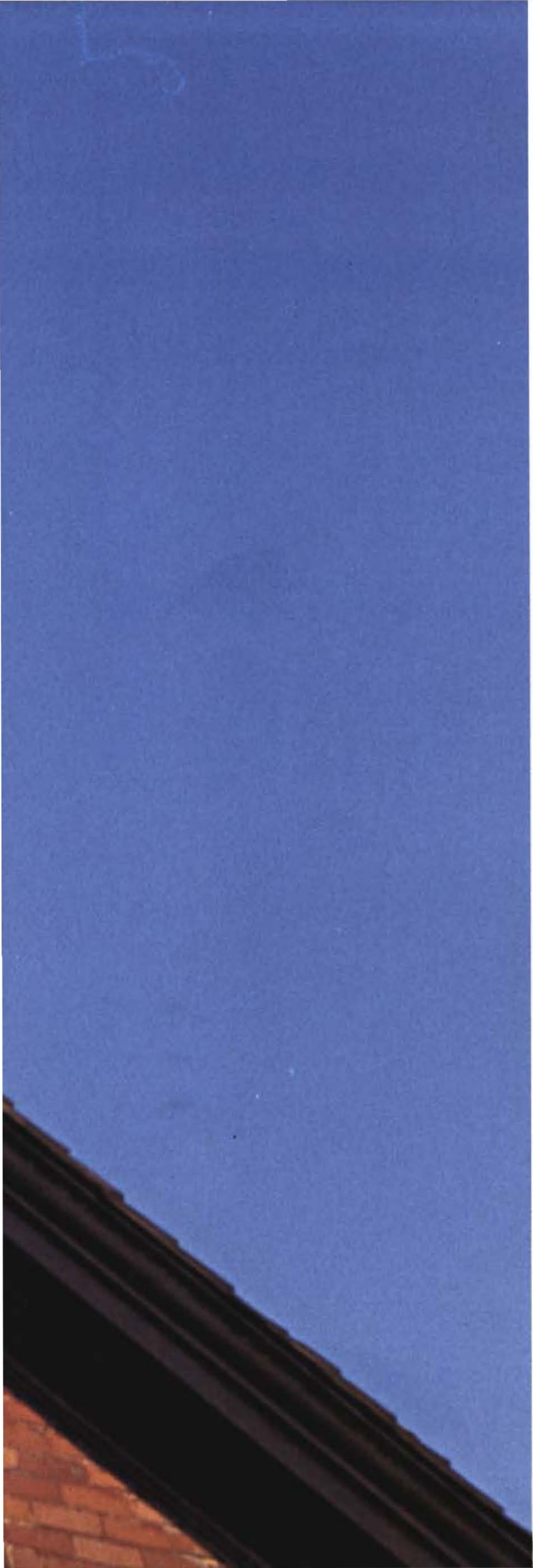


CHAPTER 6

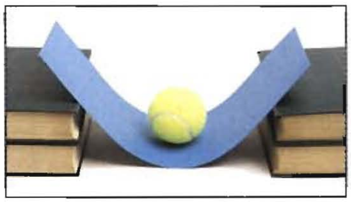
Measurement

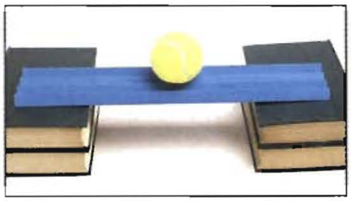
The construction of the Alexander Graham Bell Museum in Baddeck, Nova Scotia, uses a common geometric shape. Name the shape.

The photographs below show an experiment in which a ball is supported by a strip of Bristol board. Explain how the result of this experiment is related to the construction of the museum.

List 5 other examples of how the shape you identified above is used for strength.

Alexander Graham Bell Museum





Seeing Shapes

Some of the information that we receive comes from pictures that can create illusions.

The spoon looks broken because light rays bend as they pass from one material into another. For example, they bend when they pass from air into water.

Activity ❶ Length

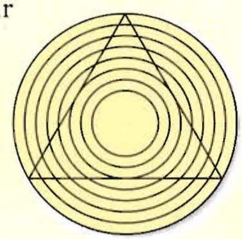
1. Horizontal and vertical lines can create an illusion involving length. In the diagram, does the horizontal or the vertical line seem longer?

2. Measure the lines and compare their lengths.

3. Draw a similar diagram in which the horizontal and vertical lines appear to be equal in length. Then, measure them to find the difference in their lengths.

Activity ❷ Perspective

1. Do the sides of the triangle appear to be straight or bent?

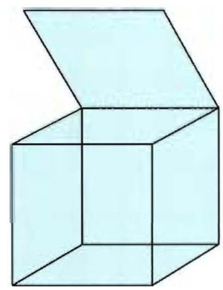


2. Describe the effect the circles have on the sides of the triangle.

Activity ❸ Reversing

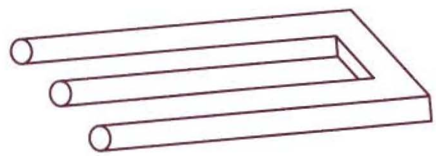
1. Focus on this box until you see it change position.

2. Describe the second position as you perceive it. Compare your description with your classmates'.



Activity ❹ Impossible Figures

1. Focus on the figure and describe what you see.



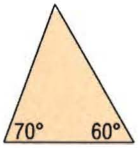
2. Is this figure possible in the real world? Why?

Warm Up

1. State the sum of the interior angles of a triangle.

Determine the unknown angle measures in each triangle.

2.



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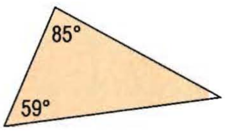

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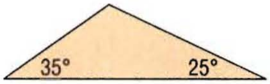

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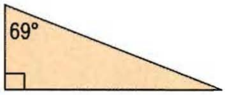

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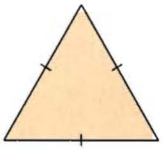

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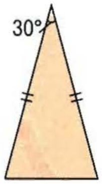

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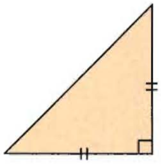

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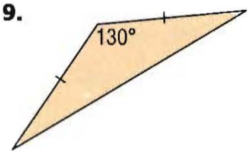


Explain the meaning of each term.

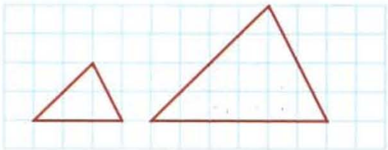
10. enlargement

11. reduction

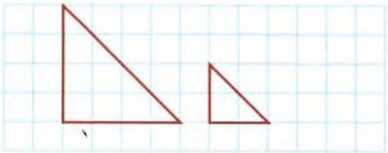
12. image

13. scale factor

14. What is the scale factor of this enlargement?



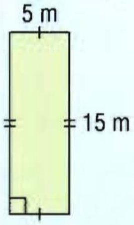
15. What is the scale factor of this reduction?



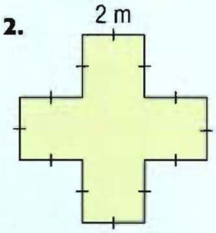
Mental Math

Calculate the perimeter of each figure.

1.



2.



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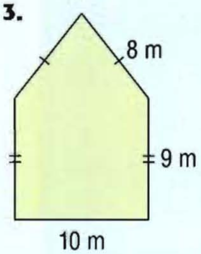

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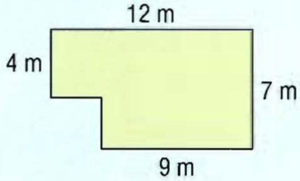
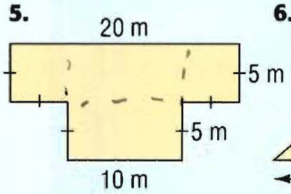


Calculate the area of each figure.

5.



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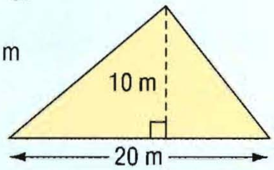

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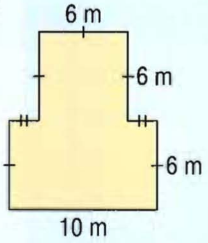

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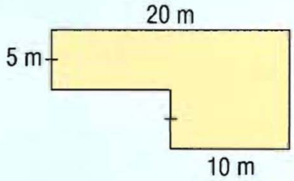


Calculate.

9. 37 + 23 + 20 + 10

10. 52 – 12 + 30 – 20

11. 84 + 16 – 25 + 5

12. 61 – 31 + 47 – 7

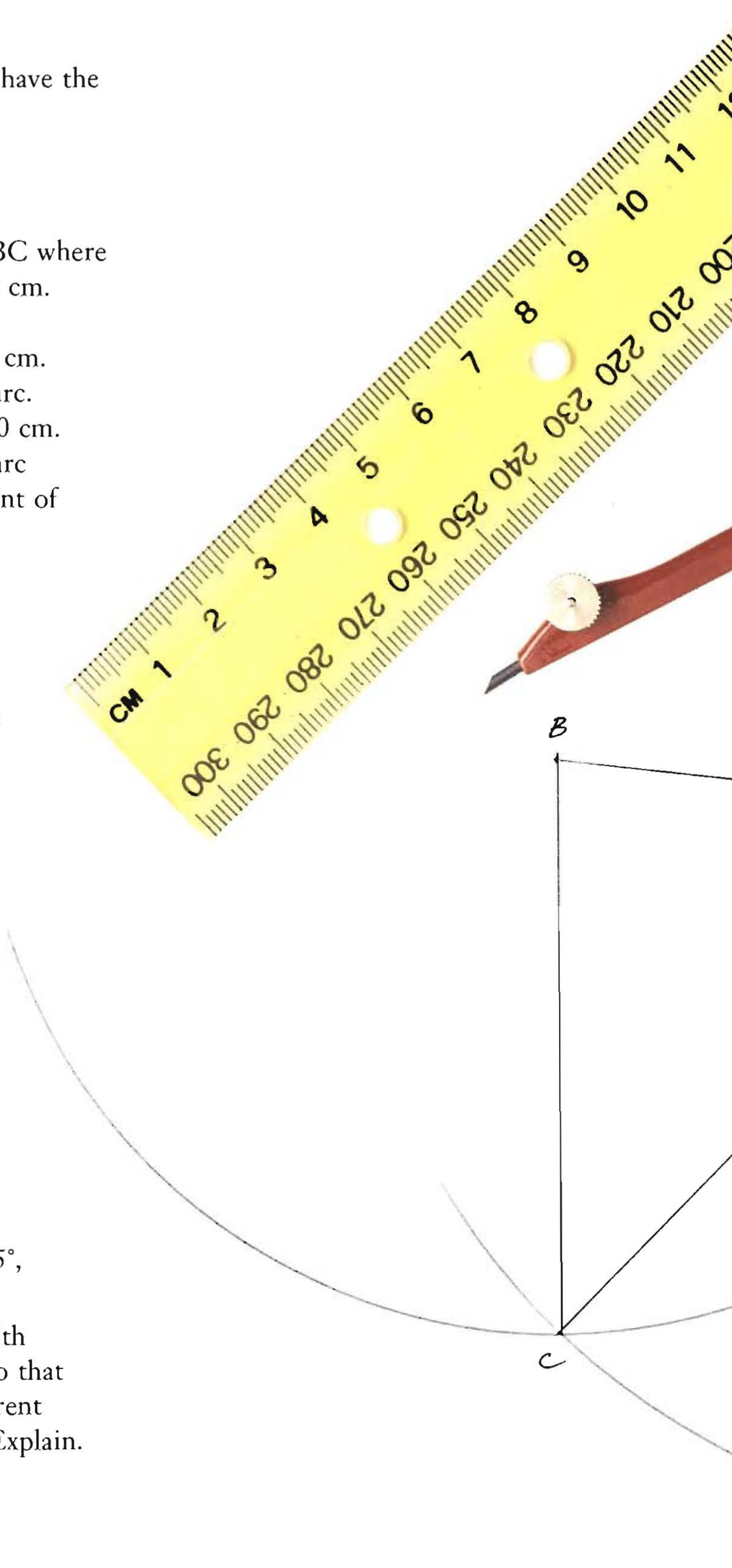
13. 92 – 12 – 35 – 15

14. 73 + 17 – 30 – 20

15. 56 + 44 – 80 + 20

Congruent Triangles

Congruent triangles are triangles that have the same size and shape.



Activity 1

1. Follow these steps to construct △ABC where AB = 7 cm, BC = 8 cm, and AC = 10 cm.

a) Draw line segment AB = 7 cm.

b) Set your compasses to a radius of 8 cm. Then, use B as a centre and draw an arc.

c) Set your compasses to a radius of 10 cm. Then, use A as a centre and draw an arc to intersect the first arc. Label the point of intersection C.

d) Draw BC and AC.

2. Is it possible to construct △DEF where DE = 7 cm, EF = 8 cm, and DF = 10 cm so that the size and shape of △DEF are different from the size and shape of △ABC? Explain.

Activity 2

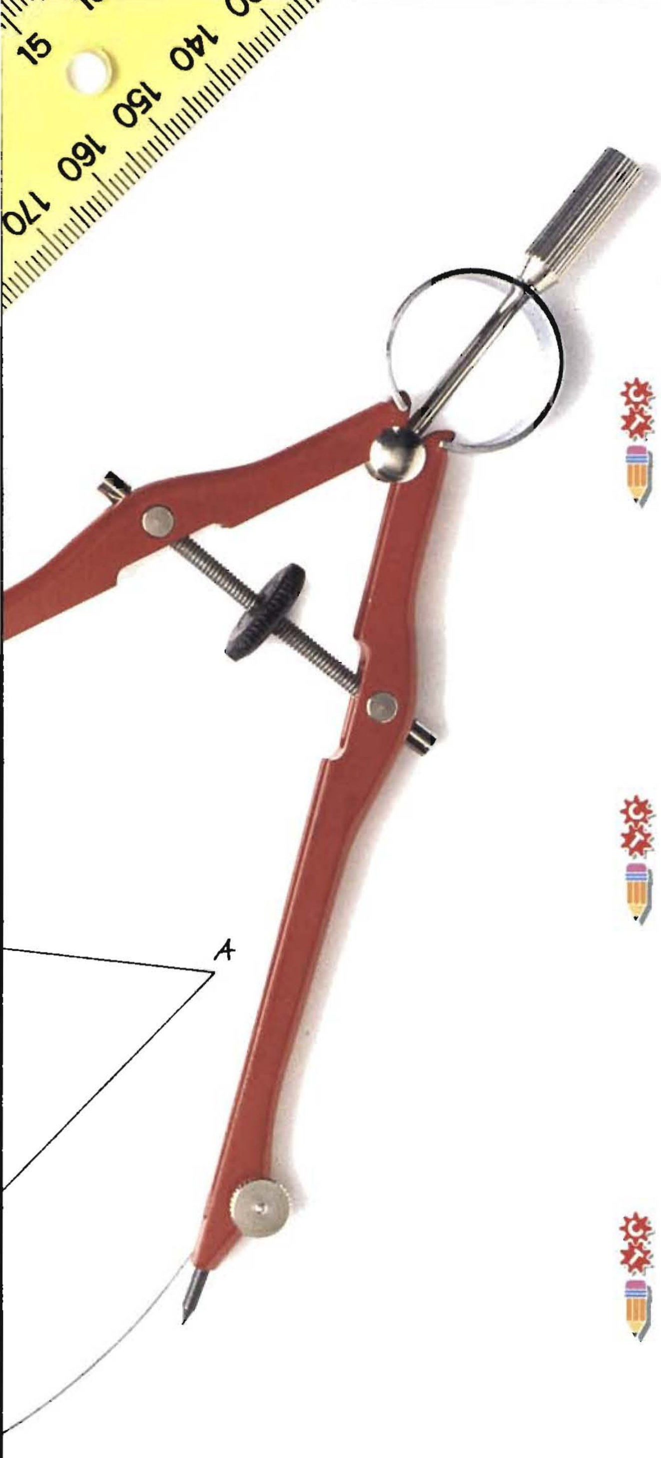
1. Construct △GHI so that GH = 5 cm and GI = 7 cm.

