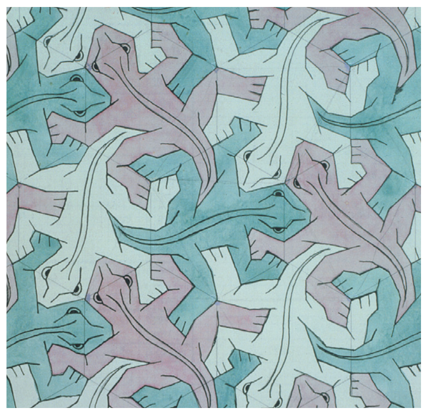
- Find another shape that forms a composite shape that tessellates.
- Tessellate with the composite shape.

8.6

14. Here is a quilt design.

Use as many different transformations as you can to describe the design. Ignore the different patterns, colours, and textures on the material. Consider only the shapes.

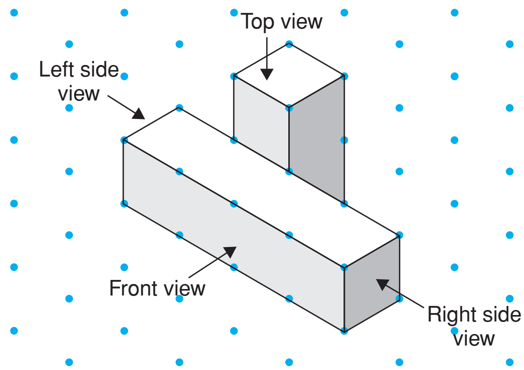
15. Look at the picture called *Reptiles*, by M.C. Escher. What transformations of the red reptile are needed to create this tessellation?



16. Draw two shapes that combine to make a composite shape that tessellates. Use the composite shape to make a tessellation. Describe the tessellation in terms of transformations and conservation of area.

Practice Test

1. Draw the front, top, and side views of this object.

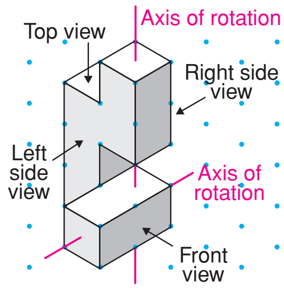


2. Use linking cubes to build the object at the right.

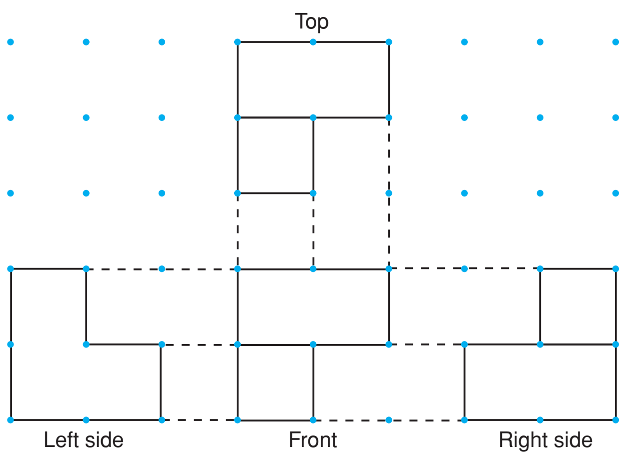
a) Draw the front, top, and side views of the object.

b) Rotate the object horizontally 90° clockwise.
Draw the new front, top, and side views of the object.

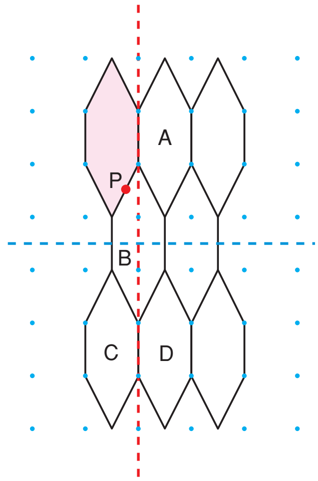
c) Rotate the object vertically 180° away from you.
Draw the new front, top, and side views of the object.



3. Use linking cubes.
Use these views to build an object.
Explain how you did this.

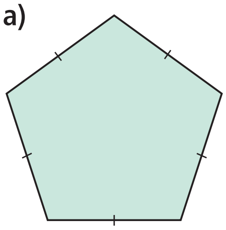


4. Use point P as a point of rotation.
Use the blue line and red line as lines of reflection.
Each of A, B, C, and D is a transformation image of the shaded shape. Identify each transformation.

