MATHPOWER™ Nine

Western Edition

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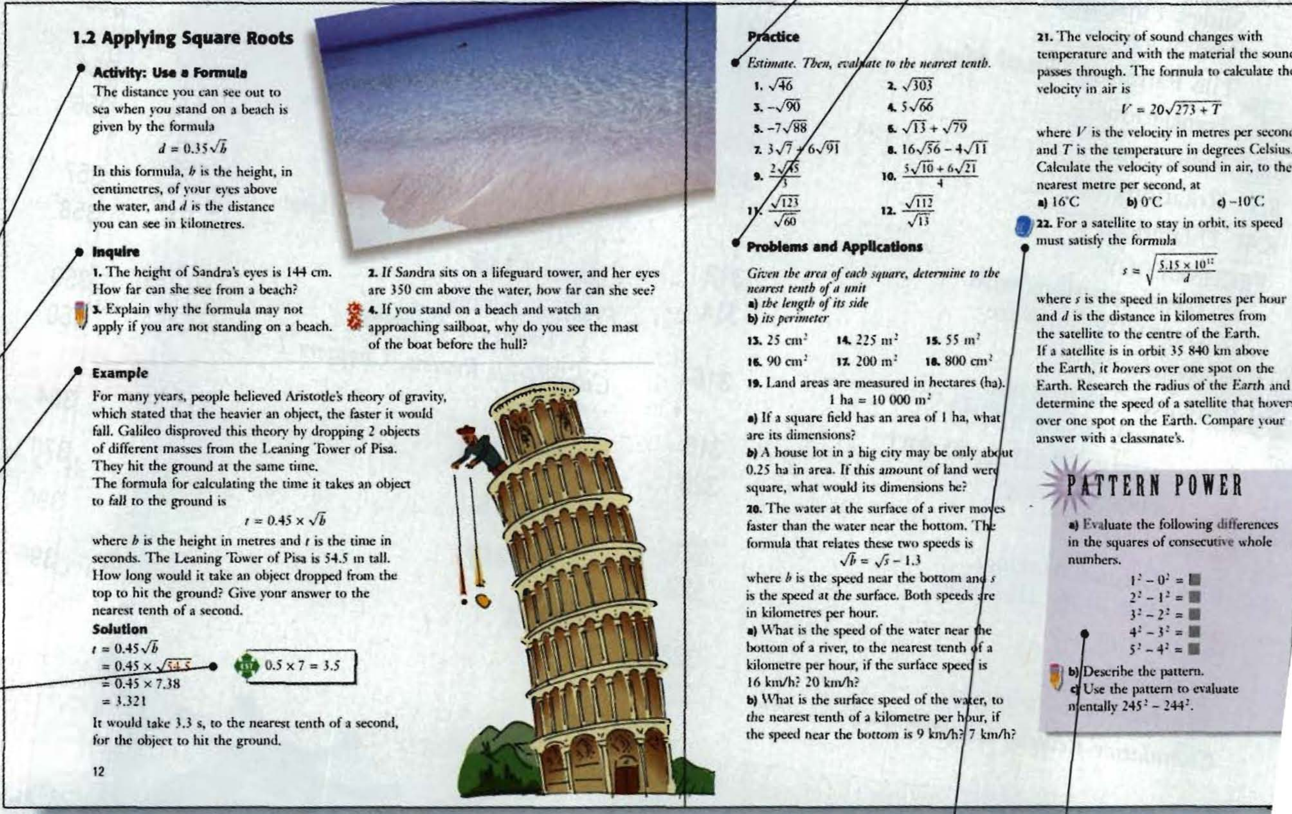
USING MATHPOWER™ 9

Each chapter contains a number of sections.
In a typical section, you find the following features.



1.2 Applying Square Roots

Activity: Use a Formula

The distance you can see out to sea when you stand on a beach is given by the formula

$$d = 0.35\sqrt{h}$$

In this formula, h is the height, in centimetres, of your eyes above the water, and d is the distance you can see in kilometres.

Inquire

1. The height of Sandra's eyes is 144 cm. How far can she see from a beach?
2. If Sandra sits on a lifeguard tower, and her eyes are 350 cm above the water, how far can she see?
3. Explain why the formula may not apply if you are not standing on a beach.
4. If you stand on a beach and watch an approaching sailboat, why do you see the mast of the boat before the hull?

Example

For many years, people believed Aristotle's theory of gravity, which stated that the heavier an object, the faster it would fall. Galileo disproved this theory by dropping 2 objects of different masses from the Leaning Tower of Pisa. They hit the ground at the same time.
The formula for calculating the time it takes an object to fall to the ground is

$$t = 0.45 \times \sqrt{h}$$

where h is the height in metres and t is the time in seconds. The Leaning Tower of Pisa is 54.5 m tall. How long would it take an object dropped from the top to hit the ground? Give your answer to the nearest tenth of a second.

Solution

$$\begin{aligned} t &= 0.45\sqrt{h} \\ &= 0.45 \times \sqrt{54.5} \\ &\doteq 0.45 \times 7.38 \\ &= 3.321 \end{aligned}$$

$\boxed{0.5 \times 7 = 3.5}$

It would take 3.3 s, to the nearest tenth of a second, for the object to hit the ground.

12

Practice

Estimate. Then, evaluate to the nearest tenth.

1. $\sqrt{46}$
2. $\sqrt{303}$
3. $-\sqrt{90}$
4. $5\sqrt{66}$
5. $-7\sqrt{88}$
6. $\sqrt{13} + \sqrt{79}$
7. $3\sqrt{7} + 6\sqrt{91}$
8. $16\sqrt{56} - 4\sqrt{11}$
9. $\frac{2\sqrt{45}}{3}$
10. $\frac{5\sqrt{10} + 6\sqrt{21}}{4}$
11. $\frac{\sqrt{123}}{\sqrt{60}}$
12. $\frac{\sqrt{112}}{\sqrt{13}}$

Problems and Applications

Given the area of each square, determine to the nearest tenth of a unit
a) *the length of its side*
b) *its perimeter*

13. 25 cm^2
14. 225 m^2
15. 55 m^2
16. 90 cm^2
17. 200 m^2
18. 800 cm^2
19. Land areas are measured in hectares (ha).

$$1 \text{ ha} = 10\ 000 \text{ m}^2$$

a) If a square field has an area of 1 ha, what are its dimensions?
b) A house lot in a big city may be only about 0.25 ha in area. If this amount of land were square, what would its dimensions be?

20. The water at the surface of a river moves faster than the water near the bottom. The formula that relates these two speeds is

$$\sqrt{b} = \sqrt{s} - 1.3$$

where b is the speed near the bottom and s is the speed at the surface. Both speeds are in kilometres per hour.
a) What is the speed of the water near the bottom of a river, to the nearest tenth of a kilometre per hour, if the surface speed is 16 km/h? 20 km/h?
b) What is the surface speed of the water, to the nearest tenth of a kilometre per hour, if the speed near the bottom is 9 km/h? 7 km/h?

21. The velocity of sound changes with temperature and with the material the soun passes through. The formula to calculate th velocity in air is

$$V = 20\sqrt{273 + T}$$

where V is the velocity in metres per secon and T is the temperature in degrees Celsius. Calculate the velocity of sound in air, to the nearest metre per second, at
a) 16°C **b)** 0°C **c)** −10°C

22. For a satellite to stay in orbit, its speed must satisfy the formula

$$s = \sqrt{\frac{5.15 \times 10^{12}}{d}}$$

where s is the speed in kilometres per hour and d is the distance in kilometres from the satellite to the centre of the Earth. If a satellite is in orbit 35 840 km above the Earth, it hovers over one spot on the Earth. Research the radius of the Earth and determine the speed of a satellite that hover over one spot on the Earth. Compare your answer with a classmate's.

PATTERN POWER

a) Evaluate the following differences in the squares of consecutive whole numbers.

$1^2 - 0^2 =$ ■
$2^2 - 1^2 =$ ■
$3^2 - 2^2 =$ ■
$4^2 - 3^2 =$ ■
$5^2 - 4^2 =$ ■

b) Describe the pattern.
c) Use the pattern to evaluate mentally $245^2 - 244^2$.

1
You start with an activity to generate your own learning.

2
The inquire questions help you learn from the activity.

3
The example shows you how to use what you have learned.

4
The EST logo in a solution indicates an estimate.

5
These questions let you practise what you have learned.

6
These questions let you apply and extend what you have learned.

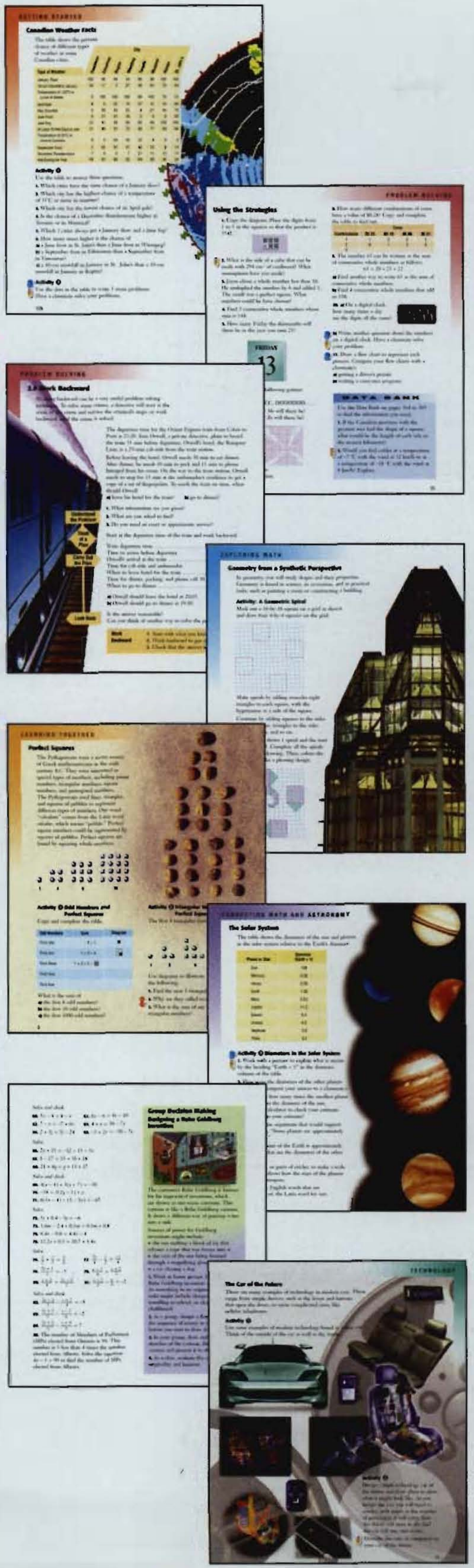
7
These logos indicate special kinds of questions.

The pencil logo tells you that you will be writing about math.

The critical thinking logo indicates that you will need to think carefully before you answer a question.

The working together logo shows you opportunities for working with a classmate or in a larger group.

8
The power questions are challenging and fun. They encourage you to reason mathematically.



In the first 4 chapters of the book, there are 12 PROBLEM SOLVING sections that help you to use different problem solving strategies.

At or near the end of each chapter is a page headed PROBLEM SOLVING: Using the Strategies. To solve the problems on this page, you use the strategies you have studied. In all chapters, this page ends with DATA BANK questions. To solve them, you need to look up information in the Data Bank on pages 364 to 369.

There are 14 EXPLORING MATH pages before chapter 1. The activities on these pages let you explore 14 mathematical standards that will be essential for citizens of the twenty-first century.

A GETTING STARTED section begins each chapter. This section reviews, in a fun way, what you should know before you work on the chapter. Each GETTING STARTED section includes a set of Mental Math questions.

In LEARNING TOGETHER sections, you learn by completing activities with your classmates.

The TECHNOLOGY sections show you some uses of technology and how you can apply technology to solve problems.

Some sections are headed CONNECTING MATH AND.... In these sections, you apply math to other subject areas, such as art, language, science, and the environment.

Each chapter includes sets of questions called REVIEW and CHAPTER CHECK, so that you can test your progress.

At the end of each review is a column headed GROUP DECISION MAKING. Here, you work with your classmates to research careers and do other projects.

Chapters 4 and 8 end with sets of questions headed CUMULATIVE REVIEW. These reviews cover the work you did in chapters 1–4 and 5–8. A CUMULATIVE REVIEW that covers chapters 1–9 follows chapter 9.

The GLOSSARY on pages 390 to 395 helps you to understand mathematical terms.

On pages 370 to 389 there are ANSWERS to most of the problems in this book.

A Problem Solving Model

The world is full of mathematical problems. A problem exists when you are presented with a situation and are unable to make sense of it. To solve any problem, you must make decisions.

MATHPOWER™ 9 will help you to become actively involved in problem solving by providing the experiences and strategies you need.

George Polya was one of the world's best problem solvers. The problem solving model used in this book has been adapted from a model developed by George Polya.

The problem solving model includes the following 4 stages.

Understand the Problem

First, read the problem and make sure that you understand it. Ask yourself these questions.

- Do I understand all the words?
- What information am I given?
- What am I asked to find?
- Can I state the problem in my own words?
- Am I given enough information?
- Am I given too much information?
- Have I solved a similar problem?

Think of a Plan

Organize the information you need. Decide whether you need an exact or approximate answer. Plan how to use the information. The following list includes some of the problem solving strategies that may help.

- Act out the problem.
- Manipulate materials.
- Work backward.
- Account for all possibilities.
- Change your point of view.
- Draw a diagram.
- Look for a pattern.
- Make a table.
- Use a formula.
- Guess and check.
- Solve a simpler problem.
- Use logical reasoning.

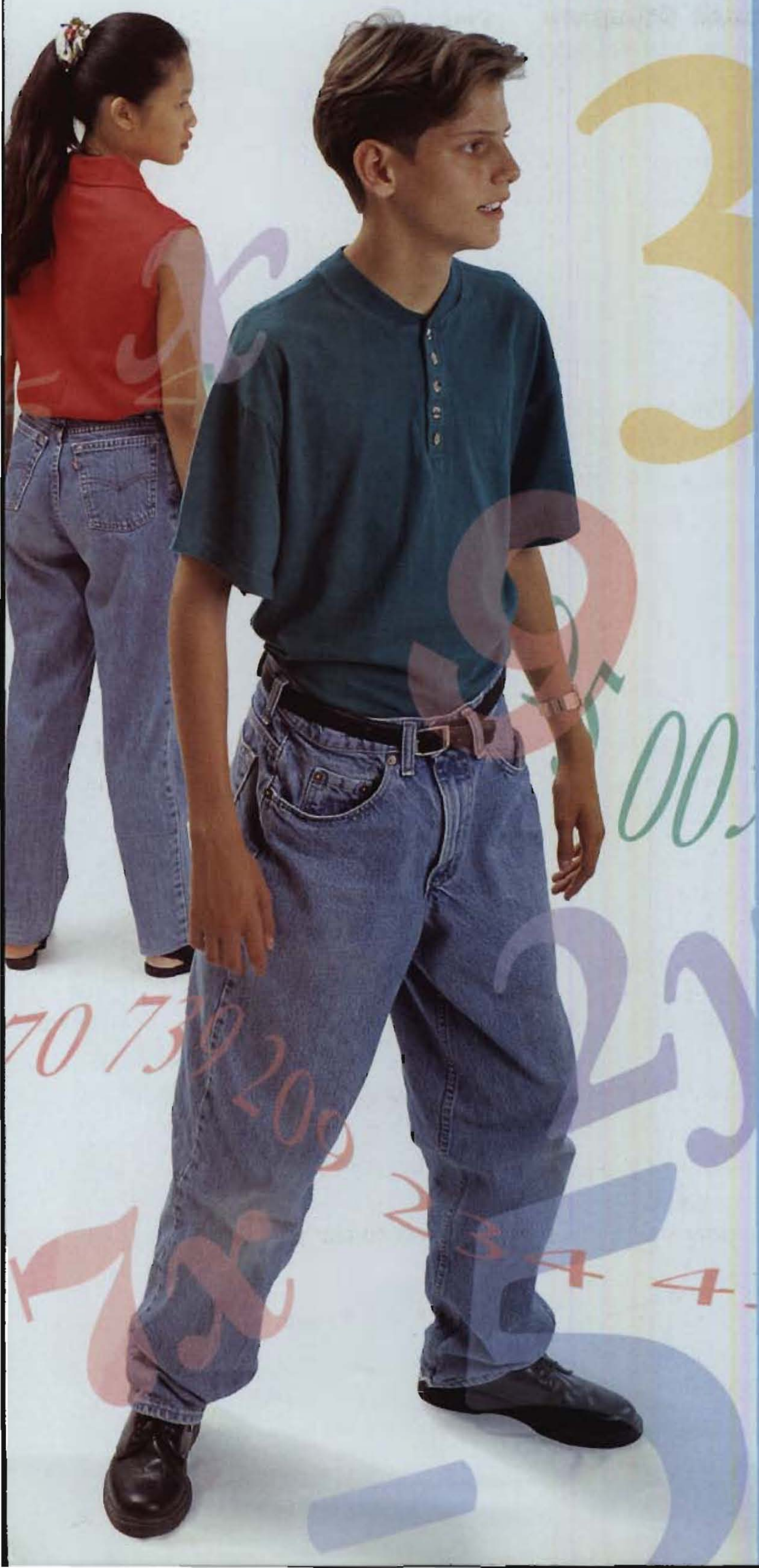
Carry Out the Plan

Choose the calculation method you will use to carry out your plan. Estimate the answer to the problem. Carry out your plan, using paper and pencil, a calculator, a computer, or manipulatives. Write a final statement that gives the solution to the problem.

Look Back

Check all your calculations. Check your answer against the original problem. Is your answer reasonable? Does it agree with your estimate? Look for an easier way to solve the problem.

EXPLORING MATH



The following 14 pages of activities will let you explore 14 mathematical standards that will be essential for citizens of the twenty-first century.

Problem Solving

Being a good problem solver is an important life skill. *MATHPOWER*™ *9* will help you develop this skill.

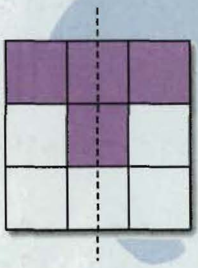
Activity

Solve these problems.
In each case, describe the method you used.

1. How many different, even 4-digit house numbers can you make with the digits 2, 3, 4, and 5?

2. A basketball team is standing in a circle. The team members are evenly spaced and numbered in order, clockwise, from 1 up. Player number 5 is directly opposite player number 12. How many players are in the circle?

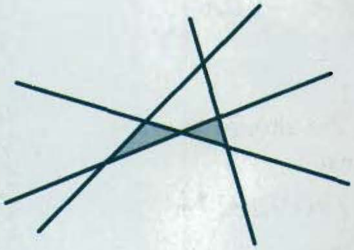
3. The diagram shows how to shade 4 of the 9 squares to give just 1 line of symmetry. In how many other ways can you shade 4 of the 9 squares to give 1 line of symmetry?



4. The number that represents the area of a square is twice the number that represents the sum of its sides. What is the side length of the square?

5. Take 6 dominoes, the 0/0, 0/1, 0/2, 1/1, 1/2, and 2/2. Arrange them in a square so that each side of the square contains the same number of dots.

6. The diagram shows how to make 2 non-overlapping triangles with 4 straight lines. How many non-overlapping triangles can you make with 5 straight lines?



Mathematics as Communication

As you develop new ideas in any field, it is important to be able to communicate these ideas to others.

Activity ❶

There are 2 research camps located in the desert at A and B. Each camp has a jeep. A water station is to be located so that the total distance the 2 jeeps must travel to the water and back is as short as possible.

1. What is the total distance the jeeps travel if the water station is placed at position 3? at position 4? at position 1? at A? at B?

2. Where should the water station be placed so that the total distance travelled by the jeeps is the shortest?

Activity ❷

There are now 3 research camps, A, B, and C, in the desert. Each camp has a jeep.

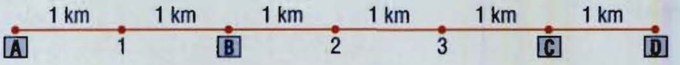
1. What is the total distance travelled by the jeeps to get water if the water station is placed at position 3? at position 2? at A? at B? at C?

2. Where should the water station be placed so that the total distance travelled by the jeeps is the shortest?

Activity ❸

Where should the water station be placed so that the total distance travelled by the jeeps is the shortest in each diagram?

1.



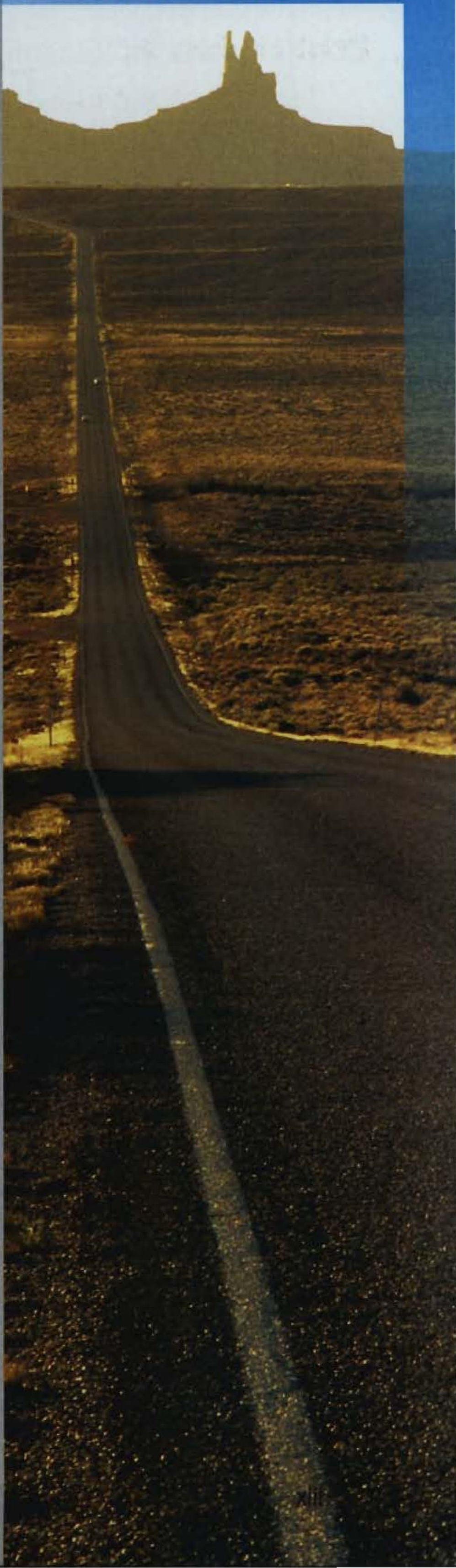
2.

Activity ❹

1. Write a rule for finding where the station should be placed if there is an even number of camps.

2. Write a rule for finding where the station should be placed if there is an odd number of camps.

3. Test your rules for 6 camps and 7 camps.



Mathematics as Reasoning

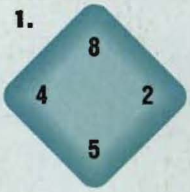
3

The ability to reason logically, to think your way through a problem, is a skill you can develop.

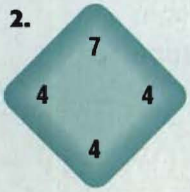
Activity ❶

For each card, use all the numbers only once. Use as many of the operations +, −, ×, ÷, and brackets ( ) as you wish. Write a number sentence with the value 24 for each card.

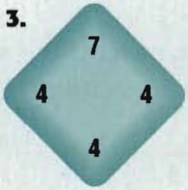
1.



2.



3.



Activity ❷

There are three books on the counter. The books are blue, green, and black. The titles of the books are *MATHPOWER™*, *SCIENCEPOWER*, and *COMPUTERPOWER*. The books belong to Sari, Terri, and Dmitri. Use the clues to reason who owns which book.

1. Sari's book is not about computers.

2. Dmitri's book is not blue or green.

3. *SCIENCEPOWER* is not green.

4. *MATHPOWER™* is a blue book.

Copy and complete these tables to sort out the owners, colours, and book titles.

	Colour of Book		
	Blue	Green	Black
Sari			
Terri			
Dmitri			

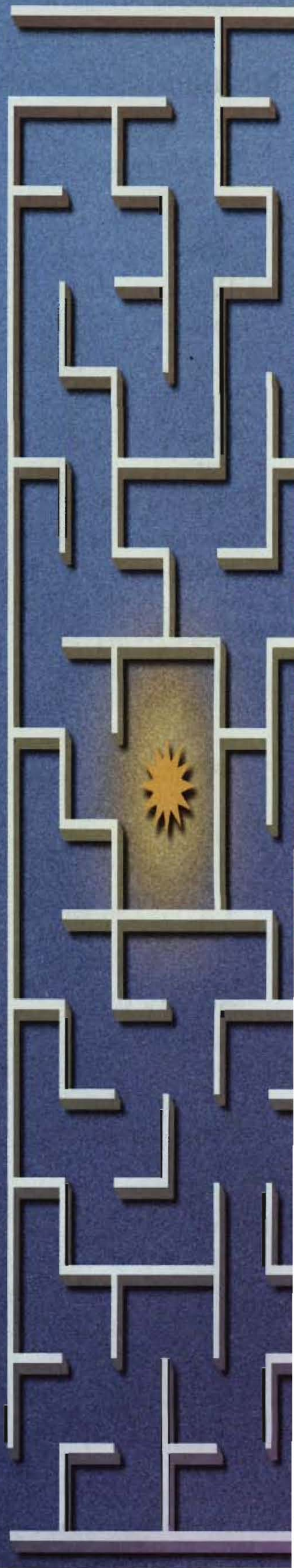
	Colour of Book		
	Blue	Green	Black
MATHPOWER™			
SCIENCEPOWER			
COMPUTERPOWER			

	Colour of Book	Title of Book
Sari		
Terri		
Dmitri		

Activity ❸

Use the table to make up the clues to a problem that can be solved by reasoning.

	Bicycles	
Jim	green	3-speed
Ali	blue	5-speed
Sue	red	10-speed



Mathematical Connections

Mathematics can be found in many places. The following activities explore some of them.

Activity ①

Work with a classmate to plan a 2-week vacation.

You will be travelling by car. Decide where you want to go and the route you are going to take to get there. You will need some maps. Assume that you can travel for a maximum of 8 h/day and that you will sleep in motels or hotels. You will want to spend at least 7 days at your vacation spot.

Keep a list of what you will spend every day for gas, food, lodging, and entertainment. Determine the total cost of the vacation.

Activity ②

1. Select a sport and list 5 ways in which mathematics is used in this sport.

2. Describe how the sport would be different if it did not include any mathematics.

Activity ③

The people responsible for playing the music you listen to on the radio use mathematics every day. List some of the ways in which mathematics is used at a radio station.

Activity ④

There are many geometric shapes that you see and use every day.