2. Is it possible to construct △JKL with JK = 5 cm and JL = 7 cm so that the size and shape of △JKL are different from the size and shape of △GHI? Explain.

Activity 3

1. Construct △MNO so that ∠M = 35°, ∠N = 65°, and ∠O = 80°.

2. Is it possible to construct △PQR with ∠P = 35°, ∠Q = 65°, and ∠R = 80° so that the size and shape of △PQR are different from the size and shape of △MNO? Explain.



Activity ❹

1. Construct $\triangle RST$ so that $\angle R = 70°$, $RS = 6$ cm, and $RT = 7$ cm.

2. Is it possible to construct $\triangle XYZ$ with $\angle X = 70°$, $XY = 6$ cm, and $XZ = 7$ cm so that the size and shape of $\triangle XYZ$ are different from the size and shape of $\triangle RST$? Explain.

Activity ❺

1. Construct $\triangle ABC$ so that $\angle A = 40°$, $AB = 7$ cm, and $\angle B = 55°$.

2. Is it possible to construct $\triangle DEF$ with $\angle D = 40°$, $DE = 7$ cm, and $\angle E = 55°$ so that the size and shape of $\triangle DEF$ are different from the size and shape of $\triangle ABC$? Explain.

Activity ❻

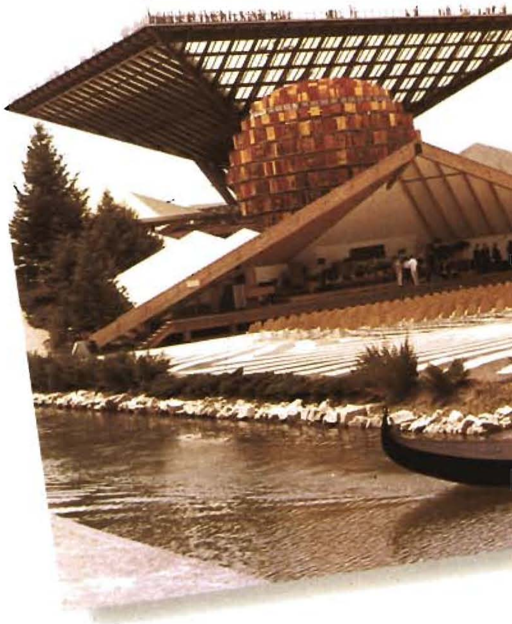
1. Construct $\triangle PQR$ so that $\angle P = 35°$ and $PQ = 7$ cm.

2. Is it possible to construct $\triangle XYZ$ with $\angle X = 35°$ and $XY = 7$ cm so that the size and shape of $\triangle XYZ$ are different from the size and shape of $\triangle PQR$? Explain.

Activity ❼

1. Use the results of the first 6 activities to decide which sets of 3 facts need to be given so that only 1 triangle can be constructed.

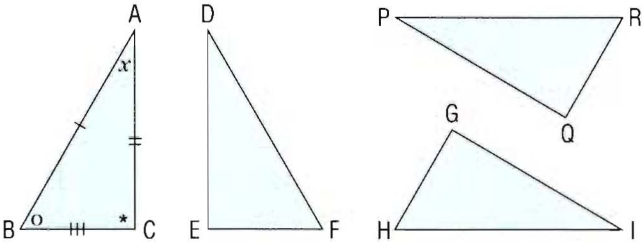
6.1 Congruent Triangles



Many beautiful and unusual structures are built using congruent triangles. One such structure is shown in this photograph of Katimavik, built for Expo 67 in Montreal.

Activity: Use the Diagrams

Each of these 4 triangles is positioned differently but they all have the same size and shape. Copy or trace them into your notebook.



Inquire

1. Mark AB with a single tick as shown in the diagram above. Then, in the other 3 triangles, use a single tick to mark the sides that are the same length as AB.

2. Mark AC with 2 ticks. Then, in the other 3 triangles, use two ticks to mark the sides that are the same length as AC.

3. Use 3 ticks to mark each of the remaining equal sides.

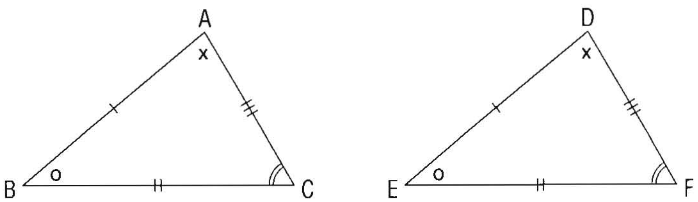
4. Mark ∠A with an x. Then, in the other 3 triangles, use an x to mark the angles that have the same measure as ∠A.

5. Mark ∠B with an o. Then, in the other 3 triangles, use an o to mark the angles that have the same measure as ∠B.

6. Use an asterisk, *, to mark the remaining equal angles.

A triangle has 6 parts, 3 angles and 3 sides. Two triangles are **congruent** if you can match up their vertices so that all pairs of corresponding angles and corresponding sides are equal.
In △ABC and △DEF, the following corresponding parts are equal.

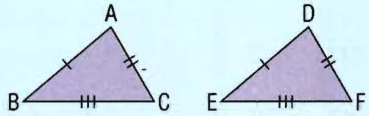
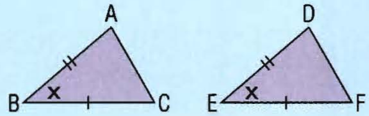
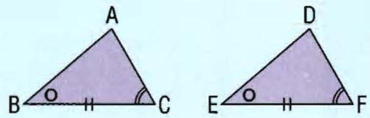
∠A = ∠D	AB = DE
∠B = ∠E	BC = EF
∠C = ∠F	AC = DF



We say △ABC ≅ △DEF. The symbol for congruence, ≅, is read "is congruent to." Note that the vertices of each triangle are listed in the same order as their corresponding angles.

You do not need to know that all 6 parts of 1 triangle are equal to the 6 corresponding parts of another triangle to prove congruency. Knowing that 3 parts of one triangle are equal to 3 corresponding parts of another triangle may be sufficient to say that the triangles are congruent.

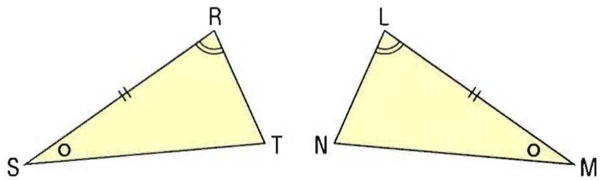
The chart gives you 3 ways to state that 2 triangles are congruent.

Congruence Condition	Example	Case
3 sides of one triangle are respectively equal to 3 sides of another triangle.		SSS (side, side, side)
2 sides and the contained angle of one triangle are respectively equal to 2 sides and the contained angle of another triangle.		SAS (side, angle, side)
2 angles and the contained side of one triangle are respectively equal to 2 angles and the contained side of another triangle.		ASA (angle, side, angle)

Example 1

a) State why $\triangle RST \cong \triangle LMN$.

b) List the other equal sides and angles.



Solution

a) In $\triangle RST$ and $\triangle LMN$,

$\angle R = \angle L$
$RS = LM$
$\angle S = \angle M$

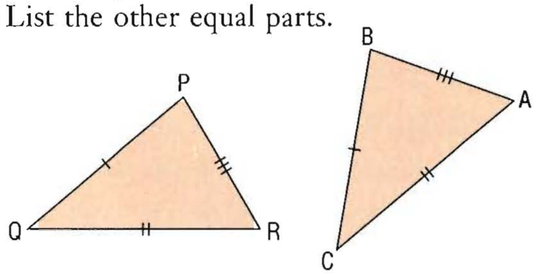
Therefore, $\triangle RST \cong \triangle LMN$ **(ASA)**

b) $RT = LN$
$ST = MN$
$\angle T = \angle N$

Example 2

a) State why $\triangle PQR \cong \triangle BCA$.

b) List the other equal parts.



Solution

a) In $\triangle PQR$ and $\triangle BCA$,

$PQ = BC$
$QR = CA$
$PR = BA$

Therefore, $\triangle PQR \cong \triangle BCA$ **(SSS)**

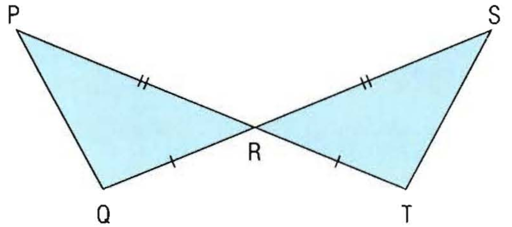
b) $\angle P = \angle B$
$\angle Q = \angle C$
$\angle R = \angle A$

CONTINUED

Example 3

a) State why $\triangle PQR \cong \triangle STR$.

b) List the other corresponding equal parts.



Solution

a) In $\triangle PQR$ and $\triangle STR$,

$PR = SR$

$\angle PRQ = \angle SRT$ (opposite angles)

$QR = TR$

Therefore, $\triangle PQR \cong \triangle STR$ **(SAS)**

b) $PQ = ST$

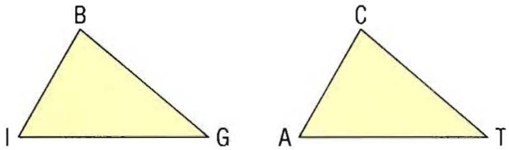
$\angle P = \angle S$

$\angle Q = \angle T$

Practice

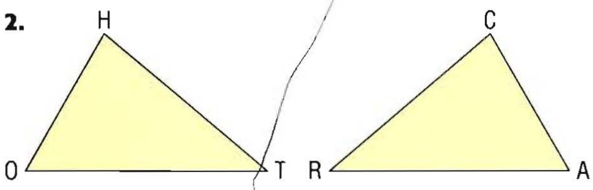
Name the equal sides and angles for these pairs of congruent triangles.

1.



$\triangle BIG \cong \triangle CAT$

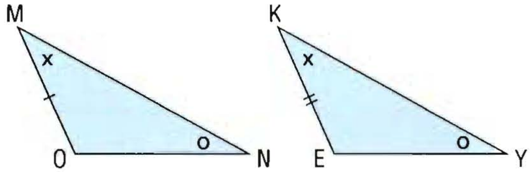
2.



$\triangle HOT \cong \triangle CAR$

Is each pair of triangles congruent in questions 3–9? If they are, what case did you use?

3.



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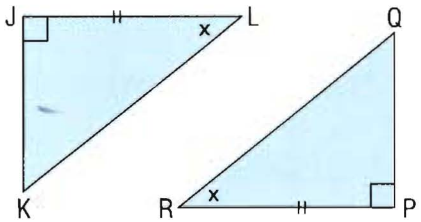

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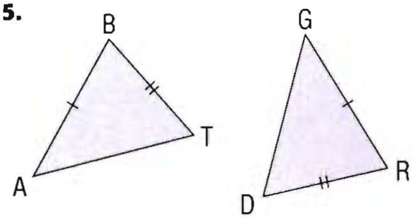

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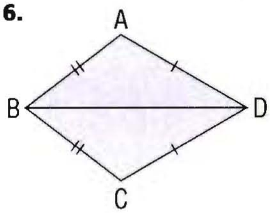

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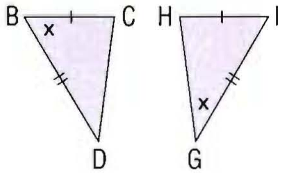

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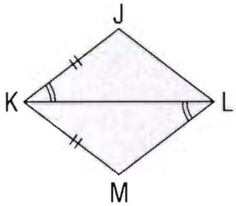

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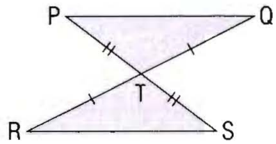
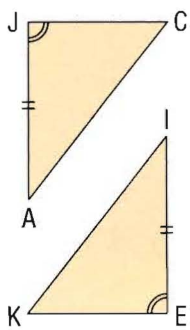
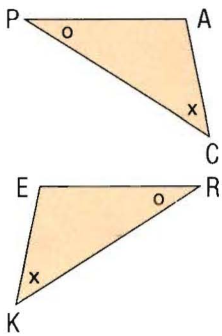


What is the fewest number of other parts that must be equal before you can state that the following pairs of triangles are congruent? Explain.

10.

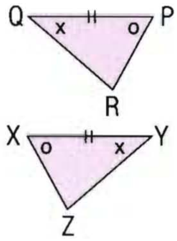


11.

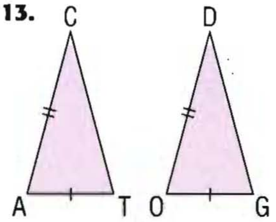


Are these pairs of triangles congruent? If they are, give the case and list all the corresponding equal parts.

12.



13.



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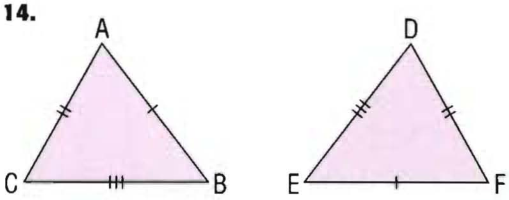
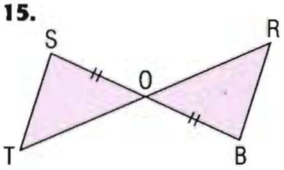


15.



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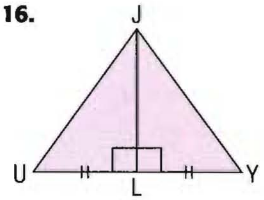

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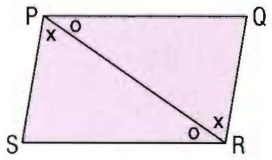

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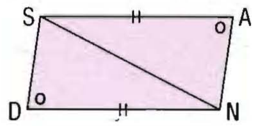

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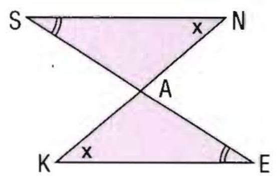
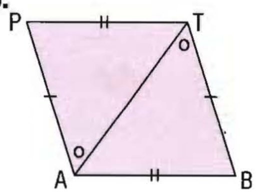


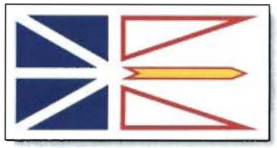
20.



