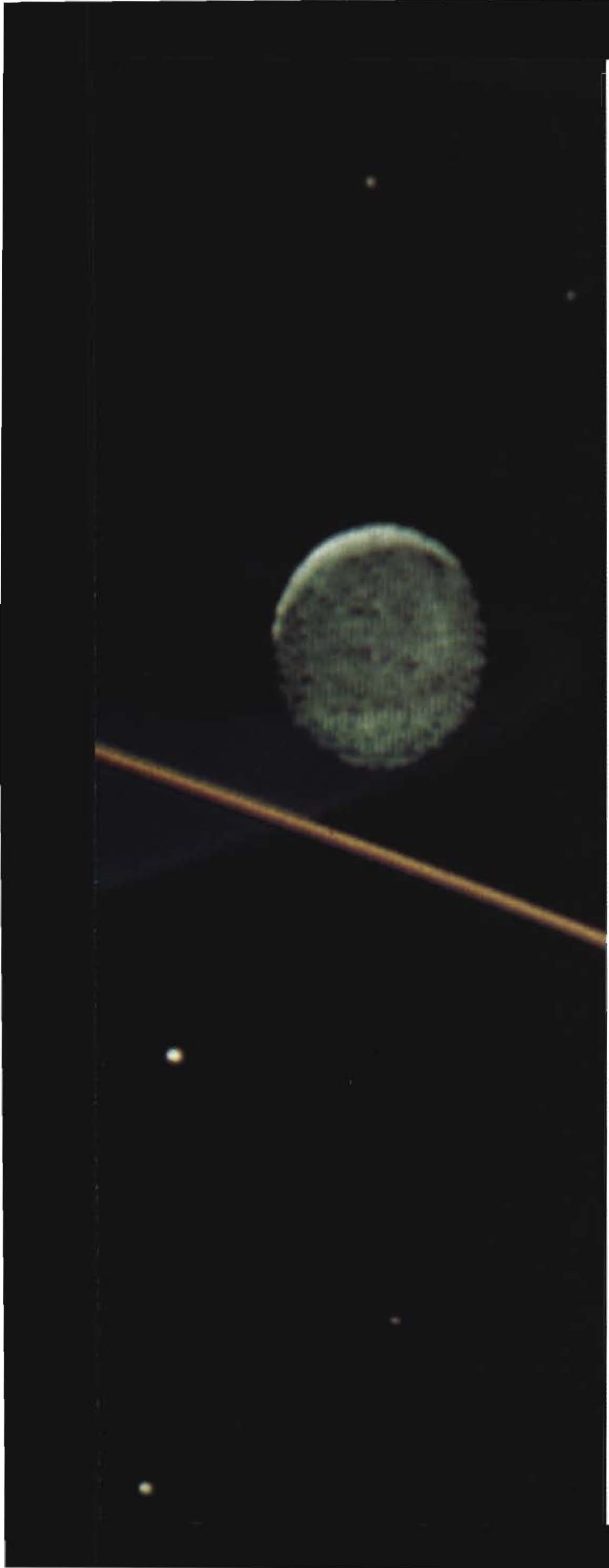


CHAPTER 5

Special Products and Factoring

The formula $v = 1.15 \times 10^{-5} \times \sqrt{\frac{m}{r}}$ gives the minimum velocity that space probes, such as *Voyager 1* and *Voyager 2*, must have to escape the Earth's gravitational pull. In this formula, v is the minimum velocity in metres per second, m is the mass of the Earth in kilograms, and r is the radius of the Earth in metres.

The mass of the Earth is 5.98×10^{24} kg, and the radius of the Earth is 6.38×10^{6} m.

Use these data to calculate the escape velocity in metres per second. What is the escape velocity in kilometres per hour?

GETTING STARTED

Activity ❶ Factors

List the whole-number factors for these numbers.

1. 12 **2.** 15 **3.** 21
4. 8 **5.** 45 **6.** 36
7. 52 **8.** 17 **9.** 100
10. 24 **11.** 80 **12.** 39

Write each number as a product of prime factors. For example, $12 = 2 \times 2 \times 3$.

13. 8 **14.** 15 **15.** 24
16. 20 **17.** 36 **18.** 42
19. 28 **20.** 19 **21.** 26

22. Copy and complete this table.

First Number	Second Number	Sum	Product
3	4	7	12
–2	–3		
7	–5		
–9	2		
–5		–9	
	–6		–18
	10		–10
–3		1	
		9	20
		–7	12
		12	20
		–11	30

Determine the missing factor in each of the following expressions.

23. $6a = (3a)($■$)$
24. $-7ab = (-7a)($■$)$
25. $4x = ($■$)(2x)$
26. $4x^2 = (-2x)($■$)$
27. $15a^2 = (5a)($■$)$
28. $-18xy = (9y)($■$)$
29. $12xy = (3x)($■$)$
30. $-4x^2 = (x)($■$)$
31. $12xy = ($■$)(6xy)$
32. $36xy^3 = ($■$)(-6y)$
33. $7x^2 = ($■$)(7x^2)$
34. $20abc = ($■$)(-5bc)$

Activity ❷ Division by Monomials

Divide.

1. $x^5 \div x^2$
2. $p^3 \div p^2$
3. $a^7 \div a$
4. $x^5 \div x$
5. $\frac{-5t^5}{t^2}$
6. $\frac{a^4b^2}{a^2b^2}$
7. $\frac{-12x^4}{4x}$
8. $\frac{8a^4}{-2a}$
9. $\frac{-30x^3y^2}{-6xy^2}$
10. $\frac{72m^2n^2}{12mn}$

11. The area of a rectangle is $45a^2b^2$ and its length is $9ab$. Find an expression for its width.

Use this example to simplify each expression.

$$\frac{4a^2b - 6ab^2}{2ab} = \frac{4a^2b}{2ab} - \frac{6ab^2}{2ab}$$
$$= 2a - 3b$$

12. $\frac{6a + 12}{6}$
13. $\frac{9x - 6y}{3}$
14. $\frac{6t - 18}{6}$
15. $\frac{33xy - 22y}{11y}$
16. $\frac{27m - 18n + 9}{9}$
17. $\frac{4x^3 - 10x^2 + 6x}{2x}$
18. $\frac{6a^2b + 4ab^2}{2ab}$
19. $\frac{12xy^4 - 9x^3y}{3xy}$

Activity ❸ Square Roots and Squares

State each square root. Assume that variables are positive.

1. $\sqrt{25}$
2. $\sqrt{121}$
3. $\sqrt{36}$
4. $\sqrt{x^2}$
5. $\sqrt{4a^2}$
6. $\sqrt{49x^2}$

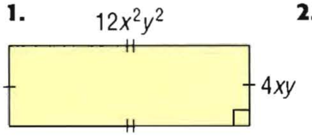
Square.

7. 2^2
8. $(-5)^2$
9. 9^2
10. $(2x)^2$
11. $(-3a)^2$
12. $(5x^3)^2$

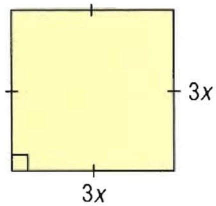
Activity ❹ Area

Write an expression for each area.

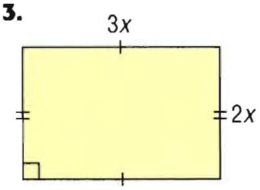
1.



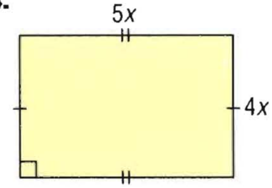
2.



3.

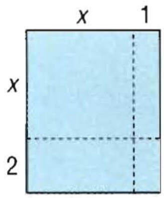


4.

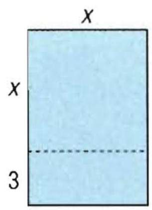


Add the areas of the smaller rectangles to write an expression for the area of each complete figure.

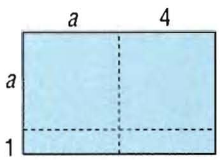
5.



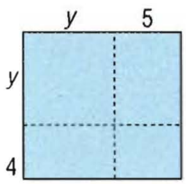
6.



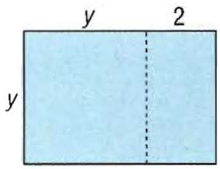
7.



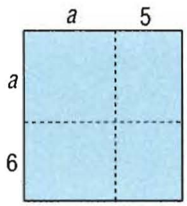
8.



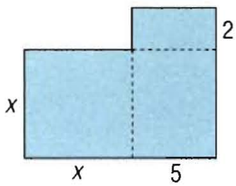
9.



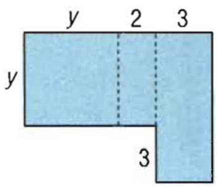
10.



11.



12.



Mental Math

Calculate.

1. $12 + 6 - 3$
2. $20 - 12 + 6$
3. $24 \div 6 \times 4$
4. $18 \times 2 \div 4$
5. $12 \times 3 - 4 \div 2$
6. $24 - 18 \div 3 \times 4$
7. $3 + 8 \times 3 \div 4$
8. $(10)(3^2)$
9. $(2 + 3)^2$
10. $(4)(5^2)$
11. $3^2 + 4^2$
12. $2^3 + (4 + 1)^2$

Calculate.

13. $(8 + 4) \div 2$
14. $(20 - 8) \div 4$
15. $6 \times (5 + 4)$
16. $3 \times 4 + 6 \times 4$
17. $8 \times (12 - 9) + 2$
18. $15 \times (10 - 8) + 1$
19. $(20 + 10) \div 2 - 3 + 1$
20. $40 + 24 \div 8 - 3 + 1$
21. $(40 + 24) \div 8 - 3 + 1$

Simplify.

22. $\frac{1}{2} \times \frac{1}{3}$
23. $\frac{1}{4} \times \frac{1}{5}$
24. $\frac{3}{4} \times 4$
25. $\frac{2}{3} \times 3$
26. $\frac{1}{3} + \frac{1}{3}$
27. $\frac{5}{8} + \frac{1}{8}$
28. $\frac{1}{2} \div \frac{1}{2}$
29. $\frac{1}{2} \div \frac{1}{4}$
30. $\frac{2}{3} - \frac{1}{3}$
31. $\frac{5}{8} - \frac{1}{8}$
32. $\frac{3}{4} + \frac{3}{4}$
33. $\frac{3}{5} + \frac{4}{5}$
34. $\frac{1}{4} \times \frac{1}{3}$
35. $\frac{1}{2} \div 2$
36. $4 \div \frac{1}{2}$
37. $\frac{1}{4} + \frac{1}{2}$
38. $\frac{1}{2} - \frac{1}{4}$
39. $\frac{7}{8} - \frac{3}{4}$
40. $\frac{1}{3} - \frac{1}{5}$
41. $\frac{1}{4} - \frac{1}{8}$
42. $\frac{2}{5} - \frac{1}{10}$
43. $3 \div \frac{1}{3}$
44. $\frac{1}{2} \times \frac{2}{3}$
45. $\frac{1}{3} + \frac{1}{6}$

Activity ❶ Patterns on a Calendar

Examine the following monthly calendar.
What month might it be?

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

1. Copy any 3-by-3 block, such as the one shaded in the table.

2. Choose 3 numbers by circling only 1 number from each column and each row of the block.

3. Record the sum of the 3 numbers.

4. Choose 3 other numbers in the same block. Again, choose only 1 number from each column and each row. Record the sum of the 3 numbers.

5. Compare your results from steps 3 and 4.

6. Describe how you can use 1 number in the block to find the sum of the 3 numbers you chose.

7. Compare your findings with your classmates'.

8. How many 3-by-3 blocks can you make from the above calendar month?

9. How many 3-by-3 blocks could you make if the first of the month was a Sunday?

10. Complete these 3-by-3 calendar blocks to show the patterns.

a)

x	$x+1$	$x+2$
$x+7$		

b)

	x	

c)

x		

11. Investigate the historical development of the calendar.

Activity ❷ Patterns in Tables

1. Copy and complete the table.

Addition				
+	3	5	7	9
2				
4				
6				
8				

2. Choose 4 sums by circling only 1 sum in each row and each column of the table.

3. Add the 4 numbers you circled.

4. Compare the sum of your 4 numbers with that of a classmate. Is the result always the same?

5. Copy and complete the following table. Then carry out steps 2 and 3 above. Explain why the sum is always the same.

+	b	d	f	h
a				
c				
e				
g				

6. Work with a partner to rewrite steps 1, 2, 3, and 4 to deal with multiplication, instead of addition.

7. Copy and complete the table.

Multiplication				
×	1	2	3	4
1				
2				
3				
4				

8. Use your instructions to investigate the products in the table. Compare your results with your classmates'.

5.1 Common Factors and the GCF

On the average, Yellowknife has blowing snow on 10 days of the year, and Halifax has blowing snow on 14 days of the year.

The number 10 has 4 factors: 1, 2, 5, and 10.
The number 14 also has 4 factors: 1, 2, 7, and 14.
The **greatest common factor** (GCF) of the numbers 10 and 14 is 2.