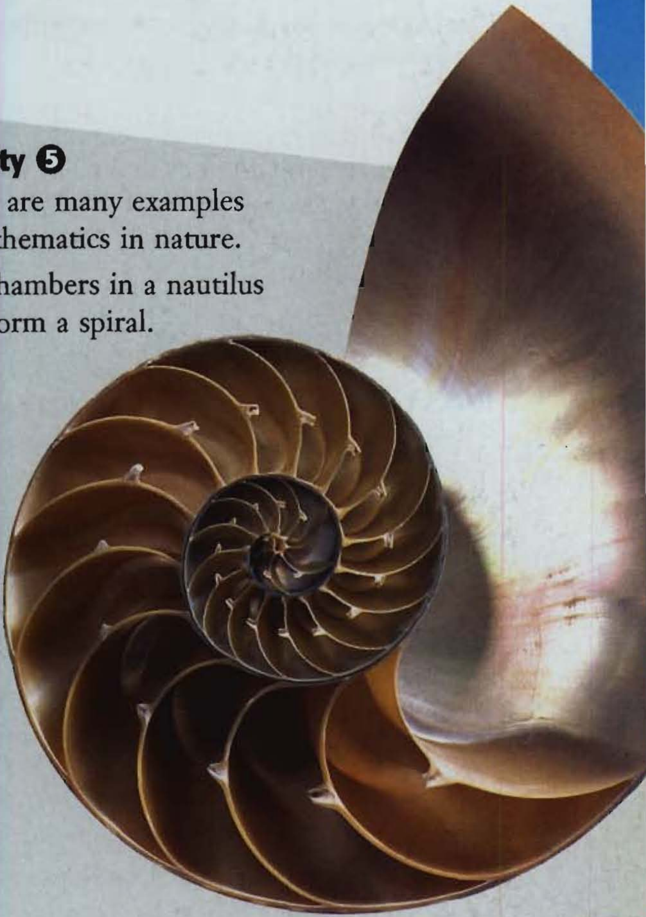
1. List 5 geometric shapes found in your classroom.

2. List 5 geometric shapes you see on your way to school.

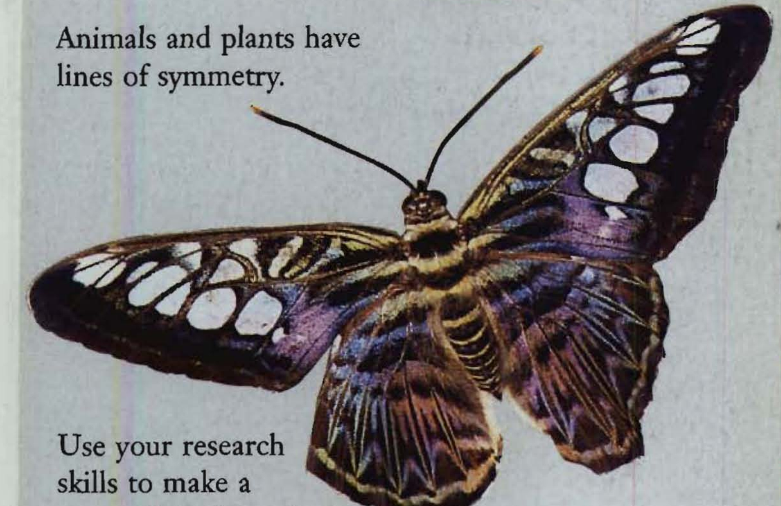
Activity ⑤

There are many examples of mathematics in nature.

The chambers in a nautilus shell form a spiral.



Animals and plants have lines of symmetry.



Use your research skills to make a poster that shows how mathematics is found in nature.

Algebra

Algebra is the language of mathematics.

Activity ❶

The concept of a variable is one of the most important in algebra. A variable can represent different numbers at different times.

In the following equation there are two variables, a triangle and a square.

▲ + ▲ + ■ = ■ + ■

If you replace the ▲ with 1 and the ■ with 2, you get

$$1 + 1 + 2 = 2 + 2$$

This is a true statement.

If you replace the ▲ with 2 and the ■ with 3, you get

$$2 + 2 + 3 = 3 + 3$$

This is not a true statement.

1. a) Find 3 other pairs of whole numbers that can replace the ▲ and the ■ to make a true statement. Put the pairs in a table.

▲	■

b) Describe the pattern.

2. Repeat the activity for the following equations. If you cannot find 3 pairs of whole numbers that make true statements, explain why.

a) ▲ + ▲ + ▲ + ■ = ■ + ■

b) ▲ + ▲ = 2 × ■

c) ▲ + 2 = ■ + ■

d) ▲ + ▲ = ■ + ■ + ■

e) ▲ + ■ = ▲ × ■

f) ▲ × ▲ × ■ = ▲ + ▲ + ■

Activity ❷

Find the value of the letter to make each equation true.

1. $m + 8 = 15$

2. $x - 5 = 7$

3. $6 - t = 4$

4. $x + x + 5 = 17$

5. $n \div 2 = 10$

6. $2 \times t + 7 = 11$

Activity ❸

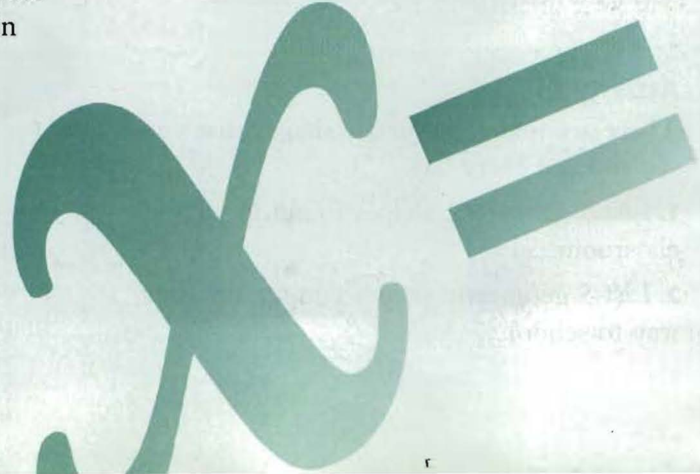
1. $s + u = 15$
$r + s + t = 15$
If $r = 6$ and $t = 4$, find u.

2. $y + z = 5$
$w + x = 7$
$x + y = 6$
If $w = 3$, find z.

3. $a = b$
$x = a + b + c + d$
$c = d$
If $x = 10$ and $b = 2$, find d.

4. $w = y$
$x = w + z$
$z = y + m$
If $w = 3$ and $x = 10$, find m.

5. $y + w = u$
$x = y - z$
If $x = 8$, $z = 3$, and $u = 11$, find w.

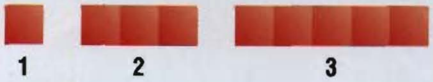


Functions

In much of the mathematics you do, you study patterns. There are many patterns in nature.

Activity ❶

1. a) Draw the next figure in the sequence.



b) Copy and complete the table for the sequence.

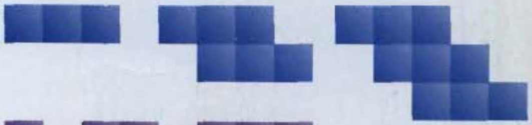
Figure	Perimeter
1	4
2	8
3	12
4	
5	
10	
100	

c) Describe the pattern in words.

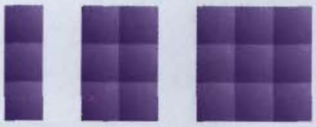
2. Repeat question 1 for these sequences.

a)

b)



c)



Activity ❷

Copy each table.

Describe the rule that lets you find y if you know x or find x if you know y.

Use the rule to complete each table.

1.

x	y
9	1
15	7
22	
8.1	
	11
	3.3

2.

x	y
8	4
36	18
40	
6.4	
	8.1
	9.7

3.

x	y
3	17
9	11
13	7
4	
18	
	6.2
	15.5

4.

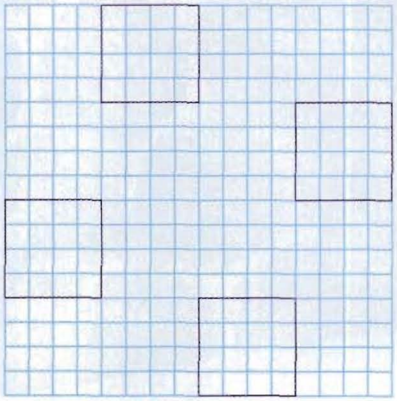
x	y
1	3
5	11
9	19
6	
12	
2.5	
	9

Geometry from a Synthetic Perspective

In geometry, you will study shapes and their properties. Geometry is found in science, in recreation, and in practical tasks, such as painting a room or constructing a building.

Activity: A Geometric Spiral

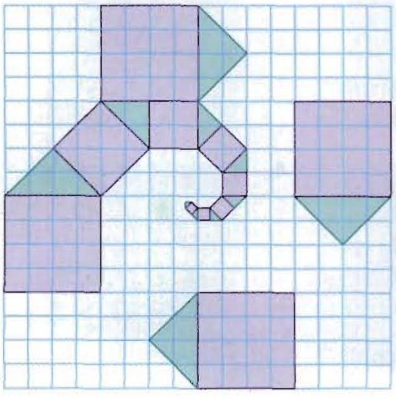
Mark out a 16-by-16 square on a grid as shown and draw four 4-by-4 squares on the grid.



Make spirals by adding isosceles right triangles to each square, with the hypotenuse as a side of the square.

Continue by adding squares to the sides of the triangles, triangles to the sides of the squares, and so on.

The diagram shows 1 spiral and the start of the other 3. Complete all the spirals in your own drawing. Then, colour the figures to make a pleasing design.

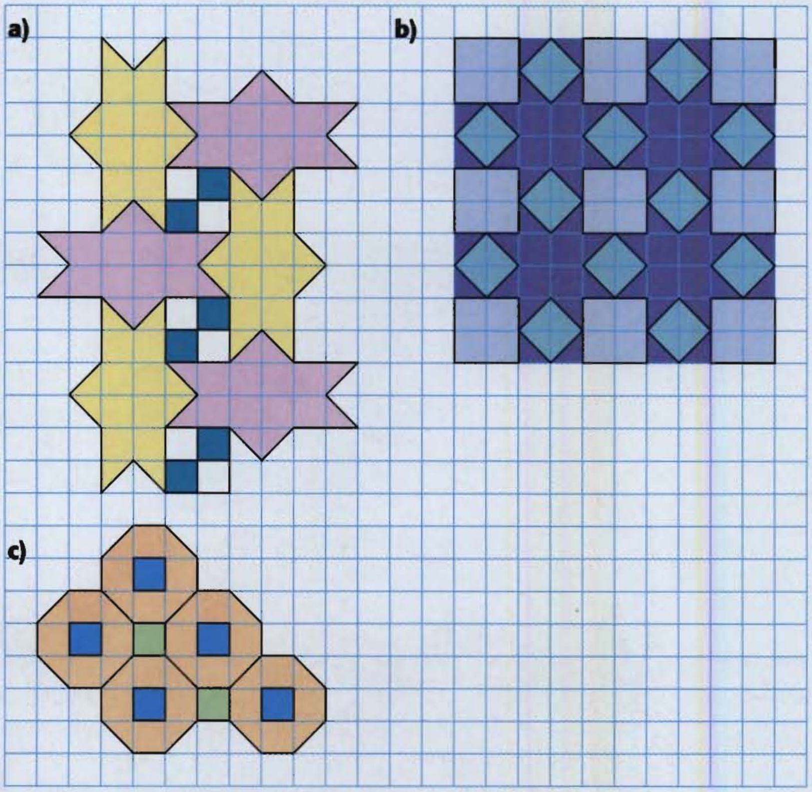


Geometry from an Algebraic Perspective

The study of geometry through the use of transformations has changed geometry from static to dynamic. For example, geometry can now be used to make moving images on film or videotape.

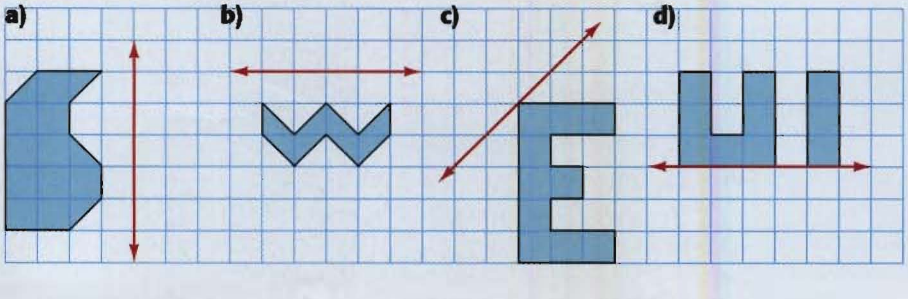
Activity ❶ Patterns

Copy each pattern onto a grid and continue it.



Activity ❷ Reflections

Copy each diagram onto grid paper and draw its reflection in the reflection line.

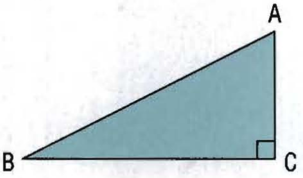


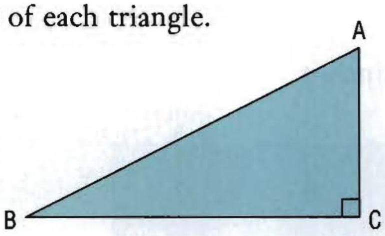
Trigonometry

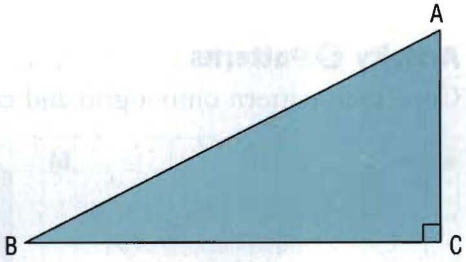
Trigonometry is the study of the measures of triangles. Trigonometry has many uses, including surveying and navigation.

Activity

1. In each of these right triangles, ∠B is 27° and ∠A is 63°. Measure sides BC and AC of each triangle.







∠A	63°
∠B	27°
∠C	90°
AB	
BC	
AC	

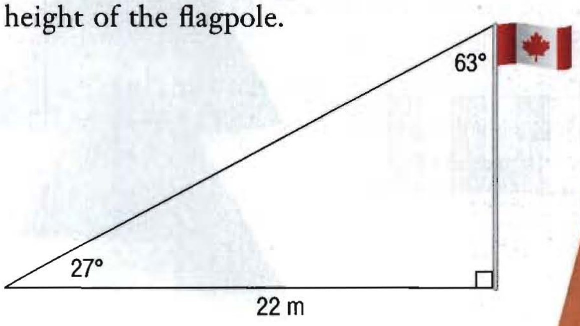
∠A	63°
∠B	27°
∠C	90°
AB	
BC	
AC	

∠A	63°
∠B	27°
∠C	90°
AB	
BC	
AC	

2. a) Calculate the quotient $\frac{BC}{AC}$ to the nearest hundredth for each triangle.

b) What do you notice about the values of the quotient?

3. Use your result from question 2 and the information in the diagram to find the height of the flagpole.



Statistics

Statistics play a very important part in our lives. They are found everywhere, even in the machines we use and the games we play.

Activity ❶ The QWERTY Keyboard

The picture shows a manual typewriter first manufactured in 1872. The keys were arranged in this way to avoid the locking of the type bars of the most frequently used letters. This keyboard is often called the QWERTY keyboard.

The table gives the average number of times a letter appears in 100 letters of written material.

A	8.2	H	5.3	O	8	V	0.9
B	1.4	I	6.5	P	2	W	1.5
C	2.8	J	0.1	Q	0.1	X	0.2
D	3.8	K	0.4	R	6.8	Y	2
E	13	L	3.4	S	6	Z	0.05
F	3	M	2.5	T	10.5		
G	2	N	7	U	2.5		

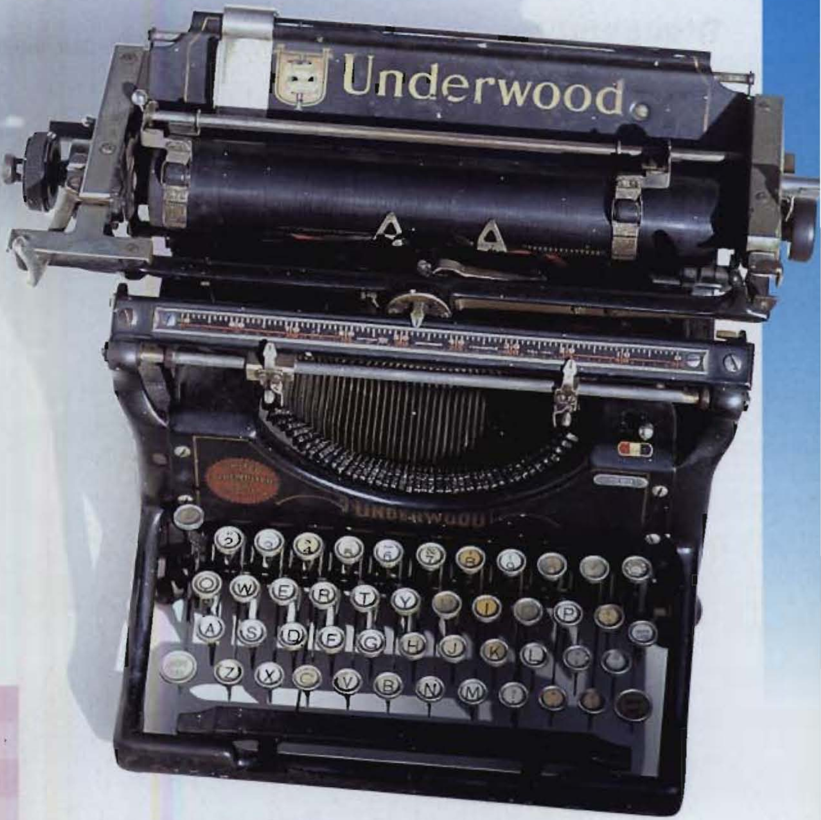
1. Draw a QWERTY keyboard.

2. On each key, write the number of times that letter will appear in 100 letters of written material.

3. Explain the arrangement of the letters on the keyboard.

4. How might you have placed the letters differently?

5. Why is the QWERTY keyboard still used today?



Activity ❷ Breaking Codes

Codes or puzzles are constructed by letting one letter stand for another.

1. How could the table in Activity 1 be used to help decipher codes or puzzles?

2. Make up a coded message and have a classmate decipher it.

Activity ❸ SCRABBLE®

1. List the letters of the alphabet. Beside each letter, write how many tiles of that letter are found in a SCRABBLE® game and the point value of that letter.

2. Explain why you think the makers of the game used these numbers of tiles and this point system.

3. Choose any 7 tiles to make the word that will give you the most points. Compare your word with your classmates'.

Probability

Probability has been called the mathematics of chance. Knowing about probability will help you make informed decisions about the likelihood of events.

Activity ❶

Suppose there are 2 blue marbles, 3 red marbles, and 5 white marbles in a bag. You select 1 marble, look at the colour, and return it to the bag.

1. a) The probability of picking a red marble is $\frac{3}{10}$. Why?

b) What is the probability of picking a blue marble? a white marble?

2. What percent of the time should you pick a white marble?

3. If you selected a marble 100 times, how many times should you pick a white marble?

Activity ❷

1. Estimate the number of times a baseball cap will land right side up if you toss it 10 times.

2. Toss a baseball cap 10 times and compare the result with your estimate.

3. What is the probability that a baseball cap will land right side up?

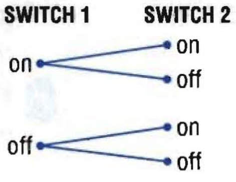
Activity ❸

Some homes have automatic garage doors that are operated by a garage-door opener. An opener has 8 switches. Each switch can be set in 2 positions, on and off.

What is the chance that 2 people have the same code for their garage-door openers? To find out, start with simple cases. What if there is only 1 switch that can be set to on or off? In this case, there are only 2 possibilities. This means that there is 1 chance out of 2 that both people will have the same code.

What if there are 2 switches?

When switch 1 is on, switch 2 can be on or off. When switch 1 is off, switch 2 can be on or off. This means that there are 4 possibilities in total, so the chance is now 1 in 4. What if there are 3 switches? 4 switches? 8 switches?

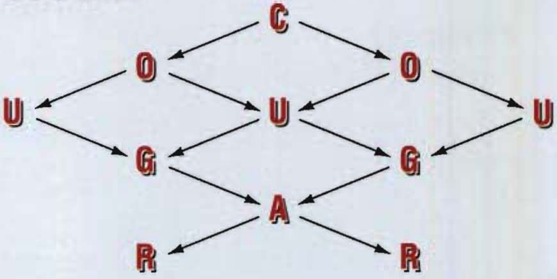


Mathematics and Counting (Discrete Mathematics)

Discrete mathematics is the study of things that can be counted.

Activity ❶

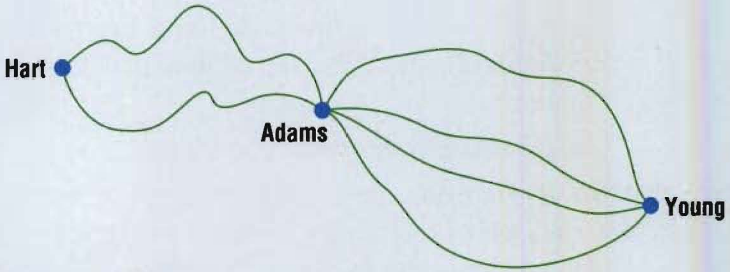
1. If you follow the arrows, how many different paths spell COUGAR?



2. Make up your own path puzzle using the name of a rock group. Ask a classmate to solve your puzzle.

Activity ❷

There are 2 roads from Hart to Adams.
There are 4 roads from Adams to Young.
How many different routes are there from Hart to Young?



Activity ❸

Four people, Lisa, Ben, Ona, and Dino, are going on a trip in a car. The car will seat 2 in the front and 2 in the back. Only Lisa and Dino can drive. In how many different ways can the 4 people sit in the car?

Activity ❹

Copy each sequence and predict the next 3 terms.

1. 2, 6, 10, 14, ■, ■, ■

2. 3, 7, 15, 31, ■, ■, ■

3. 486, 162, 54, 18, ■, ■, ■

4. 3, 4, 7, 11, 18, ■, ■, ■

Investigating Limits

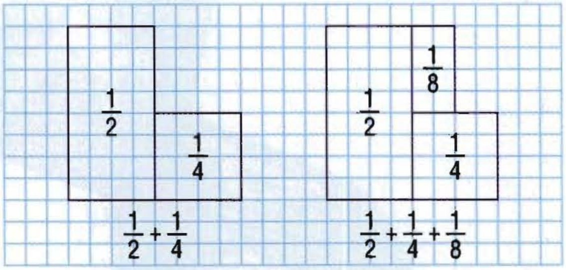
A very important branch of mathematics is known as calculus. It is applied in many fields, including the sciences, the social sciences, and business. The study of limits plays an essential part in understanding calculus.

Activity ❶

Consider this series.

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$

The series can be summed in parts as shown.



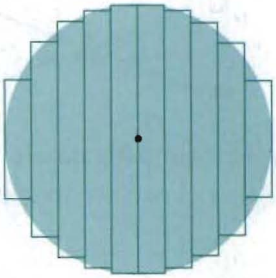
1. Draw a 16-by-16 grid and continue the pattern.

2. What number does the sum of the series approach? Check your answer with a calculator.

Activity ❷

1. Draw a circle with radius 5 cm.

2. Divide the circle into 10 rectangles, each 1 cm wide.



3. Measure the length of each rectangle and find the area of each rectangle.

4. Find the sum of the areas of the rectangles.

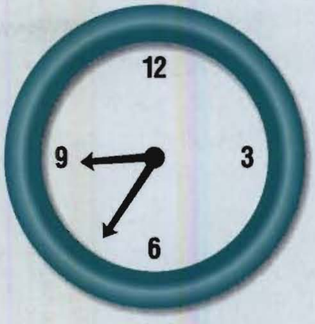
5. Find the area of the circle using the formula $A = \pi r^2$. You can assume that π is about 3.14.

6. Compare your answers from steps 4 and 5.

7. If you made the rectangles narrower, would the sum of their areas be closer to the area found using the formula? Explain.

Mathematical Structure

When is $10 + 4 = 2$?
When is $10 + 10 = 8$?
When you are telling time.
If it is now 10 o'clock, it will be 2 o'clock in 4 h, and 8 o'clock in 10 h.
Clock arithmetic is an example of modular arithmetic.



Activity ❶

A scientist is doing an experiment that takes 41 h to complete. The experiment was started at 12 noon.

1. At what time will the experiment be completed?

2. How long had the experiment been running at 8 a.m. on the second day?

3. Complete the following calculations using the arithmetic of the 12-h clock.

a) $3 + 5$
b) $9 + 9$
c) $13 + 15$
d) $15 + 22$
e) 3×6
f) 4×7
g) $5 - 7$
h) $3 - 8$

Activity ❷

On the 4-h clock shown, the hours are 1, 2, 3, and 0.
Use the 4-h clock for the following.

1. State whether the expressions in each pair are equal.

a) $2 + 3$ and $3 + 2$
b) $3 - 2$ and $2 - 3$
c) $2 + 2$ and $2 - 2$
d) $3 + 3$ and $3 - 3$

2. Simplify.

a) $2 \times (3 + 2)$
b) $(2 + 3) \times 3$
c) $(2 \times 3) \times 2$
d) $(2 \times 3) \times 3$
e) $3 \times 3 - 2 + 1$

3. State whether the expressions in each pair are equal.

a) $3 \times (3 + 2)$ and $3 \times 3 + 3 \times 2$
b) $(3 + 1) \times (2 + 3)$ and
$3 \times 1 + 3 \times 2 + 1 \times 2 + 1 \times 3$

CHAPTER 1

Numbers

A square engraving has sides of length 4 cm.

The sides of a photographic enlargement of the engraving are 3 times as long as the sides of the engraving. How do the areas of the photograph and the engraving compare?

A reduced image of the engraving is used on a postage stamp. The area of the image on the stamp is 4 times smaller than the area of the engraving. What is the length of a side of the image on the stamp?

GETTING STARTED

Activity ❶ Prime Numbers

The number 10 has 4 **factors**, 1, 2, 5, and 10. Each factor divides 10 evenly. A **prime number** is a whole number with exactly 2 factors, itself and 1. The number 1 is not considered to be prime. The first 4 prime numbers are 2, 3, 5, and 7.

1. The numbers 5 and 7 are called **twin primes**. They are consecutive odd numbers that are also prime. Find all the twin primes less than 100.

2. Prime triplets are 3 consecutive odd numbers that are also prime. What is the only set of prime triplets?

3. In 1742, the mathematician Goldbach stated that every even number, except 2, can be written as the sum of 2 prime numbers.

$$8 = 3 + 5$$
$$18 = 7 + 11 \text{ or } 18 = 5 + 13$$

Write each of the following as the sum of 2 prime numbers.

a) 24 **b)** 30 **c)** 42 **d)** 100

4. Until 1903, it was thought that $2^{67} - 1$ was a prime number. That year, a mathematician named Cole received a standing ovation at a meeting of the American Mathematical Association. He multiplied out 2^{67} and subtracted 1, then moved to another chalkboard and multiplied the following numbers.

$$\begin{array}{r} 761\ 838\ 257\ 287 \\ \underline{\times\ 193\ 707\ 721} \end{array}$$

He got the same result from both calculations. Cole had factored a number previously thought to be prime.

a) Estimate $2^{67} - 1$. Compare your method with your classmates'.

b) Investigate why mathematicians find prime numbers so interesting.

Activity ❷ Squares, Cubes, and Factors

Study the following steps.

- List the factors of 6.

1, 2, 3, 6

- Find the number of factors each factor of 6 has.

Factor of 6	1	2	3	6
Number of Factors	1	2	2	4

- Find the sum of the cubes of the numbers of factors.

Number of Factors	1	2	2	4
Number of Factors Cubed	1	8	8	64

$$1 + 8 + 8 + 64 = 81$$

- Find the square of the sum of the numbers of factors.

$$1 + 2 + 2 + 4 = 9 \text{ and } 9^2 = 81$$

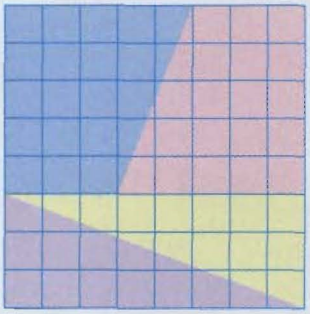
1. Write a statement about the relationship between "the sum of the cubes of the numbers of factors" and "the square of the sum of the numbers of factors" for the factors of a number.