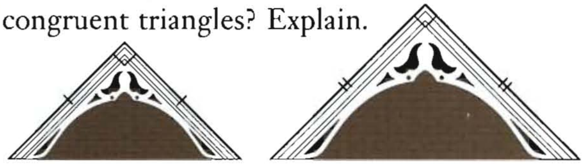
Problems and Applications

21. Alexi thinks that two triangles look congruent. Find two different ways he could find out if they are congruent without cutting the triangles out. Explain your reasoning.

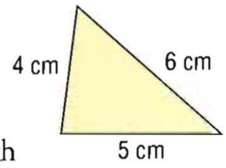
22. How many pairs of congruent triangles are there on the flag of Newfoundland and Labrador?



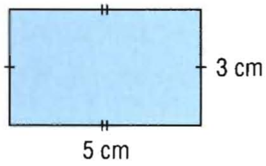
23. Do the gables on these 2 roofs form congruent triangles? Explain.



24. a) Triangles are used in construction because they hold their shape. To change the shape of this triangle, you have to change the length of at least 1 of the sides. Why?



b) To change the shape of this figure, you do not have to change the length of a side. Why not?



25. Work with a classmate to decide which of the following are always congruent, sometimes congruent, or never congruent. Illustrate your answer with diagrams.

a) 2 triangles with the same perimeter

b) 2 rectangles with the same area

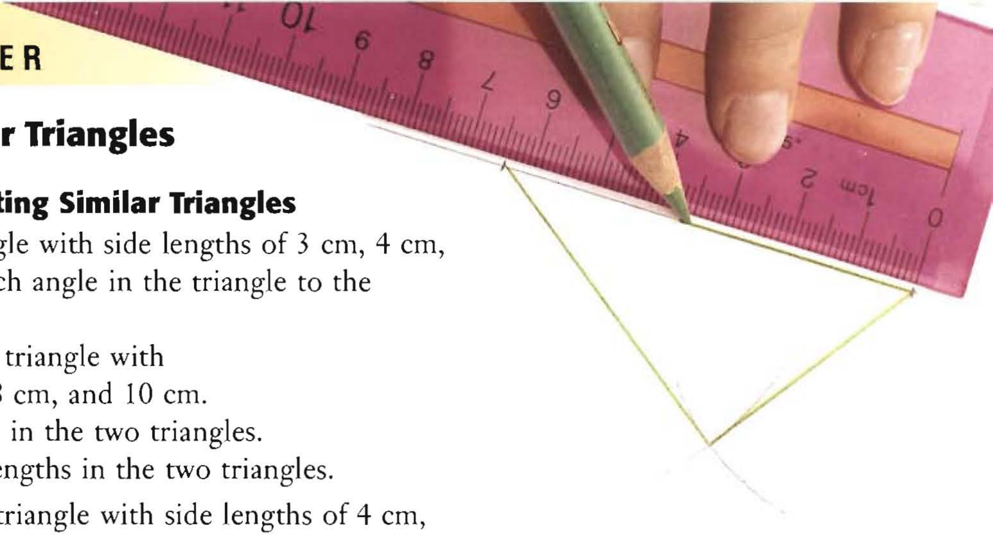
c) 2 squares with the same perimeter

d) 2 rectangles with the same perimeter

WORD POWER

a) Louisiana is 1 of 6 American states in which the name contains the consecutive letters IS. What are the other 5 states?

b) Which 2 Canadian provinces also contain the consecutive letters IS?



Investigating Similar Triangles

Activity ❶ Constructing Similar Triangles

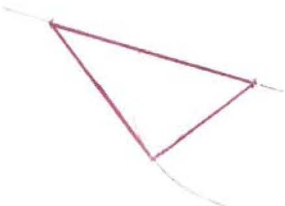
1. a) Construct a triangle with side lengths of 3 cm, 4 cm, and 5 cm. Measure each angle in the triangle to the nearest degree.

b) Repeat part a) for a triangle with side lengths of 6 cm, 8 cm, and 10 cm.

c) Compare the angles in the two triangles.

d) Compare the side lengths in the two triangles.

2. Repeat step 1 for a triangle with side lengths of 4 cm, 5 cm, and 6 cm, and a triangle with side lengths of 6 cm, 7.5 cm, and 9 cm.



3. The two triangles in step 1 are said to be **similar triangles**. So are the two triangles in step 2. Write a rule for identifying similar triangles.

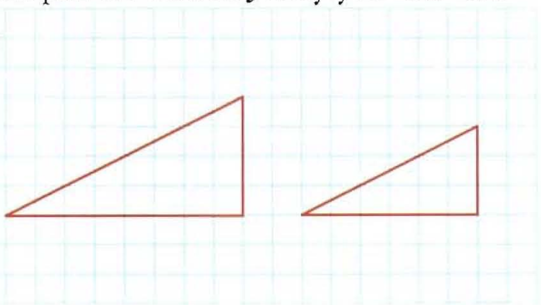
4. How do the shapes of the similar triangles compare?

5. How do the sizes of the similar triangles compare?

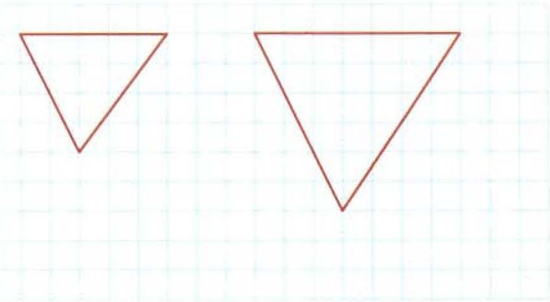
Activity ❷ Identifying Similar Triangles

Draw the triangles on grid paper. State whether the triangles in each pair are similar. Justify your answers.

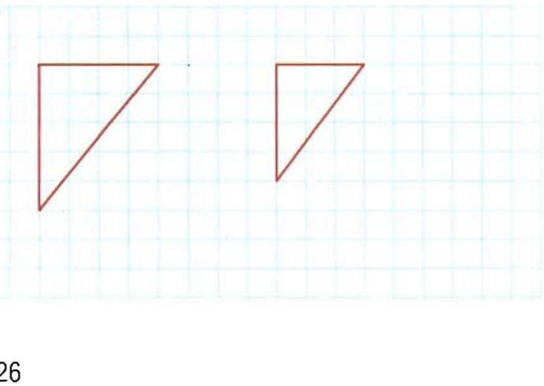
1.



2.



3.



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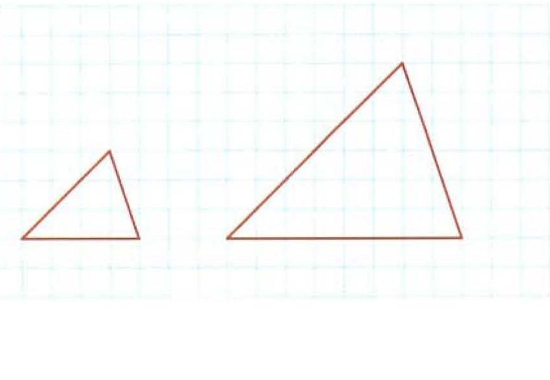
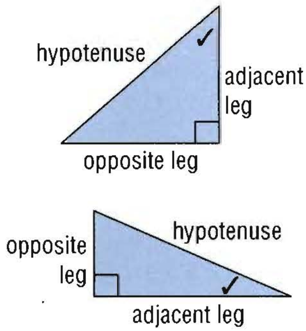


Right Triangles

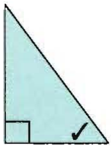
Activity ❶ Naming the Legs of Right Triangles

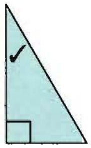
The legs of right triangles are named in relation to the acute angle being considered.

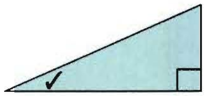


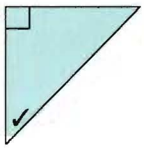
1. Study the two right triangles shown. Describe how the legs are named in relation to the marked angle.

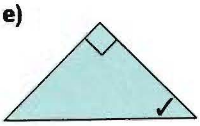
2. Copy the following triangles into your notebook. Label the sides of each triangle as the hypotenuse, the opposite leg, and the adjacent leg in relation to each marked angle.

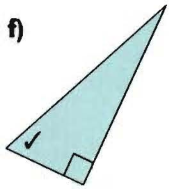
a)


b)


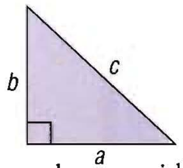
c)


d)


e)


f)


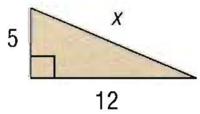
Activity ❷ The Pythagorean Theorem

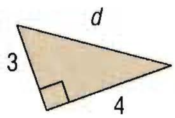


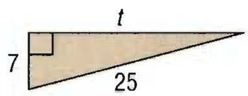
1. Write the Pythagorean Theorem in words.

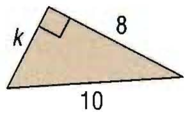
2. Use the letters in the diagram to write the Pythagorean Theorem.

3. Use the Pythagorean Theorem to calculate the unknown side of each triangle.

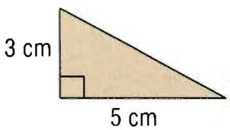
a)


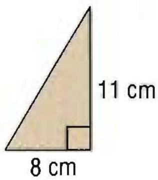
b)


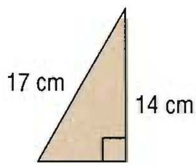
c)


d)


4. Use the Pythagorean Theorem to calculate the unknown side to the nearest tenth of a centimetre.

a)


b)


c)


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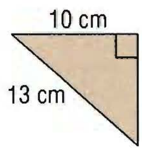


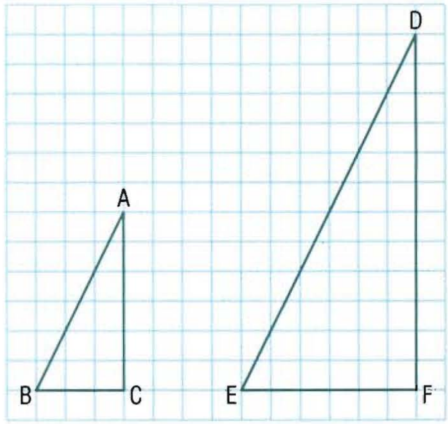
6.2 Similar Triangles

Before constructing a new plane, aeronautical engineers build and test a model. The model plane is the same shape as, or is similar to, the real plane. **Similar figures** have the same shape but not necessarily the same size.

Model Aircraft of 767 in old Air Canada livery.

Activity: Construct the Triangles

Draw △ABC and △DEF on grid paper. Measure the angles and sides in each triangle and complete the table.



△ABC		△DEF	
∠A =	AB =	∠D =	DE =
∠B =	BC =	∠E =	EF =
∠C =	AC =	∠F =	DF =

Inquire

1. How do the measures of the corresponding angles compare?

2. Find the ratios of the lengths of the corresponding sides, AB:DE, BC:EF, and AC:DF.

3. How do the ratios of the corresponding sides compare?

4. a) In what way are △ABC and △DEF the same as congruent triangles?
b) In what way are they different from congruent triangles?

5. What is the ratio of the lengths of corresponding sides in congruent triangles?

6. a) △DEF is an enlargement of △ABC. Explain why.
b) What is the scale factor?

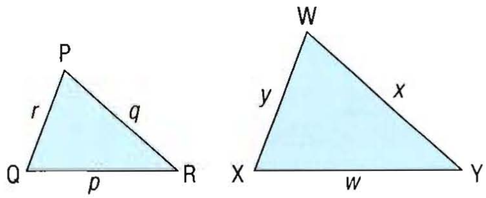
△PQR and △WXY are similar. This means that

- The corresponding angles are equal.

$\angle P = \angle W$ $\quad\quad$ $\angle Q = \angle X$ $\quad\quad$ $\angle R = \angle Y$

- The ratios of the corresponding sides are equal.

$$\frac{PQ}{WX} = \frac{QR}{XY} = \frac{PR}{WY} \quad \text{or} \quad \frac{r}{y} = \frac{p}{w} = \frac{q}{x}$$



We write △PQR ~ △WXY.

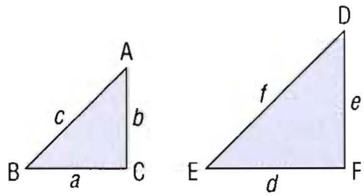
~ means "is similar to"

You can state that 2 triangles, △ABC and △DEF, are similar if you know that the corresponding pairs of angles are equal.

$\angle A = \angle D$ $\quad\quad$ $\angle B = \angle E$ $\quad\quad$ $\angle C = \angle F$

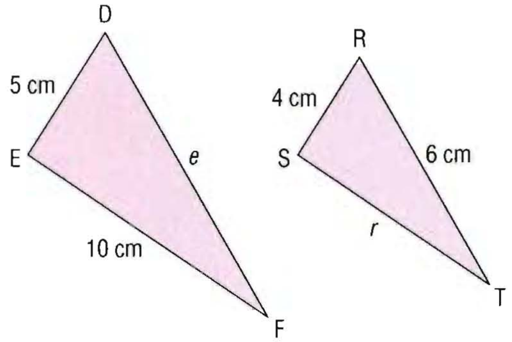
You can state that 2 triangles, △ABC and △DEF, are similar if you know that the ratios of the corresponding sides are equal.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \quad \text{or} \quad \frac{c}{f} = \frac{a}{d} = \frac{b}{e}$$



Example 1

$\triangle DEF \sim \triangle RST$. Find the values of r and e.



Solution

Since the triangles are similar, the ratios of the corresponding sides are equal.

$$\frac{DE}{RS} = \frac{EF}{ST} = \frac{DF}{RT}$$

Substitute. $\frac{5}{4} = \frac{10}{r} = \frac{e}{6}$

Take the ratios 2 at a time.

$$\frac{5}{4} = \frac{10}{r} \qquad \frac{5}{4} = \frac{e}{6}$$

Use the cross-product rule.

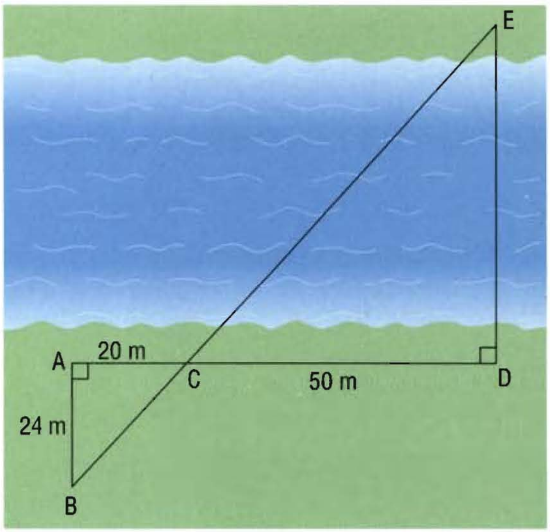
$$5 \times r = 10 \times 4 \qquad 5 \times 6 = e \times 4$$
$$5r = 40 \qquad 30 = 4e$$
$$r = 8 \qquad 7.5 = e$$

So, r is 8 cm, and e is 7.5 cm.