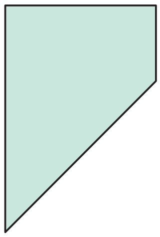


5. Use copies of each polygon.
Which of these polygons tessellate?
Use angle measures to justify your answer.

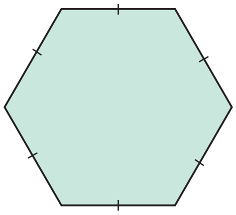
a)



b)



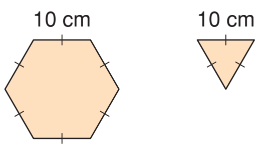
c)



6. Choose a polygon in question 5 that tessellates.
Create the tessellation.
Colour the tessellation.
Describe the tessellation in terms of transformations and conservation of area.
Try to describe the tessellation in two different ways.

7. Three students looked at a tessellation.
Igal used translations to describe the tessellation.
Shaian used rotations to describe the tessellation.
Cherie used reflections to describe the tessellation.
All three students were correct. What might the tessellation look like?
Draw a diagram to show your thinking.

8. Julie will use both of these tiles to cover her floor.
Use isometric dot paper.
Draw 2 different tessellations Julie could use.
For each tessellation, use transformations to explain how to create the tessellation using a composite shape.



Unit Problem Creating Tessellating Designs

M.C. Escher was a famous Dutch graphic artist.
He designed many different tessellations.

You will create two designs in the Escher style.
The first design is in the style of *Knights on Horseback*, on page 477.

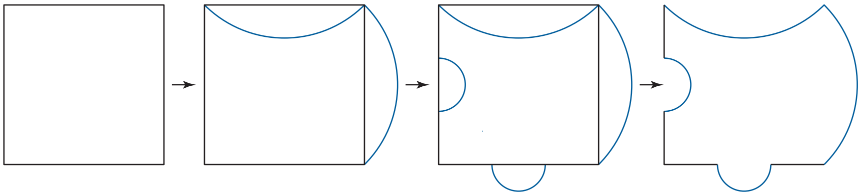
Part 1

Use square dot paper or grid paper.
Tessellate with a shape of your choice.
Sketch a design on one shape.
Repeat the sketch until every shape in the plane has the design.
Use transformations to describe how to create the tessellation beginning with one shape.

Part 2

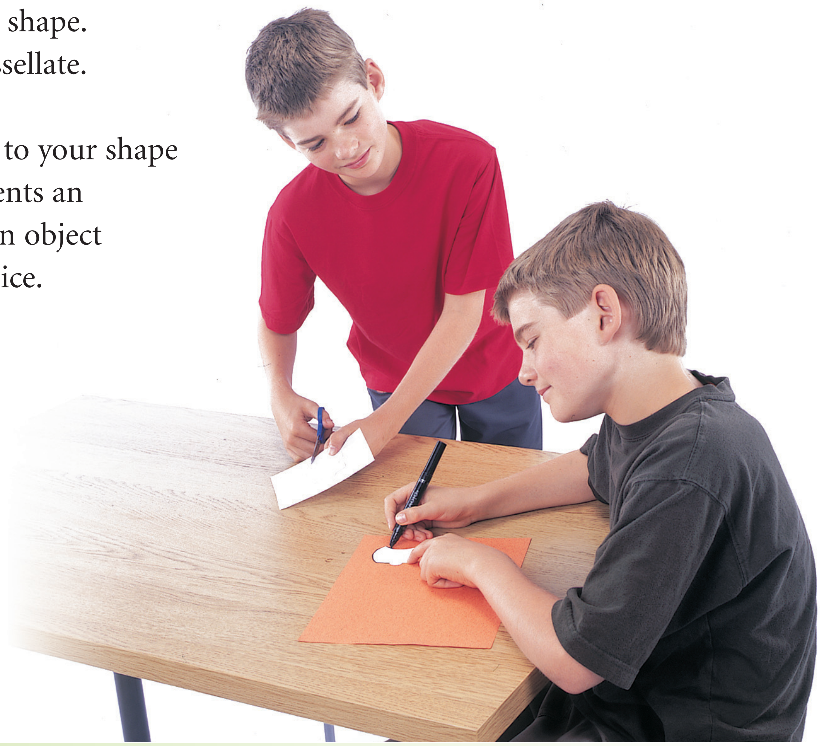
Instead, you could start with a rectangle, parallelogram, or regular hexagon.