2. Test your statement, starting with these numbers.

a) 12 **b)** 20

Activity ❸ Amazing Square

Copy the square onto grid paper.



Cut out the 4 pieces and rearrange them to form a rectangle.

1. What was the area of the original square?

2. What is the area of the rectangle?

3. Can you explain the difference in the areas, or is there some magic in squares?

Mental Math

Calculate.

1. 5 × 23

2. 16 × 9

3. 7^2

4. 11^2

5. 66 × 5

6. 5 × 2 × 8

7. 19 × 7

8. 103 × 4

9. 100 × 16

10. 2323 × 3

Calculate.

11. 85 ÷ 5

12. 123 ÷ 10

13. 217 ÷ 7

14. 366 ÷ 6

15. 6684 ÷ 2

16. 5000 ÷ 100

17. 950 ÷ 1000

18. 1200 ÷ 20

19. 1500 ÷ 50

20. 800 ÷ 25

Calculate.

21. 356 + 222

22. 140 + 555

23. 199 + 299

24. 234 + 587

25. 435 − 225

26. 810 − 105

27. 367 − 202

28. 287 − 369

29. 450 + (−301)

30. −215 − 680

Calculate.

31. 17 + 23 − 8

32. 9 − 8 − 3

33. 27 + 14 + 38

34. 15 − 16 + 20

35. 3 + 22 − 16

36. −4 + 15 − 18

37. 22 + (−15) + 13

38. −10 + 35 + 12

39. 42 + 16 + (−10)

40. −17 + 26 − 35

Calculate.

41. 2×5^2

42. 100 ÷ (6 + 4)

43. 5 × 2 × 8

44. 73 − 7 × 5

45. 4×7^2

46. $5^2 + 3^2$

47. $(17 - 11)^2$

48. $(11 - 17)^2$

49. 15 × (65 − 55)

50. 10 × 5 − 70

The Rational Numbers

Most numbers belong to several different sets of numbers at the same time.

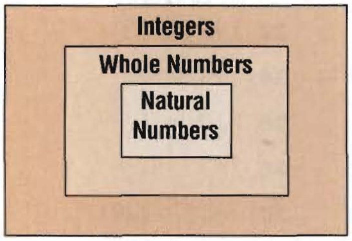
Activity ❶ Natural Numbers, Whole Numbers, and Integers

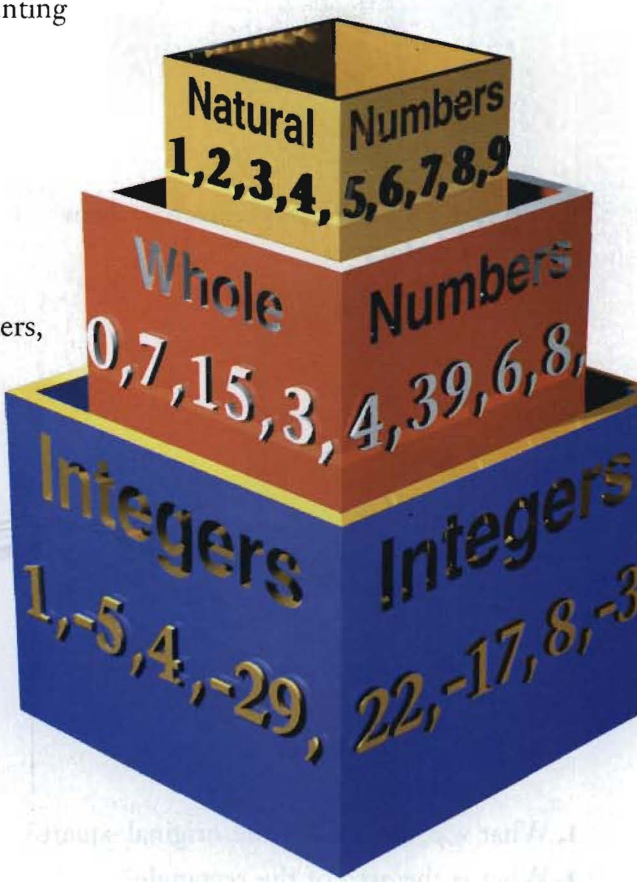
1. Which of these sets of numbers is the set of counting numbers or **natural numbers**?

a) 2, 4, 6, 8, 10, 12, 14, 16, 18, 20
b) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
c) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

2. a) Is the set of numbers you chose in question 1 also made up of whole numbers?
b) Are these numbers also integers? Explain.

3. The 3 nested boxes show how the natural numbers, the whole numbers, and the integers are related. Explain the meaning of the diagram.





Activity ❷ Terminating Decimals

1. Complete this table of rational numbers. Write the fraction in lowest terms.

2. What is meant by a terminating decimal?

3. a) When a rational number is written as a fraction, is it the quotient of 2 integers? Explain.
b) Examine the decimal forms of the rational numbers in the table. Are they integers? Why?
c) Can the decimal form of a rational number ever be an integer? If so, give an example.

Decimal Form	Fraction Form
0.5	
0.25	
0.625	
−0.2	
−0.75	
−0.85	

Activity ❸ Repeating Decimals

1. Copy and complete this table of rational numbers.

Fraction Form	Decimal Form
$\frac{2}{3}$	
$-\frac{7}{11}$	
$\frac{1}{6}$	
$-\frac{5}{9}$	

2. a) Does the repeating part of each decimal ever terminate?
b) What special mark can be written over the first digit or first set of digits that repeat?

3. Rewrite each decimal using the special mark.

4. Draw 4 boxes that nest inside one another. Label each box with the label "natural numbers," "whole numbers," "integers," or "rational numbers" to show how these sets of numbers are related. Compare your diagram with a classmate's.

5. Write a definition of the rational numbers.

Activity ❹ Classify the Numbers

1. Which of the following statements are true? Explain.
a) The rational number $-\frac{3}{8}$ is made up of two integers, one positive and one negative.
b) The decimal form of $\frac{1}{4}$ is an integer.
c) The number -5 is a natural number.
d) The number 0.232 425 262 is a repeating decimal.
e) The square root of 4, $\sqrt{4}$, is a rational number.
f) The square root of 2, $\sqrt{2}$, is a rational number.

2. Explain why 2 is a natural number, a whole number, an integer, and a rational number.

3. Explain why -10 is a rational but not a whole number.

4. Write one example of a number that is an integer but not a whole number.

5. The ratio of the circumference of a circle to its diameter is π. Is π a rational number? Explain.

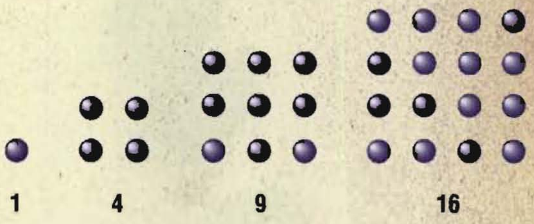
Activity ❺ Numbers and Variables

1. State the rational number that multiplies each variable.
a) $\frac{1}{3}a^2$ **b)** $\frac{1}{8}y^3$
c) $-\frac{1}{10}w^5$ **d)** $-\frac{1}{7}x^2$

2. State the rational number that multiplies each variable.
a) $\frac{q^2}{2}$ **b)** $\frac{x^4}{3}$
c) $-\frac{b^7}{5}$ **d)** $-\frac{x^9}{8}$

Perfect Squares

The Pythagoreans were a secret society of Greek mathematicians in the sixth century B.C. They were interested in special types of numbers, including prime numbers, triangular numbers, square numbers, and pentagonal numbers. The Pythagoreans used lines, triangles, and squares of pebbles to represent different types of numbers. Our word "calculate" comes from the Latin word *calculus*, which means "pebble." Perfect square numbers could be represented by squares of pebbles. Perfect squares are found by squaring whole numbers.



Activity ① Odd Numbers and Perfect Squares

Copy and complete the table.

Odd Numbers	Sum	Diagram
First one	1 = 1	■
First two	1 + 3 = 4	□□ □■
First three	1 + 3 + 5 = ■	
First four		
First five		

What is the sum of

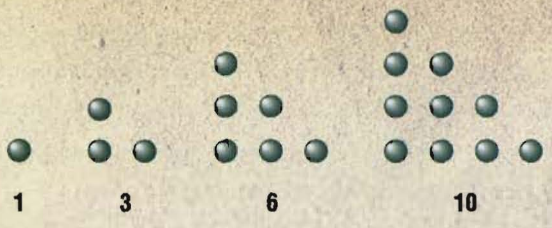
a) the first 8 odd numbers?

b) the first 10 odd numbers?

c) the first 1000 odd numbers?

Activity ② Triangular Numbers and Perfect Squares

The first 4 triangular numbers are shown.



Use diagrams to illustrate your solutions to the following.

1. Find the next 3 triangular numbers.

2. Why are they called triangular numbers?

3. What is the sum of any 2 consecutive triangular numbers?

Activity ❸ The Locker Problem

Southridge School has a very long hallway, with 1000 lockers along one side. They are numbered from 1 to 1000.

On April Fool's Day, each of Southridge's 1000 students walks down the hallway to leave the school. Every student walks in the same direction, past locker 1 first and locker 1000 last.

The first student closes every locker door. The second student opens every second locker door. The third student changes the state of every third locker door. This means that, if the locker door is open, the student closes it. If it is closed, the student opens it. The fourth student changes the state of every fourth locker door.

This pattern continues until the thousandth student leaves and changes the state of the thousandth locker door.

What are the numbers of the locker doors that are closed after the last student leaves?

Hint: Solve a simpler problem. You may want to make a table to show the states of some locker doors after the first few students walk down the hall. Use the table to predict which of the 1000 doors are closed after 1000 students leave. Another way is to use cards or counters as lockers and act out the problem.

		Locker Number																			
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Student	**1**	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
	2		O		O		O		O		O		O		O		O		O		O
	3			O			C			O			C			O			C		
	4																				
	5																				

Activity ❹ Fermat and Perfect Squares

French mathematician Pierre de Fermat (1601–1675) stated that any whole number can be written as the sum of, at most, 4 perfect squares.

$21 = 4^2 + 2^2 + 1^2$ or $21 = 3^2 + 2^2 + 2^2 + 2^2$

$39 = 5^2 + 3^2 + 2^2 + 1^2$ or $39 = 6^2 + 1^2 + 1^2 + 1^2$

1. Write each of the following as the sum of, at most, 4 perfect squares.

a) 33 **b)** 42 **c)** 77 **d)** 88

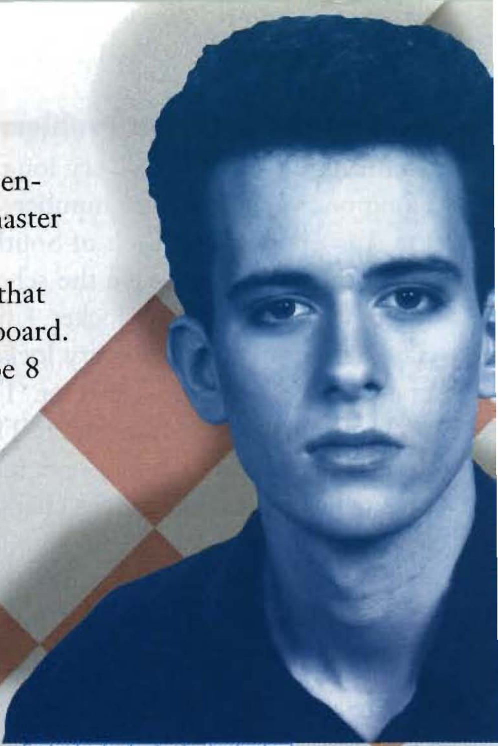
e) 153 **f)** 212 **g)** 208 **h)** 903

2. Compare your answers with a classmate's.

1.1 Estimating and Calculating Square Roots

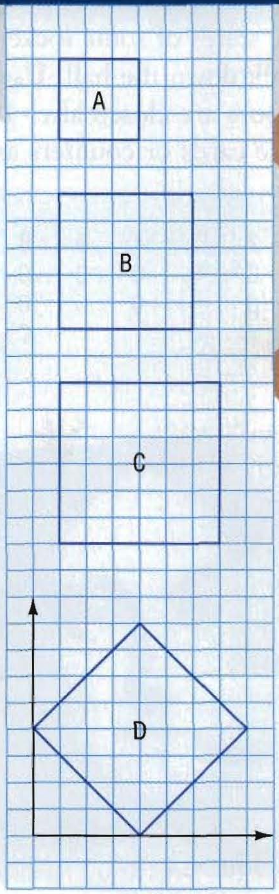
The world's top chess players are called Grandmasters. Thirteen-year-old Alexandre Lesiège of Quebec beat a Russian Grandmaster at the World Open in 1989.

A chess board is a square. It is covered by 64 smaller squares that are equal in area. There are 8 squares along each side of the board. We can say that 64 is the square of 8, but how can we describe 8 in relation to 64?



Activity: Use the Diagrams

Draw figures A, B, C, and D on 1-cm grid paper or construct them with elastics on a geoboard. Determine the area of each square.



Inquire

1. The length of each side of a square is the **square root** of its area. What is the square root of area A? area B? area C?

2. Count squares to determine the area of square D. Measure the side of square D. What is the approximate square root of area D?

3. Write your definition of the square root of a number.

4. a) What is the square root of 64?

b) Is any other integer also the square root of 64? Explain.

5. A Snakes and Ladders game board is a square with 100 smaller squares marked on it.

a) How many smaller squares lie along each side of the board?

b) What are the square roots of 100?

c) Which of the square roots of 100 could not be used to describe the number of squares along each side of the board or the length of a side of a square?

Since $5 \times 5 = 25$ and $(-5) \times (-5) = 25$, then 5 and -5 are the square roots of 25.

Since $1.2 \times 1.2 = 1.44$ and $(-1.2) \times (-1.2) = 1.44$, then 1.2 and -1.2 are the square roots of 1.44.

The **radical sign**, $\sqrt{\ }$, is used to represent the positive square root of a number. Thus, $\sqrt{25} = 5$, and $\sqrt{1.44} = 1.2$.

The positive square root is also called the **principal square root**. To avoid confusion, mathematicians do not use the radical sign for negative square roots.

Example 1

Evaluate.

a) $\sqrt{81}$

b) $\sqrt{0.36}$

c) $-\sqrt{6400}$

Solution

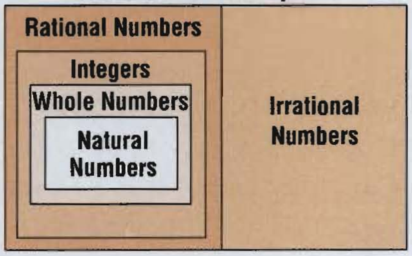
a) $9 \times 9 = 81$ so $\sqrt{81} = 9$

b) $0.6 \times 0.6 = 0.36$ so $\sqrt{0.36} = 0.6$

c) $80 \times 80 = 6400$ so $-\sqrt{6400} = -80$

Many numbers, such as $\sqrt{2}$ and $\sqrt{3}$, cannot be written as a fraction or a terminating decimal. These square roots are non-terminating, non-repeating decimals, or **irrational numbers**. Together with the natural numbers, whole numbers, integers, and rational numbers, the irrational numbers form the set of **real numbers**. To determine the approximate values of such square roots as $\sqrt{2}$ and $\sqrt{3}$, we estimate or use a calculator.

The Real Number System



Example 2

Estimate $\sqrt{356}$.

Solution

Start with numbers whose squares you know.
$30 \times 30 = 900$ (too high), $20 \times 20 = 400$ (too high),
$10 \times 10 = 100$ (too low)
Since 356 is between 100 and 400, and is closer to 400, a good estimate for $\sqrt{356}$ is $\sqrt{400}$ or 20.

Another way to estimate the square root of a number is to divide the number into groups of 2 digits, starting at the decimal point. Then, for numbers greater than 1, estimate the square root of the group furthest from the decimal point and add 1 zero for each other group. Thus, for $\sqrt{1535}$, we consider the two groups 15 and 35.
$\sqrt{15} \doteq 4$ ← square root of the group furthest from the decimal point.
There is one other group of digits, so we add one zero.
So, $\sqrt{1535} \doteq 40$.

$\doteq$ means "approximately equals"

For numbers less than 1, estimate the square root of the non-zero group closest to the decimal point. Thus, for $\sqrt{0.086}$, we consider the two groups of 08 and 60. $\sqrt{08} \doteq 3$, so $\sqrt{0.086} \doteq 0.3$.

Example 3

Estimate the square roots.

a) $\sqrt{567}$

b) $\sqrt{12\ 300}$

c) $\sqrt{0.45}$

d) $\sqrt{0.0067}$

Solution

a) $\sqrt{567}$	**b)** $\sqrt{12\ 300}$	**c)** $\sqrt{0.45}$	**d)** $\sqrt{0.0067}$
$\sqrt{5}\quad 67$	$\sqrt{1}\quad 23\quad 00$	$0\ .\ \sqrt{45}$	$0\ .\ 00\ \sqrt{67}$
$\downarrow\quad\downarrow$	$\downarrow\quad\downarrow\quad\downarrow$	$\downarrow\quad\quad\downarrow$	$\downarrow\quad\downarrow\quad\downarrow$
$2\quad 0$	$1\quad 0\quad 0$	$0\ .\ 7$	$0\ .\ 0\quad 8$
$\sqrt{567} \doteq 20$	$\sqrt{12\ 300} \doteq 100$	$\sqrt{0.45} \doteq 0.7$	$\sqrt{0.0067} \doteq 0.08$

The square root key on your calculator may appear as √ or √x.

To find the square root of a number, enter the number, then press the square root key.

CONTINUED ►

Example 4

Evaluate to the nearest tenth.

a) $\sqrt{42}$

b) $-\sqrt{164}$

Solution

a) Press 42 $\boxed{\sqrt{\ }}$
Display 42 6.4807407

EST $6 \times 6 = 36$, so $\sqrt{42} \doteq 6$

So, $\sqrt{42}$ is 6.5 to the nearest tenth.

b) Press 164 $\boxed{\sqrt{\ }}$
Display 164 12.806248

EST $13 \times 13 = 169$, so $-\sqrt{164} \doteq -13$

So, $-\sqrt{164}$ is -12.8 to the nearest tenth.

Example 5

Evaluate to the nearest tenth.

a) $5\sqrt{3} + 7\sqrt{2}$

b) $\dfrac{6\sqrt{11}}{7}$

Solution

a) $5\sqrt{3}$ means $5 \times \sqrt{3}$ and $7\sqrt{2}$ means $7 \times \sqrt{2}$

$\boxed{C}$ **5** $\boxed{\times}$ **3** $\boxed{\sqrt{\ }}$ $\boxed{+}$ **7** $\boxed{\times}$ **2** $\boxed{\sqrt{\ }}$ $\boxed{=}$ $\boxed{18.559749}$

So, $5\sqrt{3} + 7\sqrt{2} = 18.6$ to the nearest tenth.

EST
$$5\sqrt{3} + 7\sqrt{2} \doteq 5 \times 2 + 7 \times 1 = 17$$

b) $\boxed{C}$ **6** $\boxed{\times}$ **11** $\boxed{\sqrt{\ }}$ $\boxed{\div}$ **7** $\boxed{=}$ $\boxed{2.8428212}$

So, $\dfrac{6\sqrt{11}}{7} = 2.8$ to the nearest tenth.

EST
$$\frac{6\sqrt{11}}{7} \doteq \frac{6 \times 3}{6} = 3$$

Practice

Find the square roots of each number.

1. 49 **2.** 81 **3.** 121 **4.** 625

5. 0.64 **6.** 0.01 **7.** 1.96 **8.** 0.25

Evaluate.

9. $\sqrt{25}$ **10.** $\sqrt{100}$ **11.** $\sqrt{225}$

12. $\sqrt{256}$ **13.** $\sqrt{169}$ **14.** $\sqrt{0.36}$

15. $\sqrt{0.04}$ **16.** $\sqrt{1.21}$ **17.** $\sqrt{0.81}$

Estimate.

18. $\sqrt{30}$ **19.** $\sqrt{66}$ **20.** $\sqrt{92}$

21. $\sqrt{765}$ **22.** $\sqrt{989}$ **23.** $\sqrt{3245}$

24. $\sqrt{7800}$ **25.** $\sqrt{56\ 000}$ **26.** $\sqrt{880\ 000}$

Estimate.

27. $\sqrt{0.8}$ **28.** $\sqrt{0.77}$

29. $\sqrt{0.05}$ **30.** $\sqrt{0.067}$

31. $\sqrt{0.0382}$ **32.** $\sqrt{0.0023}$

33. $\sqrt{0.009}$ **34.** $\sqrt{0.0006}$

35. $\sqrt{0.000\ 22}$ **36.** $\sqrt{0.000\ 34}$

Estimate. Then, calculate to the nearest tenth.

37. $\sqrt{31}$ **38.** $\sqrt{44}$

39. $\sqrt{62}$ **40.** $\sqrt{79}$

41. $\sqrt{101}$ **42.** $\sqrt{206}$

43. $\sqrt{1123}$ **44.** $\sqrt{20\ 183}$

45. $\sqrt{86\ 003}$ **46.** $\sqrt{202\ 183}$

Evaluate to the nearest tenth.

47. $\sqrt{3} + \sqrt{7}$ **48.** $\sqrt{10} - \sqrt{5}$

49. $3\sqrt{11}$ **50.** $6\sqrt{23}$

51. $(\sqrt{3})(\sqrt{2})$ **52.** $7\sqrt{12} - 6$

53. $\sqrt{14} \div \sqrt{5}$ **54.** $\dfrac{\sqrt{20} - \sqrt{10}}{\sqrt{31}}$

Problems and Applications

55. Evaluate for $a = 5$ and $b = -2$.

a) $\sqrt{4a + 2b}$ **b)** $\sqrt{\frac{125}{a}}$

c) $\sqrt{-18b}$ **d)** $\sqrt{a^2 - 12b}$

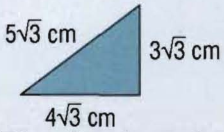
e) $-\sqrt{(ab)^2}$ **f)** $\sqrt{10a - 5b + 4}$

g) $\sqrt{-10ab^3}$ **h)** $-3\sqrt{a^2 + 2ab + b^2}$

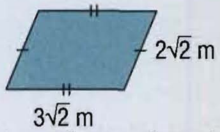
i) $7.3\sqrt{a^2 - 2ab + b^2}$

56. Calculate the perimeter of each figure to the nearest tenth of a unit.

a)



b)



57. The Greek mathematician Heron found the following formula for the area of a triangle.

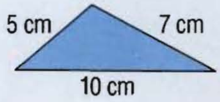
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where a, b, and c are the side lengths, and s is half the perimeter.

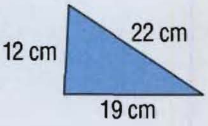
$$s = \frac{a+b+c}{2}$$

Use Heron's formula to calculate the area of each triangle.

a)



b)



58. The area, a, of an equilateral triangle is given by the formula $a \doteq 0.43s^2$, where s is the side length.
For each area, find the side length to the nearest tenth of a unit.

a) 15.6 cm^2 **b)** 44 m^2

c) 346.9 mm^2 **d)** 876.4 cm^2

59. The world's largest city square is Tiananmen Square in Beijing, China. It has an area of 0.396 km^2. What is the length of a side of the square to the nearest metre?

60. a) Evaluate $2\sqrt{2}$. **b)** Evaluate $\sqrt{8}$.

c) Compare your answers and explain your findings.

61. How are the square roots of a perfect square related?

62. A square has an area of 169 cm^2. What is the radius of the largest circle you can fit inside it? Explain.

63. Try to evaluate $\sqrt{-9}$ on your calculator. What is the result? Why?

64. One corner of a square is at (0, 0). It has an area of 49 square units. Find the possible coordinates of the other vertices.

65. To find the square root of a perfect square by subtraction, subtract the odd numbers in increasing order until the result is 0.

25 − 1 = 24
24 − 3 = 21
21 − 5 = 16
16 − 7 = 9
9 − 9 = 0

We need 5 subtractions to reach 0 from 25, so $\sqrt{25} = 5$.

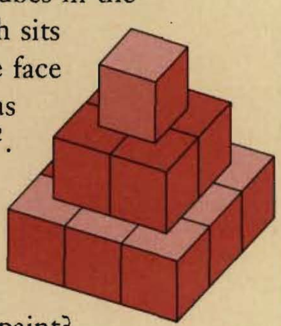
a) Use this method to find the following.

$\sqrt{289}$ $\sqrt{576}$ $\sqrt{784}$ $\sqrt{1089}$

b) Explain why this method works. Compare your explanation with your classmates'.

LOGIC POWER

There are 14 cubes in the structure, which sits on a table. The face of each cube has an area of 1 m^2. If you paint the exposed surface, how many square metres do you paint?



1.2 Applying Square Roots

Activity: Use a Formula

The distance you can see out to sea when you stand on a beach is given by the formula

$$d = 0.35\sqrt{h}$$

In this formula, h is the height, in centimetres, of your eyes above the water, and d is the distance you can see in kilometres.

Inquire

1. The height of Sandra's eyes is 144 cm. How far can she see from a beach?

2. If Sandra sits on a lifeguard tower, and her eyes are 350 cm above the water, how far can she see?

3. Explain why the formula may not apply if you are not standing on a beach.

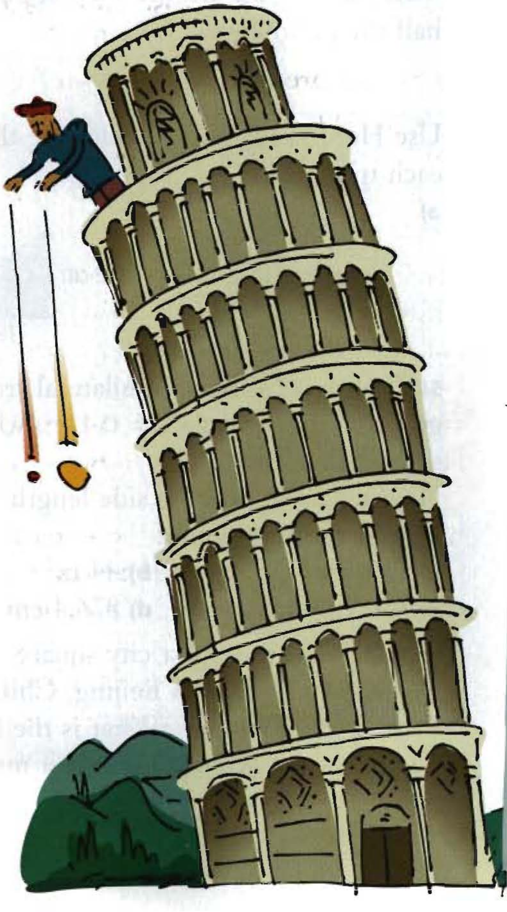
4. If you stand on a beach and watch an approaching sailboat, why do you see the mast of the boat before the hull?

Example

For many years, people believed Aristotle's theory of gravity, which stated that the heavier an object, the faster it would fall. Galileo disproved this theory by dropping 2 objects of different masses from the Leaning Tower of Pisa. They hit the ground at the same time.
The formula for calculating the time it takes an object to fall to the ground is

$$t = 0.45 \times \sqrt{h}$$

where h is the height in metres and t is the time in seconds. The Leaning Tower of Pisa is 54.5 m tall. How long would it take an object dropped from the top to hit the ground? Give your answer to the nearest tenth of a second.