Similar triangles can be used to find distances that are difficult to measure directly.

Example 2

The diagram shows how surveyors can lay out 2 triangles to find the width of the Bow River near Banff, Alberta. Use the triangles to calculate the width of the river, DE.



Solution

If $\triangle ABC$ is similar to $\triangle DEC$, you can write ratios of corresponding sides to find DE.

In $\triangle ABC$ and $\triangle DEC$,

$\angle CAB = \angle CDE$ (both 90°)

$\angle ACB = \angle DCE$ (opposite angles)

If 2 angles in one triangle are equal to 2 angles in another triangle, the third angles in each triangle must be equal.

$\angle ABC = \angle DEC$

Since corresponding angles are equal, $\triangle ABC \sim \triangle DEC$.

Since the triangles are similar, the ratios of corresponding sides are equal.

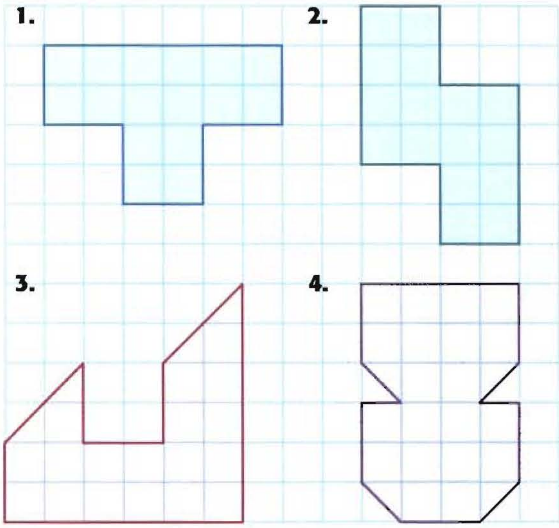
$$\frac{AC}{DC} = \frac{AB}{DE}$$
$$\frac{20}{50} = \frac{24}{DE}$$
$$20 \times DE = 24 \times 50$$
$$20 \times DE = 1200$$
$$DE = 60$$

The width of the river is 60 m.

CONTINUED

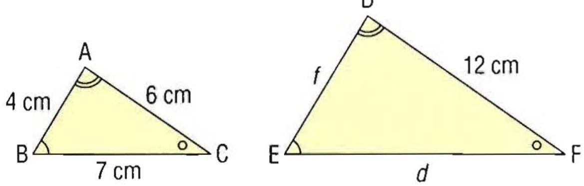
Practice

Use grid paper to make a figure similar to, but not congruent to, each of the following figures.

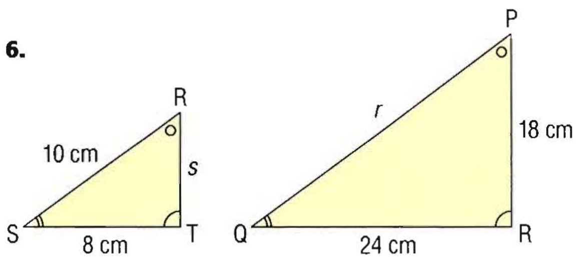


The triangles in each pair are similar. Find the unknown side lengths.

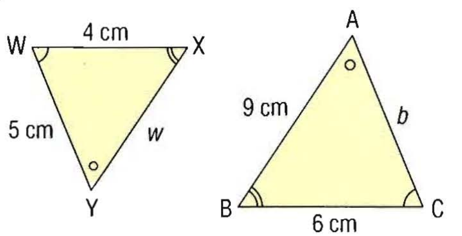
5.



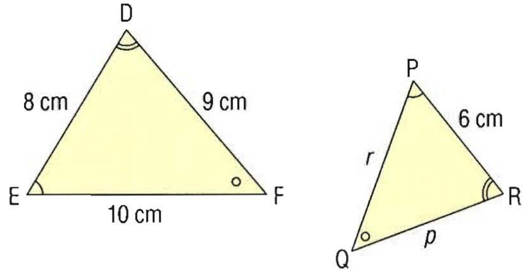
6.



7.



8.



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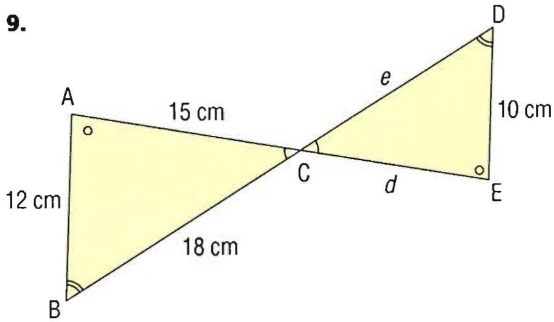
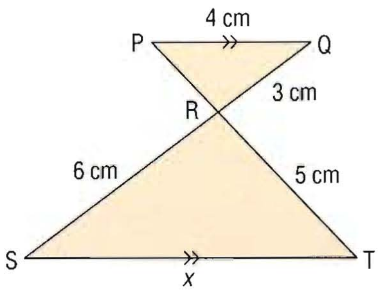
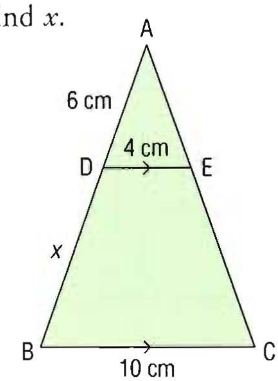


10. Find x.

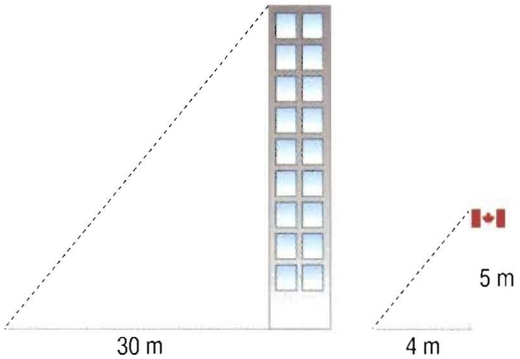


11. Find x.

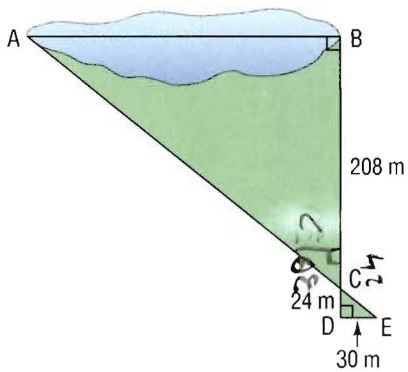


Problems and Applications

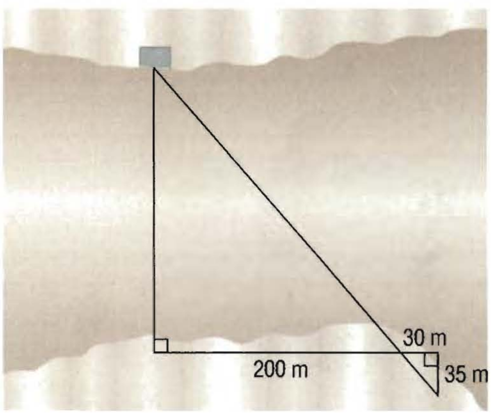
12. The 5-m flagpole casts a 4-m shadow at the same time of day as a building casts a 30-m shadow. How tall is the building?



13. Surveyors have laid out triangles to find the length of the lake. Calculate this length.



14. Surveyors have laid out these triangles to calculate the width of a canyon. Find this width to the nearest metre.



15. Pierre thinks that two triangles on a page look similar. Describe two different methods he could use to find out if the triangles are similar. Explain your reasoning.

16. Serena made a scale drawing of her triangular flower garden. Two sides of her garden are 5 m and 6 m long, and form an angle of 40°. Serena drew a 40° angle on paper, marked points at 10 cm and 12 cm on the arms, and joined these points. She measured the third side to be 9.5 cm. How long is the third side of the garden?

17. Is each of the following statements true or false? Use examples to explain your reasoning.

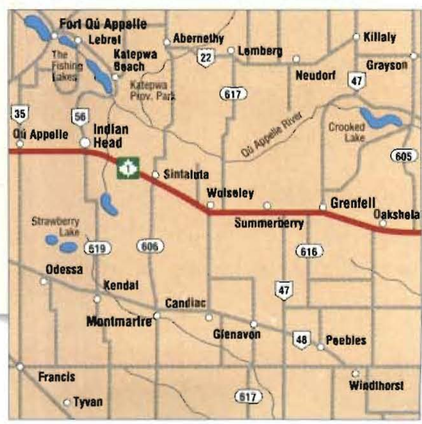
a) All similar triangles are congruent.

b) All congruent triangles are similar.

18. Decide on 3 outdoor objects whose heights you want to calculate. Your choices might include a street lamp, a flagpole, and the school. At the same time of day, find the length of the shadow cast by a metre-stick and the length of the shadow of each chosen object. Calculate the height of each object.

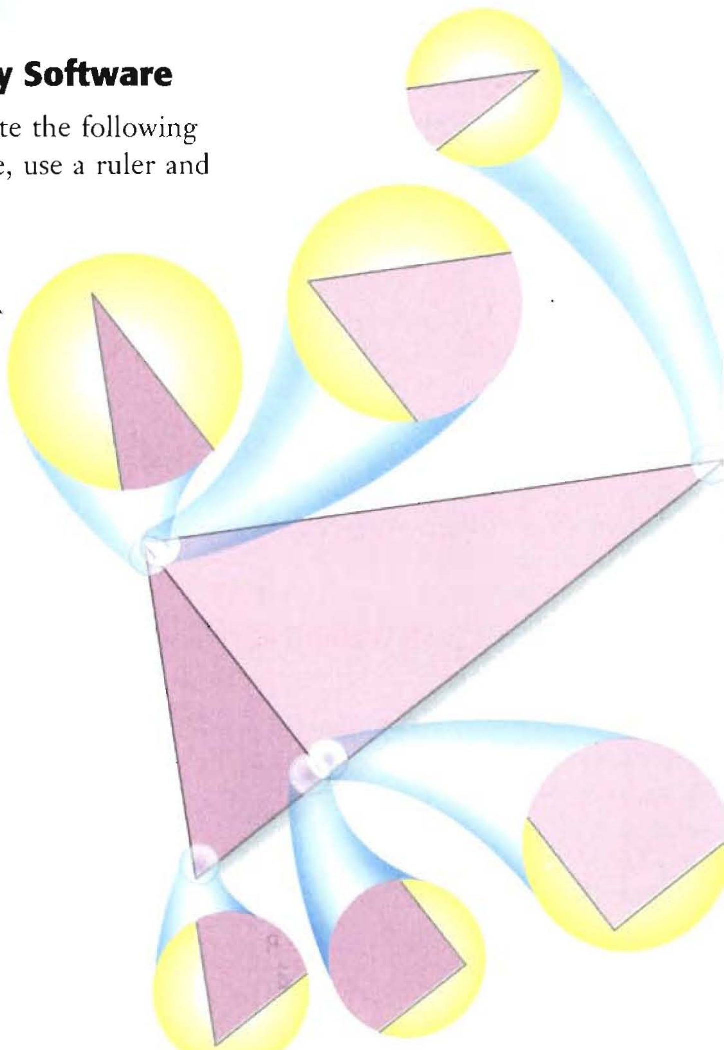
LOGIC POWER

Use your knowledge of geography to identify the province these towns are in. Then, check by finding this area on a map.



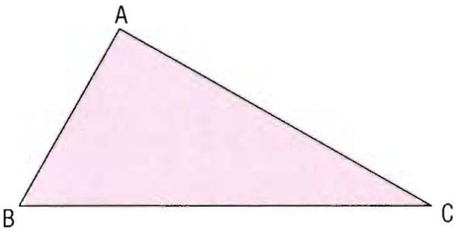
Comparing Triangles Using Geometry Software

Use a geometry software package to complete the following activities. If geometry software is unavailable, use a ruler and protractor.



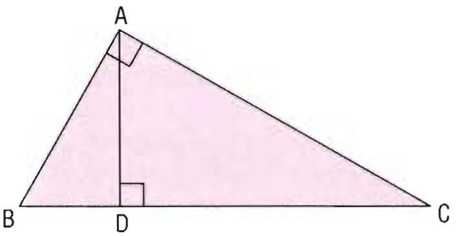
Activity 1

1. Construct a right triangle ABC. Make ∠A the right angle. Make ∠B and ∠C different from each other.



2. Measure ∠B and ∠C.

3. Construct the perpendicular AD to side BC.



4. Measure ∠DAB and ∠DAC.

5. Using all the angle measures you know, decide how △ABC, △DBA, and △DAC are related.

6. Use your result from step 5 to predict as many equal ratios of side lengths of the triangles as possible.

7. Measure the side lengths you included in step 6. Calculate the ratios to check your predictions. Compare your findings with a classmate's.

Activity 2

Refer to the triangles in Activity 1. Assume that ∠A remains a right angle and that AD remains perpendicular to BC.

1. What would have to be true about △ABC to make △ABD and △ACD congruent? Explain.

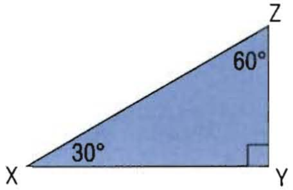
2. If △ABD and △ACD are congruent, how is each of these triangles related to △ABC?

Ratios of Side Lengths in Right Triangles

Use a geometry software package to complete the following activities. If geometry software is unavailable, use a ruler and protractor.

Activity ❶

1. Construct the right triangle XYZ with the angles shown and with side lengths of your choice.



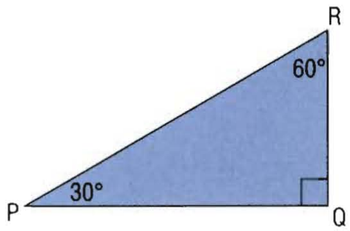
2. Measure

a) XY **b)** XZ **c)** YZ

3. Determine each of the following ratios to 2 decimal places.

a) $\frac{YZ}{XY}$ **b)** $\frac{YZ}{XZ}$ **c)** $\frac{XY}{XZ}$

4. Construct a similar triangle PQR in which the side lengths are different from those in $\triangle XYZ$.



5. Measure

a) PQ **b)** PR **c)** QR

6. Determine each of the following ratios to 2 decimal places.

a) $\frac{QR}{PQ}$ **b)** $\frac{QR}{PR}$ **c)** $\frac{PQ}{PR}$

7. a) Compare your results from steps 3 and 6.

b) Compare your results with those of your classmates.