Activity: Discover the Relationship

Write all the factors of each number.

a) 6 **b)** 12 **c)** 18

Inquire

1. Which prime factors are common to all 3 numbers?

2. What is the GCF of the 3 numbers?

3. Write a rule for finding the GCF of a set of numbers from their prime factors.

4. Write 3 different numbers that have a GCF of

a) 3 **b)** 8 **c)** 10

5. Write 4 different numbers that have a GCF of

a) 4 **b)** 5 **c)** 12

Example 1

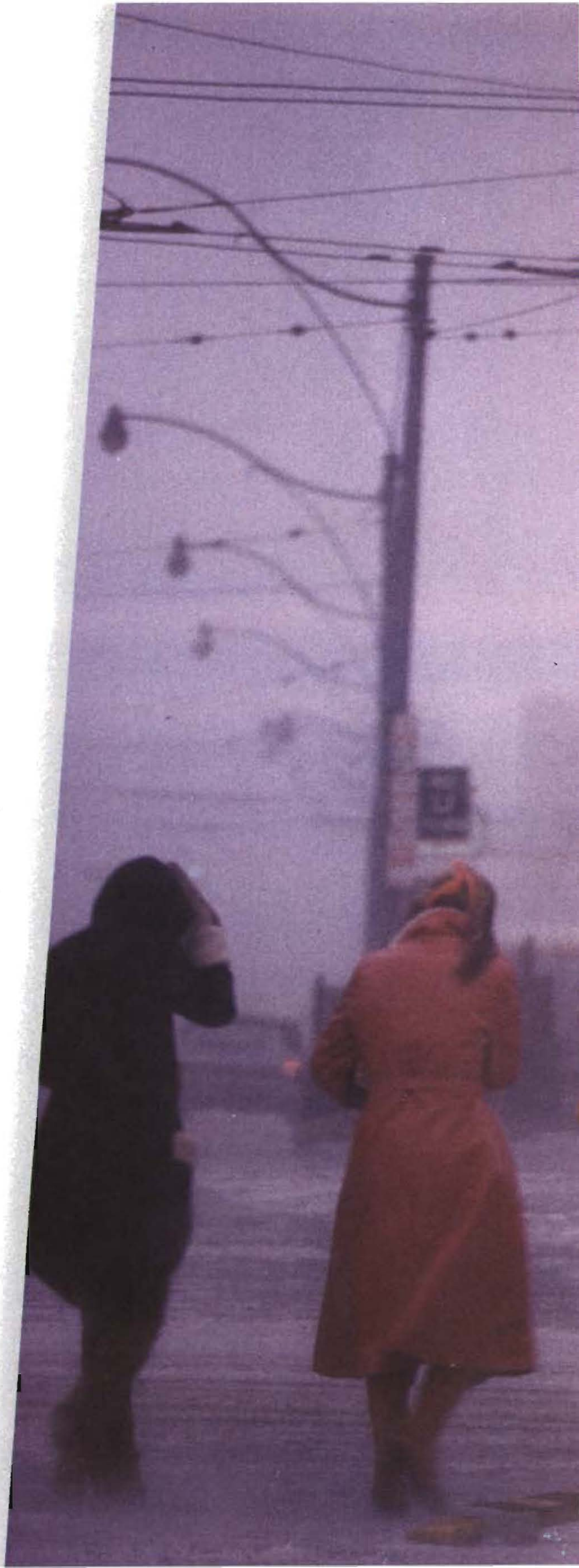
Find the common factors of 15 and 18.

Solution

Write each number as a product of its prime factors.

$15 = 3 \times 5$
$18 = 2 \times 3 \times 3$
The numbers 15 and 18 have only 1 common factor, 3.

Note that the factor 1 is common to all numbers. Therefore, we do not include it as a common factor.



Example 2

Determine the GCF of each pair of numbers.

a) 24 and 48 **b)** 36 and 42

Solution

a) Write each number as a product of its prime factors.

$24 = 2 \times 2 \times 2 \times 3$

$48 = 2 \times 2 \times 2 \times 2 \times 3$

Multiply the common factors to calculate the GCF. The GCF of 24 and 48 is $2 \times 2 \times 2 \times 3$ or 24.

b) $36 = 2 \times 2 \times 3 \times 3$

$42 = 2 \times 3 \times 7$

The GCF of 36 and 42 is 2×3 or 6.

Example 3

a) Determine the common factors of the monomials $2x^2$ and $4x$.

b) Write their GCF.

Solution

For algebraic expressions, the variable must also be factored. Write each expression as a product. Write the factors common to both.

a) $2x^2 = 2 \times x \times x$

$4x = 2 \times 2 \times x$

The common factors of $2x^2$ and $4x$ are 2 and x.

b) The GCF of $2x^2$ and $4x$ is $2x$.

Example 4

Find the GCF of $2x^3y$, $4x^2y^2$, and $2x^2y$.

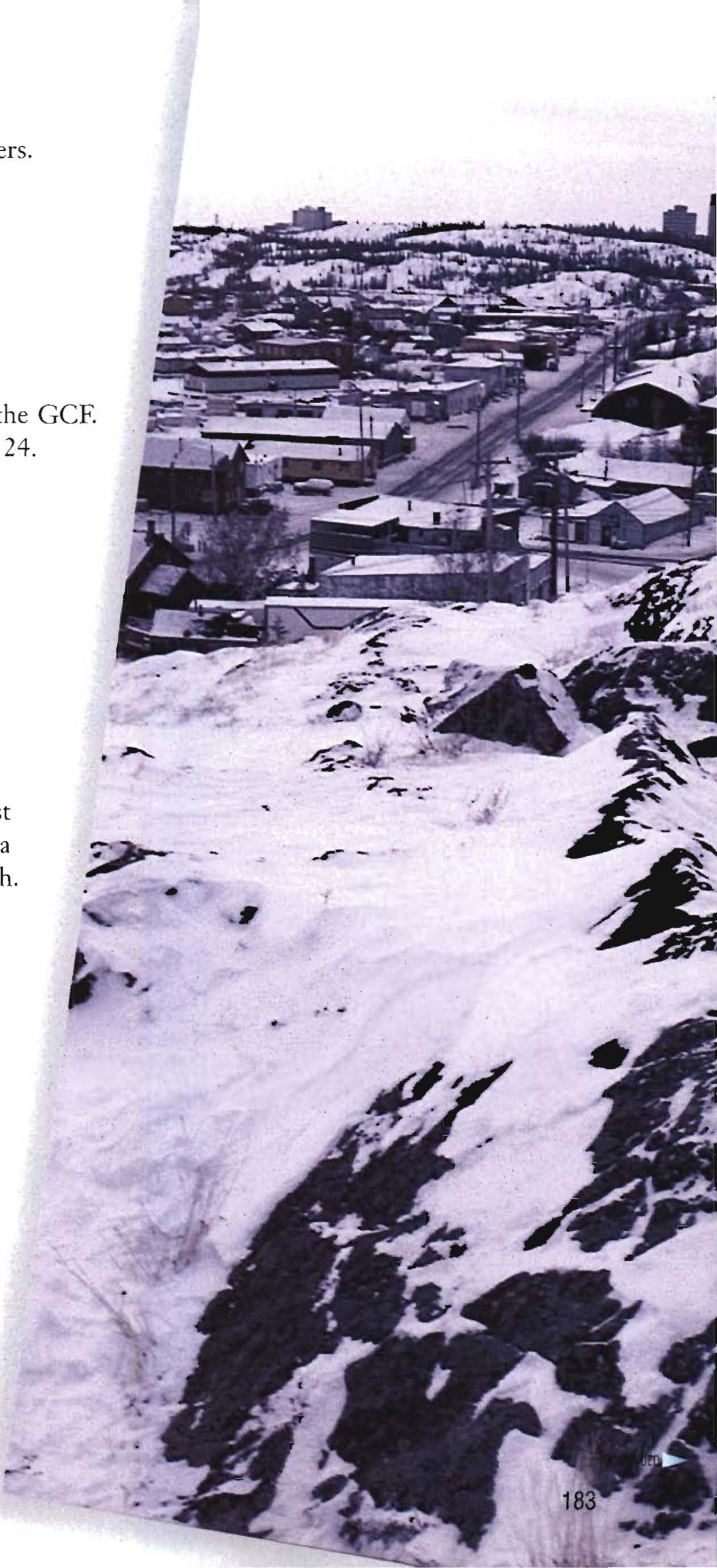
Solution

$2x^3y = 2 \times x \times x \times x \times y$

$4x^2y^2 = 2 \times 2 \times x \times x \times y \times y$

$2x^2y = 2 \times x \times x \times y$

The GCF of $2x^3y$, $4x^2y^2$, and $2x^2y$ is $2 \times x \times x \times y$ or $2x^2y$.



Practice

Write the prime factors of each number.

1. 12 **2.** 16 **3.** 28

4. 63 **5.** 144 **6.** 225

Factor fully.

7. $4xy^2$ **8.** $18a^2b^3$ **9.** $36x^2yz^2$

10. $10x^2y$ **11.** $54x^5$ **12.** $125a^4b^2$

Determine the GCF of each pair.

13. 15, 20 **14.** 16, 24 **15.** 27, 36

16. 28, 42 **17.** 48, 72 **18.** 64, 96

Determine the GCF of each pair.

19. $4a$, $6a$ **20.** $2x^2$, $3x$

21. $12m^3$, $10m^2$ **22.** $12abc$, $3abc$

23. $2x$, $4y$ **24.** $14a$, $7b$

25. $5x^2$, $10x$ **26.** $4xy$, $5xy$

27. $9mn^2$, $8mn$ **28.** $2a^3$, $8a^2$

29. $15bc$, $25b^2c$ **30.** $6x^2y^2$, $9xy$

Determine the GCF of each set.

31. $5xyz$, $10abc$, $25pqr$

32. $20x$, $10x^3$, $8x^2$

33. $12abc$, $18ab$, $6ac$

34. $10x^2y$, $15xy^2$, $25xyz$

35. $21a^2b$, $35a^2b^2c$, $49ab^2c$

36. $12xy$, $16x^2y$, $20xyz$

37. $56abc$, $64a^2b$, $36ab^2c$

Find the GCF.

38. x^2y^2, x^2y^3, x^3y^4

39. $2x^3y$, $4x^2y^4$, $2x^2y^4$

40. $3x^2y^3$, $3x^3y^2$, $6xy^2$

41. $4a^3b^3$, $8a^2b^3$, $16ab^3$

42. $10s^4t^5$, $5s^5t^4$, $15s^3t^4$

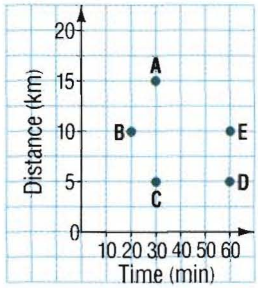
Problems and Applications

43. Montreal has 162 wet days a year, whereas Beijing has 66. What is the GCF of these numbers?

44. Two rectangles are attached as shown.

What is the length of the common side? Explain.

45. Suppose students can get to school by walking at 5 km/h, riding a bike at 10 km/h, or being driven in a car at 30 km/h. Bob rides a bike, as does Karin. Bob lives the same distance from the school as Collette, who walks. Gustav and Shirley come by car. Karin and Gustav live the same distance from school. The graph gives the times the five students take to get to school one day.

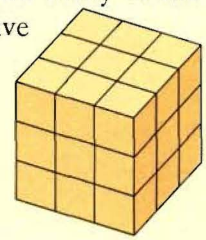


a) Which person does each point represent? Give reasons for your answers.

b) Draw a map of the area, showing where each student could live. Compare your map with a classmate's.

LOGIC POWER

If the outside of the large cube is painted red, how many of the smaller cubes have the following numbers of red faces?



a) 3 **b)** 2

c) 1 **d)** 0

5.2 Factoring Expressions with Common Factors

The world's narrowest commercial building is in Vancouver, British Columbia. The Sam Kee building is 30 m long by 1.8 m wide. In how many ways can you determine the perimeter?

Activity: Discover the Relationship

Examine the diagram of an x-tile.

Inquire

1. What is the sum of the length and width of the x-tile?

2. Write an expression for twice the sum of the length and width. Do not expand.

3. Add the sides of the x-tile to find the perimeter. Collect like terms.

4. How do the quantities represented in questions 2 and 3 compare?

5. Write a rule for removing a common factor from the terms in a polynomial.

6. Use your rule to remove a common factor from each of the following.

a) $3x + 6$ **b)** $4x + 4$ **c)** $2x - 8$

Example

a) Factor the expression $6x^2 - 14x$.
b) Check your answer by expanding.

Solution

a) Determine the GCF of both terms.
Then, divide both terms by the GCF.

$6x^2 = 2 \times 3 \times x \times x$
$14x = 2 \times 7 \times x$

The GCF is $2x$.

The second factor is $\frac{6x^2}{2x} - \frac{14x}{2x}$ or $3x - 7$.

The factors of $6x^2 - 14x$ are $2x$ and $3x - 7$.

Therefore, $6x^2 - 14x = 2x(3x - 7)$.

b) $2x(3x - 7) = 2x(3x - 7)$
$= 6x^2 - 14x$

Checks!

CONTINUED

Practice

State the missing factor.

1. $12x + 18y = (\blacksquare)(2x + 3y)$

2. $3x^2 - 5x = (\blacksquare)(3x - 5)$

3. $4ab + 3ac = (\blacksquare)(4b + 3c)$

4. $5x^2 + 10x = (\blacksquare)(x + 2)$

5. $8abc - 12ab = (\blacksquare)(2c - 3)$

Copy and complete.

6. $3y^2 + 18y = 3y(y + \blacksquare)$

7. $14a - 12b = 2(\blacksquare - 6b)$

8. $4a^3 - 8a^2 = 4a^2(\blacksquare - 2)$

9. $10x^3 - 5x^2 + 15x = 5x(2x^2 - \blacksquare + \blacksquare)$

Copy and complete.

10. $33ab - 22b = 11b(\blacksquare - \blacksquare)$

11. $4a^3 - 10a^2 + 6a = 2a(\blacksquare - \blacksquare + \blacksquare)$

12. $27a^2b^2 - 18ab + 9b = 9b(\blacksquare - \blacksquare + \blacksquare)$

13. $6x^2y - 4xy^2 = 2xy(\blacksquare - \blacksquare)$

14. $9a^3b - 12ab^4 = 3ab(\blacksquare - \blacksquare)$

Factor each binomial.

15. $10x + 15$

16. $28y - 14$

17. $2mn - n$

18. $5x^2 + 10x$

19. $8x^2 + 4x^3$

20. $9a^3b^2 - 6a^2b$

21. $4x^2y^2 - 6xy^2z^2$

22. $14a^2b^4 - 21b^2c^2$

23. $6x^2y^3z + 12xy^2z$

24. $15a^2b^5 - 9b^4c^5$

Problems and Applications

Factor each trinomial.

25. $9a - 6b + 3$

26. $4a - 8b + 16$

27. $12x^3 - 6x^2 + 24x$

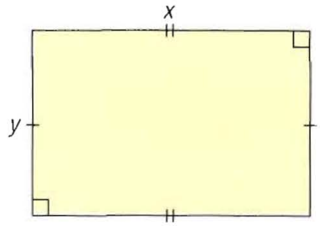
28. $10x^3 - 5x^2 + 15x$

29. $24x^4y - 18x^3y + 12x^2y^2$

30. $8a^2b + 16ab - 24a$

31. $25m^3n - 15m^2n^2 + 5mn^3$

32. a) Write the rectangle's perimeter as the sum of 2 different products and as the product of a number and a sum.



b) Which of the 2 forms in part a) is the factored form?