Solution

$$\begin{aligned} t &= 0.45\sqrt{h} \\ &= 0.45 \times \sqrt{54.5} \\ &\doteq 0.45 \times 7.38 \\ &= 3.321 \end{aligned}$$

EST $0.5 \times 7 = 3.5$

It would take 3.3 s, to the nearest tenth of a second, for the object to hit the ground.

Practice

Estimate. Then, evaluate to the nearest tenth.

1. $\sqrt{46}$

2. $\sqrt{303}$

3. $-\sqrt{90}$

4. $5\sqrt{66}$

5. $-7\sqrt{88}$

6. $\sqrt{13} + \sqrt{79}$

7. $3\sqrt{7} + 6\sqrt{91}$

8. $16\sqrt{56} - 4\sqrt{11}$

9. $\frac{2\sqrt{45}}{3}$

10. $\frac{5\sqrt{10} + 6\sqrt{21}}{4}$

11. $\frac{\sqrt{123}}{\sqrt{60}}$

12. $\frac{\sqrt{112}}{\sqrt{13}}$

Problems and Applications

Given the area of each square, determine to the nearest tenth of a unit

a) *the length of its side*

b) *its perimeter*

13. 25 cm^2

14. 225 m^2

15. 55 m^2

16. 90 cm^2

17. 200 m^2

18. 800 cm^2

19. Land areas are measured in hectares (ha).

$$1 \text{ ha} = 10\ 000 \text{ m}^2$$

a) If a square field has an area of 1 ha, what are its dimensions?

b) A house lot in a big city may be only about 0.25 ha in area. If this amount of land were square, what would its dimensions be?

20. The water at the surface of a river moves faster than the water near the bottom. The formula that relates these two speeds is

$$\sqrt{b} = \sqrt{s} - 1.3$$

where b is the speed near the bottom and s is the speed at the surface. Both speeds are in kilometres per hour.

a) What is the speed of the water near the bottom of a river, to the nearest tenth of a kilometre per hour, if the surface speed is 16 km/h? 20 km/h?

b) What is the surface speed of the water, to the nearest tenth of a kilometre per hour, if the speed near the bottom is 9 km/h? 7 km/h?

21. The velocity of sound changes with temperature and with the material the sound passes through. The formula to calculate the velocity in air is

$$V = 20\sqrt{273 + T}$$

where V is the velocity in metres per second and T is the temperature in degrees Celsius. Calculate the velocity of sound in air, to the nearest metre per second, at

a) 16°C **b)** 0°C **c)** −10°C

22. For a satellite to stay in orbit, its speed must satisfy the formula

$$s = \sqrt{\frac{5.15 \times 10^{12}}{d}}$$

where s is the speed in kilometres per hour and d is the distance in kilometres from the satellite to the centre of the Earth. If a satellite is in orbit 35 840 km above the Earth, it hovers over one spot on the Earth. Research the radius of the Earth and determine the speed of a satellite that hovers over one spot on the Earth. Compare your answer with a classmate's.

PATTERN POWER

a) Evaluate the following differences in the squares of consecutive whole numbers.

$1^2 - 0^2 =$ ■
$2^2 - 1^2 =$ ■
$3^2 - 2^2 =$ ■
$4^2 - 3^2 =$ ■
$5^2 - 4^2 =$ ■

b) Describe the pattern.

c) Use the pattern to evaluate mentally $245^2 - 244^2$.

1.3 Guess And Check

You can solve many problems by guessing the answer and then testing it to see if it is correct. If it is wrong, you can keep guessing until you get the right answer.

Jeannine wanted to raise $10 000 for charity by holding a massed bands concert in a 400-seat auditorium. She decided to sell reserved seats for $40 each and general admission seats for $16 each. How many of each type of ticket did she have printed?

1. What information are you given?

2. What are you asked to find?

Guess at the number of reserved tickets to be printed. This number will give the number of general admission tickets. Then, calculate the total receipts to see if they equal $10 000. If not, try another guess.

GUESS **CHECK**

Number of Reserved Tickets	Number of General Admission Tickets	Total Revenue ($)	Is the total $10 000?
100	300	$100 \times 40 + 300 \times 16$ $= 8800$	Too low
200	200	$200 \times 40 + 200 \times 16$ $= 11\,200$	Too high
150	250	$150 \times 40 + 250 \times 16$ $= 10\,000$	$10\,000 = 10\,000$

CHECKS!

Jeannine had 150 reserved seating tickets and 250 general admission tickets printed.

Check the answer against the given information.

Does the answer seem reasonable?

Guess and Check

1. Guess an answer that fits one of the facts.
2. Check the answer against the other facts.
3. If necessary, adjust your guess and check again.

Problems and Applications

1. Multiplying a number by 7 and adding 13 gives 55. What is the number?

2. Multiplying a number by 14 and subtracting 12 gives 114. What is the number?

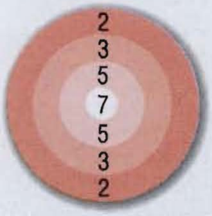
3. What 3 consecutive numbers have a sum of 237?

4. What 4 consecutive even numbers have a sum of 196?

5. The square of the number of students in the band is close to 3000. About how many students are in the band?

6. The 7 Trudeaus went to the fair. Adult tickets cost $8. Children's tickets cost $5. The total cost was $47. How many adults were there?

7. In how many ways could 4 darts hit the board to give a total score of 19?



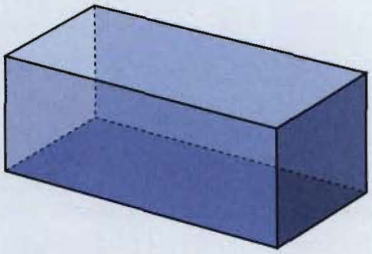
8. The perimeter of a rectangle is 60 m. The length is 4 m more than the width. What are the dimensions of the rectangle?

9. Karen wants to cut 56 m of television cable into 2 pieces, so that one is 6 m longer than the other. How long should she make each piece?

10. The cube of the number of people on the swim team is close to 4000. About how many are on the swim team?

11. Justine has some rare quarters and nickels. She has 8 more nickels than quarters. The face value of the coins is $2.50. How many are quarters?

12. The dimensions of a rectangular solid are whole numbers. The areas of the faces are 42 cm^2, 48 cm^2, and 56 cm^2. What are the dimensions of the solid?



13. Piero works at the Snack Shack. He has been asked to make a mixture of 24 kg of cashews and peanuts that will cost $7.25/kg. Cashews cost $14/kg and peanuts cost $5/kg. Copy and complete the table to find how many kilograms of cashews and how many kilograms of peanuts should be mixed together so that the cost of the mixture is $7.25/kg.

GUESS **CHECK**

Mass of Cashews (kg)	Mass of Peanuts (kg)	Total Cost of Mixture ($)	Cost per Kilogram ($)	Does cost equal $7.25/kg?

14. Canada's top honour is the Order of Canada. There are 3 categories— Companion, Officer, and Member. A total of up to 165 awards can be given in any year. If all the awards are given, the number of new Members is twice the number of new Officers. There are 35 more new Officers than new Companions. How many awards are made in each category?

15. Write a problem that can be solved using the guess and check strategy. Ask a classmate to solve the problem.

1.4 Exponents and Powers

Exponents are used as a short way to write repeated multiplication.

standard form

$$81 = \underbrace{3 \times 3 \times 3 \times 3}_{\text{repeated multiplication}} = 3^4$$

exponential form

$$81 = 3^4$$

exponent

base

power

Activity: Do an Experiment

When you repeatedly fold a piece of paper in half, the number of layers increases with the number of folds. Fold a standard piece of paper, and copy and complete the table.

Number of Folds	Number of Layers
1	$2 = 2$ or 2^1
2	$2 \times 2 = 4$ or 2^2
3	
4	
5	
6	

Inquire

1. If you were to fold the piece of paper the following numbers of times, how many layers would you have? Express each answer in exponential form.

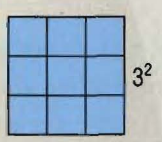
a) 5 **b)** 7 **c)** 50

2. Explain how you found your answers to question 1.

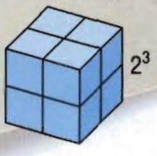
3. If 10 layers of paper are about 1 mm thick, how thick is a piece of paper after 10 folds?

4. What is the maximum number of times you can fold a piece of paper?

$3 \times 3 = 3^2$ 3^2 is read as "three to the second" or more commonly "three squared" because it can be pictured as a square.



$2 \times 2 \times 2 = 2^3$ 2^3 is read as "two to the third" or "two cubed" because it can be pictured as a cube.



Exponents are also used with variables.

$3y^4$ means $3 \times y \times y \times y \times y$ xy^2 means $x \times y \times y$

A number that multiplies a variable is known as a **coefficient**. In $3y^4$, the coefficient is 3.

When an exponent is outside a pair of brackets, the exponent is applied to everything inside the brackets.

$$(3y)^4 \text{ means } (3y) \times (3y) \times (3y) \times (3y)$$
$$= 3 \times 3 \times 3 \times 3 \times y \times y \times y \times y$$
$$= 81y^4$$

$$(xy)^2 \text{ means } (xy) \times (xy)$$
$$= x \times x \times y \times y$$
$$= x^2y^2$$

Example 1

If $x = 2$ and $y = -3$, evaluate $5x^4 + 6xy$.

Solution

Substitute the values of x and y into the expression.

$$\begin{aligned} 5x^4 + 6xy &= 5(2)^4 + 6(2)(-3) \\ &= 5(16) - 36 \\ &= 80 - 36 \\ &= 44 \end{aligned}$$

Practice

State the base and the exponent.

1. 5^3 **2.** 10^7 **3.** x^5 **4.** t^2

State the coefficient.

5. $2a$ **6.** $-7d^2$ **7.** $13x^5$ **8.** $-t^3$

Write in exponential form.

9. $4 \times 4 \times 4 \times 4 \times 4 \times 4$

10. $6 \times 6 \times 6 \times 6$

11. $m \times m \times m \times m \times m$

12. $r \times r \times r$

Write as a repeated multiplication.

13. 5^2 **14.** 1^6 **15.** 2^5 **16.** 10^4

17. 0^3 **18.** y^4 **19.** $5x^3$ **20.** $(2m)^3$

21. x^2y **22.** xy^3 **23.** $(xy)^3$ **24.** $(ab)^4$

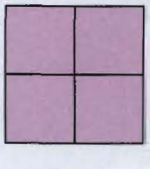
Evaluate.

25. the third power of 2 **26.** 3 to the fourth

27. 5 cubed **28.** 10 to the fifth

What power does each figure represent?

29.



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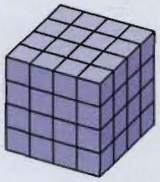

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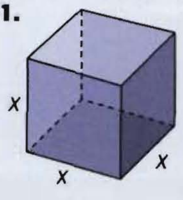

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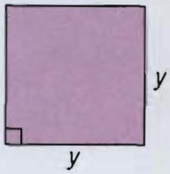


Write each number as a power of 10.

33. 100 **34.** 1000

35. 100 000 **36.** 1 000 000

37. 100 000 000 **38.** 10 000 000

Write in standard form.

39. 2^5 **40.** 5^3 **41.** 4^4 **42.** 7^3 **43.** 10^7

44. 3^6 **45.** 0.5^2 **46.** 1.1^3 **47.** 0.1^4

Write as a power of 2 or 3 and draw the square or cube represented by it.

48. 4 **49.** 27 **50.** 16

State which is smaller.

51. 5^3 or 3^5 **52.** 2^5 or 5^2

53. 4^2 or 2^4 **54.** 2^3 or 3^2

Evaluate.

55. $7^2 + 3^2$ **56.** $4^3 - 2^4$

57. 3×2^3 **58.** $4^4 \div 2^5$

59. 100×0.1^3 **60.** $0.8^2 \times 0.2^3$

61. 1000×0.2^4 **62.** 0.1×0.1^2

Evaluate.

63. $7^2 + 3^3$ **64.** $4^3 - 2^2$

65. $2^2 \times 2^3$ **66.** $4^2 \div 2^2$

67. Evaluate for $x = -4$.

a) x^3 **b)** $x^2 - 5$

c) $5x^2 - 7$ **d)** $(2x)^2$

68. Evaluate for $t = -3$ and $s = 2$.

a) $t^2 + s^2$ **b)** $(t + s)^3$ **c)** $t^3 - s^3$

d) $2t^2 - s^2$ **e)** $6s^3 - 2st$ **f)** $(3t)^2 - 4st$

CONTINUED

Problems and Applications

69. A scientist found that the number of bacteria in a culture doubled every hour. If there were 1000 bacteria at 08:00, how many were there at the following times?

a) 09:00 **b)** 11:00 **c)** 14:00

70. The ancestors of Ling Ling, the giant panda, included 2 parents and 4 grandparents. Her grandparents were 2 generations before her.

a) How many ancestors were in the seventh generation before Ling Ling? Express your answer in exponential form.

b) How many ancestors were in the tenth generation before Ling Ling? Express your answer in exponential form.

71. If a ball is thrown straight up at a speed of 30 m/s, its height in metres after t seconds is given by the formula $h = 30t - 5t^2$.

a) What is the height of the ball after 1 s? 3 s? 4 s?

b) After how many seconds will the ball hit the ground? State your assumptions.

72. a) Describe the pattern in this series.

$$1, 4, 27, 256, \ldots$$

b) Predict the fifth and tenth numbers in the series.

73. The product of the ages of a set of quadruplets is 16 times the sum of their ages. How old are they?

74. The product of the ages of a set of triplets is 12 times the sum of their ages. How old are they?

75. If $f(x) = 3x^2 + 1$

$$\begin{aligned} \text{then } f(2) &= 3(2)^2 + 1 \\ &= 3 \times 4 + 1 \\ &= 13 \end{aligned}$$

Find the following.

a) $f(4)$ **b)** $f(10)$ **c)** $f(0)$

76. Decide whether each statement is always true, sometimes true, or never true for whole numbers greater than 1. Explain your reasoning.

a) Twice a number is smaller than the number squared.

b) The cube of a number is greater than the square of the number.

c) Powers with the same base but different exponents are equal.

77. A story says that the inventor of chess asked his ruler to give him 1 grain of wheat on the first square of the chessboard, 2 on the second, 4 on the third, 8 on the fourth, and so on for all 64 squares.

Suppose you could stack loonies on a chessboard in the same manner.

a) How many loonies would you stack on the fourth square? the fifth square? the sixth square? the seventh square?

b) About how high would the stack be on the sixty-fourth square? Compare your result with your classmates'.

CALCULATOR POWER

Many calculators have a Y^x key to evaluate powers.

To evaluate 2^{12}

Press	2	Y^x	12	=
Display	2	2	12	4096

Use a calculator to evaluate the following.

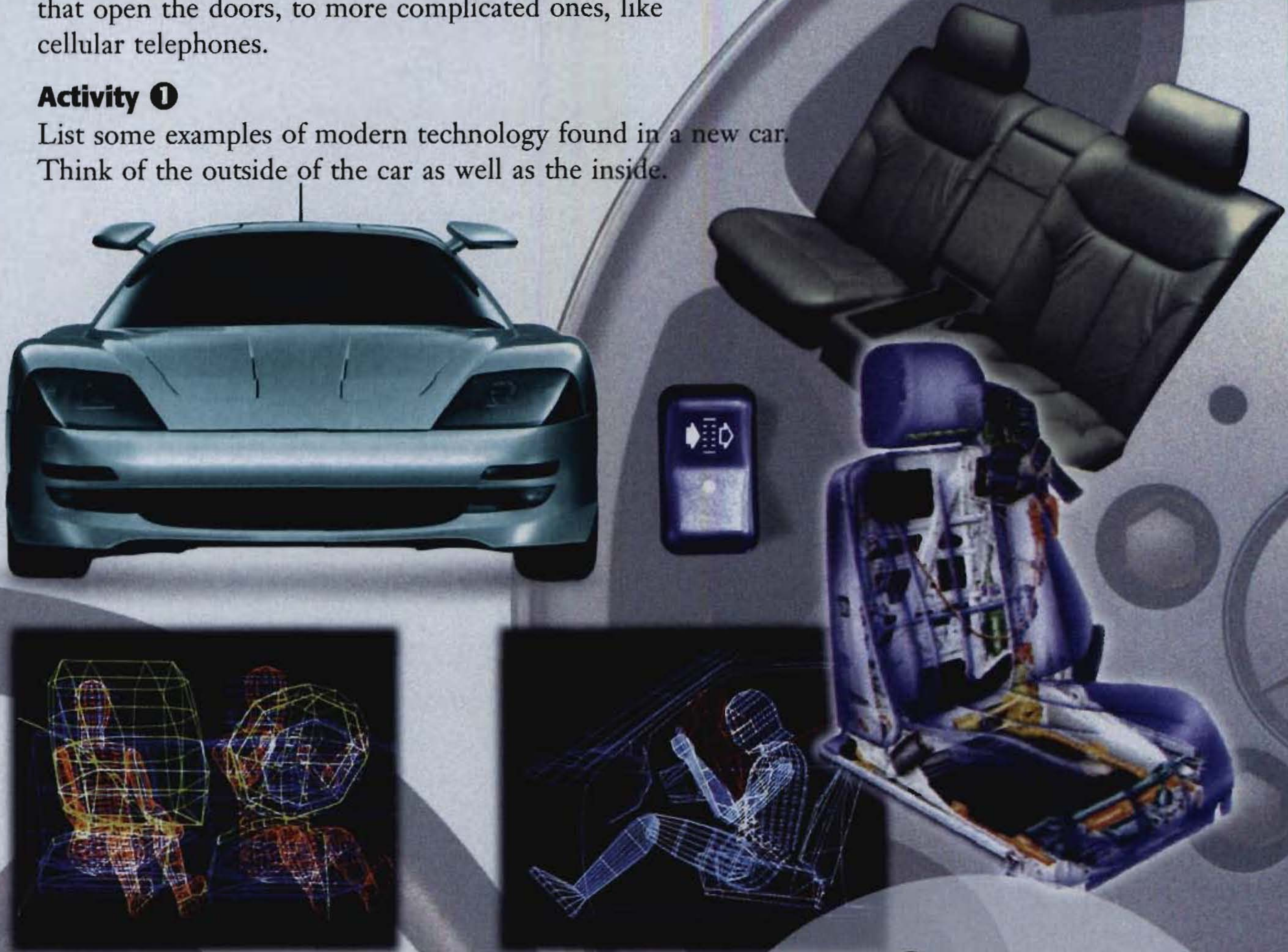
1. 2^6 **2.** 3^7 **3.** 4^8 **4.** 0.3^3

The Car of the Future

There are many examples of technology in modern cars. These range from simple devices, such as the levers and buttons that open the doors, to more complicated ones, like cellular telephones.

Activity ❶

List some examples of modern technology found in a new car. Think of the outside of the car as well as the inside.



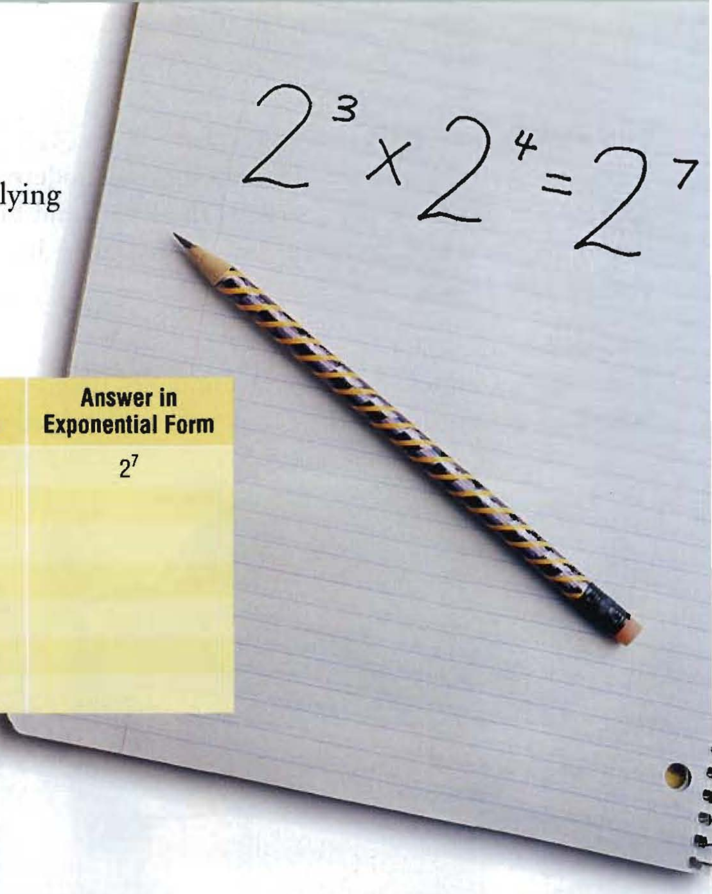
Activity ❷

Design a high technology car of the future and draw plans to show what it might look like. As you design the car, you will need to resolve such issues as the number of passengers it will carry, how the driver will steer it, the fuel the car will use, and so on.

Describe the uses of computers in your car of the future.

1.5 The Exponent Rules

The exponent rules are short cuts for multiplying and dividing powers with the *same base.*



Activity: Discover the Relationship

Copy and complete the table.

Exponential Form	Standard Form	Answer in Standard Form	Answer in Exponential Form
$2^3 \times 2^4$	8×16	128	2^7
$3^2 \times 3^2$			
$2^2 \times 2^3$			
$10^3 \times 10^3$			
$2^5 \div 2^2$			
$3^4 \div 3^2$			
$10^5 \div 10^3$			

Inquire

1. For the products, how are the exponents in the first column related to the exponents in the last column?

2. Write a rule for multiplying powers with the same base.

3. For the quotients, how are the exponents in the first column related to the exponents in the last column?

4. Write a rule for dividing powers with the same base.

Example 1

Simplify $2^5 \times 2^3$.

Solution

Method 1

$$\begin{aligned} & 2^5 \times 2^3 \\ &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^8 \end{aligned}$$

Method 2

To multiply powers with the same base, add the exponents.

$$\begin{aligned} x^m \times x^n &= x^{m+n} \\ 2^5 \times 2^3 &= 2^{5+3} \\ &= 2^8 \end{aligned}$$

Example 2

Simplify $y^7 \div y^2$.

Solution

Method 1

$$\begin{aligned} & y^7 \div y^2 \\ &= \frac{y \times y \times y \times y \times y \times y \times y}{y \times y} \\ &= y \times y \times y \times y \times y \\ &= y^5 \end{aligned}$$

Method 2

To divide powers with the same base, subtract the exponents.

$$\begin{aligned} x^m \div x^n &= x^{m-n} \\ y^7 \div y^2 &= y^{7-2} \\ &= y^5 \end{aligned}$$

Example 3

Simplify.

a) $(2^4)^3$ **b)** $(y^2)^4$

Solution

a) $(2^4)^3 = 2^4 \times 2^4 \times 2^4$
$= 2^{4+4+4}$
$= 2^{12}$

b) $(y^2)^4 = y^2 \times y^2 \times y^2 \times y^2$
$= y^{2+2+2+2}$
$= y^8$

To raise a power to a power, multiply the exponents.
$(x^m)^n = x^{m \times n}$

Practice

Simplify.

1. $5^3 \times 5^4$
2. $2^3 \times 2^7$
3. $7^5 \times 7^5$
4. $10^6 \times 10$
5. $4^2 \times 4^9$
6. 3×3^6
7. $y^2 \times y^4$
8. $x^3 \times x^6$
9. $a \times a^6$

Find the value of x.

10. $5^3 \times 5^x = 5^7$
11. $3^4 \times 3^x = 3^6$
12. $8^x \times 8^2 = 8^8$
13. $6^x \times 6^5 = 6^6$
14. $4^3 \times 4^x = 4^6$
15. $m^7 \times m^x = m^9$
16. $t^x \times t^3 = t^6$
17. $y^x \times y^4 = y^5$

Simplify.

18. $4^5 \div 4^3$
19. $3^7 \div 3^6$
20. $9^2 \div 9^2$
21. $10^6 \div 10^5$
22. $4^7 \div 4$
23. $5^8 \div 5^8$
24. $m^5 \div m^4$
25. $x^3 \div x$

Find the value of x.

26. $3^6 \div 3^x = 3^2$
27. $6^7 \div 6^x = 6^5$
28. $7^x \div 7^4 = 7^3$
29. $2^x \div 2^2 = 2^8$
30. $9^5 \div 9^x = 9^4$
31. $m^x \div m^2 = m^7$
32. $m^x \div m = m^3$
33. $y^6 \div y^x = y$

Simplify.

34. $(2^3)^4$
35. $(3^5)^2$
36. $(4^2)^7$
37. $(10^5)^3$
38. $(5^4)^4$
39. $(x^5)^4$
40. $(y^3)^3$
41. $(t^6)^7$
42. $(m^1)^5$

Find the value of x.

43. $(2^3)^x = 2^6$
44. $(3^x)^4 = 3^{12}$
45. $(5^x)^2 = 5^8$
46. $(7^5)^x = 7^{10}$
47. $(x^3)^x = x^9$
48. $(m^x)^5 = m^{15}$
49. $(t^4)^x = t^{20}$
50. $(z^x)^5 = z^5$

Problems and Applications

51. The approximate size of a quantity, expressed as a power of 10, is known as an **order of magnitude**. To the nearest orders of magnitude, the mass of the Earth is 10^{25} kg and the mass of the sun is 10^{30} kg. About how many times greater is the mass of the sun than the mass of the Earth?

52. On a test, a student wrote that $2^3 \times 3^2 = 6^5$.

a) What mistake did the student make?
b) What is the value of $2^3 \times 3^2$?

53. A student said that $6^3 \div 2^2 = 3^1$.

a) What mistake did the student make?
b) What is the value of $6^3 \div 2^2$?

54. Work with a partner. Use each of the digits from 1 to 6 only once. Write the largest and the smallest power possible.