Start with a square. Draw congruent curves on 2 sides.
A curve that goes "in" on one side must go "out" on the other side.
Draw different congruent curves on the other two sides.



Trace the new shape on cardboard.
Cut out the shape.
Use it to tessellate.

Add details to your shape so it represents an animal or an object of your choice.



Check List

Your work should show:

- ✓ the initial shape you created for each tessellation
- ✓ the tessellations you created
- ✓ how you used transformations to create the tessellations
- ✓ the correct use of mathematical language

Reflect on Your Learning

You have worked in three dimensions with objects and in two dimensions with shapes.
Which do you prefer? Give reasons for your choice.
Include a paragraph about what you have learned.

Investigation A Population Simulation

Materials:

- a paper cup
- 30 to 40 two-sided counters
- 0.5-cm grid paper

Work with a partner.

An animal population changes from year to year depending on the rates of birth and death, and on the movement of the animals.
A **simulation** is a model of a real situation. You will use a simulation to investigate how an animal population might decline.
As you complete this *Investigation*, include your completed table, graph, and written answers to the questions. You will hand these in.

➤ The counters are the animals in your population.
Count the number of animals.
Record that number for Year 0 in the *Population* table.

➤ Put the counters in the cup.
Choose which colour will be “face up.”
Pour the counters from the cup.
Counters that land face up represent animals that died or moved away during the first year. Set them aside.
Count the number of animals left.

Record that number as the population for Year 1.
Calculate and record the decrease in population.
Write the decrease as a fraction of the previous year's population.
Write the fraction as a percent.

➤ Place the counters representing live animals back in the cup.
Repeat the experiment for Year 2.
Record the data for Year 2 in the table.

➤ Continue the simulation. Record your data for up to 8 years, or until you run out of animals.

➤ Graph the population data.
Plot *Year* horizontally and *Number of Animals* vertically.
Explain your choice of graph.
What trends do you see in the graph?

Population									
Year	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Number of animals									
Decrease in population									
Change as a fraction									
Change as a percent									

➤ Describe the patterns you see in the table. Approximately what fraction and percent of the population remain from year to year?

➤ Use the patterns to predict the population for Year 9.

➤ How long does it take the population of animals to decrease to one-half its original size?

➤ What would happen to a population if there were no births to add to the population each year?

Take It Further

➤ Suppose you repeated this experiment, beginning with more animals. Would you see the same pattern? Explain.
Combine your data with data of other students to find out.

➤ Which environmental factors may cause an animal population to change?

Units 1–8 Cumulative Review

UNIT

1

1. Jasmine and Logan are swimming in a rectangular pool. The pool has dimensions 23 m by 9 m. They decide to race from one corner of the pool to the opposite corner. Jasmine swims diagonally across the pool. Logan is not a good swimmer so he swims along two sides of the pool.

a) Who swims farther?

b) How much farther does that person swim?

2. Is each statement true or false? Justify your answers.

a) $\sqrt{5} + \sqrt{2} = \sqrt{7}$

b) $\sqrt{46}$ is between 6 and 7, and closer to 7.

c) $\sqrt{36} + \sqrt{64} = 14$

2

3. Vernon used his allowance to buy lunch at school each day. Suppose Vernon spent \$4 each day for one week. How much more allowance money did he have one week ago?

4. Evaluate. Show all steps.

a) $(-4) \times (-3) + 3(-6)$

b) $\dfrac{-18}{(-4) + (-25) \div 5}$

c) $\dfrac{[11-(-5)] \div [2 \times (-2)]}{4 \div (-2)}$

d) $-12 + (-21) \div 3 + 3 \times 6$

3

5. Shantel worked a $4\frac{1}{2}$-h shift at the coffee shop. She spent $\frac{5}{6}$ of this time cleaning tables. How many hours did Shantel spend cleaning tables during her shift?