33. The perimeter of a rectangle is 46 cm. The length is 1 cm longer than the width. What are the rectangle's dimensions?

34. Find the GCF of each expression and factor fully.

a) $(a + b)x + (a + b)y$

b) $x(x - 2) + 3(x - 2)$

c) $x(2x - 3) - 5(2x - 3)$

d) $2a(a - b) + b(a - b)$

35. If you stand on the Earth and jump up at 5 m/s, your approximate height in metres above the ground after t seconds is given by the expression $5t - 5t^2$.

a) Factor the binomial.

b) Evaluate for $t = 0.4$ s.

PATTERN POWER

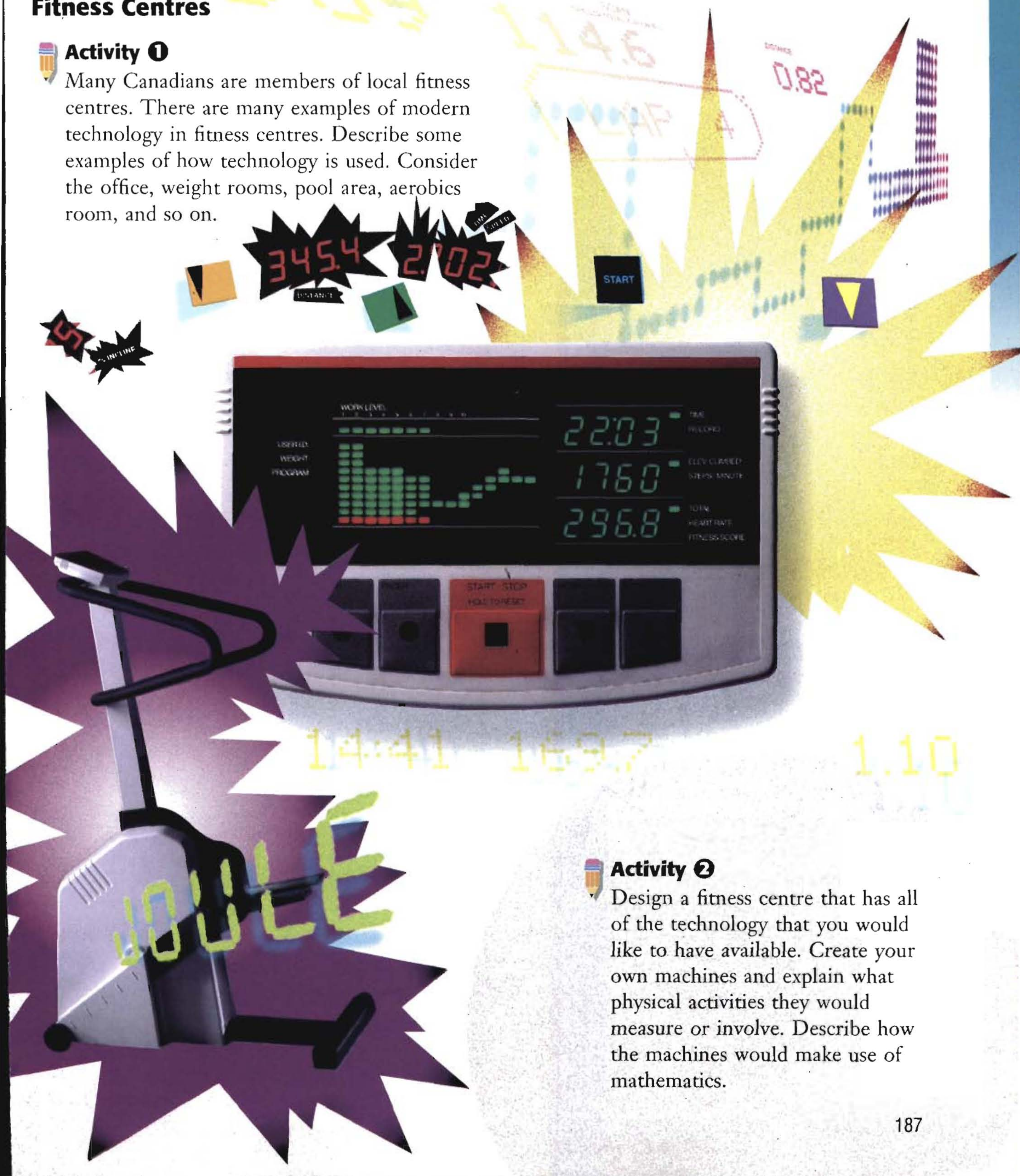
Find the missing number.

4	7	8	5
3		3	4
2	5	7	3
2	3	5	2

Fitness Centres

Activity ❶

Many Canadians are members of local fitness centres. There are many examples of modern technology in fitness centres. Describe some examples of how technology is used. Consider the office, weight rooms, pool area, aerobics room, and so on.



Activity ❷

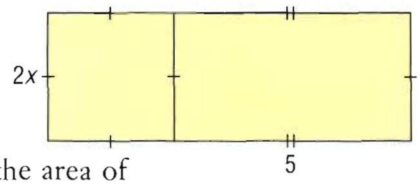
Design a fitness centre that has all of the technology that you would like to have available. Create your own machines and explain what physical activities they would measure or involve. Describe how the machines would make use of mathematics.

5.3 Multiplying a Polynomial by a Monomial

Northern Dancer was one of the fastest thoroughbred horses that ever raced. He was born, raised, and trained at Winfields Farm in Oshawa. The 500 ha of Winfields Farm contains many different sizes and shapes of rectangular fields attached to each other, like the rectangles shown below.

Activity: Explore the Pattern

Copy or trace this diagram onto a blank sheet of paper.



Inquire

1. In simplest form, what is the area of

a) the square? **b)** the rectangle attached to the square?

2. Write the sum of the areas for the square and the rectangle.

3. Consider the larger rectangle that includes the square.

a) What is the length of the larger rectangle?

b) What is the width of the larger rectangle?

c) Use your answers to parts a) and b) to write an expression for the area.

d) How can you multiply the monomial in your expression by the binomial in your expression to arrive at the answer you gave in question 2?

4. Write a rule for multiplying a polynomial by a monomial.

Example 1

Expand and simplify.

a) $2x(x^2 - 2x + 5)$

b) $4x(x - 3) - 2(x + 3)$

Solution

Use the distributive property to expand each expression. Then, collect like terms and simplify.

a) $2x(x^2 - 2x + 5)$

$= 2x(x^2 - 2x + 5)$

$= 2x^3 - 4x^2 + 10x$

b) $4x(x - 3) - 2(x + 3)$

$= 4x(x - 3) - 2(x + 3)$

$= 4x^2 - 12x - 2x - 6$

$= 4x^2 - 14x - 6$

Example 2

Expand and simplify $2(3x^2 - 4x + 5) - 2x(x - 3)$.

Solution

$2(3x^2 - 4x + 5) - 2x(x - 3)$

$= 2(3x^2 - 4x + 5) - 2x(x - 3)$

$= 6x^2 - 8x + 10 - 2x^2 + 6x$

$= 4x^2 - 2x + 10$

Practice

Expand.

1. $x(x+2)$ **2.** $x(x-3)$ **3.** $a(a+1)$

4. $t(t-1)$ **5.** $y(y+4)$ **6.** $m(m+5)$

7. $x(x-5)$ **8.** $y(y-7)$ **9.** $a(a-10)$

Expand.

10. $3x(x+2)$ **11.** $4b(b-11)$

12. $5t(t+3)$ **13.** $2x(3+x)$

14. $7y(y-5)$ **15.** $-2x(x+4)$

16. $-x(x+2)$ **17.** $-y(y-3)$

Expand and simplify.

18. $x(x+3)-x(x-2)$

19. $y(2+y)+y(y-1)$

20. $m(m-1)+m(m-1)$

21. $x(x+2)-(2x-2)$

22. $y(y-4)-y(3-2y)$

23. $a(2a-1)+a(a+1)$

24. $x(x-2)-x(x+1)$

Expand and simplify.

25. $3x(x+2)+2x(x+5)$

26. $2x(x-3)-x(x-5)$

27. $3x(2x+1)+x(3x+2)$

28. $-2y(y-3)-y(y+1)$

29. $2a(a+3)+3a(a-2)$

30. $-x(3x-4)-2x(1-x)$

31. $4x(x+2)+2x(7-2x)$

Expand.

32. $x(x^2+2x+3)$ **33.** $3(x^2+2x-5)$

34. $5x(x^2+2x-7)$ **35.** $-(x^2-3x-1)$

36. $4m(m^2-5m+6)$ **37.** $3y(2y^2-4y+3)$

38. $-3b(3b^2-5b+1)$ **39.** $-5z(z^2-2z-5)$

Problems and Applications

Expand and simplify.

40. $3(x^2+2x-5)-x(x+1)$

41. $5(x^2+2x-7)+3x(x+1)$

42. $-(x^2-3x-1)+x(3x+2)$

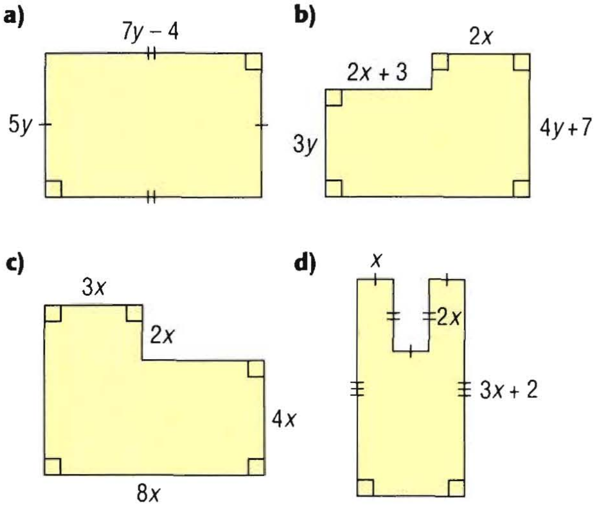
43. $4(2x+3)+3x(x^2-x+3)$

44. $3m(m-2)+4(m^2-5m+6)$

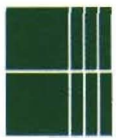
45. $5y(1-y)+3(2y^2-4y+3)$

46. $-3x(x+2)+2x(2x-1)$

47. Write, expand, and simplify an expression for the area of each figure.



48. Explain why the area of the algebra tiles models the product $2x(x+3)$.



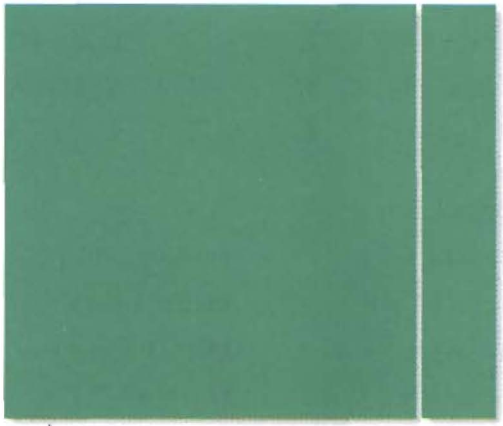
49. Use algebra tiles to model each of the following products. Expand each expression to check that the model is correct.

a) $4x(x+1)$ **b)** $x(3x+2)$ **c)** $2x(2x+1)$

50. Write a problem similar to those in question 47. Have a classmate solve your problem.

5.4 Dividing Polynomials by Monomials

An x^2-tile has an area of x^2 square units.
An x-tile has an area of x square units.



Activity: Use Algebra Tiles

Use an x^2-tile and an x-tile to model this rectangle.

Inquire

1. What is the area of the rectangle?

2. What is the length of the rectangle?

3. What is the width of the rectangle?

4. Describe how you can divide the area by the width to give the length.

5. Write a rule for dividing a polynomial by a monomial.

The distributive property applies to division as well as to multiplication.

$$\frac{a+b}{c} = \frac{1}{c}(a+b) = \frac{a}{c} + \frac{b}{c}$$

Example 1

Simplify.

a) $\dfrac{5xyz + 10xy}{5xy}$ **b)** $\dfrac{8x^5 + 12x^3 - 4x^2}{4x}$

Solution

Divide each term of the polynomial by the monomial.

a) $\dfrac{5xyz + 10xy}{5xy} = \dfrac{5xyz}{5xy} + \dfrac{10xy}{5xy}$
$= z + 2$

b) $\dfrac{8x^5 + 12x^3 - 4x^2}{4x} = \dfrac{8x^5}{4x} + \dfrac{12x^3}{4x} - \dfrac{4x^2}{4x}$
$= 2x^4 + 3x^2 - x$

Example 2

Simplify.

a) $\dfrac{15y^4 - 12y^5 + 9y^3}{-3y^2}$ **b)** $\dfrac{16a^5b^3 - 20a^4b^4 + 24a^5b^3}{4a^3b^3}$ **c)** $\dfrac{9a^4b^7z^5 - 15ab^3z^2}{-3ab^3z}$

Solution

a) $\dfrac{15y^4 - 12y^5 + 9y^3}{-3y^2}$
$= \left(\dfrac{15y^4}{-3y^2}\right) - \left(\dfrac{12y^5}{-3y^2}\right) + \left(\dfrac{9y^3}{-3y^2}\right)$
$= -5y^2 + 4y^3 - 3y$
$= 4y^3 - 5y^2 - 3y$

b) $\dfrac{16a^5b^3 - 20a^4b^4 + 24a^5b^3}{4a^3b^3}$
$= \dfrac{16a^5b^3}{4a^3b^3} - \dfrac{20a^4b^4}{4a^3b^3} + \dfrac{24a^5b^3}{4a^3b^3}$
$= 4a^2 - 5ab + 6a^2$

c) $\dfrac{9a^4b^7z^5 - 15ab^3z^2}{-3ab^3z}$
$= \dfrac{9a^4b^7z^5}{-3ab^3z} - \left(\dfrac{15ab^3z^2}{-3ab^3z}\right)$
$= -3a^3b^4z^4 - (-5z)$
$= -3a^3b^4z^4 + 5z$

Practice

Divide.

1. $\dfrac{12xy}{4xy}$
2. $\dfrac{24ab}{-4ab}$
3. $\dfrac{-12mn}{3m}$
4. $\dfrac{-30xy}{-5xy}$
5. $\dfrac{11ab}{ab}$
6. $\dfrac{5xy}{5x}$
7. $\dfrac{24x^2y}{6xy}$
8. $\dfrac{15ab^3}{-5ab}$
9. $\dfrac{36x^3y^2}{-6x^2y^2}$

Divide.