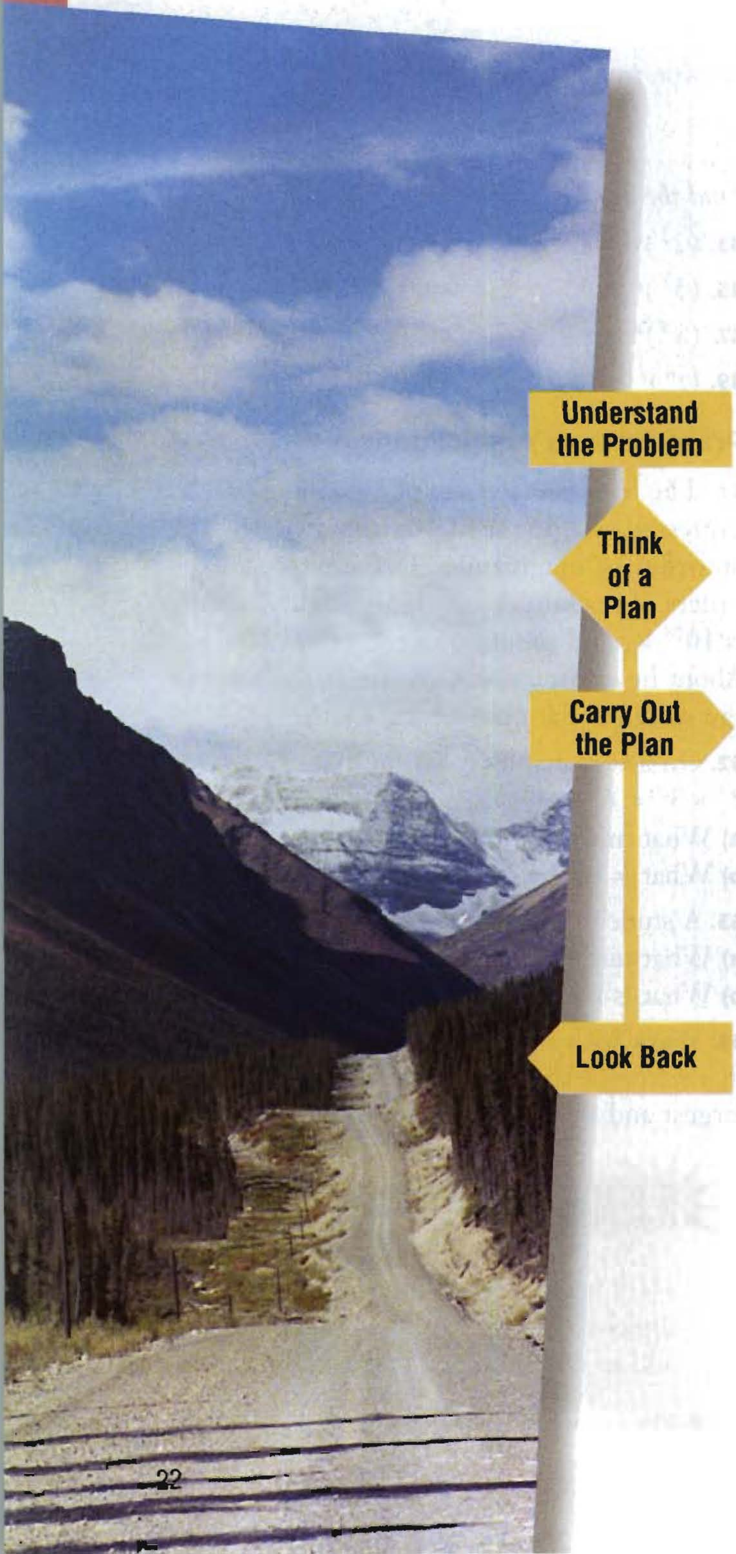
NUMBER POWER

Here is one way to use the digits 1, 2, 3, 5, 7, and 9 to add to 648. Find 3 other ways.

$$\begin{array}{r}251\\+397\\\hline 648\end{array}$$

PROBLEM SOLVING

1.6 Make Assumptions



To solve some problems, you must make assumptions.

The Alaska Highway runs from Dawson Creek, B.C., to Fairbanks, Alaska. It is 2400 km long. The speed limit is 80 km/h. The De Marco family is leaving Dawson Creek at 09:00 on a Wednesday to drive to Fairbanks. They plan to stop for 10 h to eat and sleep on Wednesday night and on Thursday night. At what time on what day should they arrive in Fairbanks?

1. What information are you given?
2. What are you asked to find?
3. What assumptions should you make?

Assume that the De Marco family will drive at the speed limit. Calculate the time they will spend driving. Add 20 h to this time for eating and sleeping.

If the De Marco family drives at the 80 km/h speed limit, the time needed to drive 2400 km is found by dividing 2400 km by 80 km/h.

$$\frac{2400}{80} = 30$$

Thirty hours from 09:00 on Wednesday is 15:00 on Thursday. Add 20 h for eating and sleeping.

The De Marco family should arrive in Fairbanks at 11:00 on Friday.

How could you use subtraction and multiplication to check your answer?

Make Assumptions

1. Decide what assumption(s) to make.
2. Use your assumption(s) to solve the problem.
3. Check that your answer is reasonable.

Problems and Applications

Solve the following problems and state each assumption that you make.

1. Frank earned $215.75 in the first month at his part-time job. How much can he expect to earn in a year?

2. The drama club sells oranges to raise money for charity. Last year, each member of the club sold 20 cases. How many cases can the 43 members of the club expect to sell this year?

3. Clara surveyed 40 students in the school. Ten of them said they would attend the dance. If the school has 800 students, how many can Clara expect to attend the dance?

4. The patrol boat travels at a speed of 15 km/h. How far can the boat travel in 5 h?

5. John trained for 5 weeks and reduced his time in the 100-m dash from 12.0 s to 11.5 s. He reasoned that with 20 more weeks of training he would be able to run 100 m in 9.5 s and break the world record. What assumption did he make? Is he necessarily correct? Explain.

6. Soo Lin surveyed 100 people who bought new bikes. Of those surveyed, 80 said the town needed new bike trails. There are 10 000 people in the town, so Soo Lin reported that 8000 people wanted new bike trails. What assumption did she make? Explain.

7. Six tents are placed 5.2 m apart in a straight line. What is the distance from the first tent to the last tent?

8. How many cuts must you make in a rope to make the following number of pieces?

a) 3 **b)** 4

9. How many seconds are there in February?

10. The distance from Eagle's Nest to Brewsterville is 425 km. The first part of the trip is 150 km of highway, where the speed limit is 100 km/h. The rest of the trip is along a country road, where the speed limit is 50 km/h.

a) How long will it take Paulina to drive from Eagle's Nest to Brewsterville?

b) If she leaves Eagle's Nest at 16:45, at what time will she arrive in Brewsterville?

11. Identify the next 3 terms in each of the following.

a) 200, 100, 50, 25, …

b) 14, 17, 21, 26, …

c) 3, 7, 15, 31, …

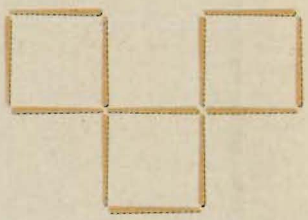
12. The world's longest sneezing fit lasted 977 days. In the first year of the fit, Donna Griffiths sneezed about a million times. How many seconds did she average between sneezes?

13. The price for a carton of juice doubles every 2 years for 50 years. What would a carton cost in 50 years? Find an alternative way of solving this problem using exponents.

14. Write a problem in which the solution requires at least one assumption. Have a classmate solve your problem and state the assumption(s).

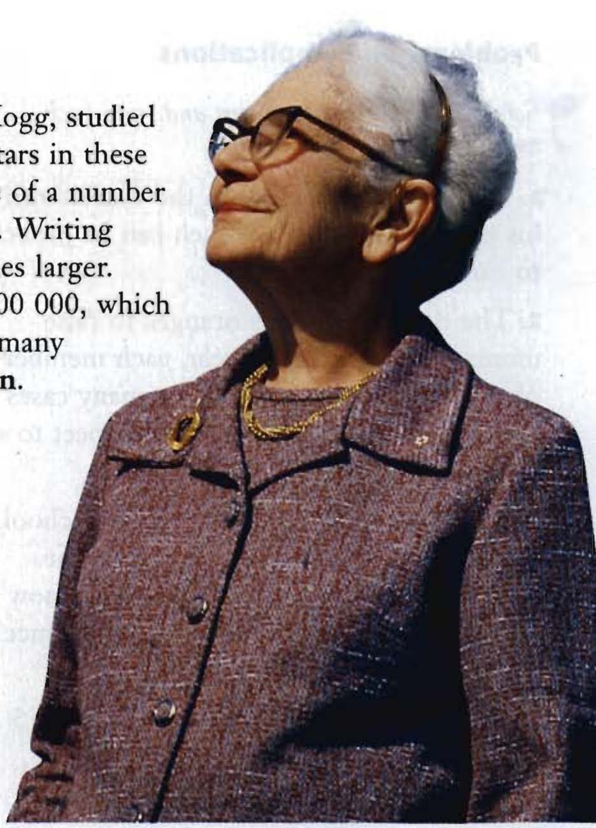
LOGIC POWER

Move 3 sticks to form 4 squares of the same size.



1.7 Scientific Notation: Large Numbers

A leading Canadian astronomer, Dr. Helen Sawyer Hogg, studied huge balls of stars, known as globular clusters. The stars in these clusters are about 14 000 000 000 years old. The size of a number like 14 000 000 000 depends on the number of zeros. Writing another zero gives 140 000 000 000, which is ten times larger. Removing a zero from 14 000 000 000 gives 1 400 000 000, which is ten times smaller. To avoid mistakes when writing many zeros, we express large numbers in **scientific notation.**



Activity: Look for a Pattern

Copy and complete the table.

Standard Form	Product Form	Scientific Notation
410	4.1 × 100	4.1×10^2
4 100	4.1 × ■	■
41 000	4.1 × ■	■
410 000	4.1 × ■	■
4 100 000	4.1 × ■	■
41 000 000	4.1 × ■	■

Inquire

1. Use the completed table to write the rule for expressing numbers in scientific notation.

2. Why would you not write 2356 in scientific notation?

Example 1

Mercury is about 58 000 000 km from the sun. Write 58 000 000 in scientific notation.

Solution

In scientific notation, a number has the form $x \times 10^n$, where x is greater than or equal to 1 but less than 10, and 10^n is a power of 10.

The decimal point starts here.

$\underbrace{58\ 000\ 000}$

Move the decimal point 7 places.

$= 5.8 \times 10\ 000\ 000$

$= 5.8 \times 10^7$

Notice that the number of places you moved the decimal point to the left is the exponent in the power of 10.

Example 2

Calculate $(4.5 \times 10^3) \times (8 \times 10^5)$. Write your answer in scientific notation.

Solution

$$\begin{aligned}(4.5 \times 10^3) \times (8 \times 10^5) &= 4.5 \times 8 \times 10^3 \times 10^5 \\ &= 36 \times 10^{3+5} \\ &= 36 \times 10^8 \\ &= 3.6 \times 10^9\end{aligned}$$

Practice

1. Copy and complete the chart. The first line has been completed for you.

7 200	7.2×1000	7.2×10^3
45 000	■	■
■	$8.5 \times 10\,000$	■
■	$1.1 \times 100\,000$	■
■	■	9.78×10^8
■	■	2.03×10^7

State each value of n.

2. $6000 = 6 \times 10^n$

3. $7\,100\,000 = 7.1 \times 10^n$

4. $35\,000 = 3.5 \times 10^n$

5. $54\,000\,000 = 5.4 \times 10^n$

6. $145\,000 = 1.45 \times 10^n$

7. $460\,000\,000 = 4.6 \times 10^n$

Write the number that is 10 times as large as each of the following.

8. 77 000

9. 6700

10. 7.6×10^5

11. 9.8×10^7

Write the number that is one-tenth as large as each of the following.

12. 350

13. 2.3×10^5

14. 6.7×10^4

15. 1 300 000

Estimate, then calculate. Write each answer in scientific notation.

16. $(3.4 \times 10^5) \times (5 \times 10^4)$

17. $(4 \times 10^8) \times (1.2 \times 10^7)$

18. $(6.7 \times 10^3) \times (8.9 \times 10^{10})$

Round each number to the highest place value, then calculate using scientific notation.

19. $23\,000\,000 \times 341\,000$

20. $870\,600\,000 \times 710\,000$

21. $92\,000\,000 \div 56\,000$

22. $83\,000\,000\,000 \div 777\,000\,000$

Problems and Applications

23. Express each number in scientific notation.

a) Canada is the country with the longest coastline at about 91 000 km.

b) The distance across the universe has been estimated at 800 000 000 000 000 000 000 000 km.

c) Do you want to hunt for sunken treasure? The *HMS Edinburgh* went down with $95 000 000 in gold on board.

24. How far can you see? The Andromeda Constellation is the furthest object we can see with the unaided eye. It is about 22 000 000 000 000 000 km from Earth.

a) Express this distance in scientific notation.

b) Light travels in space at a speed of about 300 000 km/s. Calculate how many years light takes to reach Earth from the Andromeda Constellation.

25. Why is 56×10^6 not in scientific notation?

26. Work with a classmate.

a) Find the area of Canada in square metres.

b) Find the population of Canada.

c) Calculate how many square metres there are for each Canadian.

CALCULATOR POWER

1. Multiply $1\,400\,000 \times 3\,000\,000$ using your calculator. How does your calculator display the answer?

2. Scientific calculators display the answer as `4.2          12`. What does it mean?

3. Many calculators have an EE key. To input 2.3×10^7, press **2** · **3** EE **7**. Use the EE key to find

a) $(4.5 \times 10^4) \times (5.1 \times 10^7)$

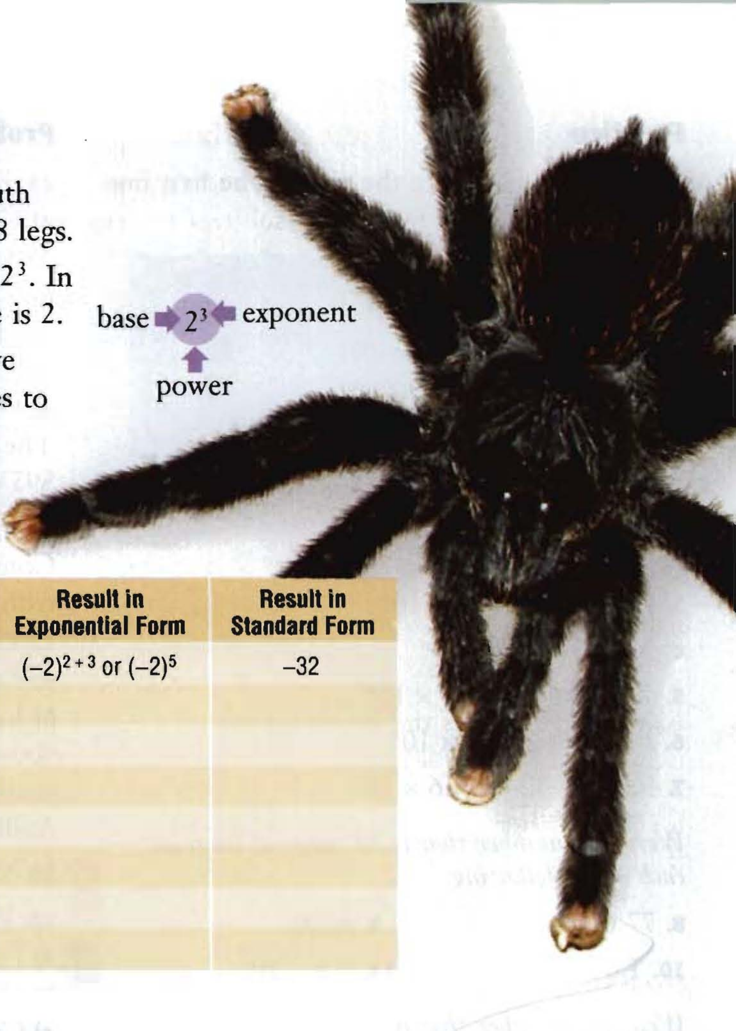
b) $(7.8 \times 10^{11}) \times (6.7 \times 10^9)$

1.8 Working with Exponents

The leg span of the giant spider from South America is 25 cm. Like all spiders, it has 8 legs.

We can write the number 8 as the power 2^3. In this power, the exponent is 3 and the base is 2.

The base of a power can also be a negative number. We can extend the exponent rules to simplify powers with integers as bases.



Activity: Look for a Pattern

Copy and complete the table.

Exponential Form	Result as a Repeated Multiplication	Result in Exponential Form	Result in Standard Form
$(-2)^2 \times (-2)^3$	$(-2) \times (-2) \times (-2) \times (-2) \times (-2)$	$(-2)^{2+3}$ or $(-2)^5$	-32
$(-2)^3 \times (-2)^4$			
$(+3)^2 \times (+3)^3$			
$(-2)^8 \div (-2)^3$			
$(-3)^4 \div (-3)^2$			
$(+5)^3 \div (+5)$			
$(-4)^3 \times (-4)^0$			
$(-3)^4 \div (-3)^0$			

Inquire

1. Write the exponent rule for multiplying powers with the same integral base.

2. Write the exponent rule for dividing powers with the same integral base.

3. Describe the difference between

a) $(1+3)^2$ and 1^2+3^2?

b) $(1-3)^2$ and 1^2-3^2?

c) $(-3)^2$ and -3^2?

Example 1

Simplify.

a) $(-0.25)^3 \times (-0.25)^5$

b) $(-y)^2 \times (-y)^3$

Solution

To multiply powers with the same base, add the exponents.

$$x^m \times x^n = x^{m+n}$$

a)
$$\begin{aligned}&(-0.25)^3 \times (-0.25)^5\\ &= (-0.25)^{3+5}\\ &= (-0.25)^8\end{aligned}$$

b)
$$\begin{aligned}&(-y)^2 \times (-y)^3\\ &= (-y)^{2+3}\\ &= (-y)^5\end{aligned}$$

Example 2

Simplify.

a) $(-6)^5 \div (-6)^2$

b) $\left(\frac{t}{2}\right)^3 \div \left(\frac{t}{2}\right)$

Solution

To divide powers with the same base, subtract the exponents.

$x^m \div x^n = x^{m-n}$

a)
$$\begin{aligned}&(-6)^5 \div (-6)^2\\ &= (-6)^{5-2}\\ &= (-6)^3\end{aligned}$$

b)
$$\begin{aligned}&\left(\frac{t}{2}\right)^3 \div \left(\frac{t}{2}\right)\\ &= \left(\frac{t}{2}\right)^{3-1}\\ &= \left(\frac{t}{2}\right)^2\end{aligned}$$

Example 3

Simplify.

a) $((-2)^2)^3$

b) $((-m)^4)^2$

Solution

a)
$$\begin{aligned}&((-2)^2)^3\\ &= (-2)^2 \times (-2)^2 \times (-2)^2\\ &= (-2)^{2+2+2}\\ &= (-2)^6\end{aligned}$$

b)
$$\begin{aligned}&((-m)^4)^2\\ &= (-m)^4 \times (-m)^4\\ &= (-m)^{4+4}\\ &= (-m)^8\end{aligned}$$

Example 3 suggests the following rule. To raise a power with an integral base to a power, multiply the exponents. $(x^m)^n = x^{m \times n}$

Example 4

Evaluate $(-5)^4(-5)^3$.

Solution

$$\begin{aligned}(-5)^4(-5)^3 &= (-5)^{4+3}\\ &= (-5)^7\\ &= -78\ 125\end{aligned}$$

C 5 +/− Yˣ 7 = −78 125.

Example 5

Evaluate $\left(-\frac{1}{2}\right)^3 \times \left(-\frac{1}{3}\right)^2$.

Solution

Because the bases are not the same, evaluate each power before multiplying.

$$\begin{aligned}&\left(-\frac{1}{2}\right)^3 \times \left(-\frac{1}{3}\right)^2\\ &= -\frac{1}{8} \times \frac{1}{9}\\ &= -\frac{1}{72}\end{aligned}$$

EST

$$\begin{aligned}&\left(-\frac{1}{10}\right) \times \left(\frac{1}{10}\right)\\ &= -\frac{1}{100}\end{aligned}$$

Example 6

Evaluate for $x = -2$ and $y = -1$.

a) $x^2 + y^3$

b) $\frac{x^3}{3} - \frac{y^2}{2}$

Solution

a)
$$\begin{aligned}&x^2 + y^3\\ &= (-2)^2 + (-1)^3\\ &= 4 - 1\\ &= 3\end{aligned}$$

b)
$$\begin{aligned}&\frac{x^3}{3} - \frac{y^2}{2}\\ &= \frac{(-2)^3}{3} - \frac{(-1)^2}{2}\\ &= \frac{-8}{3} - \frac{1}{2}\\ &= \frac{-16-3}{6}\\ &= -\frac{19}{6}\end{aligned}$$

CONTINUED

Practice

State the base.

1. $\left(\frac{1}{2}\right)^6$

2. $(-5)^2$

3. -1^4

4. $(-9)^3$

State the exponent.

5. -2^5

6. 4^2

7. $(-4)^0$

8. -5

Write in exponential form.

9. $\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$

10. $(-3)(-3)(-3)(-3)(-3)$

11. $p \times p \times p \times p \times p$

12. $(-n)(-n)(-n)(-n)$

13. $3 \times 3 \times 3 \times (-2) \times (-2) \times 3 \times (-2)$

Write as a repeated multiplication.

14. $(-2)^5$

15. -2^5

16. $\left(\frac{-1}{x}\right)^3$

Write in standard form.

17. 3^2

18. $(-3)^2$

19. $(-1)^4$

20. -1^5

21. $(-5)^3$

22. -5^3

23. $(-0.5)^3$

24. 1.1^4

25. $(-2.5)^2$

Simplify.

26. $5^3 \times 5^6$

27. $(-8)^2 \times (-8)^3$

28. $(-2)^3(-2)^4$

29. $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^4$

30. $(-2.1)^5(-2.1)^3$

31. $(-0.2)^3(-0.2)^2$

Simplify.

32. $5^4 \div 5^3$

33. $6^8 \div 6^2$

34. $\frac{(-0.4)^5}{(-0.4)^3}$

35. $\frac{(-9)^7}{(-9)^2}$

Simplify.

36. $(2^3)^2$

37. $((-3)^7)^4$

38. $\left(\left(-\frac{1}{5}\right)^2\right)^3$

39. $((-6)^5)^3$

40. $((-4)^6)^7$

41. $((-2.3)^3)^4$

Simplify.

42. $x^4 \times x^2$

43. $\left(\frac{1}{y}\right)^{12} \div \left(\frac{1}{y}\right)^5$

44. $z^8 \div z$

45. $(-m)^6(-m)^4$

46. $(s^2)^4$

47. $((-r)^3)^2$

Simplify, then calculate.

48. $(-5)^2 \times (-5)^3$

49. $6^2 \times 6^5$

50. $(-2)^3(-2)^5(-2)^2$

51. $(-1)^5(-1)^7$

52. $(-3.1)^5(-3.1)^3$

53. $(-3)^6 \div (-3)^4$

54. $(-10)^5 \div (-10)^1$

55. $(-4)^6 \div (-4)^5$

Calculate.

56. $2^8 \div 2^4$

57. $(-3)^7 \div (-3)^1$

58. $(-5)^2 \times (-5)^3$

59. $(3^2)^3$

60. $\frac{(-4)^3 \times (-4)^5}{(-4)^5}$

61. $\frac{4^9}{4^3 \times 4^2}$

62. $(-2)^3(-2)^5$

63. $(-3)^0(-3)^5$

64. $((-2)^3)^2$

65. $(6^2)^3 \div (6^2)^2$

Evaluate.

66. $(-8)^2$

67. $6(-4)^3$

68. $(-3)^2(6)^2$

69. $(-1)^5 + 3^3$

70. $4^5 - 3^5$

71. $(-2)^5 \times (-3)^4$

72. $9^2 \div (-2)^3$

73. $(-5)^2(-4)^4$

74. $(1.3)^2(-2)^4$

75. $(1.5)^2 \div (-5)^3$

76. Evaluate for $n = 3$.

a) $\frac{1}{5n^2}$ **b)** $-\frac{n^3}{6}$ **c)** $1 + 7n^5$ **d)** $n^3 - 6n$

77. Evaluate for $x = -2$ and $y = 3$.

a) x^3 **b)** $5y^4$ **c)** $\frac{x^2}{2} + \frac{y^2}{3}$

d) $\frac{x^3y^3}{3}$ **e)** $(x - y)^3$ **f)** $(y - x)^2$

g) $-\frac{x^2y^3}{8}$ **h)** $4x^3 - 5y$ **i)** $(3x^2)(-2y^2)$

Problems and Applications

78. Use the guess-and-check strategy to find the value of x.

a) $3^x = 81$

b) $(-2)^x = -512$

c) $x^5 = 1024$

d) $(-x)^3 = -1000$

e) $-5^x = -625$

f) $-x^2 = -1.69$

g) $(0.2)^x = 0.0016$

h) $x^3 = -0.216$

79. Find the value of x.

a) $x^2 \times x^3 = 32$ **b)** $x \times x^2 = 27$
c) $x^4 \div x^2 = 36$ **d)** $x^2 \div x = 64$
e) $(x^2)^2 = 81$ **f)** $(x^2)^3 = 64$

80. The formula for the volume, V, of a cube is

$$V = s^3$$

where s is the side length. Copy the chart and use the formula to complete it.

Side Length (cm)	Volume (cm³)
5	
7	
4.2	
	1000
	512

81. There are 10 bacteria in a culture at the beginning of an experiment. The number of bacteria doubles every 24 h. The table shows the number of bacteria present over the first few days.

Elapsed Time (days)	Number of Bacteria
1	10×2 $= 10 \times 2^1$ $= 20$
2	$10 \times 2 \times 2$ $= 10 \times 2^2$ $= 40$
3	$10 \times 2 \times 2 \times 2$ $= 10 \times 2^3$ $= 80$

a) Complete the table up to the end of day 8. How many bacteria are there after 8 days?
b) Use the pattern to calculate the number of bacteria after 2 weeks.
c) How many bacteria are there after 40 days?

82. If the base of a power is negative, and the exponent is even, is the standard form of the number positive or negative? Explain.

83. If the base of a power is negative, and the exponent is odd, is the standard form of the number positive or negative? Explain.

84. Any positive number has 2 square roots, but you can calculate only 1 side length if you are given the area of a square. Explain why.

85. a) Are the products $(-4)^3 \times (-4)^2$ and $(-4)^2 \times (-4)^3$ the same? Explain.
b) Are the quotients $(-4)^3 \div (-4)^2$ and $(-4)^2 \div (-4)^3$ the same? Explain.
c) Are the powers $((-4)^3)^2$ and $((-4)^2)^3$ the same? Explain.

86. A new student in your class does not know how to evaluate such powers as $(-2)^4$ and -2^4. Write an explanation and compare it with your classmates'.

PATTERN POWER

Many calculators have the bracket keys (and) to use with calculations involving the order of operations.

1. Estimate $(2.1 \times 3.2)^3$.

2. Calculate the expression in question 1 using this keying sequence:

C (2.1 × 3.2) y^x 3 =

3. Does your calculator give you the same answer if you omit the bracket keys in the keying sequence in question 2? Explain.

4. Why do some calculators give different answers for $(-2)^6$ and -2^6?

5. Estimate, then calculate.
a) $(4.2 \times 1.2)^3$
b) $(1.6 \times 1.3)^4$
c) $(2.5 \times 2.5)^3$

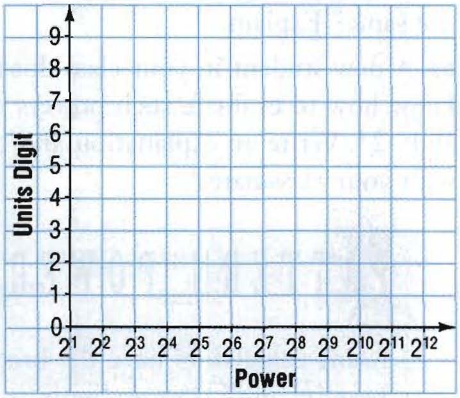
Patterns and Powers

What is the units digit of 2^{95}? To solve this problem, you might try using a calculator. What answer does your calculator give for 2^{95}? Another way to find the answer is to look for a pattern.

Activity ❶

To find the units digit of 2^{95}, complete the table and draw the graph.

$2^1 = 2$
$2^2 = 4$
$2^3 =$ ■
$2^4 =$ ■
$2^5 =$ ■
$2^6 =$ ■
$2^7 =$ ■
$2^8 =$ ■
$2^9 =$ ■
$2^{10} =$ ■
$2^{11} =$ ■
$2^{12} =$ ■



1. Describe the pattern in the units digits.

2. What is the units digit for the powers with exponents that are divisible by 4?

3. What is the units digit of

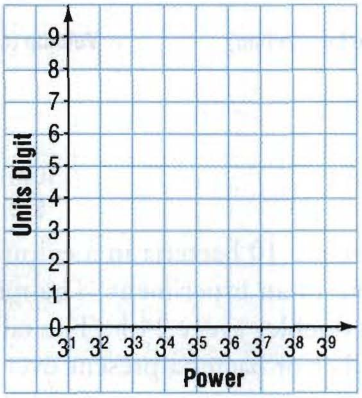
a) 2^{92}? **b)** 2^{93}? **c)** 2^{94}?