6. Find each quotient. Use number lines to illustrate the answers.

a) Austin worked for $2\frac{1}{3}$ h at the pet store and cleaned 4 fish tanks. How long did Austin spend cleaning each tank?

b) Riley has $5\frac{1}{4}$ cups of chocolate chips. She needs $\frac{3}{4}$ cup of chocolate chips to make one batch of cookies. How many batches of chocolate-chip cookies can Riley make?

4

7. A rectangular prism has three equal dimensions. Sketch the prism. Show the equal dimensions. What is another name for this prism?

8. Identify each object from the description of its net.

a) 2 congruent triangles and 3 rectangles

b) 2 congruent circles and 1 rectangle

c) 2 congruent hexagons and 6 congruent rectangles

9. Use 60 linking cubes. Assume the face of each cube has an area of 1 square unit.

a) Use all the cubes. Create a rectangular prism with the greatest possible surface area.

b) Use all the cubes. Create a rectangular prism with the least possible surface area.

Explain the strategy you used each time.

UNIT 5

10. Write each percent as a fraction and as a decimal.

a) 63.25% b) $1\frac{1}{8}\%$

c) 0.28% d) 0.7%

11. The cost of an adult bus ticket in 2006 was \$2.00. Because of the rising cost of fuel, the price was increased by $7\frac{1}{2}\%$ in 2007. The cost is expected to increase by about 12% in 2008.

a) Calculate the cost of a bus ticket in 2008. Describe the strategy you used.

b) Could you have found the cost by finding $119\frac{1}{2}\%$ of the cost of a ticket in 2006? Justify your answer.

12. There are 3 goldfish and 5 guppies in Tank A. There are 5 goldfish and 7 guppies in Tank B.

a) What is the ratio of goldfish to all the fish in each tank?

i) Tank A ii) Tank B

b) Write each ratio in part a as a fraction.

c) Suppose all the fish were moved into one large tank. What is the ratio of goldfish to all the fish in the large tank?

13. Scott delivers 14 newspapers in 10 min. This is 28% of his round.

a) How many more papers does Scott have to deliver?

b) Scott continues to deliver papers at the same rate. How long will it take him to deliver all his papers?

14. Express as a unit rate.

a) Paula picked 120 apples in 15 min.

b) Vince painted 6 fence posts in 30 min.

c) Jay ran 42 km in 3.5 h.

6

15. Expand.

a) $4(13 + 3d)$ b) $-7(5 - 6c)$

c) $8(-9d + 7)$ d) $6(8e - 1)$

16. Felix used algebra to solve this equation:

$3x + 5 = -7$

Look at Felix's work.

$$3x + 5 = -7$$
$$3x + 5 + 7 = -7 + 7$$
$$3x + 12 = 0$$
$$3x + 12 - 12 = 0 - 12$$
$$3x = -12$$
$$\frac{3x}{3} = \frac{-12}{3}$$
$$x = -4$$

a) Felix made an unnecessary step at the beginning of the solution. What was this step?

b) Did Felix get the correct solution? How can you find out?

c) Explain why the unnecessary step did or did not affect the solution.

17. Copy and complete each table of values.

a) $y = -3x$

x	−2	−1	0	1	2
y					

b) $y = -x + 3$

x	−2	−1	0	1	2
y					

UNIT

18. Troy is making baseball-cap organizers to sell at a craft sale. He needs 8 clothespins for each organizer. An equation for this relation is $c = 8n$, where n represents the number of organizers, and c represents the number of clothespins needed.

a) Use the equation to create a table of values.

b) Suppose Troy makes 12 organizers. How many clothespins does he need?

c) Suppose Troy has 144 clothespins. How many organizers can he make?

d) Construct a graph for the data in part a.

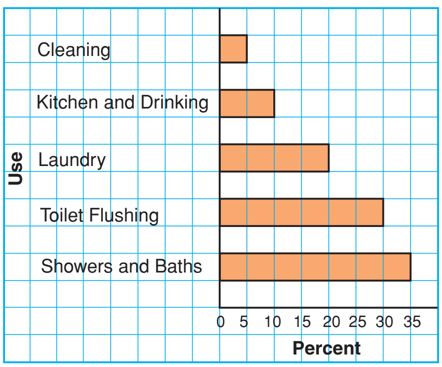
e) Describe the relationship between the variables in the graph.

f) Find the ordered pair on the graph that shows how many organizers can be made with 48 clothespins.