10. $\dfrac{12xy - 15y^2 + 24y}{3y}$
11. $\dfrac{5x^3 + 10x^2 - 15x}{5x}$
12. $\dfrac{7y^4 + 7y^3 - 21y}{-7y}$
13. $\dfrac{4m^3 + 8m^2 - 12m}{4m}$
14. $\dfrac{9x^3 - 24x^2 - 15x}{-3x}$
15. $\dfrac{6j^5 + 12j^4 + 18j^3}{-6j}$

Divide.

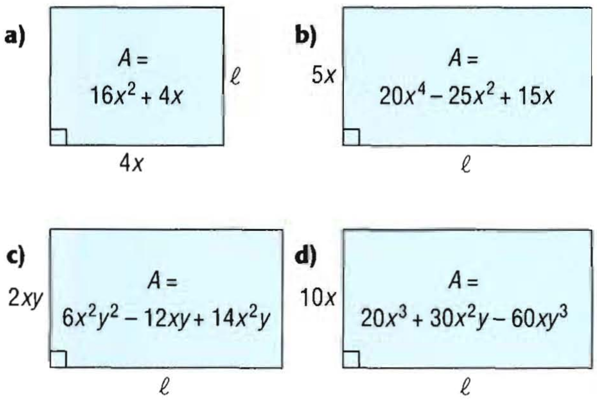
16. $\dfrac{10x^4 + 5x^3 - 15x^2}{-5x^2}$
17. $\dfrac{-21m^2 + 14m^3 - 21m^4}{-7m^2}$
18. $\dfrac{10p^2q^2 - 15pq^3 + 25p^3q^4}{-5pq^2}$
19. $\dfrac{-12a^3b^2 + 9a^2b^3 + 24a^4b^4}{3a^2b^2}$

Divide.

20. $\dfrac{-20x^3yz + 30x^2y^2z - 40xy^3z}{-10xyz}$
21. $\dfrac{8a^3b^2c^3 - 12a^2b^2c^2 + 16a^2b^3c}{4a^2b^2}$
22. $\dfrac{-12x^4y^6 - 16x^5y^5 - 24x^6y^4}{-4x^4y^4}$
23. $\dfrac{30m^3n^5 - 36m^4n^4 - 30m^5n^3}{6m^3n^3}$
24. $\dfrac{25a^3b^3c^5 - 40a^4b^3c^4 + 35a^6b^4c^3}{-5a^2b^2c^2}$

Problems and Applications

25. Determine the length, l, of the unknown side, given the area and the length of one side.



26. Write a problem similar to those in question 25. Have a classmate solve your problem.

LOGIC POWER

Four students entered a problem solving contest. Each student represented a different zone of the town. Use the clues to determine which zone each student represented and in which order the students finished.

1. David came second, just behind the student from the west zone.

2. Petra represented neither the east nor the west zones.

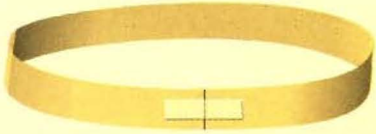
3. The student from the north zone finished second last, just ahead of Frank.

4. David and Jarvi represented opposite zones in the town.

The Möbius Strip

Activity ❶ Making a Möbius Strip

1. Take a strip of paper about 30 cm long and 5 cm wide and tape it together to make a ring. You could paint this ring with 2 colours, red on the inside and blue on the outside, because the ring has 2 sides. To get from one side to the other, you have to cross an edge.



2. Take another strip of paper about 30 cm long and 5 cm wide. Turn 1 end of the paper to make a half twist. Then, tape the 2 ends together. The result is called a Möbius strip after the German mathematician, Augustus Möbius, who discovered it.

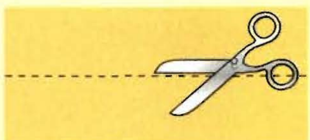
3. Use a coloured pencil and start "painting" the strip. You will notice you can "paint" the whole strip with just one colour. You never have to cross an edge. The strip has only one side.

Activity ❷ Cutting a Möbius Strip in Half

If you cut a paper ring around the middle, you get 2 paper rings, each one-half the width of the original.

1. Predict what you will get when you cut a Möbius strip around the middle.

2. Cut the Möbius strip around the middle and describe the results. How many sides does the resulting figure have? Is it a Möbius strip? Explain.

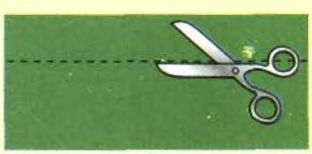


Activity ❸ Cutting a Möbius Strip into Thirds

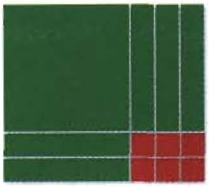
If you cut a paper ring into thirds, you get 3 paper rings, each one-third the width of the original.

1. Predict what you will get if you cut a Möbius strip into thirds.

2. Cut a Möbius strip starting one-third the way in from its edge. The scissors will make 2 trips around the strip. Describe the result. How many sides does each of the resulting figures have? Are they both Möbius strips? Explain.



5.5 Binomial Products



Activity: Use Algebra Tiles

Use algebra tiles to model this rectangle.

Inquire

1. Write an expression for the area of the rectangle by counting the numbers of x^2-tiles, x-tiles, and 1-tiles. Collect like terms and write the expression in descending powers of x.

2. Write an expression for the length of the rectangle.

3. Write an expression for the width of the rectangle.

4. Use the expressions for the length and width to write an expression for the area of the rectangle. Do not expand.

5. How do the quantities you represented in questions 1 and 4 compare?

6. Write a rule for multiplying 2 binomials.

7. Use your rule to multiply the following. Check your results by modelling with algebra tiles.

a) $(x+1)(x+3)$ **b)** $(x+2)(x+4)$

8. Check your results in question 7 by substituting 1 for x.

Example

Find the product of these binomials.

a) $(x+1)(x+2)$ **b)** $(3x-4)(5x+1)$

Solution

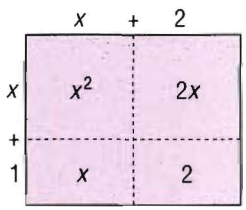
To expand a binomial, use the distributive property.

a) $$\begin{aligned}(x+1)(x+2) &= x(x+2)+1(x+2)\\ &= x^2+2x+x+2\\ &= x^2+3x+2\end{aligned}$$

b) $$\begin{aligned}(3x-4)(5x+1) &= 3x(5x+1)-4(5x+1)\\ &= 15x^2+3x-20x-4\\ &= 15x^2-17x-4\end{aligned}$$

The same result can be obtained by multiplying each term in the first binomial by each term in the second binomial.
You can remember this method with the acronym FOIL, which stands for First terms, Outside terms, Inside terms, and Last terms.

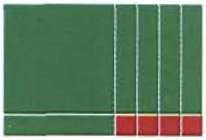
$$\begin{aligned}(x+1)(x+2) &= (x+1)(x+2)\\ &= \underset{F}{x^2}+\underset{O}{2x}+\underset{I}{x}+\underset{L}{2}\\ &= x^2+3x+2\end{aligned}$$



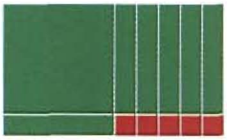
CONTINUED

Practice

Express each area as a product and in expanded form.

1.


2.


3.


4.


Expand.

5. $3(x-5)$ **6.** $x(2x+3)$

7. $-7(2x-6)$ **8.** $4x(3x-1)$

9. $5a^2(3a-4b)$ **10.** $-2x(3x+5y)$

Find the product.

11. $(x+1)(x+2)$ **12.** $(x+4)(x+3)$

13. $(a+4)(a+4)$ **14.** $(y+5)(y+6)$

15. $(x-4)(x-3)$ **16.** $(a-4)(a-2)$

17. $(b-1)(b-5)$ **18.** $(y-9)(y-9)$

19. $(x-6)(x+3)$ **20.** $(c+2)(c-8)$

21. $(t+10)(t-10)$ **22.** $(q-2)(q+5)$

Expand.

23. $(c+3)(c-4)$ **24.** $(x+2)(x-5)$

25. $(y+6)(y-2)$ **26.** $(a+9)(a-5)$

27. $(x-3)(x+3)$ **28.** $(b-7)(b+10)$

29. $(y-12)(y+3)$ **30.** $(x-7)(x+1)$

Multiply.

31. $(x+5)(2x+1)$ **32.** $(3y+1)(y+2)$

33. $(x-1)(2x-1)$ **34.** $(a-3)(-2a-5)$

35. $(5y-7)(y+3)$ **36.** $(x-5)(4x+3)$

37. $(-2x+3)(2x+1)$ **38.** $(5y-2)(3y-4)$

39. $(4x+1)(3x-5)$ **40.** $(2y-9)(5y+2)$

41. $(7y-3)(2y-7)$ **42.** $(-3x-2)(8x+5)$

Problems and Applications

Multiply.

43. $(x+0.5)(x+2)$ **44.** $(x-1.2)(x+3)$

45. $(x-2.5)(x-10)$ **46.** $(x-3)(x+2.1)$

Expand the following.

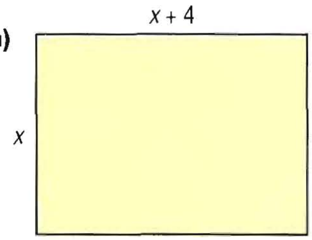
47. $2(x+3)(x+5)$ **48.** $4(x-9)(x+5)$

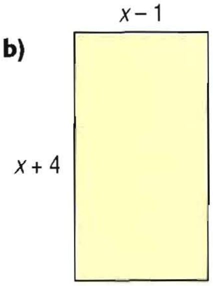
49. $-1(a+3)(a-2)$ **50.** $10(x+7)(x-5)$

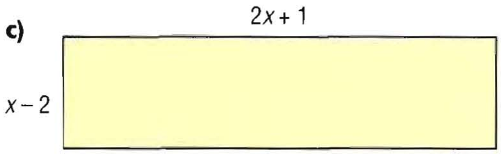
51. $3(2x-1)(3x-2)$ **52.** $2x(x+7)(x-10)$

53. $0.5(x-1)(x+3)$ **54.** $1.8(x+1)(x+1)$

55. Evaluate each area in 2 different ways for $x = 3$ cm.

a)


b)


c)


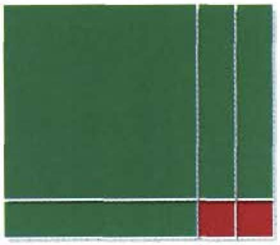
56. a) Verify that $(x+6)(x+2) \neq x^2 + 12$ by substituting 1 for x.
b) Expand $(x+6)(x+2)$ correctly.

57. A square building of side x metres is extended by 10 m on one side and 5 m on the other side to form a rectangle.
a) Express the new area as the product of 2 binomials.
b) Evaluate the new area for $x = 20$.

58. After 2 years, an investment of \$1000 compounded annually at an interest rate r grows to the amount $1000(1+r)^2$ in dollars.
a) Expand the expression.
b) Evaluate for $r = 0.08$.

5.6 Factoring Trinomials: $x^2 + bx + c$

Many trinomials can be written as a product of 2 binomials.



Activity: Use Algebra Tiles

Use algebra tiles to model this rectangle.

Inquire

1. Write an expression for the area of the rectangle by counting tiles. Collect like terms and write the expression in descending powers of x.

2. Use expressions for the length and width to write an expression for the area of the rectangle. Do not expand.

3. How are the quantities represented by your 2 expressions related?

4. How is the numerical term in the expression from question 1 related to the numerical terms in the expression from question 2?

5. How is the coefficient of the x-term in the expression from question 1 related to the numerical terms in the expression from question 2?

6. How is the x^2-term in the expression from question 1 related to the x-terms in the expression from question 2?

7. Use your findings to factor each of the following.

a) $x^2 + 5x + 6$ **b)** $x^2 - 3x + 2$ **c)** $x^2 - x - 2$

Expanding 2 general binomials gives the following results.

$$\begin{aligned}(x + m)(x + n) &= x^2 + nx + mx + mn \\ &= x^2 + mx + nx + mn \\ &= x^2 + (m + n)x + mn\end{aligned}$$

Compare the above result to the general trinomial $x^2 + bx + c$.

$$\begin{array}{ccc} x^2 + & bx & + \; c \\ \updownarrow & \updownarrow & \updownarrow \\ x^2 + & (m + n)x & + \; mn \end{array}$$

When we make this comparison, we see that $b = m + n$ and $c = mn$.

Therefore, the general trinomial $x^2 + bx + c = x^2 + (m + n)x + mn$

$$= (x + m)(x + n)$$

CONTINUED

Example 1

Factor $x^2 - 10x + 16$.

Solution

To factor $x^2 - 10x + 16$, find m and n so that $m + n = -10$ and $mn = 16$.
This means that both factors are negative.
List pairs of factors of 16 and choose the correct pair.

Therefore, $m = -2$ and $n = -8$.

$$\begin{aligned} x^2 - 10x + 16 &= x^2 + (-2 - 8)x + (-2)(-8) \\ &= (x - 2)(x - 8) \end{aligned}$$

Pairs of Factors of 16	Sum of Pairs of Factors
1, 16	17
−1, −16	−17
2, 8	10
−2, −8	−10
4, 4	8
−4, −4	−8

Example 2

Factor $x^2 + 4x - 21$.

Solution

Find m and n so that $m + n = 4$ and $mn = -21$.

This means that one factor is positive and one factor is negative.

Therefore, $m = -3$ and $n = 7$.

$x^2 + 4x - 21 = (x - 3)(x + 7)$

Pairs of Factors of −21	Sum of Pairs of Factors
1, −21	−20
−1, 21	20
3, −7	−4
−3, 7	4

Example 3

Factor $2x^2 - 4x - 70$.

Solution

Factor the trinomial first to remove the GCF, which is 2.

$2x^2 - 4x - 70 = 2(x^2 - 2x - 35)$

Find m and n so that $m + n = -2$ and $mn = -35$.
One value is positive and one value is negative.

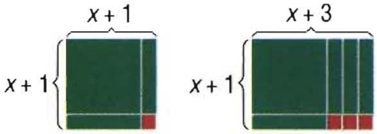
Therefore, $m = 5$ and $n = -7$.