4. What is the units digit of 2^{95}?

Activity ❷

To find the units digit of 3^{100}, complete the table and draw the graph.

$3^1 =$ ■
$3^2 =$ ■
$3^3 =$ ■
$3^4 =$ ■
$3^5 =$ ■
$3^6 =$ ■
$3^7 =$ ■
$3^8 =$ ■
$3^9 =$ ■



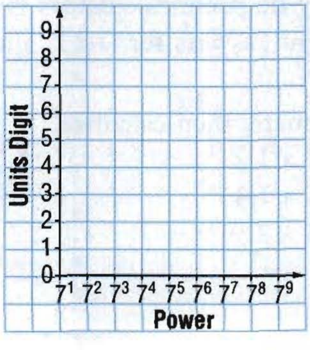
1. Describe the pattern in the units digits.

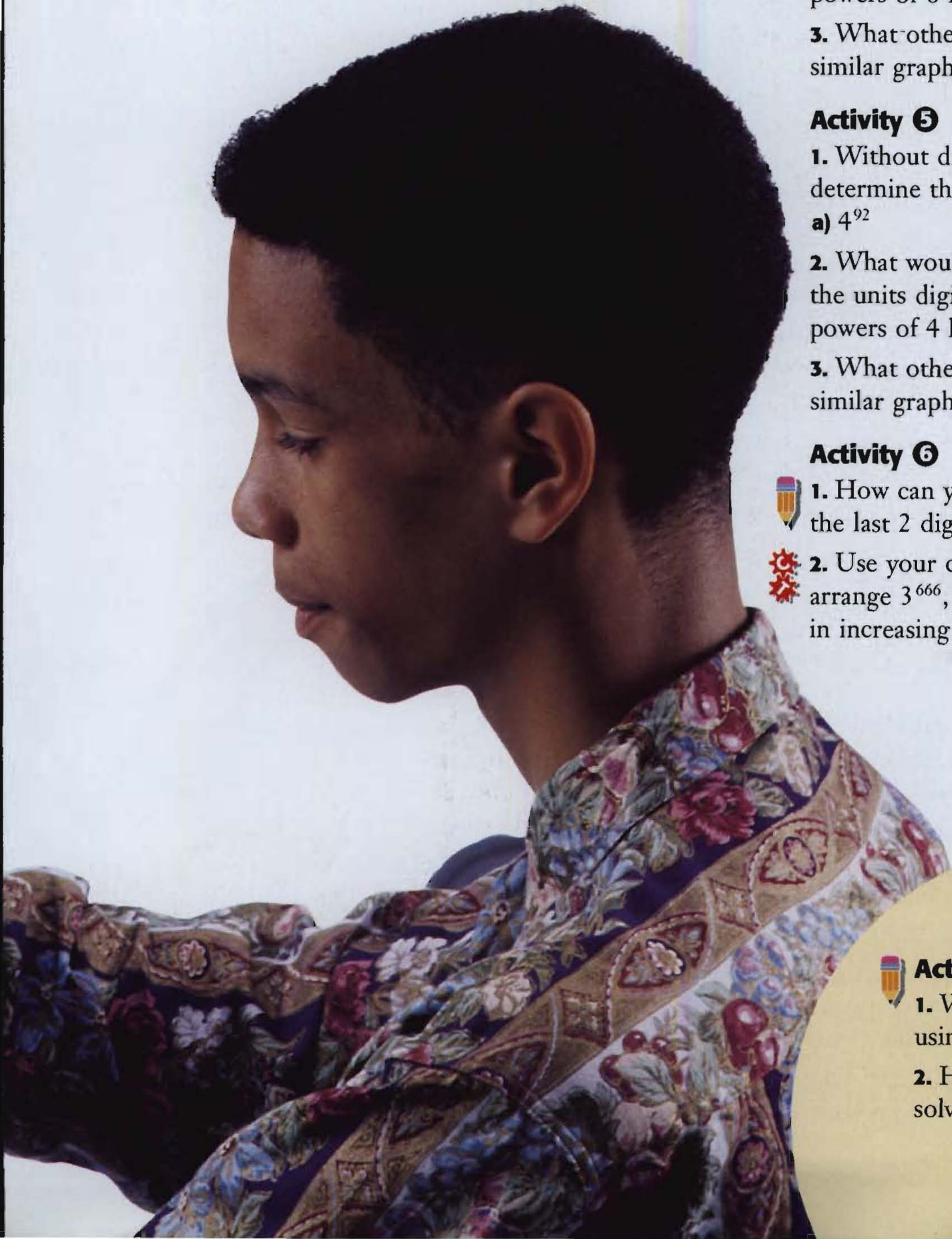
2. What is the units digit of 3^{100}?

Activity ❸

To find the units digit of 7^{111}, complete the table and draw the graph.

$7^1 =$ ■
$7^2 =$ ■
$7^3 =$ ■
$7^4 =$ ■
$7^5 =$ ■
$7^6 =$ ■
$7^7 =$ ■
$7^8 =$ ■
$7^9 =$ ■





Activity ❹

1. Without drawing a graph, determine the units digit of 6^{80}.

2. What would the graph of the units digits versus the powers of 6 look like?

3. What other bases give similar graphs?

Activity ❺

1. Without drawing a graph, determine the units digit of

a) 4^{92} **b)** 4^{93}

2. What would the graph of the units digits versus the powers of 4 look like?

3. What other bases give similar graphs?

Activity ❻

1. How can you determine the last 2 digits of 6^{1000}?

2. Use your calculator to arrange 3^{666}, 4^{555}, 5^{444}, 6^{333} in increasing order.

Activity ❼

1. Write a problem using a power of 8.

2. Have a classmate solve your problem.

1.9 Powers of Monomials

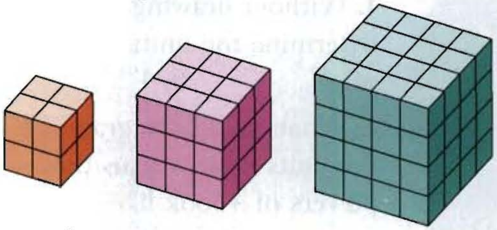
There are 35 major pyramids in Egypt. The typical Egyptian pyramid has a square base. The area of the base is given by a power of a monomial, s^2, where s is the side length.

A **monomial** is a number or a variable, or the product of numbers and variables.

$2 \qquad a \qquad -5b \qquad x^2 \qquad 15p^2q^3$

Activity: Look for a Pattern

The following cubes have been built from interlocking cubes.



Inquire

1. Copy the table. Work with a partner to complete it. Consider each interlocking cube to have a side length of 1 unit.

Side Length of Large Cube	Total Number of Interlocking Cubes
2	
3	
4	
5	
10	
100	

2. To what exponent is the side length of each cube raised to express the volume?

3. What algebraic expression represents the volume of a cube with side length x?

4. What algebraic expression represents the volume of a cube with side length $2x$?

5. The volume of a cube with side length $2x$ can also be found by multiplying the lengths of the sides.

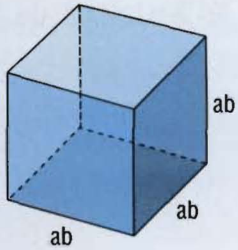
$2x \times 2x \times 2x = 8x^3$

How does the resulting expression compare with the expression you wrote in question 4?

6. Write a rule to find powers of a monomial.

The side lengths of a cube puzzle are represented by the monomial *ab*.

We can find the volume from the formula $V = s^3$, where s is the side length.



For the given cube, the volume is

$$\begin{aligned} V &= (ab)^3 \\ &= (ab)(ab)(ab) \\ &= (a \times a \times a)(b \times b \times b) \\ &= a^3b^3 \end{aligned}$$

We obtain the same result by finding the third power of each factor in the monomial.

$$\begin{aligned} (ab)^3 &= (a)^3(b)^3 \\ &= a^3b^3 \end{aligned}$$

In general, $(xy)^m = x^my^m$

The power $\left(\frac{a}{b}\right)^3 = \frac{a \times a \times a}{b \times b \times b}$

$$= \frac{a^3}{b^3}$$

In general, $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$

Example 1

Simplify.

a) $(x^3)^2$ **b)** $(y^3)^4$

Solution

a) $(x^3)^2 = x^{3\times2}$
$= x^6$

b) $(y^3)^4 = y^{3\times4}$
$= y^{12}$

Example 2

Simplify.

a) $(3x^4y^2)(-x^2y^3)^2$ **b)** $\left(\frac{2x^3}{3y^2}\right)^2$

Solution

a)

$$\begin{aligned} &(3x^4y^2)(-x^2y^3)^2 \\ &= (3x^4y^2)(-1)^2(x^2)^2(y^3)^2 \\ &= (3x^4y^2)(1)(x^4)(y^6) \\ &= (3 \times 1)(x^{4+4})(y^{2+6}) \\ &= 3x^8y^8 \end{aligned}$$

b)

$$\begin{aligned} &\left(\frac{2x^3}{3y^2}\right)^2 \\ &= \frac{(2x^3)^2}{(3y^2)^2} \\ &= \frac{(2)^2(x^3)^2}{(3)^2(y^2)^2} \\ &= \frac{4x^6}{9y^4} \end{aligned}$$

CONTINUED

Practice

Simplify.

1. $(x)^2$ **2.** $(a)^3$ **3.** $(p)^5$

4. $(n^2)^2$ **5.** $(-t^3)^2$ **6.** $(-y^2)^3$

Simplify.

7. $(x^2)^3$ **8.** $(y^3)^2$ **9.** $(m^2)^2$

10. $(n^3)^4$ **11.** $(x^3)^3$ **12.** $(y^2)^3$

13. $(z^4)^3$ **14.** $(m^4)^5$ **15.** $(p^{18})^2$

16. $(-s^{10})^2$ **17.** $(-x)^{31}$ **18.** $-(-b^0)^3$

Simplify.

19. $(xy)^2$ **20.** $(ab)^3$ **21.** $(-xy)^2$

22. $(mn)^4$ **23.** $(pq)^3$ **24.** $(-2xt)^2$

25. $(4xy)^2$ **26.** $(-2ax)^3$ **27.** $-(3rs)^3$

Simplify.

28. $(x^2y^2)^3$ **29.** $(x^2y^3)^2$ **30.** $(a^2b)^3$

31. $(ab^3)^2$ **32.** $(mn)^3$ **33.** $(-ab^2)^2$

34. $(-j^3k^4)^2$ **35.** $(x^2y)^2$ **36.** $-(s^3t^2)^0$

Simplify.

37. $(2x^2)^3$ **38.** $(3y^3)^2$ **39.** $(4x^4)^2$

40. $(5y^2)^2$ **41.** $(-m^2)^2$ **42.** $(-n^2)^3$

43. $(-2n^2)^3$ **44.** $(-3y^2)^2$ **45.** $(3pqr)^2$

46. $(-3yz)^3$ **47.** $(-4x^2y^3)^3$ **48.** $-(3xy^0)^2$

Simplify.

49. $\left(\frac{m}{2}\right)^4$ **50.** $\left(\frac{r}{t}\right)^8$ **51.** $\left(\frac{-d}{p}\right)^5$

52. $\left(\frac{2b}{5c}\right)^3$ **53.** $\left(\frac{-2x}{y^2}\right)^3$ **54.** $\left(\frac{3s^4}{2q^3}\right)^2$

Simplify.

55. $(2x^2y^3)^2(x^2y)$ **56.** $(-3xy)(-2xy)^2$

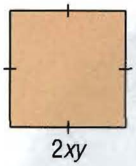
57. $(2xy^2)^3(3x^2y^2)$ **58.** $(10abc)^2(-2a^2bc)$

59. $(2a^4b^3)(-3ab)^3(10a^2b^2)$

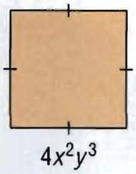
Problems and Applications

60. Write and simplify an expression for the area of each square.

a)

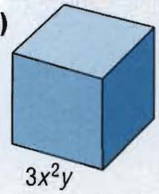


b)

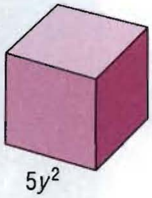


61. Write and simplify an expression for the volume of each cube.

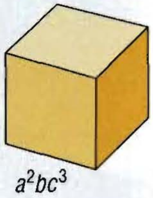
a)



b)



62. Find the volume of this cube if $a = 1$, $b = 2$, and $c = 3$.

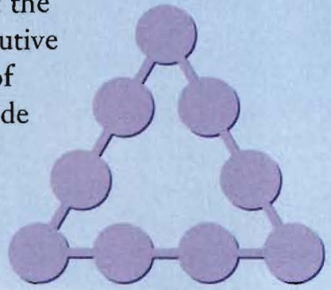


63. Does replacing the variables by their opposites, x by $-x$ and y by $-y$, result in the opposite of each monomial? Explain.

a) $(3x^2y^3)^2$ **b)** $(3x^2y^3)^3$

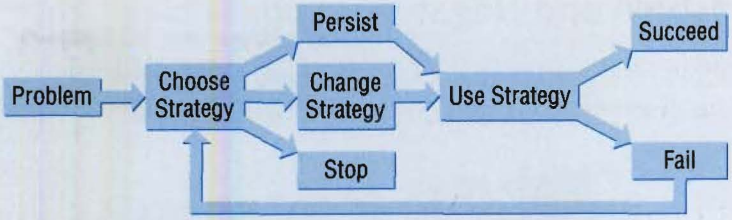
NUMBER POWER

Copy the diagram and place the numbers from 1 to 9 in the circles so that the consecutive sums of each side differ by 4.



1.10 Use a Flow Chart

A flow chart provides a way of describing or checking many processes. This flow chart shows how you might try to solve a problem.



When 15 is added to a number, and the result is squared, then multiplied by 3, the result is 243. What is the number?

Understand the Problem

1. What information are you given?
2. What are you asked to find?
3. Do you need an exact or approximate answer?

Think of a Plan

Construct a flow chart to show the operations on the original number.
Reverse the flow chart to work backward.

Carry Out the Plan

number → +15 → square → ×3 → 243

243 → ÷3 → square root → −15 → number

$243 \div 3 = 81$
The square roots of 81 are 9 and –9.
$9 - 15 = -6$ $\quad -9 - 15 = -24$
The number is either –6 or –24.

Look Back

How can you use substitution to check your answer?

Use a Flow Chart

1. Use a flow chart to represent the problem.
2. Solve the problem.
3. Check that your answer is reasonable.

Problems and Applications

1. The cube of a number is divided by 20, then 36 is subtracted. The result is 14. What is the number?

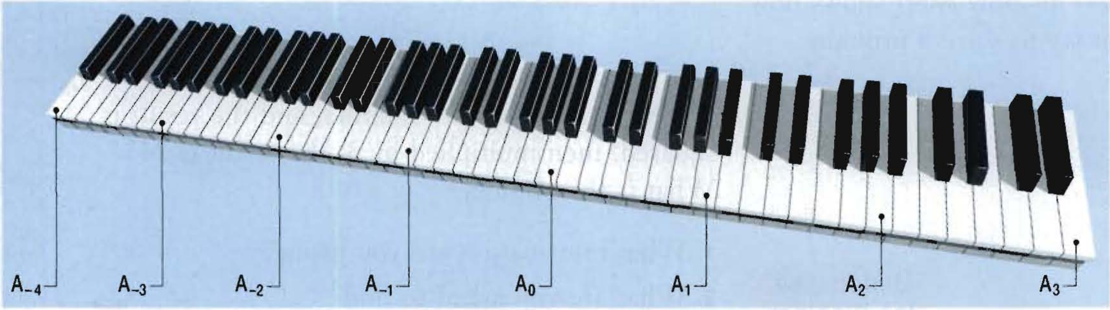
2. Saskatchewan's population is about 1 000 000. This is about 6 000 000 less than Quebec's population, which is about 1.4 times smaller than Ontario's population. Find Ontario's population to the nearest million.

3. Troy has read 104 pages of a 496-page book. If he reads 32 pages a day, how many more days will he take to finish the book?

4. Draw a flow chart to represent each process. Compare your flow charts with a classmate's.
   - a) shopping for a new jacket
   - b) phoning a friend from your home
   - c) writing an essay

1.11 Zero and Negative Exponents

There are 8 A notes on a piano. They can be named as shown in the diagram.



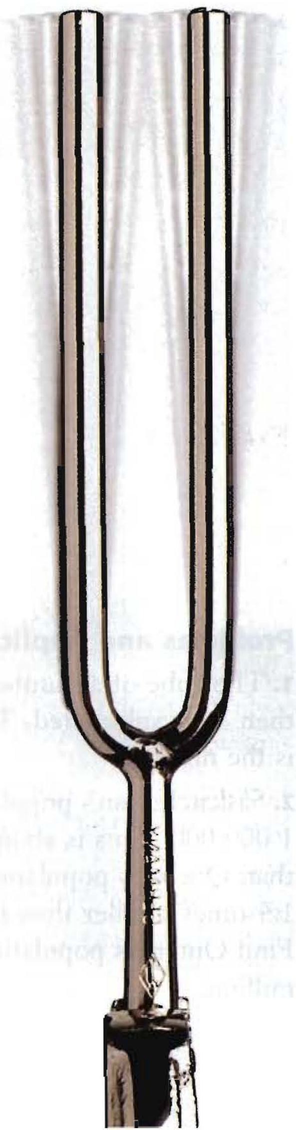
When tuning a piano, a tuner first adjusts the note A_0. This note, called "concert A," is the A above middle C. The formulas to determine the frequencies of the A notes are as follows. The units of frequency are hertz, symbol Hz.

$A_{-4} = 440 \times 2^{-4}$ $A_{-3} = 440 \times 2^{-3}$ $A_{-2} = 440 \times 2^{-2}$ $A_{-1} = 440 \times 2^{-1}$

$A_0 = 440 \times 2^0$ $A_1 = 440 \times 2^1$ $A_2 = 440 \times 2^2$ $A_3 = 440 \times 2^3$

Calculate the frequencies of notes A_1, A_2, and A_3.

To calculate the other frequencies, we need to know about zero and negative exponents.



Activity: Discover the Relationship

Copy and complete the table by doing each division twice. First expand and divide, and then use the exponent rule for division.

Division	Expand and Divide	Exponent Rule
$2^4 \div 2^4$	$\frac{2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2} = ?$	$\frac{2^4}{2^4} = 2^{4-4} = 2^?$
$3^2 \div 3^2$		
$4^3 \div 4^3$		
$10^4 \div 10^4$		
$x^5 \div x^5$		

Inquire

1. How do the 2 answers in each row compare?

2. What is the value of 5^0? 7^0? y^0?

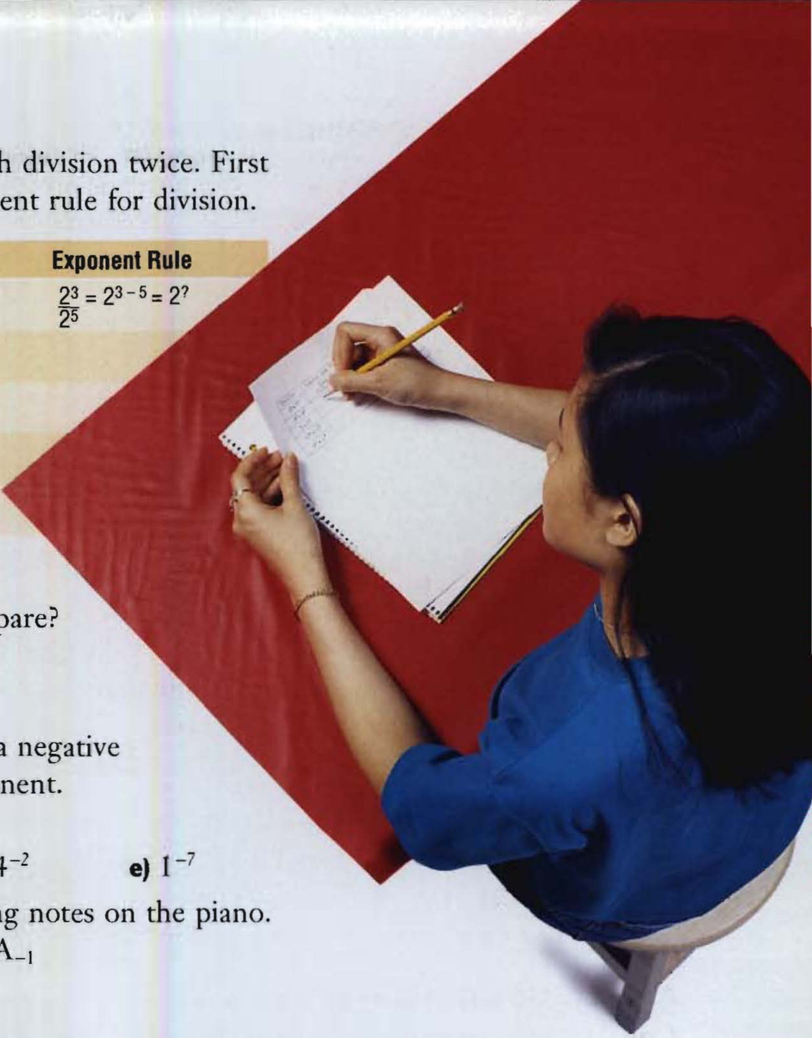
3. What is the value of any number raised to the exponent 0?

4. Calculate the frequency of A_0 on the piano.

Activity: Discover the Relationship

Copy and complete the table by doing each division twice. First expand and divide, and then use the exponent rule for division.

Division	Expand and Divide	Exponent Rule
$2^3 \div 2^5$	$\frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2^?}$	$\frac{2^3}{2^5} = 2^{3-5} = 2^?$
$3^2 \div 3^6$		
$5 \div 5^3$		
$10^4 \div 10^5$		
$x^5 \div x^9$		



Inquire

1. How do the 2 answers in each row compare?

2. State another way to write

a) 3^{-2} **b)** 4^{-3} **c)** 1^{-4}

3. Write a rule for writing a number with a negative exponent as a number with a positive exponent.

4. Evaluate.

a) 2^{-4} **b)** 5^{-2} **c)** 3^{-3} **d)** 4^{-2} **e)** 1^{-7}

5. Calculate the frequencies of the following notes on the piano.

a) A_{-4} **b)** A_{-3} **c)** A_{-2} **d)** A_{-1}

Example 1

Evaluate.

a) 4^0

b) $(-5)^0$

c) -2^0

Solution

$x^0 = 1$, where x can be any number except 0

a) $4^0 = 1$ **b)** $(-5)^0 = 1$ **c)** $-2^0 = -1$

Example 2

Evaluate.

a) 4^{-3}

b) $(-3)^{-3}$

c) $2^0 - 2^{-2}$

Solution

$x^{-m} = \frac{1}{x^m}$, where x can be any number except 0

a) $4^{-3} = \frac{1}{4^3}$

$= \frac{1}{64}$

b) $(-3)^{-3} = \frac{1}{(-3)^3}$

$= -\frac{1}{27}$

c) $2^0 - 2^{-2} = 1 - \frac{1}{2^2}$

$= 1 - \frac{1}{4}$

$= \frac{3}{4}$

CONTINUED

Example 3

Evaluate.

a) $3^{-3} \times 3^{5} \times 3^{-4}$

b) $(-2)^{-5} \div (-2)^{-2}$

c) $(4^{-1})^{2}$

d) $\frac{1}{5^{-2}}$

Solution

a)
$$\begin{aligned} &3^{-3} \times 3^{5} \times 3^{-4} \\ &= 3^{-3+5-4} \\ &= 3^{-2} \\ &= \frac{1}{3^{2}} \\ &= \frac{1}{9} \end{aligned}$$

b)
$$\begin{aligned} &(-2)^{-5} \div (-2)^{-2} \\ &= (-2)^{-5-(-2)} \\ &= (-2)^{-3} \\ &= \frac{1}{(-2)^{3}} \\ &= -\frac{1}{8} \end{aligned}$$

c)
$$\begin{aligned} &(4^{-1})^{2} \\ &= 4^{(-1)\times 2} \\ &= 4^{-2} \\ &= \frac{1}{4^{2}} \\ &= \frac{1}{16} \end{aligned}$$

d)
$$\begin{aligned} &\frac{1}{5^{-2}} \\ &= \frac{1}{\frac{1}{5^{2}}} \\ &= \frac{1}{\frac{1}{25}} \\ &= 1 \times \frac{25}{1} \\ &= 25 \end{aligned}$$

Practice

1. Write each power with a positive exponent.

a) 9^{-8} **b)** 1^{-4} **c)** $(0.5)^{-6}$

d) $(-7)^{-6}$ **e)** $\frac{1}{5^{-4}}$ **f)** $\frac{1}{(-3)^{-5}}$

2. Write in exponential form.

a) $\frac{1}{8 \times 8}$ **b)** $\frac{1}{7 \times 7 \times 7}$ **c)** $\frac{1}{9 \times 9 \times 9 \times 9}$

d) $\frac{1}{4}$ **e)** $\frac{1}{27}$ **f)** $\frac{1}{64}$ **g)** $\frac{1}{243}$

Evaluate.

3. 5^{0} **4.** 2^{6} **5.** 3^{-1}

6. $(-10)^{-3}$ **7.** $(0.1)^{-3}$ **8.** $(-3)^{-6}$

9. $\frac{1}{3^{-1}}$ **10.** $\frac{1}{4^{-2}}$ **11.** $\frac{1}{(-3)^{-1}}$

Write each expression as a power.

12. $7^{4} \times 7^{5}$ **13.** $9^{6} \times 9^{-4}$

14. $8^{-3} \times 8^{-5}$ **15.** $6^{7} \div 6^{3}$

16. $5^{-7} \div 5^{-2}$ **17.** $4^{-2} \div 4^{6}$

18. $(3^{3})^{4}$ **19.** $(9^{-2})^{4}$

20. $(8^{-1})^{-5}$ **21.** $-(2^{-3})^{-2}$

Write each expression as a power.

22. $2^{4} \times 2^{-3} \times 2^{2}$ **23.** $3^{-5} \times 3^{-3} \times 3^{2}$

24. $5^{6} \times 5^{-9} \times 5$ **25.** $8^{4} \times 8^{-5} \div 8^{-2}$

26. $(-2)^{-4} \times (-2)^{-3} \div (-2)^{-1}$

27. $(-3)^{-6} \div (-3)^{-2} \times (-3)^{4}$

Evaluate.

28. $3^{2} \times 3^{2}$ **29.** $4^{7} \div 4^{5}$

30. $5^{2} \times 5^{-4}$ **31.** $6^{-2} \div 6^{0}$

32. $7^{-4} \div 7^{-5}$ **33.** $6^{-3} \div 6^{-3}$

34. $\frac{(6^{0})^{-4}}{(10^{2})^{-2}}$ **35.** $\frac{(7^{-3})^{0}}{(0.1^{-1})^{-2}}$

Evaluate.

36. $2^{-2} \times 2^{-2}$ **37.** $3^{4} \div 3^{5} \times 3$ **38.** $3^{0} + 3^{3}$

39. $4^{2} - 2^{-1}$ **40.** $5^{3} + 3^{2}$

41. $2^{-2} + 5^{0}$ **42.** $2^{3} \times 2^{-1} + 5$

43. $(6 - 3)^{-2}$ **44.** $(9^{0} + 2^{0})^{-1}$

45. $\frac{1}{2^{-2}} + \frac{1}{3^{-1}}$ **46.** $\frac{2^{-1}}{3^{-1}}$ **47.** $\frac{-3^{-2}}{4^{-1}}$

Evaluate.