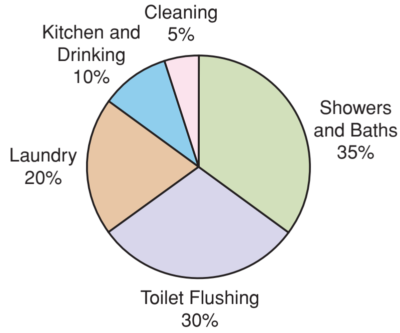
7

19. Each graph below shows how water is used in Canadian homes.

Water Use in Canadian Homes



Water Use in Canadian Homes



a) What are the advantages of each graph?

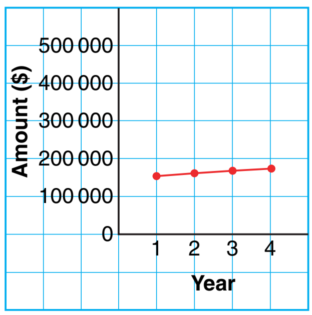
b) What are the disadvantages of each graph?

c) Which type of graph is more appropriate to display these data? Justify your choice.

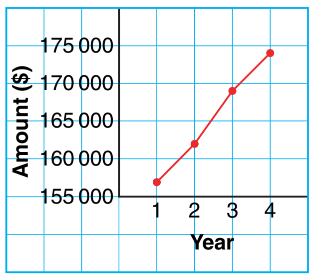
d) Could you use a line graph to display these data? Why or why not?

20. Two groups debated how much money the government was spending on social programs. Each group used the same data to draw a graph.

Spending on Social Programs



Spending on Social Programs



a) What impression does each graph give?

b) Describe how the graphs create a false impression.

c) Who might use each graph? Justify your answer.

UNIT

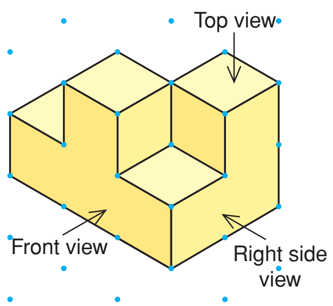
21. Gavin bought a bag of gift wrapping bows. The bag contains 4 red bows, 7 gold bows, 3 white bows, and 6 silver bows. Gavin takes one bow without looking, records the colour, then returns the bow to the bag.
The process is repeated.
Find the probability of each event:
a) 2 gold bows
b) a red bow, then a white bow
c) a gold bow, then a silver bow
d) not a gold bow, then a white bow

22. On a particular day in January, there is a 40% probability of snow in Whitehorse, a 55% probability of snow in Iqaluit, and a 35% probability of snow in Fort McMurray. What is the probability that it will snow in all 3 cities on that day?

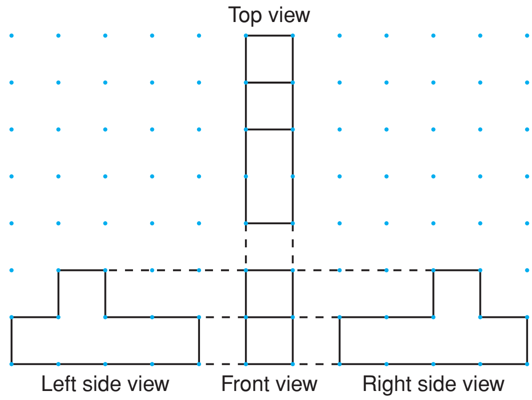
23. A true/false test has 6 questions. Jayla answered all the questions by guessing.
a) What is the probability that Jayla answered all the questions correctly?
b) Use a tree diagram to verify your answer in part a.

8

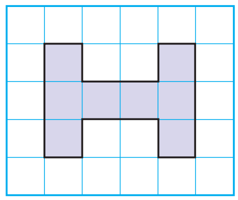
24. Sketch the top, front, and side views of this object.



25. Use linking cubes. Use this set of views to build an object.
How can you check that your object is correct?



26. Copy this shape. Use transformations to create a tessellation on grid paper.
Describe the tessellation in terms of transformations and conservation of area.



27. Here is a regular hexagon. Does this hexagon tessellate?
If your answer is yes, create the tessellation.
If your answer is no, explain how you know the shape does not tessellate. Then find a polygon that combines with the hexagon to make a composite shape that tessellates. Create the tessellation.