8. What can you conclude about the ratios of the side lengths in a right triangle with fixed angle measures?

Activity ❷

Check your conclusion from Activity 1, step 8, by repeating steps 1 to 6 for two different triangles with angles of 40°, 50°, and 90°.

Staging Rock Concerts

One of the most exciting aspects of a rock concert is the integration of the music and the lighting. The lights around the stage are held in place by lighting pipes. The process of suspending the lights is known as "flying the pipes."

Before the pipes are flown, the lighting designer must find the proper angles for the lights. To do this, the designer uses a ruler and protractor.

In the following example, the stage has a depth of 8 m. The support for the 3 pipes is to be 1 m from the back of the stage. The 3 pipes, A, B, and C, are to be 9 m, 7 m, and 5 m above the stage floor. To find the angle at which each pipe must be pointed, the designer uses the following steps.

Draw the stage floor, GE, letting 1 cm represent 1 m.

Find $\frac{1}{4}$ of the stage depth. $\frac{1}{4} \times 8 = 2$

Mark the lights' *focal point*, F, $\frac{1}{4}$ of the stage depth or 2 m from the front of the stage.

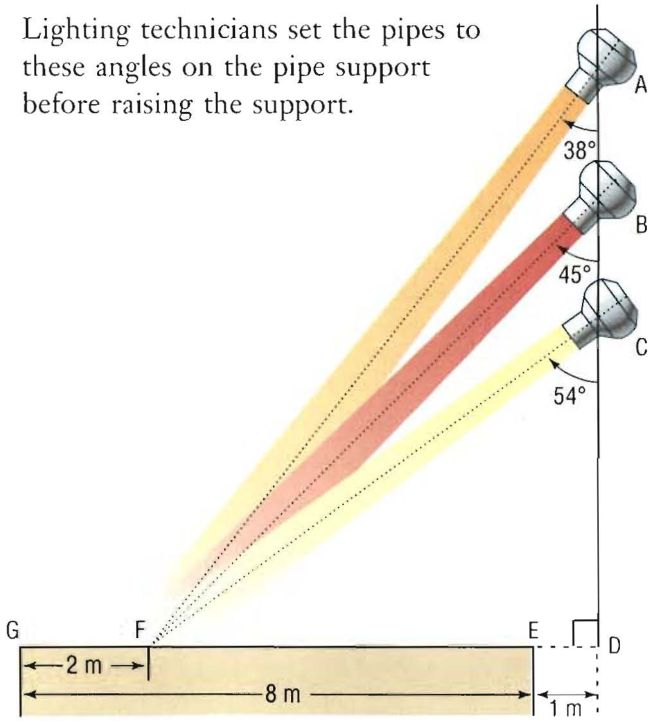
Mark a point, D, on the pipe support 1 m from the back of the stage.

Draw the pipe support and mark the lighting pipes, A, B, and C, 9 m, 7 m, and 5 m from the stage floor.

Join AF, BF, and CF to make ∠FAD, ∠FBD, and ∠FCD. Measure these 3 angles.

∠FAD = 38°, ∠FBD = 45°, and ∠FCD = 54°.

Lighting technicians set the pipes to these angles on the pipe support before raising the support.



Activity 1

These questions refer to the example on the opposite page.

1. a) Another way to determine ∠FAD is to measure ∠AFD and use the angle properties of a triangle. What is the relationship between the measures of ∠FAD and ∠AFD? Explain.

b) Why might a designer choose to measure ∠AFD, ∠BFD, and ∠CFD and to calculate ∠FAD, ∠FBD, and ∠FCD from them?

2. The measure of ∠FBD could be determined without the use of a protractor. Explain how.

Activity 2

The lighting designer adjusts the angles of the lighting pipes for stages of different sizes. Use the table to find the angles needed for pipes A, B, and C at each venue.

Venue	Stage Width (m)	Stage Depth (m)	Pipe Height (m) A	B	C	Distance of Pipe Support from Stage (m)
Maple Leaf Gardens	12	8	11	10	9	3
B.C. Place	16	12	12	10	7	1
O'Keefe Centre	8	6	9	7	6	0
Lansdowne Park	12	10	10	8	6	2

Activity 3

For some lighting designs, it is important to use the same lighting angles throughout a tour. In these cases, the height of each lighting pipe is the variable that the designer must determine.

1. Choose one of the venues in Activity 2. Use the stage depth and the distance of the pipe support from the stage to determine the heights of the pipes for angles of 35°, 45°, and 50°.

2. Describe how the problem solving process differs from the process used in the example.

Activity 4

Very few concerts have lighting equipment only behind the performers. For variety and visibility, most designers also hang lighting pipes on both sides of the stage.

1. Using the data from Activity 2, add pipes A, B, and C on stage left, and pipes X, Y, and Z on stage right for one of the venues.

2. How are the triangles you drew for stage left and the triangles you drew for stage right related?

Activity 5

1. With a partner, choose a performer or group for whom you would like to do a lighting design. Also, choose one of the venues in Activity 2.

2. Decide how many pipes to use

a) at the back of the stage

b) on each side of the stage

3. Decide how far from the stage to place the pipe supports.

4. Choose the colours of the lights.

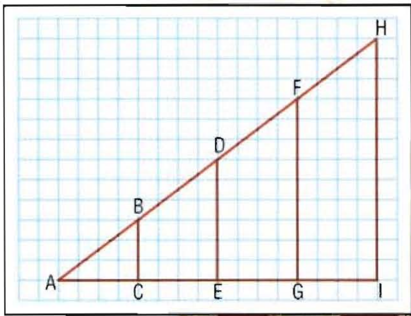
5. Use the stage dimensions to determine the angles of the lighting pipes.

6.3 Right Triangles and the Tangent Ratio

Early Canadian pioneers faced long, harsh winters. Many of their normal household needs, such as warm quilts, were handmade. The quilts are still prized for their intricate needlework. This quilt, called the *Prairie Queen*, shows different-sized, similar right triangles.

Activity: Compare the Ratios

Draw this set of nested triangles on grid paper. Then, copy and complete the table.
Express each ratio in decimal form.



Triangle	△ABC	△ADE	△AFG	△AHI
Ratio	$\frac{BC}{AC}$ =	$\frac{DE}{AE}$ =	$\frac{FG}{AG}$ =	$\frac{HI}{AI}$ =

How do the ratios compare?

Inquire

1. Why are △ABC, △ADE, △AFG, and △AHI similar?

2. Which angle is common to all 4 triangles?

3. The ratio you found is called the **tangent ratio** for the common angle. Explain the meaning of the tangent ratio by describing the positions of the 2 sides in each ratio in relation to the common angle.

4. Measure the common angle to the nearest degree.

5. The tangent ratio for an angle can be determined with the [TAN] key on a scientific calculator. Set your calculator to [DEG] mode, enter the measure of the common angle, and press the [TAN] key. Record the display to 2 decimal places.

6. The ratio from the Activity and the value from your calculator both represent the tangent ratio for the common angle. Compare the two values. If they are different, explain.

Trigonometry is the study of the relationships among the sides and angles of triangles. One such relationship is the tangent ratio, which is an example of a **trigonometric ratio**.

Example 1

Find the tangent ratio, to 3 decimal places, for each angle.

a) 40° **b)** 55° **c)** 73° **d)** 89°

Solution

a) $\tan 40° \doteq 0.839$ [C] **40** [TAN] 0.8390996

b) $\tan 55° \doteq 1.428$ **c)** $\tan 73° \doteq 3.271$ **d)** $\tan 89° \doteq 57.290$

≐ means "approximately equal"

Example 2

Find each angle measure, to the nearest degree, for each tangent ratio.

a) $\tan A = 1.782$ **b)** $\tan B = 0.509$ **c)** $\tan C = 6.895$ **d)** $\tan D = 0.063$

Solution

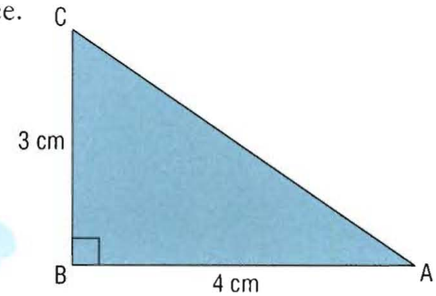
a) If $\tan A = 1.782$ C 1.782 INV TAN 60.700287 or C 1.782 2ND TAN^{-1} 60.700287

$\angle A \doteq 61°$

b) If $\tan B = 0.509$, $\angle B \doteq 27°$ **c)** If $\tan C = 6.895$, $\angle C \doteq 82°$ **d)** If $\tan D = 0.063$, $\angle D \doteq 4°$

Example 3

a) Find $\tan C$ in $\triangle ABC$. **b)** Calculate $\angle C$ to the nearest degree.



Solution

a) The tangent ratio may be found for either acute angle in a right triangle. For $\angle C$, the tangent ratio is found as follows:

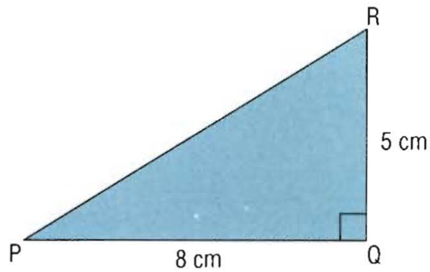
$$\tan C = \frac{\text{length of the leg opposite } \angle C}{\text{length of the leg adjacent to } \angle C}$$
$$= \frac{4}{3}$$
$$\doteq 1.333$$

Think: $\tan = \frac{\text{opposite}}{\text{adjacent}}$

b) Using a calculator, $\angle C \doteq 53°$. C 4 ÷ 3 = INV TAN 53.130102

Example 4

In $\triangle PQR$, find **a)** $\tan P$ and $\angle P$ **b)** $\tan R$ and $\angle R$



Solution

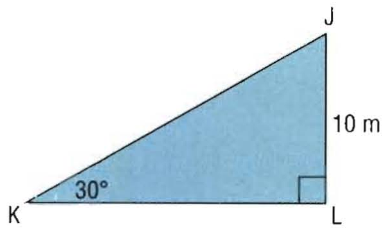
a) $\tan P = \frac{5}{8}$
$= 0.625$
$\angle P \doteq 32°$

b) $\tan R = \frac{8}{5}$
$= 1.6$
$\angle R \doteq 58°$

Example 5

In $\triangle JKL$, find the length of KL, to the nearest metre.

Solution



$$\tan K = \frac{10}{KL}$$
$$\tan 30° = \frac{10}{KL}$$
$$0.577 \doteq \frac{10}{KL}$$
$$KL \times 0.577 \doteq 10$$
$$KL \doteq \frac{10}{0.577}$$
$$KL \doteq 17.3$$

The length of KL is 17 m, to the nearest metre.

CONTINUED

Practice

Find the following to 3 decimal places.

1. tan 15° **2.** tan 62° **3.** tan 5°

4. tan 30° **5.** tan 82° **6.** tan 45°

Find ∠B to the nearest degree.

7. tan B = 0.600 **8.** tan B = 0.833

9. tan B = 3.025 **10.** tan B = 5.050

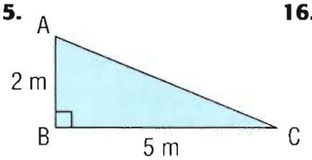
Find ∠W to the nearest degree.

11. $\tan W = \frac{4}{5}$ **12.** $\tan W = \frac{6}{7}$

13. $\tan W = \frac{7}{4}$ **14.** $\tan W = \frac{15}{9}$

Calculate tan C in each triangle.

15.



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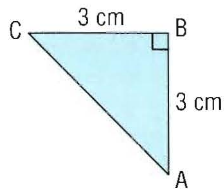
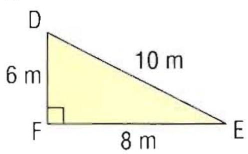


Calculate tan D, ∠D, tan E, and ∠E. Round each angle measure to the nearest degree.

17.



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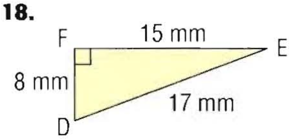

Calculate x to the nearest tenth of a metre.

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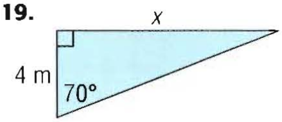

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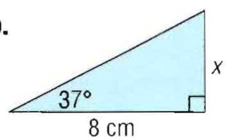

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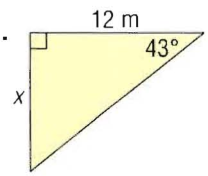

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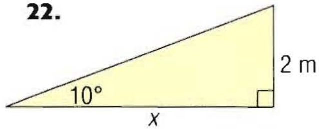

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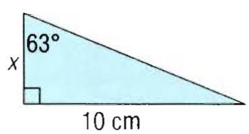

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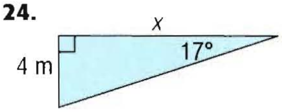
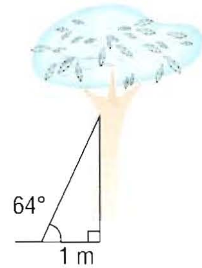


Problems and Applications

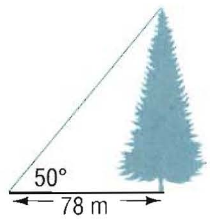
25. In a right triangle, the leg adjacent to an angle of 23° is 12 cm long. How long is the leg opposite the 23° angle, to the nearest tenth of a centimetre?

26. In a right triangle, the leg opposite the 53° angle is 4 cm long. How long is the leg adjacent to the 53° angle, to the nearest centimetre?

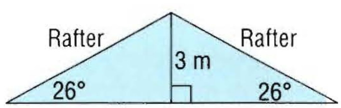
27. When a ladder is rested against a tree, the foot of the ladder is 1 m from the base of the tree and forms an angle of 64° with the ground. How far up the tree does the ladder reach, to the nearest tenth of a metre?



28. One of Canada's tallest trees is a Douglas fir on Vancouver Island. The angle of elevation measured by an observer who is 78 m from the base of the tree is 50°. How tall is this tree, to the nearest metre?

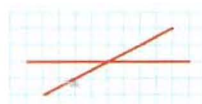


29. The angle of inclination of the rafters of the roof of a house is 26°. The roof support is 3 m high. How wide is the house, to the nearest metre?



30. Pietra walked diagonally across a rectangular school yard 45 m by 65 m. To the nearest degree, at what angle with respect to the longer side did she walk?