$2x^2 - 4x - 70 = 2(x + 5)(x - 7)$

Pairs of Factors of −35	Sum of Pairs of Factors
1, −35	−34
−1, 35	34
5, −7	−2
−5, 7	2

Practice

Natasha modelled the process of factoring $x^2 + 2x + 1$ and $x^2 + 4x + 3$ as shown.



Use algebra tiles and her method to factor these expressions.

1. $x^2 + 5x + 4$ **2.** $x^2 + 6x + 9$

State the values of m and n that satisfy the given conditions.

3.	Sum $m + n$	Product mn
a)	7	12
b)	8	15
c)	13	12
d)	18	77
e)	−8	15
f)	−10	25
g)	−7	12

4.	Sum $m + n$	Product mn
a)	−1	−12
b)	1	−12
c)	−3	−40
d)	25	150
e)	4	−5
f)	−1	−42
g)	−7	−60

Factor.

5. $x^2 + 7x + 10$ **6.** $y^2 - 8y + 15$

7. $w^2 - w - 56$ **8.** $z^2 + 3z - 40$

9. $x^2 - x - 30$ **10.** $a^2 - 17a + 16$

11. $x^2 - 9x - 10$ **12.** $x^2 + 12x + 20$

13. $x^2 + 10x + 25$ **14.** $m^2 - 9m + 18$

15. $a^2 - 6a + 9$ **16.** $y^2 + 11y + 30$

17. $x^2 + 10x + 9$ **18.** $x^2 - 15x - 16$

19. $a^2 + 6a - 16$ **20.** $x^2 + 9x + 20$

21. $a^2 - 25a + 24$ **22.** $y^2 - 9y + 14$

23. $y^2 - 7y - 18$ **24.** $x^2 - x - 72$

25. $s^2 - 2s - 80$ **26.** $a^2 - 18a + 81$

Simplify by combining like terms, if possible. Then, remove the GCF and factor fully.

27. $2x^2 - 21x + 36 + x^2$ **28.** $5x^2 - 2x - 10 - 3x$

29. $7x^2 + 35x + 42$ **30.** $b^2 + 3b + 4 + b^2 + 5b$

31. $bx^2 - 28bx + 75b$ **32.** $x^2 - 3x + 8 - 9x + x^2$

33. $5jx^2 - 40jx + 75j$ **34.** $3tx^2 + 12tx + 12t$

35. $t^3 + t^2 - 12t$ **36.** $3k^3 + 15k^2 - 18k$

Problems and Applications

Factor, if possible.

37. $x^2 + x + 1$ **38.** $a^2 - 7a - 8$

39. $b^2 + 14b + 48$ **40.** $y^2 + 7y - 12$

41. $z^2 - 20z + 100$ **42.** $m^2 - 2m + 5$

43. $x^2 - 4x - 4$ **44.** $y^2 + 8y - 20$

45. The area of a rectangle is represented by the expression $x^2 + 9x + 20$.

a) Factor the expression.

b) A smaller rectangle is 1 unit shorter on each side than the first rectangle. Write a factored expression for the area of the smaller rectangle.

c) Expand the expression for the area of the smaller rectangle.

46. Write 4 trinomials in the form $x^2 + 12x + \blacksquare$ that can be factored.

47. a) Use algebra tiles to factor $x^2 - 2x - 3$. Explain your method.

b) Use your method to factor $x^2 - 3x - 4$ using algebra tiles.

48. a) Complete the factoring by supplying the missing terms.

$x^2 + 6x + \blacksquare = (x + \blacksquare)(x + \blacksquare)$

$x^2 - 5x + \blacksquare = (x - \blacksquare)(x - \blacksquare)$

$x^2 + \blacksquare x + 12 = (x + \blacksquare)(x + \blacksquare)$

$x^2 - \blacksquare x + 5 = (x - \blacksquare)(x - \blacksquare)$

$x^2 - \blacksquare x - 12 = (x - \blacksquare)(x + \blacksquare)$

b) Compare your answers with a classmate's. Which cases have more than one solution? Explain.

49. Make up 5 trinomials in the form $x^2 + bx + c$. Make 3 factorable and 2 impossible to factor. Exchange trinomials with a classmate. Try to factor each other's trinomials.

5.7 Special Product: $(a - b)(a + b)$

How are the binomials $x + 4$ and $x - 4$ the same and how are they different?

Activity: Look for a Pattern

Copy and complete this table.

Binomials	Product	Simplified Product
$(x + 4)(x - 4)$	$x^2 + 4x - 4x + 16$	
$(a - 7)(a + 7)$		
$(k + 5)(k - 5)$		
$(x - y)(x + y)$		
$(2a - 1)(2a + 1)$		

Inquire

1. a) What do you notice about the 2 middle terms in each product?
b) What is the algebraic sum of the middle terms in each product?

2. How many terms are there in each simplified product?

3. Write a rule for expanding 2 binomials that are identical except for the signs in the middle.

Example

Expand.

a) $(x + 3)(x - 3)$ **b)** $(7x - 1)(7x + 1)$

Solution

a) $(x + 3)(x - 3) = (x + 3)(x - 3)$
$= x^2 - 3x + 3x - 9$
$= x^2 - 9$

b) $(7x - 1)(7x + 1) = (7x - 1)(7x + 1)$
$= 49x^2 + 7x - 7x - 1$
$= 49x^2 - 1$

These expanded forms represent a special case of the product of 2 binomials, called the **difference of squares**.

$(a + b)(a - b) = a^2 - b^2$

Practice

Write the missing factor.

1. $(a-7)(\blacksquare) = a^2 - 49$
2. $(x+2)(\blacksquare) = x^2 - 4$
3. $(\blacksquare)(3m-7) = 9m^2 - 49$
4. $(9x+8)(\blacksquare) = 81x^2 - 64$
5. $(x-y)(\blacksquare) = x^2 - y^2$
6. $(\blacksquare)(2a+3b) = 4a^2 - 9b^2$

Expand.

7. $(x+1)(x-1)$
8. $(a+5)(a-5)$
9. $(p+6)(p-6)$
10. $(x-9)(x+9)$
11. $(y-8)(y+8)$
12. $(t-10)(t+10)$

Multiply.

13. $(2x-1)(2x+1)$
14. $(3y+1)(3y-1)$
15. $(4a+3)(4a-3)$
16. $(6t-5)(6t+5)$
17. $(x-y)(x+y)$
18. $(10t-3)(10t+3)$
19. $(p+q)(p-q)$
20. $(3a-b)(3a+b)$
21. $(x-6y)(x+6y)$
22. $(j-10r)(j+10r)$
23. $(x^2+3)(x^2-3)$
24. $(k^2+9)(k^2-9)$

Problems and Applications

25. Factor each of the numbers as shown and complete the table.

Numbers	$(a+b)(a-b)$	Product
33×27	$(30+3)(30-3)$	891
24×16		
47×53		
62×58		

Multiply each of the following, using the method in question 25.

26. $(10+2)(10-2)$
27. $(15+3)(15-3)$
28. $(20-2)(20+2)$
29. 14×6
30. 17×23
31. 32×28

Expand.

32. $(x-1)(x+1)(x^2+1)$
33. $(a+2)(a-2)(a^2+4)$
34. $(x-9)(x+9)(x^2+81)$
35. $(3x-2)(3x+2)(9x^2+4)$
36. $(x-1)(x+1)(x^2+1)(x^4+1)$

37. If a square field is made into a rectangle by shortening 2 opposite sides by 50 m each and lengthening the other 2 sides by 50 m each, how do the areas of the original field and the new field compare? Explain.

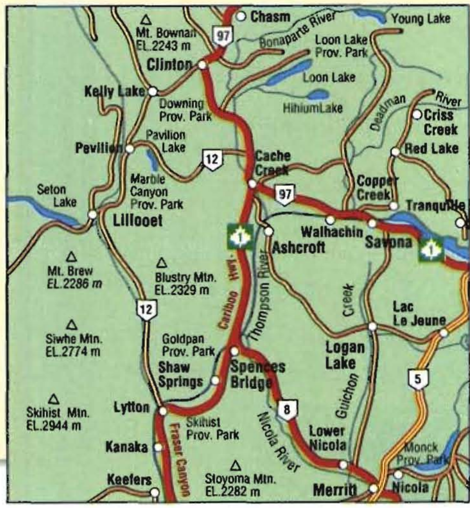
38. Which of the following numbers of terms are not possible as the product of 2 binomials?

1 term
2 terms
3 terms
4 terms
5 terms

Give reasons for your answers.

LOGIC POWER

Use your knowledge of geography to find out what province this area is in. Then find the area on a complete map.



5.8 Factoring the Difference of Squares

Activity: Complete the Table

Copy and complete the table.

Factored Form	Product
$(x-5)(x+5)$	x^2-25
$(b+8)(b-8)$	
$(z+9)(z-9)$	
	a^2-36
	y^2-100
	r^2-81

Inquire

1. How is the first term in each binomial of the factored form related to the first term in the product?

2. How is the second term in each binomial of the factored form related to the second term in the product?

3. Write a rule for working backward from the product to the factored form.

4. Use your rule to factor each of the following.

a) $x^2 - 4$ **b)** $y^2 - 49$ **c)** $4z^2 - 25$

Example

Factor.

a) $x^2 - 49$ **b)** $25a^2 - 81b^2$ **c)** $9y^2 - 81$ **d)** $12b^2 - 27c^2$

Solution

Remove any common factors.
Take the square root of each term in the difference of squares.
Write 2 binomial factors that are identical except for the signs in the middle.

a) $x^2 - 49$
$= (x+7)(x-7)$

b) $25a^2 - 81b^2$
$= (5a-9b)(5a+9b)$

c) $9y^2 - 81z^2$
$= 9(y^2 - 9z^2)$
$= 9(y+3z)(y-3z)$

d) $12b^2 - 27c^2$
$= 3(4b^2 - 9c^2)$
$= 3(2b-3c)(2b+3c)$

Practice

Determine the missing factor.

1. $x^2 - 25 = (x - 5)(\blacksquare)$
2. $w^2 - 100 = (w + 10)(\blacksquare)$
3. $k^2 - 81 = (k - 9)(\blacksquare)$
4. $4a^2 - 121 = (2a - 11)(\blacksquare)$
5. $9 - 16x^2 = (3 + 4x)(\blacksquare)$
6. $x^4 - 36 = (x^2 + 6)(\blacksquare)$

Factor, if possible, using a difference of squares.

7. $t^2 - 4$ **8.** $x^2 - 16$
9. $b^2 - 9$ **10.** $m^2 - 49$
11. $p^2 - 25$ **12.** $w^2 - 36$
13. $a^2 - 81$ **14.** $q^2 - 100$
15. $y^2 + 144$ **16.** $1 - c^2$
17. $64 - x^2$ **18.** $121 + w^2$
19. $s^2 - t^2$ **20.** $z^2 - x^2$
21. $h^2 - g^2$ **22.** $p^2 - q^2$

Factor.

23. $4a^2 - 9b^2$ **24.** $16p^2 - 81$
25. $25a^2 - 49$ **26.** $9b^2 - 25$
27. $16 - x^2$ **28.** $25 - 36b^2$
29. $16 - 49x^2$ **30.** $81 - 4a^2$
31. $16x^2 - 1$ **32.** $144 - 121x^2$
33. $169 - 100t^2$ **34.** $225 - 49w^2$

Factor, if possible.

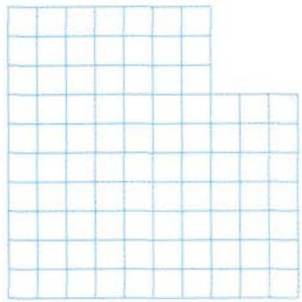
35. $6x^2 - 25$ **36.** $16y^2 - 49$
37. $9 - 4z^2$ **38.** $25a^2 - 36$
39. $x^2y^2 - 4$ **40.** $m^2 + 64$
41. $(a + b)^2 - (a - b)^2$
42. $25 - 81p^2q^2$

Remove the common factor, then factor using a difference of squares.

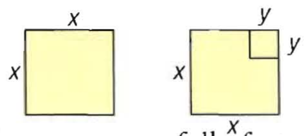
43. $2m^2 - 50$ **44.** $9x^2 - 36$
45. $20r^2 - 45$ **46.** $8a^2 - 50$
47. $10y^2 - 1000$ **48.** $x^3 - x$
49. $50y^2 - 72$ **50.** $x^3 - 9x$
51. $8y^2 - 8$ **52.** $4p^2 - 16$
53. $27k^2 - 12$ **54.** $16t^2 - 16$
55. $64 - x^2$ **56.** $50 - 18x^2$
57. $12x^2 - 75y^2$ **58.** $50x^2 - 98y^2$

Problems and Applications

59. Cut a 10-by-10 square out of grid paper. Remove a 3-by-3 square from one corner.



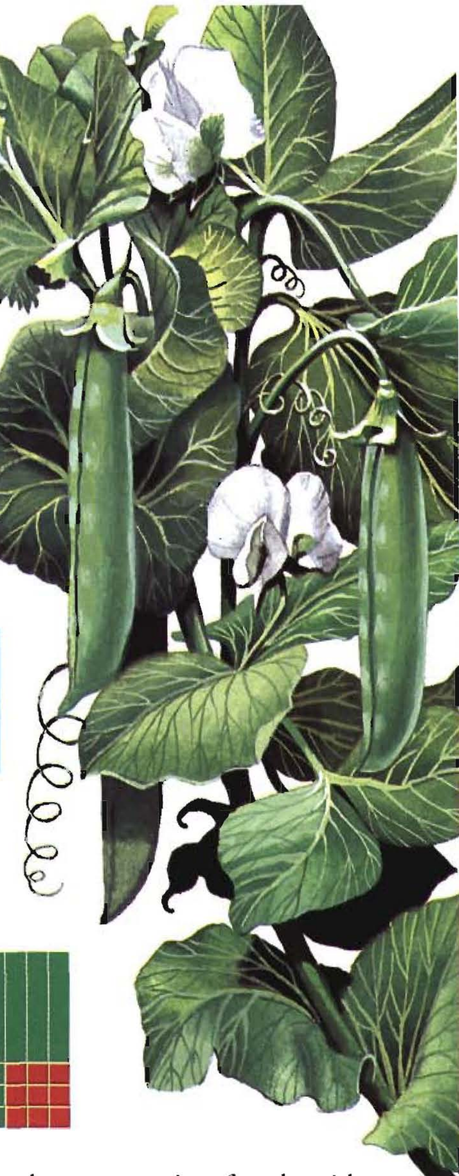
a) The area of the new figure is $10^2 - 3^2$. What is the area?
b) Cut off one of the 7-by-3 rectangles and add it to the larger rectangle to make one rectangle. What are the dimensions of this rectangle?
c) How do the dimensions of the rectangle compare with the result of factoring $10^2 - 3^2$?
d) Repeat parts a) to c) for a square that is 6-by-6, with a 2-by-2 square removed. The area of the resulting figure is $6^2 - 2^2$.
e) Repeat parts a) to c) for a starting square of side length x, with a square of side length y removed.