48. $\left(\frac{1}{3}\right)^{-1}$ **49.** $\left(\frac{-1}{5}\right)^{2}$ **50.** $\left(\frac{-7}{8}\right)^{0}$

51. $\left(\frac{1}{10}\right)^{-2}$ **52.** $\left(\frac{-2}{3}\right)^{-3}$ **53.** $\left(\frac{-3}{4}\right)^{-2}$

Simplify.

54. $x^{4} \times x^{3}$ **55.** $x^{-2} \times x^{3}$ **56.** $y^{-1} \times y^{-3}$

57. $t^{8} \div t^{4}$ **58.** $m^{6} \div m^{-2}$ **59.** $b^{-2} \div b^{-4}$

60. $(m^{4})^{2}$ **61.** $(t^{-2})^{4}$

62. $(y^{-5})^{-2}$ **63.** $m^{3} \times m^{-2} \times m^{4}$

64. $\frac{a^{-4} \times a^{-2}}{a^{2}}$ **65.** $\frac{t^{4} \times t^{3}}{t^{9}}$

66. $y^{-3} \times y^{7} \times y^{-5}$ **67.** $t^{-4} \div t^{-6} \times t^{6}$

Problems and Applications

68. a) Use your calculator to evaluate each power in this sequence.

2^5 $\quad 2^4$ $\quad 2^3$ $\quad 2^2$ $\quad 2^1$ $\quad 2^0$

b) Describe the pattern.

69. a) Evaluate each power in this sequence with a calculator.

2^3 $\quad 2^2$ $\quad 2^1$ $\quad 2^0$ $\quad 2^{-1}$ $\quad 2^{-2}$ $\quad 2^{-3}$ $\quad 2^{-4}$

b) Write each decimal answer as a fraction.

c) What is the exponential form for the next power in this sequence?

d) Without using a calculator, state the fraction that equals your answer to part c).

70. a) Use a calculator to evaluate each power in this sequence.

3^3 $\quad 3^2$ $\quad 3^1$ $\quad 3^0$ $\quad 3^{-1}$ $\quad 3^{-2}$ $\quad 3^{-3}$

b) Compare 3^3 and 3^{-3} written as a fraction. What do you notice?

c) What mathematical term describes your answer to part b)?

71. Evaluate. **a)** $\frac{6^3}{6^2} \times \frac{5^6 \times 5^{-2}}{(5^2)^2}$ **b)** $\frac{45 \times 4^{-2}}{(2^2)^2}$

72. Use the exponent laws to simplify these expressions. Leave your answers in the form ax^by^c where a, b, and c are integers.

a) $\frac{34x^{-3}y^4}{17x^3y^5}$ **b)** $\frac{39x^7y^6}{13x^{-4}y^8}$

73. Evaluate for $a = 2$ and $b = 3$.

a) a^3 **b)** b^4 **c)** a^0

d) a^{-3} **e)** b^{-2} **f)** a^{-4}

g) $(a \times b)^{-2}$ **h)** $(a - b)^{-1}$ **i)** $(b^2 - a^2)^{-2}$

74. Which is greater, 2^{-3} or 3^{-2}? Explain your reasoning. Compare your answer with your calculator answer.

75. Find the value of x.

a) $2^x = 8$ **b)** $3^x = 81$ **c)** $4^x = 1$

d) $3^x = \frac{1}{9}$ **e)** $5^x = \frac{1}{125}$ **f)** $x^3 = 27$

g) $x^{-2} = \frac{1}{4}$ **h)** $x^{-3} = \frac{1}{1000}$

76. Evaluate.

a) $\left(\frac{1}{2}\right)^0$ **b)** $\left(\frac{2}{3}\right)^{-3}$

c) $\left(\frac{1}{3}\right)^2\left(\frac{-1}{2}\right)^{-3}$ **d)** $\left(\frac{4}{5}\right)^0 - \left(\frac{3}{4}\right)^2$

e) $\left(\frac{4}{5}\right)^6\left(\frac{4}{5}\right)^{-8}$ **f)** $\left(\frac{5}{3}\right)^3 \div \left(\frac{5}{3}\right)^5$

77. The radioactivity of a sample of carbon-14 drops to $\frac{1}{2}$ or 2^{-1} of its original value in about 5700 years. After 11 400 years, the radioactivity is $\frac{1}{4}$ or 2^{-2} of its original value.

a) What fraction of the radioactivity remains after 28 500 years?

b) Write the fraction as a power with a negative exponent.

c) Write the fraction as a power with a positive exponent.

d) After how long is the radioactivity $\frac{1}{128}$ or 2^{-7} times its original value?

78. Is each statement always true, sometimes true, or never true? Explain.

a) The value of a power with a negative exponent is less than 0.

b) The value of a power with a fractional base is less than 1.

c) Two powers in which the exponents are both 0 have equal values.

79. On a test, 3 students evaluated $2^{-2} \times 2^0$ as follows.

Terry $\quad 2^{-2} \times 2^0 = 4^{-2}$
$\quad = \frac{1}{4^2}$ or $\frac{1}{16}$

Sean $\quad 2^{-2} \times 2^0 = 2^0$ or 1

Michel $\quad 2^{-2} \times 2^0 = 4^0$ or 1

a) What errors did each student make? What did each do correctly?

b) What is the correct answer?

80. Describe how to evaluate such powers as $(-3)^{-2}$ and -3^{-2}. Use your calculator to justify your descriptions. Compare your descriptions with your classmates'.

The Binary Number System

The decimal system that we use has ten digits.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

The **binary system** consists of 2 digits, 0 and 1.

A computer stores information in a **bit**, which is a short form of "binary digit." The electrical current in a bit can be either off or on. Off is represented by 0 and on by 1. Bits are combined into groups of 8 called **bytes**.

Activity ❶ Binary Numbers

Copy and complete the table to write the first 16 decimal numbers in binary. The first 5 have been done for you.

Decimal Form	Binary Form
0	0
1	1
2	10
3	11
4	100

Activity ❷ Powers of Two

1. Copy the following table. To complete it, use your results from Activity 1 and look for a pattern.

Decimal Number	Power of 2	Binary Number
1	2^0	1
2	2^1	10
4	2^2	100
8		
16		
32		
64		
128		

2. How is the exponent in the power of 2 related to the number of zeros in the binary number?

Activity ❸ Whole Numbers in Binary

You can use the results of Activity 2 to help you write whole numbers in binary form, because whole numbers can be written as sums of powers of 2. To write 43 as the sum of powers of 2, successively add the largest possible powers of 2.

$$\begin{aligned} 43 &= 32 + 11 \\ &= 32 + 8 + 3 \\ &= 32 + 8 + 2 + 1 \\ &= 2^5 + 2^3 + 2^1 + 2^0 \end{aligned}$$

In the binary system $2^5 + 2^3 + 2^1 + 2^0$

$$\begin{aligned} &= 100000 + 1000 + 10 + 1 \\ &= 101011 \end{aligned}$$

Write each of these decimal numbers in binary form.

1. 20 **2.** 37 **3.** 56 **4.** 78 **5.** 147

Activity ❹ Binary Numbers to Whole Numbers

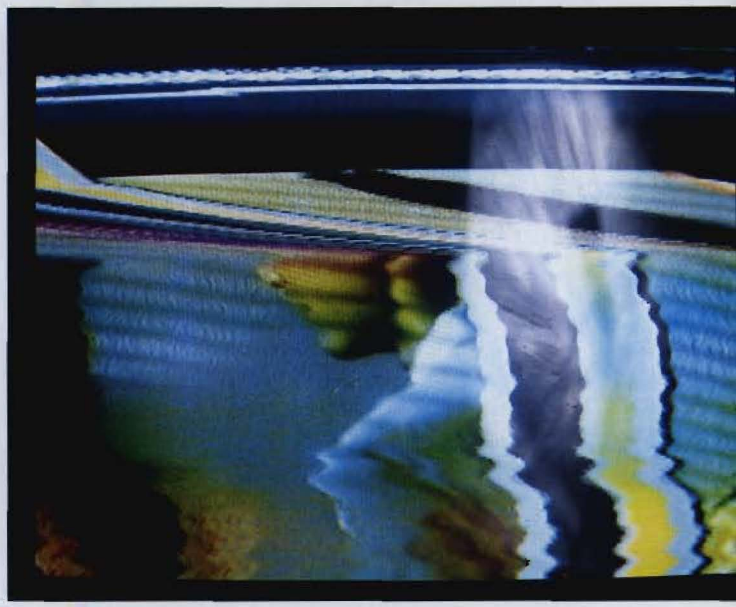
To write binary numbers as whole numbers, reverse the steps in Activity 3.

$$\begin{aligned} 110110 &= 100000 + 10000 + 100 + 10 \\ &= 2^5 + 2^4 + 2^2 + 2^1 \\ &= 32 + 16 + 4 + 2 \\ &= 54 \end{aligned}$$

Write these binary numbers as whole numbers.

1. 10101 **2.** 11001

3. 101010 **4.** 110011



Activity ❺ Binary Codes

Some television stations scramble their signals so that viewers cannot use unauthorized satellite dishes. Authorized users receive a code to activate their descramblers. The code is a binary number.

If you had 1 bit to use, you could use one of 2 codes. 0 or 1

If you had 2 bits, you could use one of 4 codes.

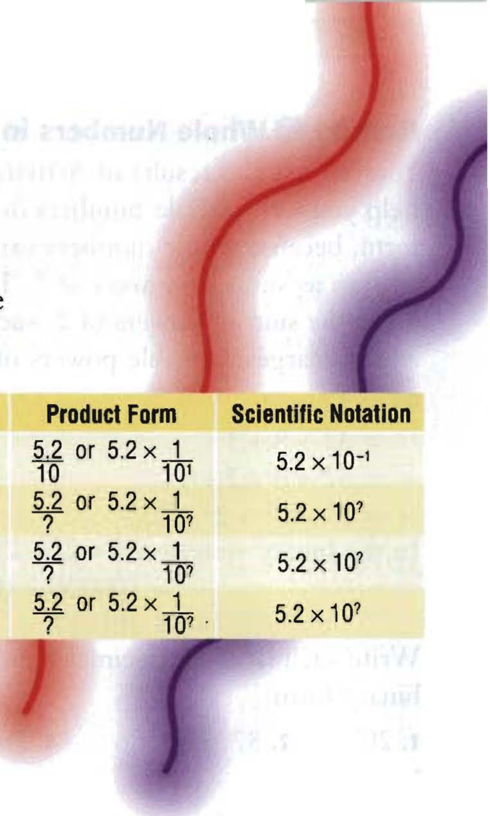
00 01 10 11

1. How many different codes could you use if you had 3 bits? 4 bits? 5 bits?

2. Television stations have 2^{56} bits available to them. Approximately how many codes can they use? If you set about finding the code by trying 1 000 000 codes a day, about how many years would it take you to find the code if you succeeded on the last try?

1.12 Scientific Notation: Small Numbers

The wavelength is the distance from one peak of a wave to the next. The wavelength of red light is 0.000 07 cm. The wavelength of violet light is 0.000 04 cm. This is the range of wavelengths we can see. To work with these small numbers, we need to extend our knowledge of scientific notation.



Activity: Look for a Pattern

Copy and complete the table.

Decimal Form	Product Form	Scientific Notation
0.52	$\frac{5.2}{10}$ or $5.2 \times \frac{1}{10^1}$	5.2×10^{-1}
0.052	$\frac{5.2}{?}$ or $5.2 \times \frac{1}{10^?}$	$5.2 \times 10^?$
0.0052	$\frac{5.2}{?}$ or $5.2 \times \frac{1}{10^?}$	$5.2 \times 10^?$
0.000 52	$\frac{5.2}{?}$ or $5.2 \times \frac{1}{10^?}$.	$5.2 \times 10^?$

Inquire

1. Write a rule for writing numbers less than 1 in scientific notation.

2. Write in scientific notation the wavelength of

a) red light **b)** violet light

3. Why would you not write 0.2347 in scientific notation?

Example 1

Write in scientific notation.

a) 5100

b) 0.0051

Solution

Write the number in the form $x \times 10^n$, where x is greater than or equal to 1 but less than 10.

a)
$$\begin{aligned} 5100 &= 5.1 \times 1000 \\ &= 5.1 \times 10^3 \end{aligned}$$

b)
$$\begin{aligned} 0.0051 &= 5.1 \times \frac{1}{1000} \\ &= 5.1 \times \frac{1}{10^3} \\ &= 5.1 \times 10^{-3} \end{aligned}$$

For 5100, the decimal point is moved 3 places to the left, and the exponent is 3. $5100 = 5.1 \times 10^3$

For 0.0051, the decimal point is moved 3 places to the right and the exponent is –3. $0.0051 = 5.1 \times 10^{-3}$

Example 2

Evaluate $(3.2 \times 10^{-2}) \times (5 \times 10^{-4})$.

Solution

$$\begin{aligned} (3.2 \times 10^{-2}) \times (5 \times 10^{-4}) &= 3.2 \times 5 \times 10^{-2} \times 10^{-4} \\ &= 16 \times 10^{(-2)+(-4)} \\ &= 16 \times 10^{-6} \\ &= 1.6 \times 10^{-5} \end{aligned}$$

Practice

Write in scientific notation.

1. 4 500 000

2. 0.089

3. 0.2

4. 0.000 055

Write in scientific notation.

5. 45×10^7

6. 0.34×10^6

7. 33×10^{-8}

8. 10^{-8}

9. 0.06×10^{-7}

10. 100×10^{-14}

Write in decimal form.

11. 2.3×10^{8}

12. 4.7×10^{-6}

13. 7×10^{-9}

14. 10^{-6}

Write the number that is 10 times as large as each of the following.

15. 2.3×10^{6}

16. 4.5×10^{-9}

17. 5×10^{-7}

18. 10^{-11}

Write the number that is one-tenth as large as each of the following.

19. 7.8×10^{9}

20. 6.8×10^{-6}

21. 8×10^{-10}

22. 10^{-7}

Estimate, then calculate. Write each answer in scientific notation.

23. $(2.5 \times 10^{-3}) \times (5 \times 10^{-4})$

24. $(8 \times 10^{-5}) \times (1.2 \times 10^{-1})$

25. $(3.2 \times 10^{-3}) \times (4.1 \times 10^{-2})$

26. $(5.2 \times 10^{3}) \div (2 \times 10^{-2})$

27. $(3.5 \times 10^{-4}) \div (7 \times 10^{-3})$

Problems and Applications

28. Order from highest to lowest.

a) 4.3×10^{-3}, 4.35×10^{-3}, 8.4×10^{-4}, 10^{-3}

b) 5.6×10^{-9}, 10^{-9}, $\frac{1}{10^{8}}$, 5.6×10^{-8}

c) $\frac{1}{1000}$, 2.1×10^{-3}, 10^{-2}, 2.12×10^{-3}

29. One water molecule has a mass of about 3×10^{-26} kg. Calculate the number of water molecules in Lake Superior, which holds about 1.2×10^{16} kg of water.

30. Energy is measured in joules, symbol J. If you tap your finger 10 times on your desk, you use about 1 J of energy. Express the following quantities of energy in scientific notation.

a) If you run for 1 h, you use about 2 000 000 J of energy.

b) When a cricket chirps, it uses about 0.0008 J of energy.

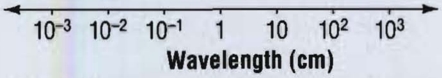
c) The food energy in a slice of apple pie is about 1 500 000 J.

31. Why is 0.23×10^{-9} not in scientific notation?

32. Is it possible to write the number 5 in scientific notation? Explain.

33. a) The waves that bombard us every day have wavelengths that range from 0.000 000 000 000 01 cm to 1 000 000 000 cm. Write these numbers in scientific notation.

b) Draw a number line showing the wavelengths in powers of 10.



Mark the range of wavelengths visible to humans.

c) Use your research skills to find out if there are creatures that can see outside the human range.

CALCULATOR POWER

1. On your calculator, multiply $0.000\,000\,4 \times 0.000\,055$. How does your calculator display the answer?

2. Scientific calculators display `2.2   - 11`. What does this answer mean?

3. Many calculators have an EE key. To input 5.2×10^{-8}, press **5** . **2** EE **8** +/-

Use the EE key to evaluate the following. Describe your calculator key strokes in each case.

a) $(3.5 \times 10^{-4}) \times (4.8 \times 10^{-7})$

b) $(1.5 \times 10^{-8}) \div (5 \times 10^{-12})$

c) $\frac{(3.6 \times 10^{6}) \times (2.4 \times 10^{-8})}{2.7 \times 10^{-3}}$

Calculators and the Order of Operations

Activity ❶ Addition, Subtraction, Multiplication, and Division

1. a) Use your calculator to evaluate this expression.

$$3.2 \times 2.0 - 8.8 \div 2.2$$

b) In what order does your calculator perform operations when addition, subtraction, multiplication, and division are involved?

2. a) Is 7 the correct value for the following expression? Explain.

$$7 - 8 \times 0.5 - 0.5$$

b) If the correct value is not 7, what is it?

3. Use your calculator to evaluate each expression.

a) $\frac{7}{10} - 0.5 \times \frac{2}{5}$

b) $\frac{9}{10} - 0.1 \div \frac{1}{10}$

c) $10 \div \frac{1}{2} \times 2.5$

d) $10 + 10 \times 0.5 \div 0.5$

Activity ❷ Brackets and Exponents

1. a) Use your calculator to evaluate the expressions $-6.2 \div (0.5)^2 - 1.6$ and $-6.2 \div 0.5^2 - 1.6$.

b) Does the use of brackets, in this case, make any difference to the order in which your calculator evaluates the expressions? Explain.

2. a) Use your calculator to evaluate the expression $-3^2 \times (4 - 1)^3$.

b) In what order does your calculator perform operations for expressions that contain exponents and brackets?

3. What does the acronym BEDMAS mean?

4. a) Is 3.4 the correct value for the following expression? Explain.

$$8.8 \div 2.2 \times 3 + (6.1 - 1.1) \div 5$$

b) If the correct value is not 3.4, what is it?

5. Use your calculator to evaluate each expression.

a) $96 - 3(4.2 - 0.2)$

b) $(19.5 - 6.5)^2 - (8.4 - 4.4) - 7 \times 10$

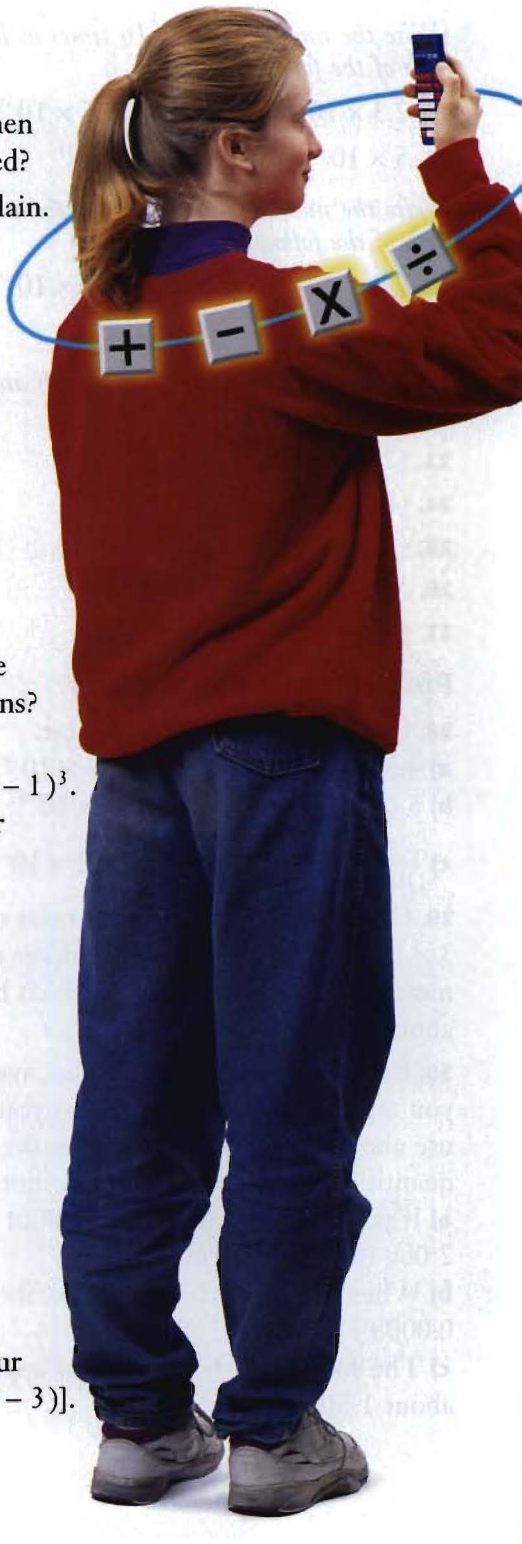
6. Add brackets to make each statement true.

a) $\frac{2}{3} + 4 \times \frac{1}{2} + \frac{1}{4} \div 3 = \frac{5}{3}$

b) $0.5^2 - 0.1 \times 8 \div 2 = 0.6$

c) $-2 \times 18.5 - 6.3 \div 4 = -6.1$

7. Some calculators do not have square bracket keys. Use your knowledge of the order of operations to evaluate $-4[8 + 7(6 - 3)]$.



Activity ❸ Using the Memory Keys

Most calculators have one memory for storing and recalling data.

1. a) If your calculator does not have $\boxed{\text{M+}}$ and $\boxed{\text{MRC}}$ keys, what are the equivalent keys on your calculator?

b) Follow this keying sequence.

Press $\boxed{\text{C}}$ 10 $\boxed{\text{M+}}$ $\boxed{\text{C}}$ $\boxed{\text{MRC}}$ $\boxed{=}$

c) Describe the functions of the $\boxed{\text{M+}}$ and $\boxed{\text{MRC}}$ keys.

2. Do not clear the memory. Follow these keying sequences.

a) Press $\boxed{\text{C}}$ 20 $\boxed{+}$ $\boxed{\text{MRC}}$ $\boxed{=}$

b) Press $\boxed{\text{C}}$ 30 $\boxed{-}$ $\boxed{\text{MRC}}$ $\boxed{=}$

c) Press $\boxed{\text{C}}$ 40 $\boxed{\times}$ $\boxed{\text{MRC}}$ $\boxed{=}$

d) Press $\boxed{\text{C}}$ 50 $\boxed{\div}$ $\boxed{\text{MRC}}$ $\boxed{=}$

3. a) Describe the use of the calculator's memory in question 2.

b) How did the calculator's memory save keystrokes?

4. a) Clear the memory by pressing $\boxed{\text{MRC}}$ $\boxed{\text{MRC}}$ when the display is zero. Then, follow this keying sequence.

Press $\boxed{\text{C}}$ 35 $\boxed{\text{M+}}$ 62 $\boxed{+}$ 20 $\boxed{+}$ $\boxed{\text{MRC}}$ $\boxed{=}$

b) How was the calculator's memory used in this example?

Activity ❹ Using The Calculator Efficiently

1. a) Describe how you would use the $\boxed{\text{M+}}$ and $\boxed{\text{MRC}}$ keys to evaluate $(31.3 - 24.7) \times (15.7 + 4.6)$, without using bracket keys.

b) How many keystrokes did you use to calculate the answer in part a)?

2. Find and describe another keying sequence that uses less keystrokes.

3. Which keystroke method is more efficient? Explain.

4. a) Use your calculator to devise one way to calculate the answer to this expression to 4 decimal places, without using bracket keys. $\frac{31.5}{11.1 \times (24.5 - 6.8)}$

b) Record the type and number of keystrokes that you used to make the calculation.

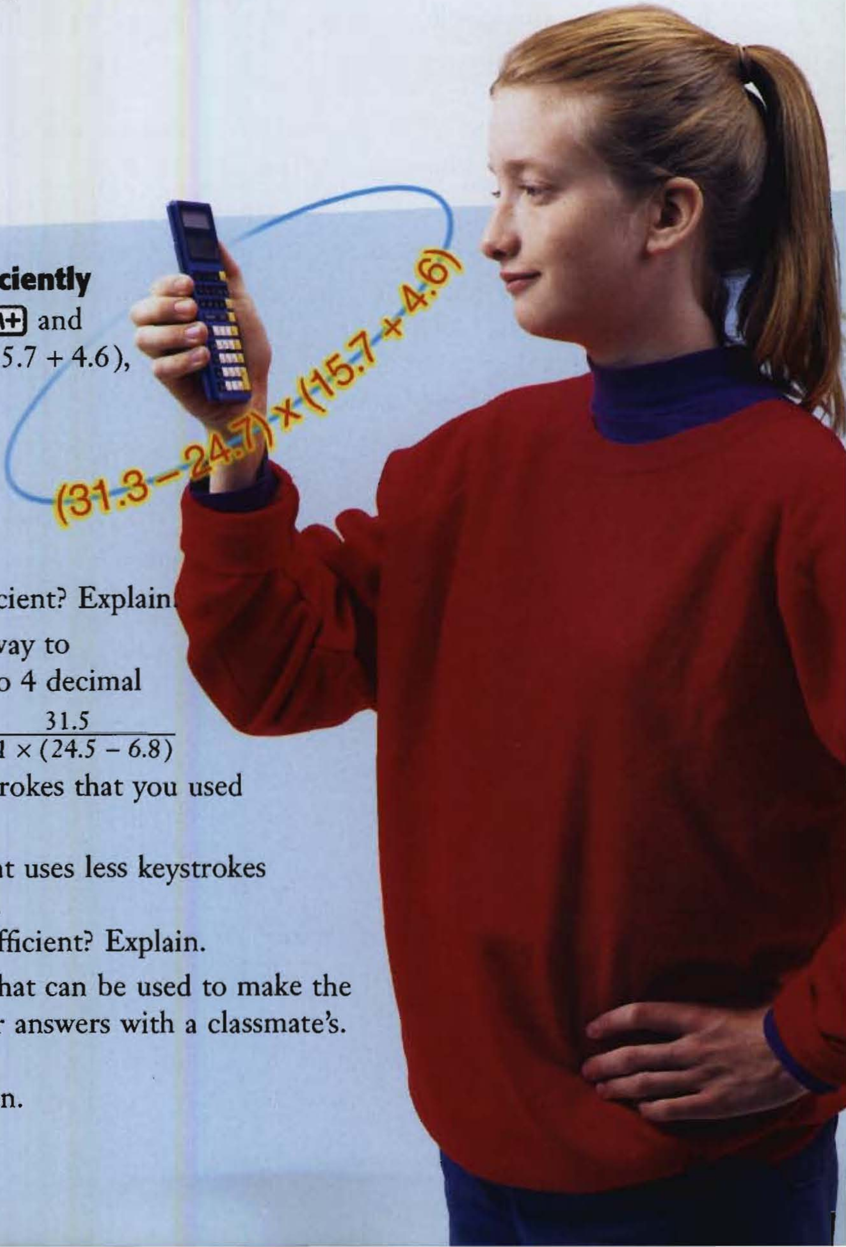
5. a) Find another calculator method that uses less keystrokes to evaluate the expression in question 4.

b) Which of the two methods is more efficient? Explain.