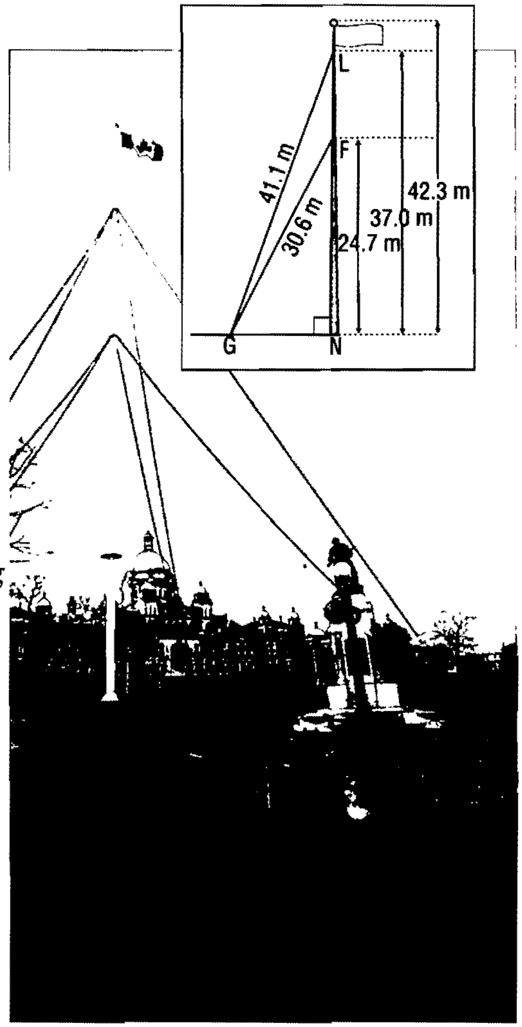
31. Calculate the acute angle, to the nearest degree, formed by these lines.



32. Write a problem similar to questions 25 and 26. Have a classmate solve it.

6.4 Right Triangles and the Sine Ratio

The flagpole in front of the provincial parliament buildings in Victoria, British Columbia, is a single Douglas fir. With a height of 42.3 m, the flagpole must be supported by guy wires. There are two sets of guy wires, one set attached at an elevation of 24.7 m, the other at an elevation of 37.0 m.



Activity: Use the Diagram

Complete the ratios in the table. Express them in decimal form to 2 decimal places.

Triangle	ΔFGN	ΔLGN
Ratio	$\frac{FN}{FG} =$	$\frac{LN}{LG} =$

Inquire

1. Each ratio you found is called a **sine ratio**. In ΔFGN, you found the sine ratio for $\angle FGN$.

a) Explain the meaning of the sine ratio for $\angle FGN$ by describing the positions of the 2 sides in the ratio in relation to $\angle FGN$.

b) Test your explanation by checking the positions of the sides used to calculate the sine ratio for $\angle LGN$ in ΔLGN.

2. With a protractor, measure to the nearest degree

a) $\angle FGN$ **b)** $\angle LGN$

3. The sine ratio for an angle can be determined with the [SIN] key on a scientific calculator.

a) Set your calculator to [DEG] mode, enter the measure of $\angle FGN$, and press the [SIN] key. Record the display to 2 decimal places.

b) Repeat part a) for $\angle LGN$.

4. Compare the ratio from the Activity with the ratio from question 3 and explain any differences for

a) $\angle FGN$ **b)** $\angle LGN$

Example 1

Find the sine ratio, to 3 decimal places, for each angle.

a) 75° **b)** 52° **c)** 17° **d)** 90°

Solution

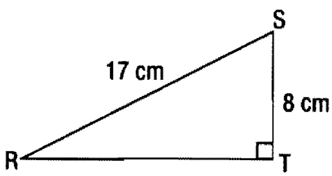
a) $\sin 75° \doteq 0.966$ [C] 75 [SIN] 0.9659258

b) $\sin 52° \doteq 0.788$ **c)** $\sin 17° \doteq 0.292$ **d)** $\sin 90° = 1.000$

Example 2

a) Find $\sin R$ in ΔRST.

b) Calculate $\angle R$ to the nearest degree.



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Solution

a) The sine ratio may be found for either acute angle in a right triangle.

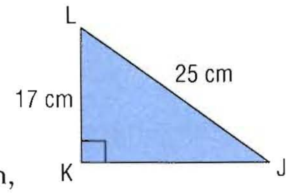
$$\sin R = \frac{\text{length of the leg opposite } \angle R}{\text{length of the hypotenuse}}$$
$$= \frac{8}{17}$$
$$\doteq 0.471$$

Think: $\sin = \frac{\text{opposite}}{\text{hypotenuse}}$

b) Using a calculator, $\angle R \doteq 28°$. C 8 ÷ 17 = INV SIN 28.072487
or C 8 ÷ 17 = 2ND SIN^{-1} 28.072487

Example 3

In $\triangle JKL$, find the length of JK, to the nearest tenth of a centimetre.



Solution 1

Using the sine ratio,

$$\sin J = \frac{17}{25}$$
$$= 0.68$$
$$\angle J \doteq 43°$$

Therefore, $\angle L = 180° - 43° - 90°$
$$= 47°$$
$$\sin L = \frac{JK}{25}$$
$$\sin 47° = \frac{JK}{25}$$
$$0.731 \doteq \frac{JK}{25}$$
$$0.731 \times 25 \doteq JK$$
$$18.275 \doteq JK$$

EST $0.7 \times 30 = 21$

The length of JK is 18.3 cm, to the nearest tenth of a centimetre.

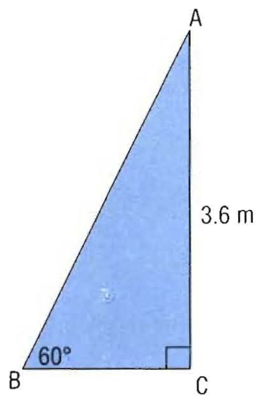
Solution 2

Using the Pythagorean Theorem,

$$JK^2 + LK^2 = JL^2$$
$$JK^2 + 17^2 = 25^2$$
$$JK^2 = 25^2 - 17^2$$
$$= 625 - 289$$
$$= 336$$
$$JK = \sqrt{336}$$
$$\doteq 18.33$$

Example 4

In $\triangle ABC$, find the length of AB, to the nearest tenth of a metre.



Solution

$$\sin 60° = \frac{3.6}{AB}$$
$$0.866 \doteq \frac{3.6}{AB}$$
$$0.866 \times AB \doteq 3.6$$
$$AB \doteq \frac{3.6}{0.866}$$
$$\doteq 4.16$$

EST $4 \div 1 = 4$

The length of AB is 4.2 m, to the nearest tenth of a metre.

Practice

Find the following to 3 decimal places.

1. $\sin 45°$ **2.** $\sin 60°$ **3.** $\sin 37°$

4. $\sin 25°$ **5.** $\sin 0°$ **6.** $\sin 89°$

Find $\angle J$ to the nearest degree.

7. $\sin J = 0.503$ **8.** $\sin J = 0.952$

9. $\sin J = 0.712$ **10.** $\sin J = 0.303$

11. $\sin J = 0.998$ **12.** $\sin J = 0.101$

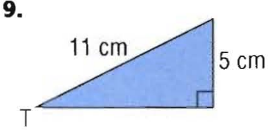
Find $\angle B$ to the nearest degree.

13. $\sin B = \frac{2}{3}$ **14.** $\sin B = \frac{3}{4}$ **15.** $\sin B = \frac{1}{2}$

16. $\sin B = \frac{2}{5}$ **17.** $\sin B = \frac{1}{8}$ **18.** $\sin B = \frac{7}{9}$

Calculate sin T. Then, find $\angle T$ to the nearest degree.

19.



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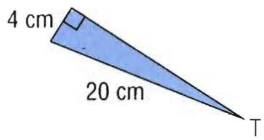
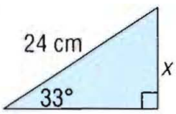


Find the value of x to the nearest tenth of a centimetre.

21.



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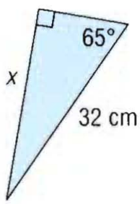

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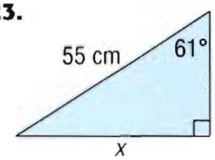

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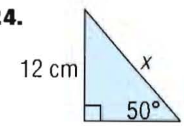

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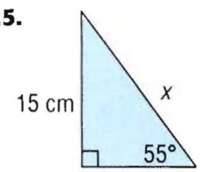

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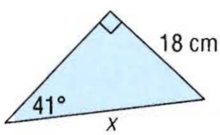


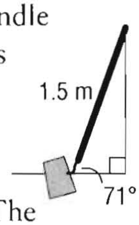
Problems and Applications

27. In $\triangle PQR$, $\angle Q = 90°$ and $PR = 20$ cm. Find PQ to the nearest tenth of a centimetre if $\angle R = 41°$.

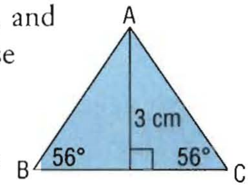
28. In $\triangle DEF$, find $\angle F$ to the nearest degree if $DE = 15$ cm, $DF = 18$ cm, and $\angle E = 90°$.

29. In $\triangle ABC$, $\angle B = 90°$. If $AB = 10$ cm and $\angle C = 38°$, find the length of AC, to the nearest tenth of a centimetre.

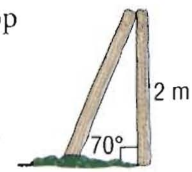
30. A 1.5-m hoe rests against the side of a garden shed. The angle the handle of the hoe forms with the ground is 71°. How far up the wall of the shed does the hoe reach, to the nearest tenth of a metre?



31. $\triangle ABC$ is an isosceles triangle. The height of the triangle is 3 cm, and the two acute angles at its base are each 56°. How long are the two equal sides, to the nearest tenth of a centimetre?

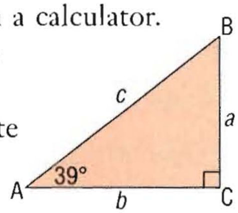


32. A tree is splintered by lightning 2 m up its trunk, so that the top part of the tree touches the ground. The angle the top of the tree forms with the ground is 70°. How tall is the tree, to the nearest tenth of a metre?



33. a) Evaluate $\sin 39°$ with a calculator. Round your answer to the nearest thousandth.

b) Use the diagram to write the ratio represented by your answer to part a).



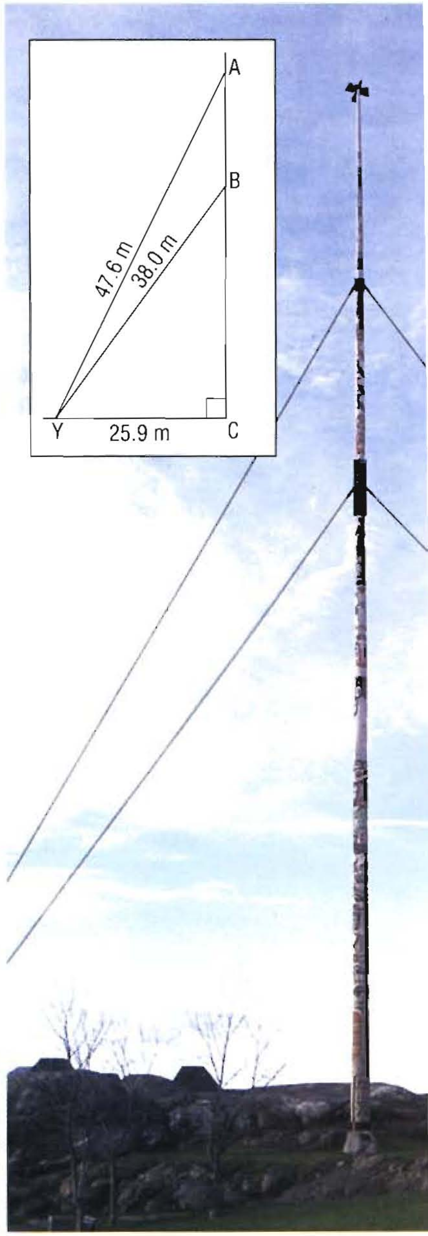
c) What is the length of a in terms of c?

d) What is the length of c in terms of a? Explain.

34. Write a problem similar to question 27, 28, or 29. Have a classmate solve it.

6.5 Right Triangles and the Cosine Ratio

To mark the fifteenth Commonwealth Games held in Victoria, the world's tallest totem pole *The Spirit of Nations* was erected at Songhees Point on Victoria Harbour. Unlike most totem poles, which are stabilized by sinking them into the ground, *The Spirit of Nations* requires anchor lines because it sits on rock.



Activity: Use the Diagram

Complete the ratios in the table and express them in decimal form to 2 decimal places.

Triangle	$\triangle$ ACY	$\triangle$ BCY
Ratio	$\frac{CY}{AY}=$	$\frac{CY}{BY}=$

Inquire

1. Each ratio you found is called a **cosine ratio**. In $\triangle$BCY, you found the cosine ratio for $\angle$BYC.

a) Explain the meaning of the cosine ratio for $\angle$BYC by describing the positions of the 2 sides in the ratio in relation to $\angle$BYC.
b) Test your explanation by checking the positions of the sides used to calculate the cosine ratio for $\angle$AYC in $\triangle$ACY.

2. With a protractor, measure to the nearest degree
a) $\angle$BYC **b)** $\angle$AYC

3. The cosine ratio for an angle can be determined with the [COS] key on a scientific calculator.
a) Set your calculator to [DEG] mode, enter the measure of $\angle$BYC, and press the [COS] key. Record the display to 2 decimal places.
b) Repeat part a) for $\angle$AYC.

4. Compare the ratio from the Activity with the ratio from question 3 and explain any differences for
a) $\angle$BYC **b)** $\angle$AYC

Example 1

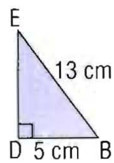
Find the cosine ratio, to 3 decimal places, for each angle.
a) 42° **b)** 9° **c)** 20° **d)** 90°

Solution

a) $\cos 42° \doteq 0.743$ [C] 42 [COS] 0.7431448
b) $\cos 9° \doteq 0.988$ **c)** $\cos 20° \doteq 0.940$ **d)** $\cos 90° = 0.000$

Example 2

a) Calculate cos B in $\triangle$BDE.
b) Calculate $\angle$B to the nearest degree.



Solution

a) The cosine ratio may be found for either acute angle in a right triangle.

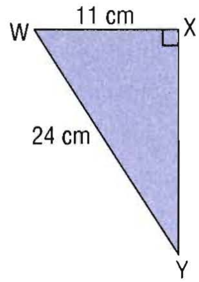
$$\cos B = \frac{\text{length of the leg adjacent to } \angle B}{\text{length of the hypotenuse}}$$
$$= \frac{5}{13}$$
$$\doteq 0.385$$

Think: $\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$

b) Using a calculator, $\angle B = 67°$. C 5 ÷ 13 = INV COS   67.380 135
or C 5 ÷ 13 = 2ND $\cos^{-1}$ 67.380 135

Example 3

In $\triangle WXY$, find the length of XY, to the nearest centimetre.



Solution 1

Using the cosine ratio,

$$\cos W = \frac{11}{24}$$
$$\doteq 0.458$$
$$\angle W \doteq 63°$$

Since $\angle W = 63°$ and $\angle X = 90°$, $\angle Y = 27°$.

$$\cos Y = \frac{XY}{24}$$
$$\cos 27° = \frac{XY}{24}$$
$$0.891 \doteq \frac{XY}{24}$$
$$0.891 \times 24 \doteq XY$$
$$21.384 \doteq XY$$

EST $1 \times 20 = 20$

The length of XY is 21 cm, to the nearest centimetre.