60. Work with a partner to fully factor the following expressions.

a) $x^4 - 1$ **b)** $x^8 - 1$ **c)** $x^4 - 625$

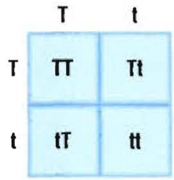
5.9 Special Products: Perfect Squares



An Austrian monk, Gregor Mendel (1822–1884), began the modern study of genetics. He found that the seeds from some tall pea plants produced dwarf plants, as well as tall ones.

We can represent Mendel's findings with a diagram called a Punnett square. Each parent plant, represented outside the square, has a dominant gene (T) for tallness and a recessive gene (t) for dwarfism. Because the tallness gene is dominant, each parent is tall.

The genes inherited by the offspring appear inside the square. Offspring with the combinations TT, Tt, and tT are tall plants. Offspring with 2 recessive genes (tt) are dwarf plants. If both tall parents have a recessive gene for dwarfism, what fraction of their offspring are dwarf plants?



A Punnett square is an example of a perfect square. Many algebraic expressions are also perfect squares.

Activity: Use Algebra Tiles

Use algebra tiles to model these squares.

Inquire

1. Count tiles to write a polynomial that represents the area of each square. Collect like terms and write each polynomial in descending powers of x.

2. Record an expression for the side length of each square.

3. Use your expression for the side length to write an expression for the area of each square. Do not expand.

4. How are the quantities represented by your expressions from questions 1 and 3 related?

5. Write a rule for finding the square of a binomial.

6. Use your rule to expand each of the following.

a) $(x + 4)^2$ **b)** $(x - 1)^2$ **c)** $(x - 2)^2$

Example

Expand the following binomials.

a) $(x + 6)^2$ **b)** $(x - 5)^2$ **c)** $(2x - 7)^2$

Solution

A **perfect square** consists of 2 identical binomial factors.
Expand, collect like terms, and simplify.

a) $(x+6)^2$
$= (x+6)(x+6)$
$= x^2 + 6x + 6x + 36$
$= x^2 + 12x + 36$

b) $(x-5)^2$
$= (x-5)(x-5)$
$= x^2 - 5x - 5x + 25$
$= x^2 - 10x + 25$

c) $(2x-7)^2$
$= (2x-7)(2x-7)$
$= 4x^2 - 14x - 14x + 49$
$= 4x^2 - 28x + 49$

In general, the following are true for the product of 2 identical binomials.

$$(a+b)^2 = (a+b)(a+b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = (a-b)(a-b) = a^2 - ab - ab + b^2 = a^2 - 2ab + b^2$$

Practice

Square.

1. $(-7)^2$ **2.** $(-9)^2$ **3.** $(-6)^2$
4. $(-12)^2$ **5.** $(2x)^2$ **6.** $(-3a)^2$
7. $(11y)^2$ **8.** $(-x)^2$ **9.** $(-4y)^2$

What is the first term in each product?

10. $(x+7)^2$ **11.** $(a-9)^2$ **12.** $(2x-1)^2$
13. $(9t+5)^2$ **14.** $(10b-3)^2$ **15.** $(3y+6)^2$
16. $(7p-2)^2$ **17.** $(4j+1)^2$ **18.** $(6q-8)^2$

What is the middle term, including its sign, in each product?

19. $(x-3)^2$ **20.** $(y+8)^2$
21. $(x+y)^2$ **22.** $(a-b)^2$
23. $(2x+3)^2$ **24.** $(4a-5)^2$
25. $(3x+2y)^2$ **26.** $(6p-7)^2$

Square.

27. $(y-10)^2$ **28.** $(3a-1)^2$
29. $(5x+2)^2$ **30.** $(3-x)^2$
31. $(5-y)^2$ **32.** $(5a+b)^2$
33. $(3x+y)^2$ **34.** $(4x-3y)^2$
35. $(7a-2b)^2$ **36.** $(4m+5n)^2$

Problems and Applications

Rewrite in the form $(a+b)^2$ *or* $(a-b)^2$.

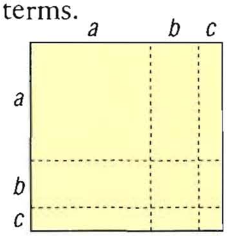
37. $x^2 + 14x + 49$ **38.** $x^2 - 16x + 64$
39. $4a^2 + 12a + 9$ **40.** $9b^2 - 24b + 16$
41. $64m^2 - 32m + 4$ **42.** $81n^2 + 90n + 25$

None of the following trinomials is a perfect square. Change 1 term to make the trinomial a perfect square.

43. $x^2 + 12x + 18$ **44.** $a^2 + 7a + 16$
45. $y^2 - 9y + 9$ **46.** $m^2 - 4m + 16$
47. $4x^2 - 4x + 2$ **48.** $9y^2 + 10y + 4$

49. Verify that $(x+3)^2 \neq x^2 + 9$ by substituting 1 for x.

50. Use the diagram to expand $(a+b+c)^2$. Collect like terms.



51. Explain how you can recognize a perfect square trinomial. You might use a sketch. Discuss terms and coefficients.

5.10 Products of Polynomials

Activity: Complete the Table

Copy and complete this table.

Product Form	Expanded Using the Distributive Property	Simplified Form
$(x+2)(x+3)$	$(x+2)x+(x+2)3$	
$(x+3)(x^2+x+2)$	$(x+3)x^2+(x+3)x+(x+3)2$	
$(x-1)(x^2-2x+4)$	$(x-1)x^2+(x-1)(-2x)+(x-1)4$	

Inquire

1. Find the products in the first column by multiplying each term of the first factor by each term of the second factor and simplifying.

2. How do the answers you found in 1 compare with the answers in the simplified form in the table?

3. Write a rule for multiplying polynomials.

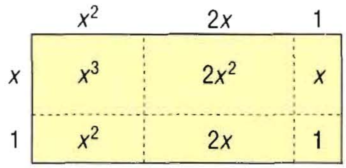
Example

Expand.

a) $(x+1)(x^2+2x+1)$ **b)** $(x-2)(x^2-3x-5)$ **c)** $(x-5)(2x^2-x+2)$

Solution

Multiply each term in the first expression by each term in the second expression and simplify.



a)
$$\begin{aligned}(x+1)(x^2+2x+1) &= (x+1)(x^2+2x+1)\\ &= x^3+2x^2+x+x^2+2x+1\\ &= x^3+2x^2+x^2+x+2x+1\\ &= x^3+3x^2+3x+1\end{aligned}$$

b)
$$\begin{aligned}(x-2)(x^2-3x-5) &= (x-2)(x^2-3x-5)\\ &= x^3-3x^2-5x-2x^2+6x+10\\ &= x^3-3x^2-2x^2-5x+6x+10\\ &= x^3-5x^2+x+10\end{aligned}$$

c)
$$\begin{aligned}(x-5)(2x^2-x+2) &= (x-5)(2x^2-x+2)\\ &= 2x^3-x^2+2x-10x^2+5x-10\\ &= 2x^3-x^2-10x^2+2x+5x-10\\ &= 2x^3-11x^2+7x-10\end{aligned}$$

Practice

Expand.

1. $3(x^2 + 3x - 5)$
2. $2(3a^2 - 5a + 7)$
3. $2(x^2 + 7x + 12)$
4. $3(a^2 + 7a + 10)$
5. $5(y^2 + 4y + 10)$
6. $x(x^2 - 10x + 25)$
7. $x(x^2 + 11x + 28)$
8. $a(2a^2 + 8a + 15)$
9. $x(4x^2 - 3x - 4)$
10. $x(6x^2 - 23x + 20)$

Expand and simplify.

11. $(x + 1)(x^2 + 2x + 3)$
12. $(x + 2)(x^2 - 3x + 1)$
13. $(x + 3)(x^2 + 2x - 3)$
14. $(x - 3)(x^2 + 6x + 5)$
15. $(x - 4)(x^2 - 8x - 7)$
16. $(x - 2)(x^2 - 5x + 6)$
17. $(x - 5)(x^2 + 10x - 11)$

Expand and simplify.

18. $(a - 1)(3a^2 + a + 5)$
19. $(b - 7)(5b^2 - b - 2)$
20. $(x - 8)(2x^2 - 3x + 3)$
21. $(x - 5)(4x^2 - 3x - 7)$
22. $(x - 2)(3x^2 - 5x - 4)$
23. $(a - 1)(7a^2 - a + 1)$
24. $(y - 5)(2y^2 + 5y + 2)$

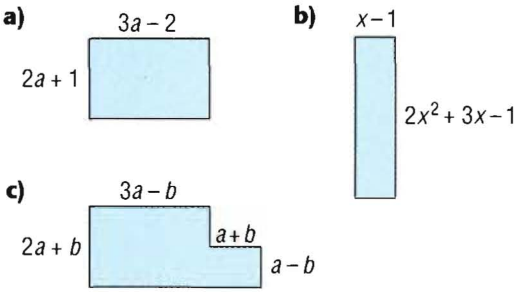
Expand and simplify.

25. $(2x^2 + 3x - 2)(x + 5)$
26. $(3a^2 - 5a - 6)(a + 3)$
27. $(5b^2 - 7b + 10)(b + 1)$
28. $(7w^2 - w - 1)(w - 1)$
29. $(2y^2 + 3y + 2)(y - 2)$
30. $(4t^2 - 2t - 1)(t - 5)$
31. $(x^2 + 2xy + y^2)(x + 1)$
32. $(x^2 - 3xy + y^2)(x - 2)$
33. $(b^2 - 3by - y^2)(x + 3)$

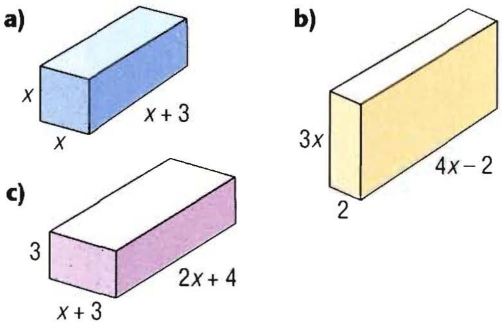
Problems and Applications

Expand and simplify.

34. $(3x + 2)(5x^2 - 2xy + y^2)$
35. $(2y - 5)(2y^3 - 3y^2 + 7)$
36. $(4x + 9)(7x^2 + x - 3)$
37. $(3a - 7)(2a^2 - 3a + 5)$
38. $(2y + 1)(3y^2 + 4y + 2)$
39. $(3a^2 - 6a - 7)(3a - 5)$
40. Determine the area of each figure.



41. Determine the volume of each box.

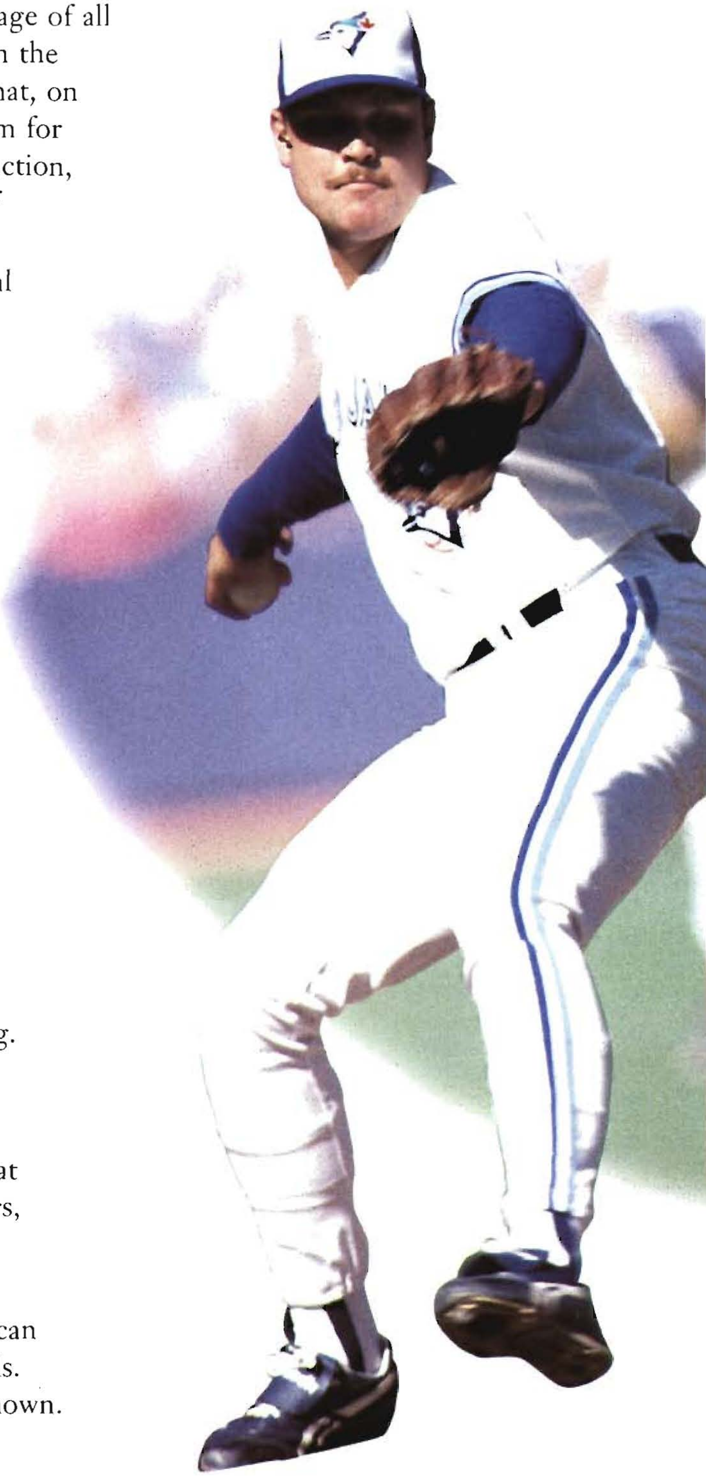


WORD POWER

Change the word NAIL to the word FILE by changing one letter at a time. You must form a real word each time you change a letter. The best solution has the fewest steps.