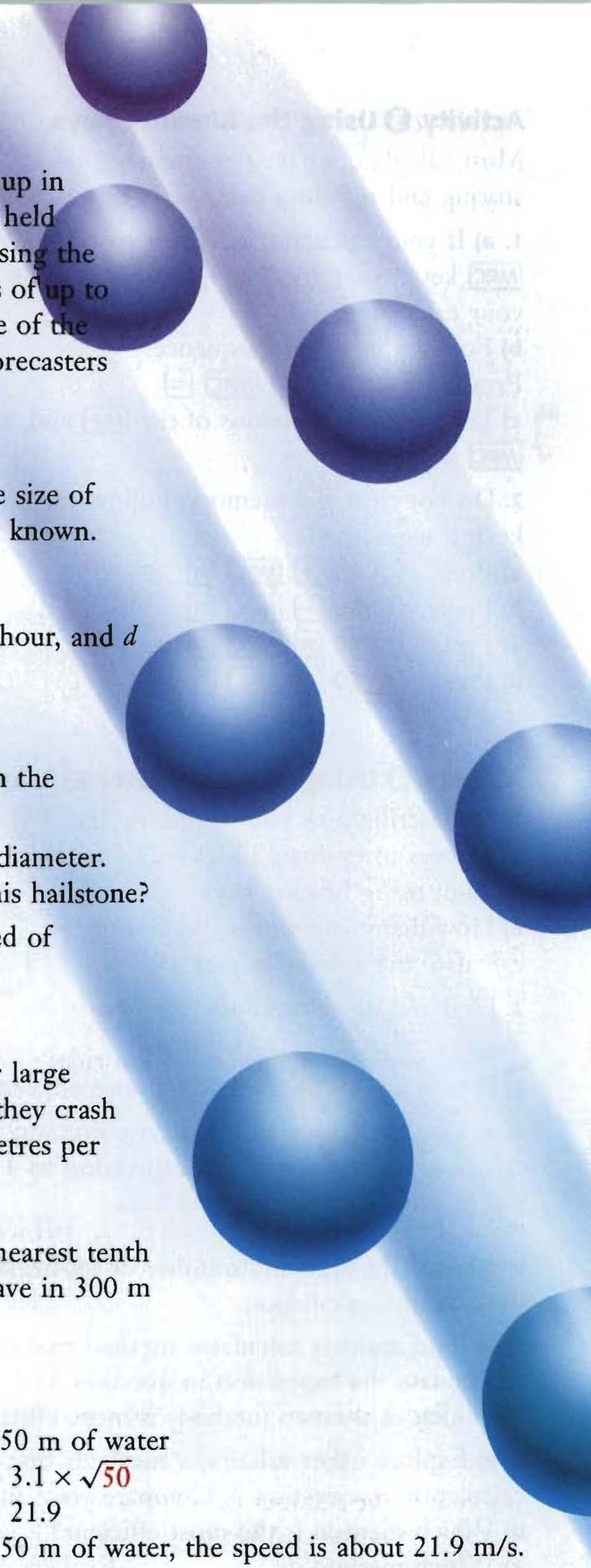
6. a) Explore other calculator methods that can be used to make the calculation in question 4. Compare your answers with a classmate's.

b) Which method is the most efficient?

c) Which method do you prefer? Explain.



1.13 Rational Numbers and Formulas



Hailstones are balls of ice that grow as they are held up in the clouds by thunderstorm updrafts. While they are held up, supercooled water drops hit them and freeze, causing the hailstones to grow. Large hailstones can fall at speeds of up to 150 km/h and can have masses of up to 2 kg. Because of the damage that can occur during a hailstorm, weather forecasters try to warn people when hail is probable.

Activity: Use a Formula

Weather forecasters can use a formula to estimate the size of hailstones if the speed of the thunderstorm updraft is known. The formula is

$$d = 0.05s$$

where s is the speed of the updraft in kilometres per hour, and d is the diameter of a hailstone in centimetres.

Inquire

1. What is the predicted diameter of a hailstone when the updraft speed is 30 km/h? 80 km/h? 100 km/h?

2. The smallest hailstone possible is about 0.5 cm in diameter. What is the approximate updraft speed to produce this hailstone?

3. How might weather forecasters determine the speed of an updraft?

Example

Tidal waves are caused by underwater earthquakes or large storms at sea. Tidal waves can be very destructive if they crash into populated areas. The speed of a tidal wave in metres per second can be found using the formula

$$s = 3.1 \times \sqrt{d}$$

where d is the depth of the ocean in metres. To the nearest tenth of a metre per second, what is the speed of a tidal wave in 300 m of water? 50 m of water?

Solution

$s = 3.1 \times \sqrt{d}$

In 300 m of water

$s = 3.1 \times \sqrt{300}$

$\doteq 53.7$

$\boxed{C}$ **3.1** $\boxed{\times}$ **300** $\boxed{\sqrt{\ }}$ $\boxed{=}$ $\boxed{53.693575}$

In 300 m of water, the speed is about 53.7 m/s.

In 50 m of water

$s = 3.1 \times \sqrt{50}$

$\doteq 21.9$

In 50 m of water, the speed is about 21.9 m/s.

Problems and Applications

1. The stopping distance a car requires in good road conditions is given by the formula

$$d = 0.4s + 0.002s^2$$

where d represents the approximate stopping distance in metres and s represents the speed in kilometres per hour. Calculate the distance required to stop at the following speeds.

a) 50 km/h **b)** 80 km/h

c) 100 km/h **d)** 120 km/h

2. The drama club decided to raise money by performing in a drama marathon. Pledges averaged $5.50 per actor for every hour of acting. Write a formula to calculate the amount each member could earn for the club. Let h represent the number of hours each member performs and v represent the amount of money earned.

3. The Purple Cab Company needs a formula to program into its computers to calculate fares. The manager has decided that a minimum fare of $3.50 must be charged. There is also a charge of $0.70 per half kilometre.

a) Write the formula for the total fare.

b) Determine the cost of a 7.5 km ride.

4. A rental company charges for lawn ornament rentals according to the formula

$$C = 2.40n + 2.00$$

where n is the number of ornaments rented and C is the cost in dollars. Calculate the cost of renting 12 pink flamingos for a birthday party.

5. The face value, f dollars, of a collection of dimes and nickels is given by the equation

$$f = 0.1d + 0.05n$$

where d is the number of dimes and n is the number of nickels. What is the face value of 35 dimes and 48 nickels?

6. The following formula calculates the time, t seconds, it takes for an object to fall from a height, h metres.

$$t = \sqrt{h \div 4.9}$$

The High Level Bridge in Lethbridge, Alberta, is the highest railway bridge of its type in the world. It stands 96 m above the river valley below. How long does it take for an object to fall from the top of the High Level Bridge to the ground below it?

7. The period of revolution of a planet around the sun is the time it takes for the planet to complete one orbit of the sun. The period, P years, is given by Kepler's third law

$$P^2 = D^3$$

where D is the average distance of the planet from the sun in astronomical units (AU). One astronomical unit is the average distance of the Earth from the sun. Use the average distance from the sun to find the period of revolution for each of the following planets to the nearest tenth of a year.

a) Jupiter; 5.20 AU

b) Mercury; 0.387 AU

c) Uranus; 19.2 AU

8. Write a problem that requires a formula. Have a classmate solve your problem.

CALCULATOR POWER

Here is a way to write the digits from 1 to 9 in order, so that when + or − signs are included the answer is 100.

$$123 + 45 - 67 + 8 - 9 = 100$$

Find 4 other ways to do this.

1.14 Use a Data Bank

You must locate information to solve some problems. There are many sources of information, including computer files, libraries, newspapers, magazines, atlases, experts, and data banks.

A grade 9 history class is going from Saint John to Ottawa, then to Montreal, and back to Saint John. The students will spend 3 days in Ottawa and 2 days in Montreal. Their bus can average 70 km/h. The students plan to leave Saint John on a Monday morning. Draw up a schedule for the trip.

1. What information are you given?

2. What are you asked to find?

3. What information do you need to locate?

4. Do you need an exact or approximate answer?

You need the driving distances between the cities. You could use an atlas, almanac, or road map, or ask at a tourist information centre. To calculate the time required for each part of the trip, divide the distance by the average speed.

Use the Data Bank on pages 364 to 369 in this book. Use the chart of "Driving Distances Between Cities."

Saint John to Ottawa	1130 km
Ottawa to Montreal	190 km
Montreal to Saint John	940 km

Saint John to Ottawa takes $1130 \div 70 \doteq 16$ h. Allow 2 days.
Ottawa to Montreal takes $190 \div 70 \doteq 3$ h. Allow $\frac{1}{2}$ a day.

Montreal to Saint John takes $940 \div 70 \doteq 13$ h. Allow $1\frac{1}{2}$ days.

Schedule: Leave Saint John Monday morning. Arrive in Ottawa Tuesday evening. Spend Wednesday, Thursday, and Friday in Ottawa. Leave Saturday morning for Montreal. Arrive in Montreal at noon on Saturday. Spend the rest of Saturday and Sunday in Montreal. Leave Montreal for Saint John on Monday morning. Arrive in Saint John on Tuesday around noon.

Does the schedule seem reasonable?

Use a Data Bank

1. Look up the information you need.
2. Solve the problem.
3. Check that the answer is reasonable.

Problems and Applications

Use the Data Bank on pages 364 to 369 of this book to solve the problems.

1. a) What is the flying distance between Vancouver and Regina?
b) What is the driving distance between Vancouver and Regina?
c) How much longer is the driving distance than the flying distance?

2. The St. Joseph's High School Band is planning a tour. The band will leave from Halifax and stop in Saint John, Quebec City, Toronto, and Montreal, in that order. The band will play one concert at a high school in each city and will then return to Halifax. The band will travel on a bus that can average 80 km/h. The band will leave Halifax on a Monday morning. Draw up a schedule for the trip.

3. Sandra flew from Vancouver to Halifax to compete in a gymnastics competition. She left Vancouver at 11:00. The flight to Toronto took 4 h. She took 1 h and 15 min to change planes in Toronto. The flight to Halifax took 1 h and 30 min. What time was it in Halifax when she landed?

4. What is the wind chill temperature if the thermometer reading is −23°C and the wind speed is 32 km/h?

5. If it is 09:00 on July 1 in Paris, France, what is the time and the date in Sydney, Australia?

6. Justine drove from Halifax to Vancouver. She drove on the Trans Canada Highway for the entire trip.
a) How far did she drive?
b) If she averaged 90 km/h, how many hours did she spend driving?
c) What major cities did she drive through?

7. The world's longest covered bridge is at Hartland, New Brunswick. The world's longest cantilevered bridge is the Pont de Quebec across the St. Lawrence River. How many times longer is the Pont de Quebec?

8. The population density of a country, city, or region is the average number of people living in each square kilometre of land. Use the area and the population of each Canadian province to work out the population density of each. Rank the population densities from highest to lowest.

9. a) You are the manager of a music group that wants to tour cities in Canada for two weeks to promote its new release. Make up a travel schedule for the group.
b) Compare your schedule with your classmates'. Decide the best features of each schedule.

NUMBER POWER

This puzzle appeared in the work of a Chinese mathematician, Sun Tzu, who lived in the 4th or 5th century A.D.

Pick a whole number less than 60 and greater than 5. Divide the whole number by 3. Call the remainder x. Divide the whole number by 4. Call the remainder y. Divide the whole number by 5. Call the remainder z.

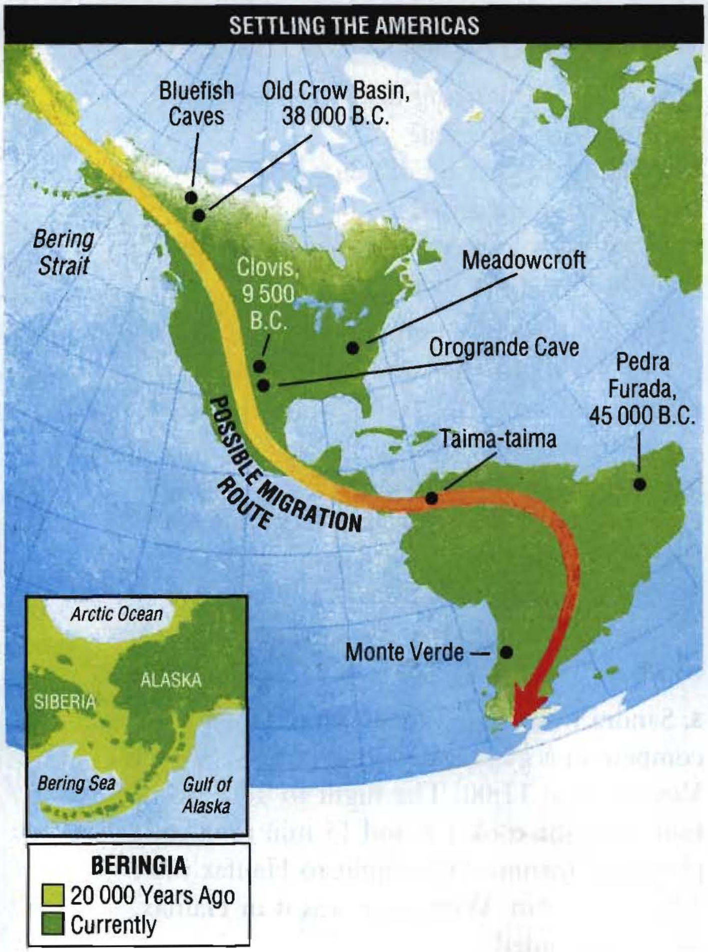
Evaluate the expression
$40 \times x + 45 \times y + 36 \times z$
and divide the value by 60.
a) What do you notice about the remainder?
b) Compare your finding with your classmates'.

CONNECTING MATH AND ARCHAEOLOGY

Radiocarbon Dating

One theory about how people first entered North and South America is that they crossed the Bering Strait over 40 000 years ago. One method that archaeologists use to establish such dates is called *radiocarbon dating*. All living plants and animals contain the same amount of radioactive carbon–14, C–14, per kilogram of mass. When they die, the C–14 slowly changes to nitrogen–14, N–14, which is not radioactive. The level of radioactivity of the dead plant or animal slowly decreases with time. By measuring the level of radioactivity, archaeologists can find how much C–14 is left.

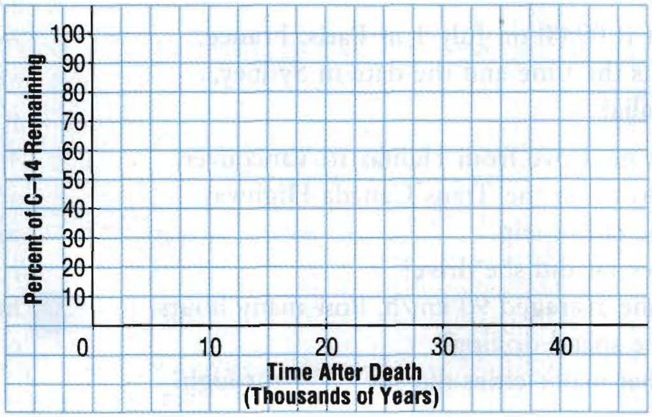
It takes about 5700 years for half the C–14 to change to N–14. Another way of saying this is that C–14 has a “half-life” of about 5700 years.



Activity ❶

Copy and complete the table. Graph the percent of C–14 remaining versus time after death.

Time After Death (Years)	Percent of C–14 Remaining
0	100
5700	50
11 400	25
17 100	
22 800	
28 500	
34 200	
39 900	



Courtesy of the Cooley-Dickinson Hospital

Spear point embedded in toe bone of a horse discovered in New Mexico.

Activity ❷

The map shows where archaeological digs have discovered evidence of human activity.

1. At the Bluefish Caves in the Yukon, Jacques Cinq-Mars of the Archaeological Survey of Canada found a caribou bone that was cut and shaped to form a tool. Six percent of the C–14 remained in the bone. How old is the bone? What was the date of the settlement at the Bluefish Caves?

2. At Monte Verde in southern Chile, dead plants from 15 species still used today for medicinal purposes were found under a peat bog. The plants came from the coast, 100 km away. The settlers probably trekked to the coast to get them. The plants had 20% of their C–14 remaining. How old are they?

3. At Taima-taima in Venezuela, the remains of a slain mastodon were found. The remains had 21% of their C–14 remaining. How old are the remains?

4. At Orogrande Cave in New Mexico, a horse's toe bone with a spear point in it had 5% of its C–14 remaining. How old is the bone?

5. In southwestern Pennsylvania, a mat woven from bark was found in a settlement that has been named Meadowcroft. The bark had 10% of its C–14 remaining. How old is the mat? What was the date of the settlement?

Review

Calculate.

1. 5^2 **2.** 7^2 **3.** 12^2

4. 31^2 **5.** $\sqrt{0.16}$ **6.** $\sqrt{81}$

7. $\sqrt{121}$ **8.** $\sqrt{400}$

9. $6\sqrt{36}$ **10.** $3^2 + 4^2 + 7^2$

11. $-8\sqrt{49}$ **12.** $\frac{6\sqrt{25} - 8}{2}$

Write both square roots of each number.

13. 25 **14.** 36 **15.** 64 **16.** 144

Estimate. Then, calculate to the nearest tenth.

17. $\sqrt{59}$ **18.** $\sqrt{372}$ **19.** $\sqrt{1273}$

20. $\sqrt{41\ 093}$ **21.** $\sqrt{4.93}$ **22.** $\sqrt{0.0187}$

Calculate to the nearest tenth.

23. $\sqrt{5} - \sqrt{7}$ **24.** $3\sqrt{17}$

25. $10\sqrt{7} - 6\sqrt{19}$ **26.** $(\sqrt{23})(\sqrt{8})$

27. $\sqrt{18} \div \sqrt{5}$ **28.** $\frac{\sqrt{30} - \sqrt{20}}{\sqrt{3}}$

Express in exponential form.

29. $2^3 \times 2^5$ **30.** $5^7 \div 5^3$ **31.** $(2^4)^2$

32. $7^8 \times 7^4$ **33.** $(3^3)^3$ **34.** $6^5 \div 6$

35. Evaluate for $c = 5, d = -6$.

a) $5c + 4d$ **b)** $3d + 4cd$ **c)** $c^2 + d^2$

Write in scientific notation.

36. 27 300 000 **37.** 0.000 000 019 3

Write in decimal notation.

38. 2.53×10^7 **39.** 9.71×10^{-4}

Calculate. Write your answer in scientific notation.

40. $(2.5 \times 10^3)(3.1 \times 10^5)$

41. $(6.2 \times 10^{-4})(3.7 \times 10^2)$

42. $(8.6 \times 10^{21}) \div (2.5 \times 10^7)$

Evaluate.

43. 3^5 **44.** 12^2 **45.** $(-5)^3$

46. $5(2)^5$ **47.** 6^0 **48.** $(-5)^0$

49. 2^{-1} **50.** 3^{-2} **51.** $\left(\frac{2}{3}\right)^3$

52. $\left(\frac{3}{5}\right)^{-2}$ **53.** $\frac{1}{3^{-2}}$ **54.** $\frac{5}{2^{-3}}$

Evaluate.

55. $3^{-1} - 3^0$ **56.** $2^{-3} + 2^{-2}$

Simplify.

57. $2^3 \times 2^{-5}$ **58.** $n^{-4} \div n^{-6}$

59. $((-3)^{-2})^4$ **60.** $((-x)^5)^{-1}$

61. $(-0.4)^{-5}(-0.4)^{-3}$ **62.** $4^2 \times 4^{-3} \times 4^{-1}$

Evaluate.

63. $5^3 \times 5^2 \times 5$ **64.** $2^0 \times 2^3$

65. $6^5 \div 6^3$ **66.** $(2^3)^4$

67. $2^{-3} \times 2^2$ **68.** $(4^2)^3 \div 4^4$

69. $(2^3)^{-2}$ **70.** $((-3)^2)^{-1}$

71. Evaluate for $x = 3$ and $y = -2$.

a) x^2y^2 **b)** y^5

c) $-6xy$ **d)** $(x - y)^3$

e) $6x^3 - 5y$ **f)** $(-x)^2(-y)^2$

Calculate. Write your answer in decimal notation.

72. 4.716×100 **73.** 16.93×10^3

74. $0.63 \div 10$ **75.** $123.94 \div 10^3$

Simplify.

76. $(-2x^2y^3)^3$ **77.** $(-5x^2y^2)^2$ **78.** $(-2abc^4)^3$

Simplify.

79. $(2x^2yz^2)(x^2y^2z^2)^3$ **80.** $(3a^3b)^3(-a^3bx^4)$

Simplify.

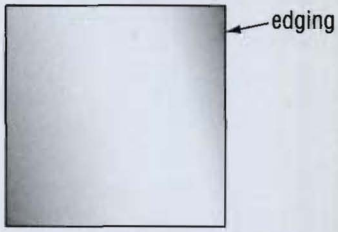
81. $\left(\frac{f}{3}\right)^3$ **82.** $\left(\frac{-r}{q}\right)^7$ **83.** $\left(\frac{5m}{7n}\right)^2$

84. A box of mirror tiles contains 25 square tiles. Each tile covers 400 cm^2.

a) What are the dimensions of each tile?

b) What is the total area covered by the 25 tiles?

c) What length of edging is needed around the mirror?



85. Quebec City has a population of about 6.4×10^5 and an area of about 3.2×10^3 km^2.

a) How many square kilometres are there per person in Quebec City? Write your answer in scientific notation.

b) If each person's area was a square, what would its side be to the nearest metre?

86. a) Why is 0.45×10^7 not in scientific notation?

b) Write 0.45×10^7 in scientific notation.

87. The velocity of sound in air may be found from the formula

$$V = 20\sqrt{273 + T}$$

where V is the velocity in metres per second and T is the temperature in degrees Celsius. Calculate the velocity of sound in air when the temperature is

a) 18°C **b)** 30°C

c) the temperature on Canada's coldest day ever

88. The formula that the Safety Taxi Co. uses to determine fares is

$$F = 2.50 + 0.125d$$

where F is the fare in dollars and d is the distance travelled in kilometres. Find the fare from Hyde Street to 34 Busy Road if the distance is 5.6 km.

Group Decision Making

Researching Earth Science Careers

To work effectively in groups.

a) Contribute ideas by speaking quietly.

b) Encourage others to participate, and listen to them carefully.

c) Respect the opinions of others.

d) Ask the teacher for help only when your whole group has the same difficulty.

1. Brainstorm as a class 6 earth science careers to investigate. They might include geographer, cartographer, geologist, paleontologist, and seismologist.

2. In home groups of 6, decide which career each person will research.

1 2 3 4 5 6	1 2 3 4 5 6
1 2 3 4 5 6	1 2 3 4 5 6

Home Groups

3. In your home group, decide what you want to know about each career. Make sure you include the ways in which math is used in each career.

4. Research your career individually.

5. Form an expert group with students who researched the same career as you. Combine your findings.

1 1 1 1	2 2 2 2	3 3 3 3
4 4 4 4	5 5 5 5	6 6 6 6

Expert Groups

6. In your expert group, prepare a report on the career. The report can take any form the group chooses.

7. In your expert group, evaluate the process and your presentation. What worked well and what would you do differently next time?

Chapter Check

Calculate.

1. $\sqrt{36}$ **2.** $5\sqrt{16}$ **3.** $-\sqrt{121}$

Estimate. Then, calculate to the nearest tenth.

4. $\sqrt{93}$ **5.** $(\sqrt{8})(\sqrt{39})$

Evaluate for $r = 4$ and $t = -5$.

6. $5r + 4t$ **7.** $3rt + t^2$

Evaluate for $x = 0.5$ and $y = 1.2$.

8. $3x + 5y$ **9.** $10xy - 2x$

Evaluate.

10. 4^2 **11.** 3^4 **12.** 5^3

Write the answer in exponential form.

13. $3^4 \times 3^5$ **14.** $2^7 \div 2^3$ **15.** $(5^2)^4$

Write in scientific notation.

16. 45 000 000 **17.** 213 000

Evaluate.

18. $(-3)^4$ **19.** $(-1)^{14}$

20. 6^0 **21.** 5^{-1}

22. $(3^{-2})^2$ **23.** $(-3)^{-2}$

24. -2^3 **25.** $(-0.7)^2$

26. $2^5 \div (-2)^3$ **27.** $5^{-2} \times 5^3 \times 5^{-2}$

28. $7^0 - 4^{-1}$ **29.** $(1 - 3)^4$

Write in scientific notation.

30. 0.0004

31. 0.000 000 000 002 31

Write in decimal form.

32. 4.6×10^{-3} **33.** 3.21×10^{-6}

Estimate, then calculate. Express your answer in scientific notation.

34. $(9.5 \times 10^{-2}) \times (5.1 \times 10^{-6})$

35. $(6.3 \times 10^{-4}) \div (1.9 \times 10^{-7})$

Simplify.

36. $(-3)^4 \div (-3)^5$

37. $s^2 \times s^{-5}$

38. $((2)^{-3})^{-1}$

39. $3^{-1} \div 3^{-2} \times 3^{-3}$

40. Evaluate for $m = -2$.

a) $3m^4$ **b)** $4m^3 + 5$

41. Forty percent of Canada's cropland is in Saskatchewan. The total area of cropland in Saskatchewan is about 134 000 km². What are the dimensions of a square that is big enough to hold all this land? Give your answer to the nearest kilometre.

Simplify.

42. $(-3x^4y^3)^3$ **43.** $(x^2y^4)(-3x^3y^2)^2$

Simplify.

44. $\left(\frac{g}{8}\right)^2$ **45.** $\left(\frac{-b}{d}\right)^9$ **46.** $\left(\frac{2y}{3k}\right)^4$

47. The cost of renting a car from a car rental agency is given by the formula

$$C = 50n + 0.25d$$

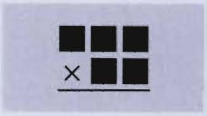
where C is the cost in dollars, n is the number of days the car is rented for, and d is the total distance driven in kilometres.

a) Calculate the cost of renting a car for 3 days and driving 350 km per day.

b) How much money do you have left for food and accommodations if you have $750 budgeted for the trip?

Using the Strategies

1. Copy the diagram. Place the digits from 1 to 5 in the squares so that the product is 3542.



2. What is the side of a cube that can be made with 294 cm^2 of cardboard? What assumptions have you made?

3. Jason chose a whole number less than 10. He multiplied the number by 6 and added 1. The result was a perfect square. What numbers could he have chosen?

4. Find 3 consecutive whole numbers whose sum is 144.

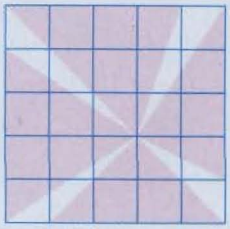
5. How many Friday the thirteenths will there be in the year you turn 21?

6. Assume that the following pattern continues.

A, BBB, CCCCC, DDDDDDD, ...

a) How many letter Ms will there be?
b) How many letter Zs will there be?

7. In the figure, the side of each small square represents 1 cm.



Find the area of
a) the shaded region.
b) the unshaded region.

8. How many different combinations of coins have a value of $0.28? Copy and complete the table to find out.

	Coins			
Combinations	**$0.25**	**$0.10**	**$0.05**	**$0.01**
1	1	0	0	3
2	0	1	2	8
3				

9. The number 63 can be written as the sum of consecutive whole numbers as follows.

$$63 = 20 + 21 + 22$$

a) Find another way to write 63 as the sum of consecutive whole numbers.

b) Find 4 consecutive whole numbers that add to 138.

10. a) On a digital clock, how many times a day are the digits all the same?

b) Write another question about the numbers on a digital clock. Have a classmate solve your problem.

11. Draw a flow chart to represent each process. Compare your flow charts with a classmate's.

a) getting a driver's permit
b) writing a computer program

DATA BANK

Use the Data Bank on pages 364 to 369 to find the information you need.

1. If the Canadian province with the greatest area had the shape of a square, what would be the length of each side to the nearest kilometre?

2. Would you feel colder at a temperature of –7 °C with the wind at 32 km/h or at a temperature of –18 °C with the wind at 8 km/h? Explain.