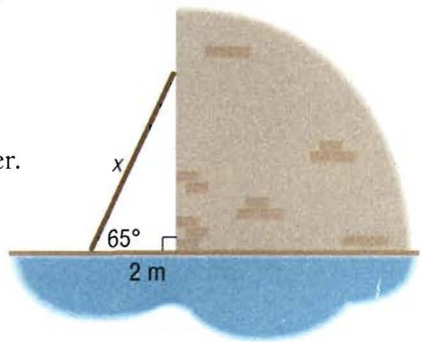
Solution 2

Using the Pythagorean Theorem,

$$WX^2 + XY^2 = WY^2$$
$$11^2 + XY^2 = 24^2$$
$$XY^2 = 24^2 - 11^2$$
$$= 576 - 121$$
$$= 455$$
$$XY = \sqrt{455}$$
$$\doteq 21.33$$

Example 4

A ladder leans against a vertical wall and makes an angle of 65° with the ground. The foot of the ladder is 2 m from the base of the wall. Calculate the length of the ladder, to the nearest tenth of a metre.



Solution

The cosine ratio can be used to calculate the length of the ladder.

$$\cos 65° = \frac{2}{x}$$
$$0.423 \doteq \frac{2}{x}$$
$$0.423 \times x \doteq 2$$
$$x \doteq \frac{2}{0.423}$$
$$\doteq 4.728$$

EST $2 \div 0.4 = 5$

The length of the ladder is 4.7 m, to the nearest tenth of a metre.

CONTINUED

Practice

Find the following to 3 decimal places.

1. $\cos 30°$ **2.** $\cos 45°$ **3.** $\cos 60°$ **4.** $\cos 89°$

5. $\cos 0°$ **6.** $\cos 5°$ **7.** $\cos 19°$ **8.** $\cos 83°$

Find $\angle P$ to the nearest degree.

9. $\cos P = 0.343$ **10.** $\cos P = 0.887$

11. $\cos P = 0.621$ **12.** $\cos P = 0.019$

13. $\cos P = 0.731$ **14.** $\cos P = 0.524$

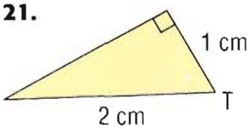
Find $\angle Q$ to the nearest degree.

15. $\cos Q = \frac{1}{6}$ **16.** $\cos Q = \frac{5}{11}$ **17.** $\cos Q = \frac{5}{9}$

18. $\cos Q = \frac{7}{8}$ **19.** $\cos Q = \frac{15}{16}$ **20.** $\cos Q = \frac{3}{14}$

Calculate cos T. Then, find $\angle T$ to the nearest degree.

21.



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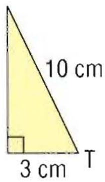
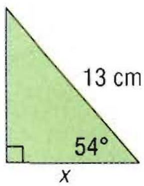


Find x to the nearest tenth of a centimetre.

23.



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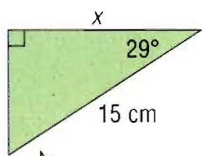

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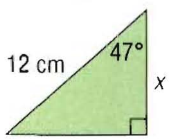

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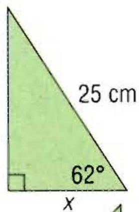

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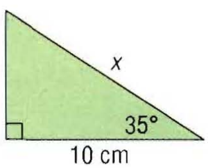

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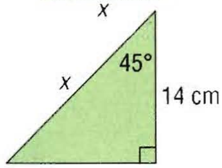

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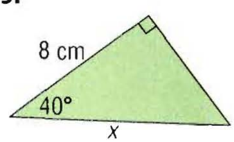

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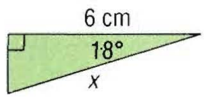
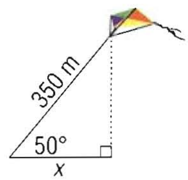


Problems and Applications

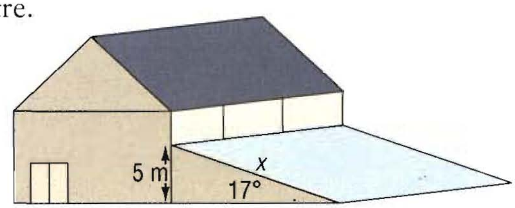
31. The leg adjacent to the 74° angle in a right triangle is 6 cm long. How long is the hypotenuse, to the nearest tenth of a centimetre?

32. The hypotenuse of a right triangle is 10 cm long. How long is the leg adjacent to the 21° angle, to the nearest tenth of a centimetre?

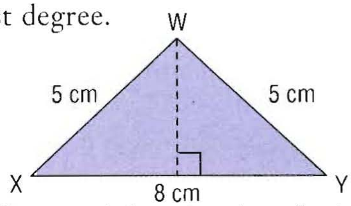
33. A kite string is 350 m long. The angle the string makes with the ground is 50°. How far from the person holding the string is a person standing directly under the kite?



34. A graded ramp is to be built to a barn loft. The ramp is to be inclined at an angle of 17°. The floor of the loft is 5 m above ground level. Use the sine ratio to find the length of the ramp, to the nearest tenth of a metre.



35. Find all the angles in $\triangle WXY$, to the nearest degree.



36. a) Does a right triangle exist in which the sine and cosine ratios of the same acute angle are equal?

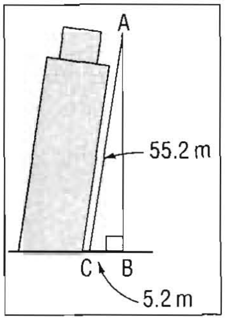
b) If such a triangle exists, explain why the ratios are equal.

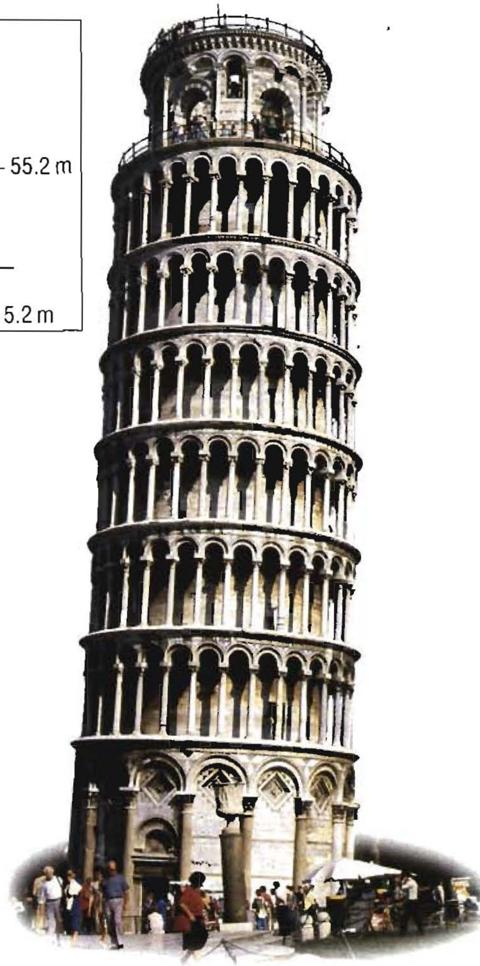
37. Write a problem similar to questions 31 and 32. Have a classmate solve it.

6.6 Solving Right Triangles

Activity: Study the Diagram

The diagram shows some approximate measurements for the Leaning Tower of Pisa. Name the unknown angles and the unknown side in the triangle.





Inquire

1. a) Describe two different methods you could use to calculate the angle that the tower makes with the ground.
b) Use a method of your choice to complete the calculations. Round your answer to the nearest degree.

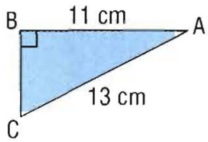
2. a) Describe two different methods you could use to calculate the vertical height of the top of the tower above the ground.
b) Use a method of your choice to complete the calculations. Round your answer to the nearest tenth of a metre.

3. Of the unknown angles and sides you listed in the Activity, which one have you not yet found? Determine its value.

To **solve** a right triangle means to find all the unknown sides and unknown angles.

Example 1

Solve △ABC. Find lengths to the nearest tenth of a centimetre and angles to the nearest degree.



Solution

The cosine ratio can be used to calculate ∠A.

$$\cos A = \frac{11}{13}$$
$$\doteq 0.846$$
$$\angle A \doteq 32°$$
$$\angle C = 180° - 32° - 90°$$
$$= 58°$$

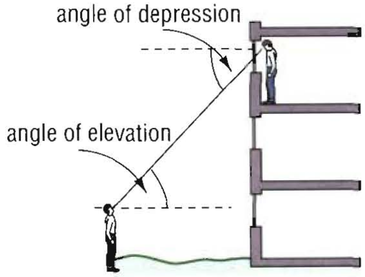
The Pythagorean Theorem can be used to calculate the length of BC.

$$BC^2 = 13^2 - 11^2$$
$$= 169 - 121$$
$$= 48$$
$$BC = \sqrt{48}$$
$$\doteq 6.928$$

EST $\sqrt{49} = 7$

The length of BC is 6.9 cm, to the nearest tenth of a centimetre.
∠A is 32° and ∠C is 58°, to the nearest degree.

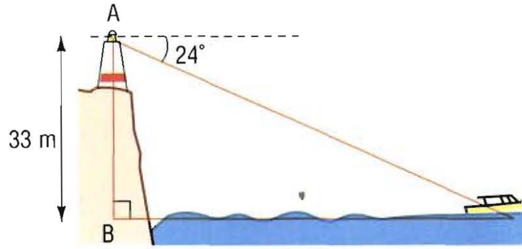
If you stand outside a building and look at an upstairs window, the angle that your line of sight makes with the horizontal is known as the **angle of elevation**.



If someone is standing at the window and looking down at you, the angle that the person's line of sight makes with the horizontal is known as the **angle of depression**.

Example 2

A lighthouse sits at the top of a sheer cliff. The top of the lighthouse is 33 m above sea level. The angle of depression to sight a small fishing boat at sea is 24°. How far from the base of the cliff is the fishing boat, to the nearest metre?



Solution

In $\triangle ABC$, $\angle BAC = 90° - 24°$

$= 66°$

The length of BC can be found from the tangent ratio.

$$\tan 66° = \frac{BC}{33}$$
$$2.246 \doteq \frac{BC}{33}$$
$$2.246 \times 33 \doteq BC$$
$$74.12 \doteq BC$$

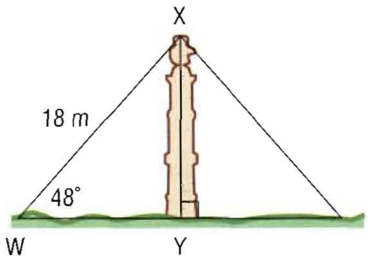
The fishing boat is 74 m from the base of the cliff, to the nearest metre.

Example 3

Ropes are used to pull a totem pole upright. Then, the ropes are anchored in the ground to hold the pole until the hole is filled. The rope holding this totem pole is 18 m long and forms an angle of 48° with the ground. Find, to the nearest metre,

a) the height of the totem pole

b) how far the anchor point is from the base of the totem pole



Solution

The height of the pole can be found using the sine ratio. Once two sides of the right triangle are known, the third side may be found using the Pythagorean Theorem.

a)
$$\sin 48° = \frac{XY}{18}$$
$$0.743 \doteq \frac{XY}{18}$$
$$0.743 \times 18 \doteq XY$$
$$13.374 \doteq XY$$

EST: $0.7 \times 20 = 14$

The height of the totem pole is 13 m, to the nearest metre.

b) Using the Pythagorean Theorem,

$$WY^2 = 18^2 - 13^2$$
$$= 324 - 169$$
$$WY = \sqrt{155}$$
$$\doteq 12.450$$

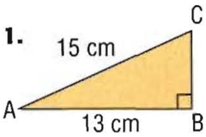
EST: $\sqrt{144} = 12$

The anchor point is 12 m from the pole, to the nearest metre.

Practice

Find all the unknown angles to the nearest degree and all the unknown sides to the nearest tenth of a unit.

1.



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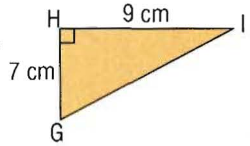

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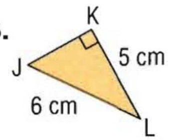

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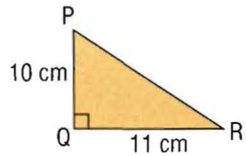

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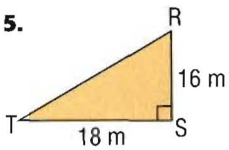

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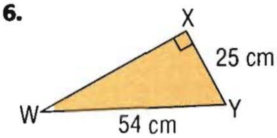
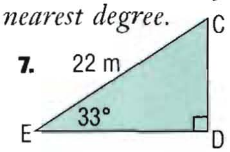


Solve each triangle. Round each side length to the nearest tenth of a unit and each angle to the nearest degree.

7.



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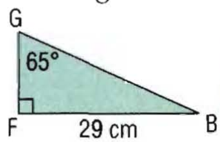

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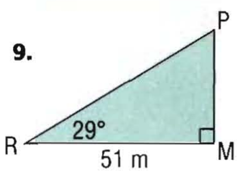

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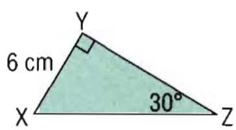

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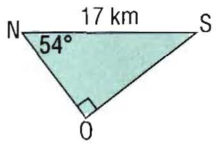

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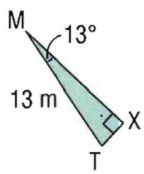
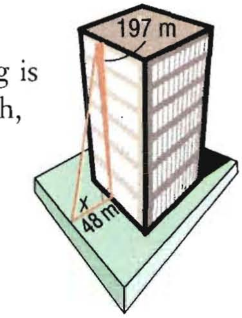


Problems and Applications

13. A kite is 32 m above the ground. The angle the kite string makes with the ground is 39°. How long is the kite string, to the nearest metre?

14. The two guy wires supporting a flagpole are each anchored 7 m from the flagpole and form an angle of 52° with the ground. What is the total length of guy wire, to the nearest metre, needed to support this flagpole?

15. Calgary's Bankers Hall building is 197 m tall. From a point level with, and 48 m from, the base of the building, what is the angle of elevation of the top of the building, to the nearest degree?



16. a) When you are standing without shoes on, what is the height of your eyes above the ground, to the nearest centimetre?

b) Calculate the angle of depression, to the nearest degree, you would use to look at a point on the ground 5 m in front of you.

c) Would the answer to part b) be greater or less for a person taller than you? Explain.

17. a) Copy and complete this table. Write each ratio as a decimal to 3 decimal places.

b) Compare each tangent value to each value in the last column. What do you notice?

c) Explain your results in part b).