5.11 Rational Expressions

Duane Ward had the lowest earned run average of all the Blue Jays' pitchers the first year they won the World Series. His ERA was 1.95, meaning that, on average, about 2 runs were scored against him for every 9 innings he pitched. The ERA is a fraction, calculated by dividing 9 times the number of runs, r, by the number of innings pitched, i. The expression $\frac{9r}{i}$ is an example of a rational expression.



Activity: Discover the Relationship

Copy and complete the table.

Fraction	$\frac{2}{5}+\frac{1}{5}$	$\frac{2}{5}-\frac{1}{5}$	$\frac{2}{5}\times\frac{1}{5}$	$\frac{2}{5}\div\frac{1}{5}$
Simplified Fraction				

Inquire

1. Substitute x for the 2 in each expression and perform the operations.

2. Substitute x for 2 and y for 1 in each expression and perform the operations.

3. Substitute x for 5 in each expression and perform the operations.

4. Substitute x for 5 and y for 2 in each expression and perform the operations.

5. Use your findings to simplify the following.

a) $\frac{3}{x}+\frac{2}{x}$ **b)** $\frac{3}{x}-\frac{2}{x}$ **c)** $\frac{3}{x}\times\frac{2}{x}$ **d)** $\frac{3}{x}\div\frac{2}{x}$

Recall that rational numbers are numbers that can be written as the quotient of two integers, $\frac{a}{b}$, where a is any integer, and b can be any integer except zero.

A **rational expression** is an expression that can be written as the quotient of two polynomials. Some examples of rational expressions are shown.

$$\frac{a}{2} \quad \frac{4}{y} \quad \frac{x}{x-2}$$

Since division by zero is not defined, restrictions must sometimes be placed on the variables.

For the expression, $\frac{4}{y}$, y cannot be zero. We write $\frac{4}{y}$, $y \neq 0$.

For the expression, $\frac{x}{x-2}$, x cannot be 2. We write $\frac{x}{x-2}$, $x \neq 2$.

Example 1

Simplify. State the restrictions on the variables.

a) $\frac{3x^2y}{4x} \times \frac{2xy^3}{5y}$ **b)** $\frac{-8x^5y^8}{4x^3y^5}$ **c)** $\frac{3x^2y}{4} \div \frac{2x}{5}$

Solution

a)
$$\begin{aligned} &\frac{3x^2y}{4x} \times \frac{2xy^3}{5y} \\ &= \frac{6x^3y^4}{20xy} \\ &= \left(\frac{6}{20}\right)\left(\frac{x^3}{x}\right)\left(\frac{y^4}{y}\right) \\ &= \frac{3}{10}x^2y^3 \\ &= \frac{3x^2y^3}{10},\ x \neq 0,\ y \neq 0 \end{aligned}$$

b)
$$\begin{aligned} &\frac{-8x^5y^8}{4x^3y^5} \\ &= \left(\frac{-8}{4}\right)\left(\frac{x^5}{x^3}\right)\left(\frac{y^8}{y^5}\right) \\ &= -2x^2y^3,\ x \neq 0,\ y \neq 0 \end{aligned}$$

c)
$$\begin{aligned} &\frac{3x^2y}{4} \div \frac{2x}{5} \\ &= \frac{3x^2y}{4} \times \frac{5}{2x} \\ &= \frac{15x^2y}{8x} \\ &= \left(\frac{15}{8}\right)\left(\frac{x^2}{x}\right)\left(\frac{y}{1}\right) \\ &= \frac{15xy}{8},\ x \neq 0 \end{aligned}$$

Example 2

Simplify.

a) $\frac{x-1}{2} + \frac{x-2}{3}$ **b)** $\frac{(x+1)}{2} - \frac{2(x-1)}{5}$

Solution

To add or subtract rational expressions with different denominators, find equivalent fractions with a common denominator.

a)
$$\begin{aligned} &\frac{x-1}{2} + \frac{x-2}{3} \\ &= \frac{3(x-1)}{3 \times 2} + \frac{2(x-2)}{2 \times 3} \\ &= \frac{3x - 3 + 2x - 4}{6} \\ &= \frac{3x + 2x - 3 - 4}{6} \\ &= \frac{5x - 7}{6} \end{aligned}$$

b)
$$\begin{aligned} &\frac{(x+1)}{2} - \frac{2(x-1)}{5} \\ &= \frac{5(x+1)}{5 \times 2} - \frac{2[2(x-1)]}{2 \times 5} \\ &= \frac{5(x+1) - 2(2x-2)}{10} \\ &= \frac{5x + 5 - 4x + 4}{10} \\ &= \frac{5x - 4x + 5 + 4}{10} \\ &= \frac{x + 9}{10} \end{aligned}$$

CONTINUED

Practice

For each pair of fractions, write the lowest common denominator.

1. $\frac{1}{4}, \frac{3}{8}$ **2.** $\frac{5}{12}, \frac{7}{24}$ **3.** $\frac{1}{3}, \frac{1}{6}$

4. $\frac{5}{3x}, \frac{1}{7x}$ **5.** $\frac{6}{5a}, \frac{2}{3a}$ **6.** $\frac{7}{8x}, \frac{5}{6x}$

Simplify. State the restrictions on the variables.

7. $\frac{12x}{4xy}$ **8.** $\frac{20}{10y}$ **9.** $\frac{9x^2}{3x}$

10. $\frac{6abc}{3ab}$ **11.** $\frac{-10x^3y^4}{5x^2y^2}$ **12.** $\frac{20a^2b^2}{-5ab}$

Simplify.

13. $\frac{x}{2} \times \frac{3}{x}$ **14.** $\frac{2}{m} \times \frac{m}{4}$ **15.** $\frac{x}{y} \times \frac{y}{x}$

16. $\frac{m}{3} \div \frac{m}{4}$ **17.** $\frac{2}{y} \div \frac{4}{y}$ **18.** $\frac{x}{y} \div \frac{x}{y}$

Multiply.

19. $\frac{x^2y}{x} \times \frac{xy}{y}$ **20.** $\frac{2xy^2}{2x} \times \frac{5x^2y}{3x}$

21. $\frac{3p^2q^2}{2p^2} \times \frac{4pq}{3qr}$ **22.** $\frac{7a^2b}{2a} \times \frac{2ab}{2}$

Divide these monomials.

23. $\frac{x^2y^3}{3} \div \frac{xy}{3}$ **24.** $\frac{2x^2y}{5} \div \frac{2xy}{5}$

25. $\frac{xy^2}{10} \div \frac{2xy^5}{5x^3y^3}$ **26.** $\frac{3x^3y^2}{2x^2y^2} \div \frac{3}{xy}$

Simplify these fractions.

27. $\frac{7}{y} + \frac{12}{y}$ **28.** $\frac{x}{a} - \frac{11}{a}$

29. $\frac{7}{12x} - \frac{4}{12x}$ **30.** $\frac{45xy}{4z} - \frac{9xy}{4z}$

Simplify.

31. $\frac{2}{x} + \frac{3-x}{x}$ **32.** $\frac{x+3}{3x^2} + \frac{4-x}{3x^2}$

33. $\frac{3}{x^2} - \frac{(7-x)}{x^2}$ **34.** $\frac{2-x}{5x^2} - \frac{1}{5x^2}$

Simplify.

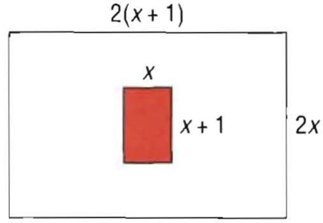
35. $\frac{x-2}{2} + \frac{x+5}{4}$ **36.** $\frac{x-2}{3} - \frac{x-6}{4}$

37. $\frac{x+2}{2} - \frac{4-x}{3}$ **38.** $\frac{2-x}{5} - \frac{3-x}{3}$

39. $\frac{x-1}{4} - \frac{2(x-3)}{6}$ **40.** $\frac{2(x-2)}{5} + \frac{x+1}{2}$

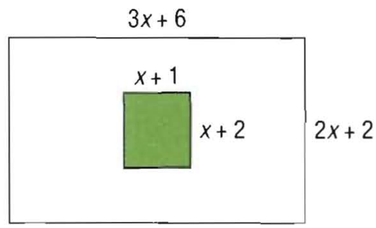
Problems and Applications

41.



a) What is the total area of the figure? Leave your answer in factored form.
b) What is the area of the red rectangle in factored form?
c) What is the quotient of the area of the red rectangle divided by the total area of the figure? Write your answer in simplest form.
d) Check your answer by substituting 1 for x.

42.



a) What is the total area of the figure in factored form?
b) What is the area of the green rectangle in factored form?
c) What is the quotient of the area of the green rectangle divided by the total area of the figure? Write your answer in simplest form.
d) Verify your answer by substituting 2 for x.

Computers and Connectivity

Connectivity means connecting a computer by telephone lines or other methods to other computers or other sources of information. The connected system can be as simple as the computer at your desk and a regular telephone. A simple system for a car is a laptop computer and a cellular telephone. Both systems allow you to send and receive information. They give you access to many resources.

Activity ❶ Electronic Bulletin Boards

Like the public bulletin boards you sometimes see in grocery stores, electronic bulletin boards are usually open to everyone. There are bulletin boards for almost every subject.

1. How would sports cards collectors use an electronic bulletin board?
2. How would mystery novelists use one?
3. What kind of service could a newspaper provide on an electronic bulletin board?
4. One of the biggest electronic bulletin boards is called CompuServe. What kind of service does it supply? How much does it cost to use the service?

Activity ❷ Commercial Services

Some businesses offer services for computer users. One type of service is travel reservations. You can get information on airline schedules and fares, then order your tickets and charge the cost to a credit card.

1. Describe how home banking would work on a computer.
2. Describe how teleshopping would work.
3. What other commercial services do you think would be useful?

Activity ❸ Electronic Mail

Electronic mail is also called E-mail. It is used within companies, so that employees can send messages to each other. To send a message, you type it into your computer and send the message to the receiver's electronic mailbox. You can also put the message in several mailboxes at once, or broadcast it to everyone on the E-mail system. To open your own mailbox, you type your password into your computer and "open" your mail.

1. What advantages does E-mail have over a telephone?
2. Why do E-mail users have passwords?
3. Would E-mail be useful in homes?

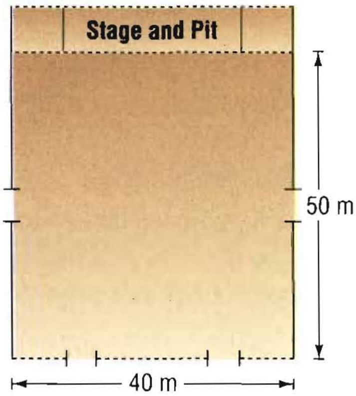
Activity 1 Designing a Theatre

You have been asked to submit a design for the seating in a theatre that will be located in an old movie house. The floor slopes to the stage and orchestra pit, so all you need to do is to place the seats and the aisles. The stage and orchestra pit are centred and measure 30 m across.

Each seat measures 50 cm by 50 cm, including arm rests. There is to be a 50-cm space in front of each seat for leg room and access. The seating area measures 40 m wide by 50 m deep. Aisles must be at least 2 m wide.

There must be an aisle across the front and the back of the seating area, plus aisles to get customers to their seats. Aisles may be curved or straight.

There should be no more than 20 seats in a row. There are 2 doors evenly spaced at the back of the theatre and a door in the middle of each side. There is no balcony.



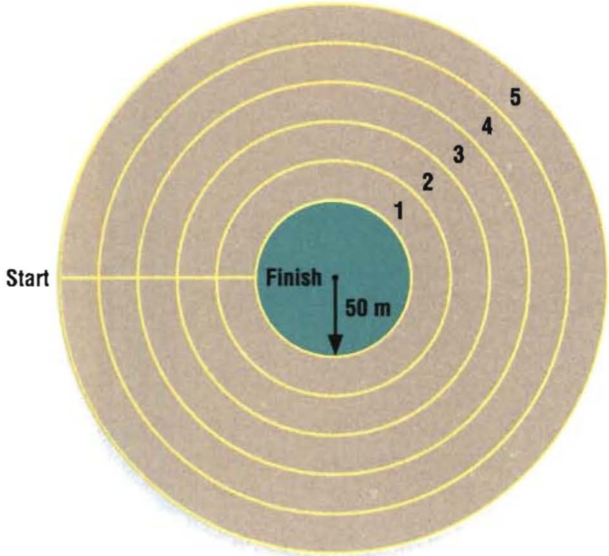
Design the placement of the seats and aisles so that people have easy access to their seats. The owners of the theatre want you to decide how many seats to put in. The owners want a pleasant, comfortable arrangement, but keep in mind that they are in the business of selling tickets. If you include too few seats, your design may not be accepted. You must label each aisle and each seat for the purpose of ticket sales.

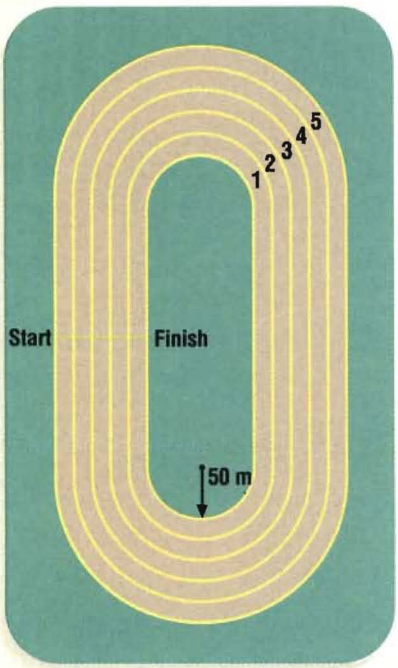
Activity ❷ Designing a Circular Track

Suppose there is to be a race between 5 people on a circular track. The track has an inside radius of 50 m. Each lane of the track is 1 m wide. The lanes are numbered 1 to 5, as shown. The start and finish line is also shown.

1. Suppose all 5 runners begin at the start line and run once around the track. Calculate the distance that each runner runs. Assume that $\pi = 3.14$.

2. To have all runners run the same distance, the start line for each lane must be staggered. Make the necessary calculations and mark a different start line for each lane. Each of the other runners must cover the same distance as the runner in lane 1 covers in 1 lap of the track from the start to the finish line.





Activity ❸ Designing an Oval Track

1. Design an oval track like the one used in the Olympic Games. The ends of the track have an inside radius of 50 m. The sides of the track are straight. There are 5 lanes and the width of each lane is 1 m. The runner in lane 1 must run 400 m in 1 lap from the start to the finish.

2. Calculate the distance each runner would run if all the runners started at the same start line and finished after 1 lap of the track.

3. Make the necessary calculations and mark the start line for each lane. Each runner must run 400 m in a lane, and the finish line is the same for all runners.

Review

1. Write the GCF of each pair.

a) 35, 40 **b)** 21, 28

c) 34, 51 **d)** 120, 96

e) $10a^2, 5a$ **f)** $16xy, 12xz$

g) $10ab^2, 18ab$ **h)** $15xy, 25x^2y^2$

2. Write the GCF of each set.

a) $21x^2y, 15xy^2, 9x^2y^2$

b) $24x^2, -16xy, 32y^2$

c) $18xy^2, -27x^2y^2, 36x^2y$

Factor.

3. $5x - 15$ **4.** $6x^2 - 18x$

5. $5ab + 10ac$ **6.** $7a^2 + 35a^3$

7. $8abc - 12bc$ **8.** $3x^2 + 9y^2$

9. $3a^2 - 6ab + a$ **10.** $2x + 6y - 10z$

Expand and simplify.

11. $2(3x + 1) + 3(x + 4)$

12. $4(a + 5) + 3(a - 2)$

13. $3y(2y - 7) + 2(y - 5)$

14. $2(m + 1) - m(m + 1)$

15. $-z(2z + 3) - (z - 5)$

16. $5x(2x - 7) - 3x(3x - 5)$

Expand.

17. $2y(-y^2 + 3y - 7)$

18. $-3t(1 - 2t - t^2)$

19. $4m(m^2 + 2m - 3)$

20. $-3x(2x^2 - 4x + 2)$

Divide.

21. $\dfrac{8a^3b^5 - 16a^4b^4 + 4a^5b^3}{4a^3b^3}$

22. $\dfrac{6a^4b^5z^5 - 12a^4b^5z^4}{-3a^3b^4z^2}$

Expand.

23. $(x + 2)(x - 3)$ **24.** $(x - 4)(x + 7)$

25. $(x + 5)(x + 2)$ **26.** $(x - 2)(x - 3)$

27. $(x + 10)(x - 2)$ **28.** $(2x - 5)(3x - 2)$

29. $(-2a - 3)(5a + 2)$ **30.** $(4x - 7)(-3x + 2)$

31. $(3x - 5)(-7x + 6)$ **32.** $(-5a - b)(2a + 3b)$

Factor.

33. $x^2 + 8x + 7$ **34.** $x^2 - 6x + 5$

35. $y^2 + 8y + 15$ **36.** $a^2 + 8a + 12$

37. $b^2 + 10b + 24$ **38.** $x^2 - 7x + 6$

39. $x^2 - 11x + 28$ **40.** $a^2 - 7a + 12$

Factor.

41. $a^2 - a - 20$ **42.** $x^2 - x - 30$

43. $x^2 - 5x - 14$ **44.** $m^2 - 6m - 40$

45. $x^2 + 4x - 21$ **46.** $x^2 + 10x - 24$

47. $x^2 + 2x - 35$ **48.** $x^2 - 2x - 15$

Factor fully.

49. $2x^2 + 24x + 40$ **50.** $5a^2 - 40a + 80$

51. $4w^2 - 4w - 120$ **52.** $3r^2 - 21r + 30$

53. $2j^2 - 6j + 8$ **54.** $3t^2 + 18t - 21$

55. $7y^2 + 7y - 140$ **56.** $3z^2 - 39z + 126$

Factor.

57. $x^2 - 1$ **58.** $y^2 - 4$ **59.** $4a^2 - 9$

60. $a^2 - 4b^2$ **61.** $4x^2 - y^2$ **62.** $4a^2 - 9b^2$

63. $9 - x^2$ **64.** $25 - 49x^2$ **65.** $2a^2 - 50$

66. $5x^2 - 20$ **67.** $4x^2 - 36$ **68.** $16a^2 - 36$

Identify the expressions that are perfect squares.

69. $x^2 - 2x + 1$ **70.** $x^2 + 9x + 3$

71. $x^2 - 8x - 16$ **72.** $x^2 + 10x + 25$

Expand.

73. $(x+2)^2$ **74.** $(x-3)^2$

75. $(y+6)^2$ **76.** $(m-5)^2$

Expand.

77. $(x-2)(x^2-3x+2)$

78. $(x-3)(x^2-5x-3)$

79. $(x+1)(2x^2-2x+1)$

80. $(x+5)(3x^2-x-1)$

81. $(x^2-x+1)(x+1)$

82. $(x^2+2x+3)(x-1)$

83. $(5x^2-5x-3)(x-4)$

84. $(4x^2-3x-7)(x+2)$

Simplify.

85. $\frac{x-1}{2}+\frac{x-4}{3}$ **86.** $\frac{x-2}{3}+\frac{1-x}{5}$

87. $\frac{3-x}{3}+\frac{2(x-2)}{4}$ **88.** $\frac{x-5}{6}+\frac{2(x+3)}{7}$

89. $\frac{1}{x^2}+\frac{1-x}{x^2}$ **90.** $\frac{3x+1}{7x^2}+\frac{2-x}{7x^2}$

Simplify.

91. $\frac{x-5}{2}-\frac{x+1}{3}$ **92.** $\frac{x+2}{3}-\frac{x-3}{4}$

93. $\frac{2(x+3)}{2}-\frac{x-1}{7}$ **94.** $\frac{x-1}{6}-\frac{2-x}{7}$

95. $\frac{1-x}{7}-\frac{2-x}{5}$ **96.** $\frac{x-2}{4}-\frac{x-3}{7}$

97. $\frac{2x+1}{2x}-\frac{x-1}{2x}$ **98.** $\frac{7-x}{x^2}-\frac{2-x}{x^2}$

99. The area of a rectangle is represented by the expression $x^2+10x+16$.

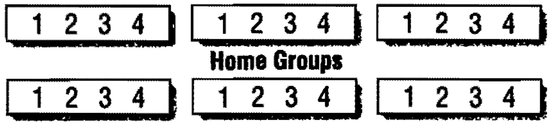
a) Factor the expression.

b) A larger rectangle is 2 units longer on each side. Write a factored expression for the area of the larger rectangle.

c) Expand and simplify the expression for the area of the larger rectangle.

Group Decision Making
Mirror, Mirror on the Wall

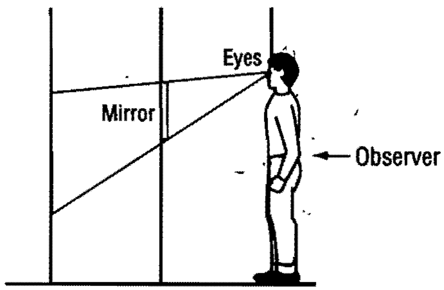
Solve this problem in home groups.



The problem is this: When you are standing in front of a mirror and you want to see more of yourself, should you move forward or backward?

1. As a group, discuss the solution to the problem. Try to find a solution that everyone agrees with. Use a mirror to test your group's solution.

2. Next, make a drawing of the problem. Use a stick figure for the person in front of the mirror and a point on the figure for the person's eyes. Remember that the reflection appears at the same distance behind the mirror as the person is in front of the mirror.



3. Make other drawings on top of the first one, with the stick figure moved forward and backward. Discuss the results.

4. Does a change in the size of the mirror change the problem?

5. Meet as a class. Present your group's solution to the problem.

Chapter Check

Factor fully.

1. $3xy - 6xz$

2. $24xy^2 + 12x^2y$

Expand and simplify.

3. $2x(x - 2) + 3(x + 3)$

4. $4x(x + 1) - 5x(x - 1)$

5. $-3x(x^2 - 2x + 1)$

6. $2m(2m^2 - 6m - 10)$

Divide.

7. $\dfrac{5y^4 - 10y^5 + 15y^6}{5y^4}$

8. $\dfrac{12a^5b^3 - 8a^4b^3 + 4a^3b^3}{4a^3b^3}$

Expand.

9. $(x - 2)(x + 4)$

10. $(x - 3)(2x + 7)$

11. $(4a - 3b)(2a - 3b)$

12. $(y + 3)(-2y + 1)$

Factor.

13. $x^2 + 7x + 10$

14. $x^2 - 9x + 18$

15. $x^2 - 3x - 10$

16. $x^2 + 2x - 35$

17. $x^2 + 8x + 16$

18. $x^2 - 18x + 17$

Factor fully.

19. $2x^2 - 8x + 8$

20. $5x^2 - 5x - 100$

Expand.

21. $(x - 2)^2$

22. $(w + 7)^2$

Factor.

23. $a^2 - 4$

24. $4x^2 - 25$

25. $81 - x^2$

26. $1 - 4b^2$

Factor.

27. $2t^2 - 200$

28. $3x^2 - 12$

29. $100 - 16t^2$

30. $2 - 18y^2$

State whether each trinomial is a perfect square.

31. $x^2 - 10x + 25$

32. $a^2 - 14a - 49$

33. $x^2 - 12x + 36$

34. $y^2 - 16y - 64$

Simplify.

35. $\dfrac{25xy}{5y} \times \dfrac{2x^2y^3}{2y}$

36. $\dfrac{10a^2b^3}{5ab^4} \div \dfrac{2ab}{5a^2b^2}$

Simplify.

37. $\dfrac{x + 3}{2} + \dfrac{5 + x}{3}$

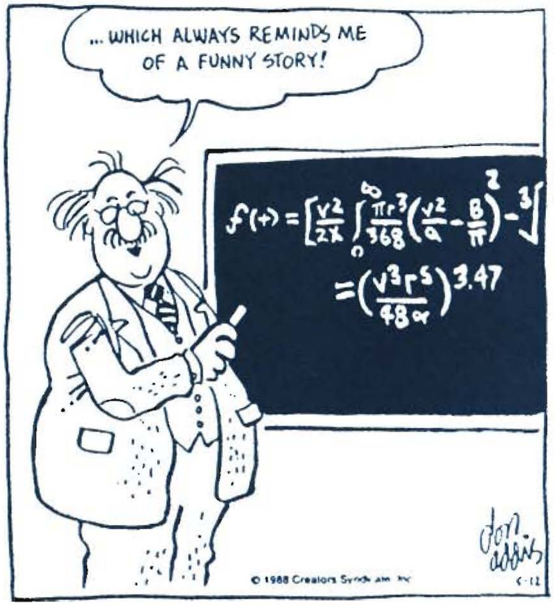
38. $\dfrac{2(x - 3)}{5} - \dfrac{x - 1}{7}$

39. A rectangle has an area of $x^2 + 13x + 30$.

a) Express the area as a product of 2 binomials.

b) The length of a second rectangle is 2 units more than the length of the first rectangle. The width of the second rectangle is 1 unit less than the width of the first rectangle. Write an expression in factored form for the area of the second rectangle.

c) Expand and simplify your expression from part b).



BENT OFFERINGS by Don Addis.
By permission of Don Addis and Creators Syndicate.

Using the Strategies

1. Using different fractions with 10s as denominators, fill in the 3-by-3 grid below so that the sum vertically, horizontally, and diagonally is $1\frac{1}{2}$.

2. a) Write any 2-digit number.
b) Keep summing the digits in your number until you obtain a single digit. For example, for 67 the sum of the digits is 6 + 7 or 13. The sum of the digits for 13 is 1 + 3 or 4.
c) Divide the 2-digit number you wrote by 9 and note the remainder. What do you notice?
d) Repeat parts a) to c) for other 2-digit numbers. Describe your results.
e) Do the results for numbers with more than 2 digits follow the same pattern?

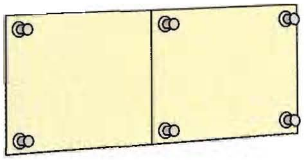
3. In the following division problem, *a* and *b* represent missing single-digit numbers. Find values for *a* and *b*.

$$\begin{array}{r} 2b7 \\ a21\overline{)79287} \end{array}$$

4. The greatest common factor of 2 numbers, m and n, is 14. If $m = 2 \times 5 \times 7^2$, name 3 numbers that could be n.

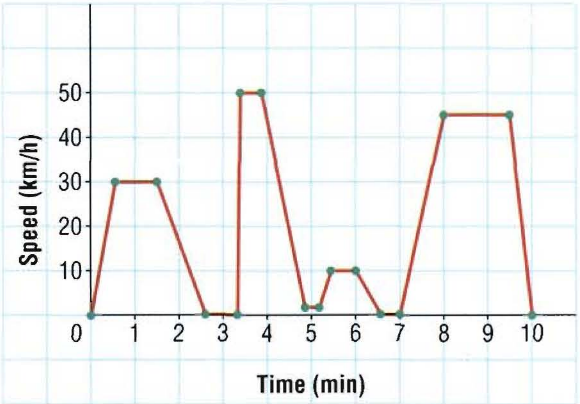
5. Kim has 2 chores at home. Every 4 days, she must clean the gerbil cage. Every 6 days, she must clean the canary cage. Last Monday, she did both jobs. On what day of the week will she next do both jobs?

6. To hang a picture on a bulletin board, Masao uses 4 thumbtacks, 1 in each corner. For 2 pictures of the same size, Masao can overlap the corners and hang 2 pictures with only 6 tacks.



a) What is the minimum number of tacks Masao needs to hang 6 pictures of the same size in a row?
b) Write an expression for finding the number of tacks needed to hang any number of pictures in a row.

7. The graph shows the speed of a car for 10 min. Write a story to explain the graph.



DATA BANK

1. Ontario and Quebec are Canada's 2 largest provinces. What percent of the total area of Canada's provinces is covered by these 2 provinces combined?

2. Use information from the Data Bank on pages 364 to 369 to write a problem. Have a classmate solve your problem.