Angle	tan	sin	cos	$\frac{\sin}{\cos}$
30°				
39°				
45°				
53°				
60°				
66°				

PATTERN POWER

1. Copy and complete the following.

$1^3 = 1$

$1^3 + 2^3 = 9$

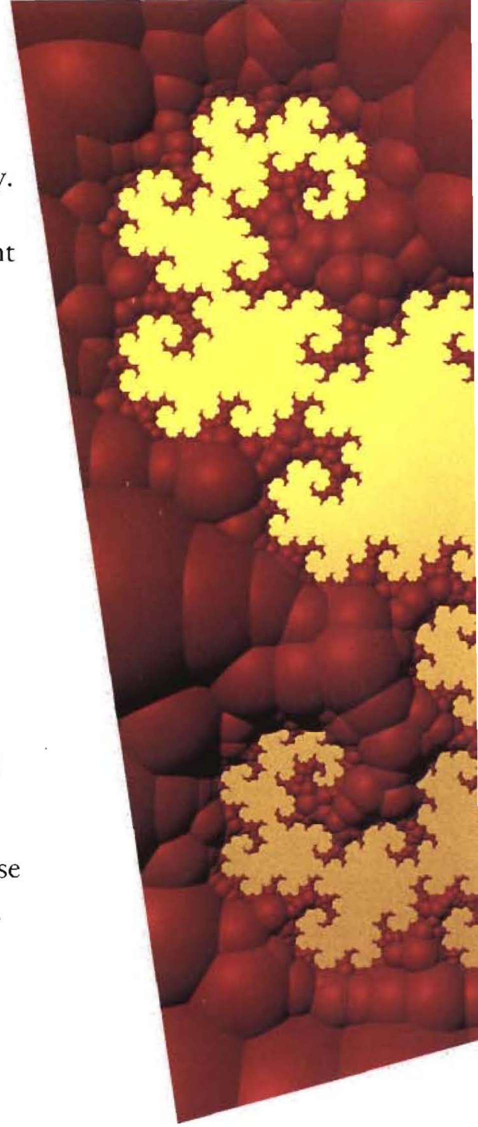
$1^3 + 2^3 + 3^3 = ■$

$1^3 + 2^3 + 3^3 + 4^3 = ■$

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = ■$

2. Describe the pattern in words.

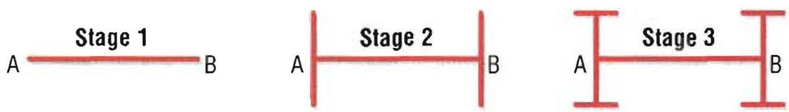
Fractal Geometry



One of the newest branches of mathematics is **fractal geometry**. The word "fractal" comes from the Latin verb *frangere*, which means to break. The study of fractals has led to the development of new computer graphics programs and may give scientists a way to describe complex things in nature.

Activity ❶ The H Fractal

The diagrams show the first 3 stages of an H Fractal with a reduction factor of $\frac{1}{2}$.



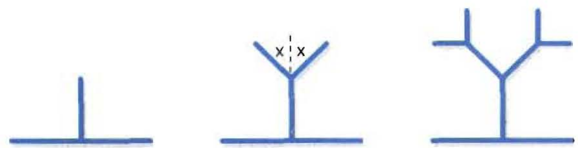
In stage 1, begin with a horizontal line segment AB of length 1 unit. In stage 2, add 2 vertical line segments, each of length $\frac{1}{2}$, perpendicular to AB at A and B. In stage 3, add 4 horizontal line segments, each of length $\frac{1}{4}$, perpendicular to the 2 vertical line segments. This H fractal has a reduction factor of $\frac{1}{2}$ because each new set of line segments is half as long as the previous set.

1. On grid paper, draw the first 4 stages of an H fractal with a reduction factor of $\frac{1}{2}$.

2. On grid paper, draw the first 4 stages of an H fractal with a reduction factor of $\frac{3}{4}$.

Activity ❷ The Fractal Tree

The diagrams show the first 3 stages of a fractal tree with a reduction factor of 0.7.



To produce the second stage from the first, 2 perpendicular branches are added to the end of the previous branch. Notice that if we continued the previous branch, it would bisect the right angle formed by the new branches. We can construct a third stage from the second in the same way.

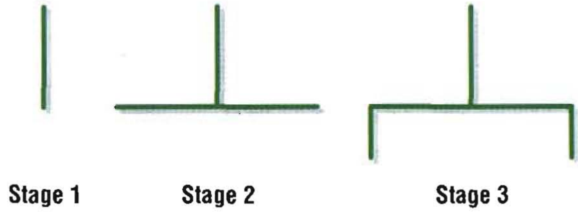
Draw the first 4 stages of a fractal tree using a reduction factor of your choice.

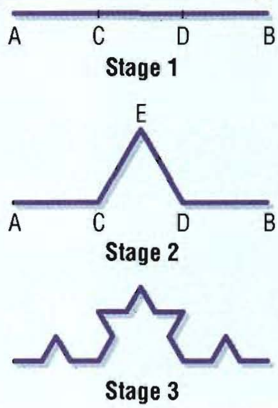
Activity ❸ The Binary Tree

The diagrams show the first 3 stages of a binary tree with a reduction factor of $\frac{1}{2}$.

Begin with a vertical branch of length 1. Each horizontal branch is twice the length of the vertical branch above it. Each vertical branch is $\frac{1}{2}$ the length of the vertical branch above it.

Draw the first 6 stages of a binary tree using a reduction factor of your choice.





Activity ❹ The "Snowflake" Fractal

This is the most famous fractal. The first 3 stages are shown.

Begin with a line segment AB and divide it into 3 equal parts. In stage 2, an equilateral triangle CED is constructed on the middle line segment. The middle segment CD is then removed. The process can be repeated as many times as you wish.

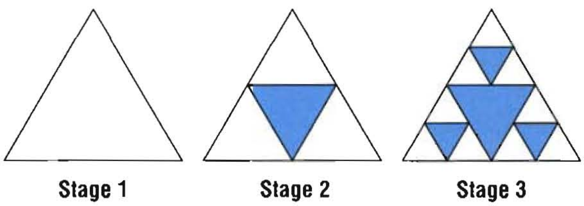
Construct a fractal curve by drawing squares instead of triangles. 

Activity ❺ Sierpinski's Sieve

The diagrams show the first 3 stages of Sierpinski's Sieve.

Begin with an equilateral triangle. Divide the triangle into 4 smaller congruent equilateral triangles, and remove the inner triangle. At each stage, a triangle is removed from the centre of each triangular region.

Draw the first 4 stages of Sierpinski's Sieve.



Activity ❻ Uses of Fractals

1. Use your research skills to find out how computer graphics programs based on fractal geometry are used in movies such as *Star Trek II*.

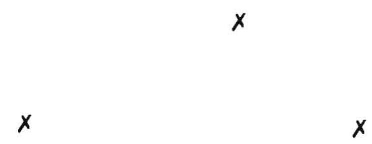
2. Can you find any other applications of fractals? If so, describe them.

Automobiles

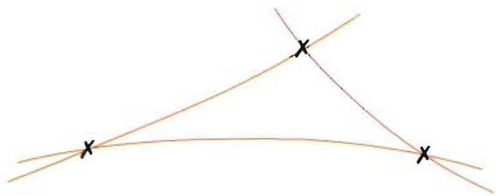
Automobiles are designed to be visually appealing to potential buyers. They are also designed so that there will be less air resistance at high speeds.

Activity ❶

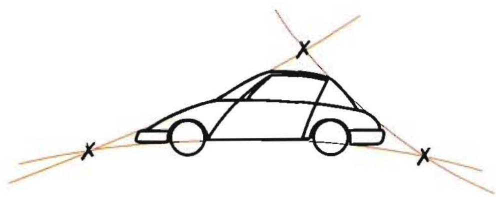
1. Draw 3 reference points on a blank piece of paper, as shown below.



2. Join your 3 reference points using curved lines.



3. Draw the shape of an automobile within the curved lines.



4. How would you locate the reference points to design a low sportscar? a family sedan?

5. Compare your answers to question 4 with your classmates'.

Activity ❷

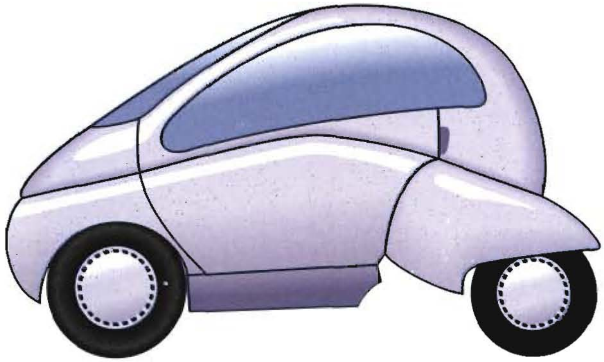
1. Trace the automobile on the right onto a sheet of blank paper.

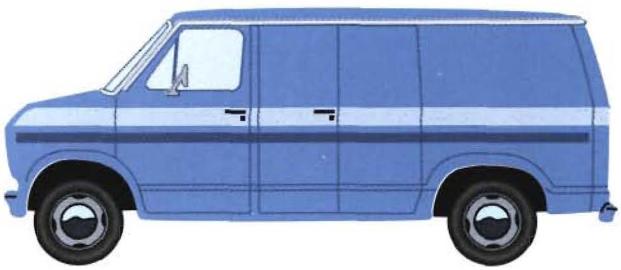
2. Draw its 3 reference points and the curved lines.

3. Trace the 3 reference points onto another sheet of blank paper and trade them for a classmate's.

4. Design a new automobile with your classmate's reference points.

5. How similar are your designs to each other's and to the original design?





Activity ❸

1. Examine the pictures of the sportscar, racing car, and van shown on the left.

2. Rank these 3 vehicles in terms of their maximum speed, from lowest to highest.

3. Which of the vehicles is the most air resistant? Explain.

4. What shapes offer the least air resistance? Explain why this is so.

5. Trace each picture onto a blank sheet of paper.

6. Locate the 3 reference points for each type of vehicle.

7. Trade your 3 sets of reference points for a classmate's.

8. Design 3 new vehicles from your classmate's reference points.

9. Compare your designs with the original designs.

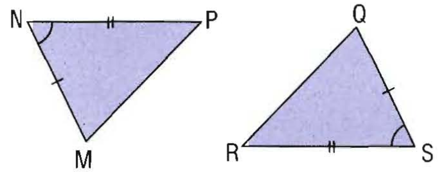
Activity ❹

1. Experiment with the design of an automobile using 4 and 5 reference points. Compare your results with your classmates'.

2. Research computer programs that are available to design automobiles.

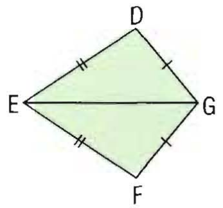
Review

1. State why these triangles are congruent and list all the corresponding equal parts.



Explain why the pairs of triangles are congruent.

2.



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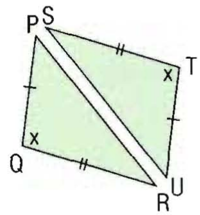

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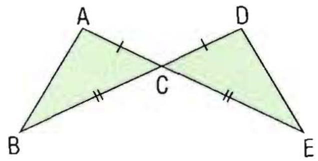

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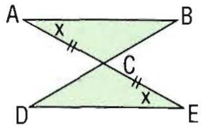
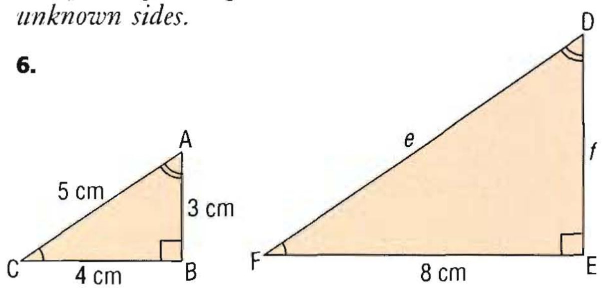


The pairs of triangles are similar. Find the unknown sides.

6.



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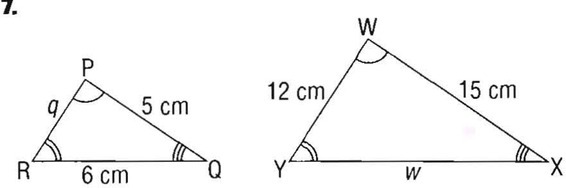

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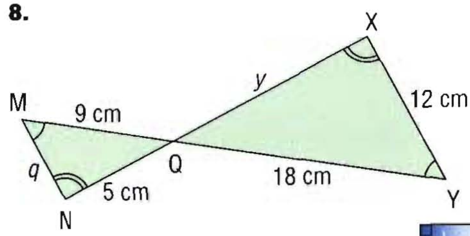
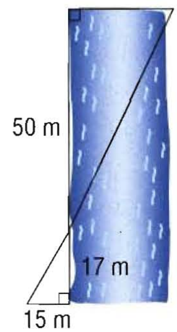
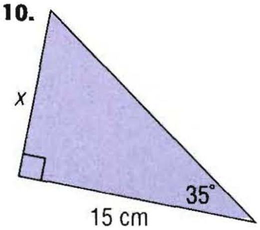


9. Use the dimensions of the surveyors' triangles to find the width of the river, to the nearest metre.

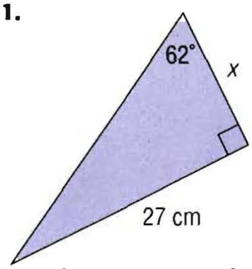


Use the tangent ratio to find x to the nearest tenth of a centimetre.

10.

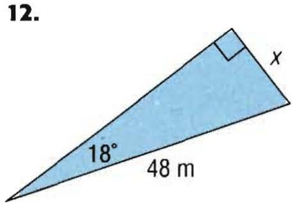


11.

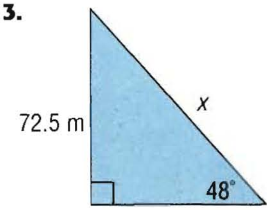


Use the sine ratio to find x to the nearest tenth of a metre.

12.

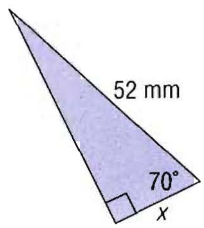


13.

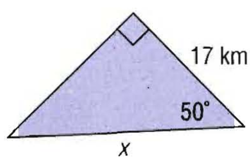


Use the cosine ratio to find x to the nearest unit.

14.

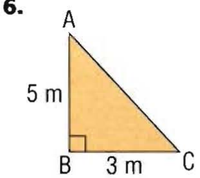


15.

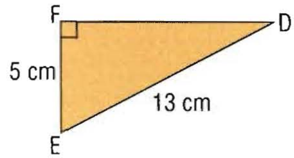


Solve each triangle. Round each side length to the nearest tenth of a unit and each angle to the nearest degree.

16.



17.



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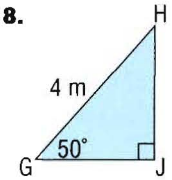

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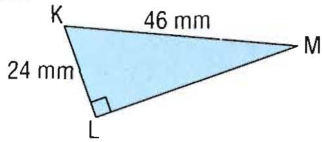
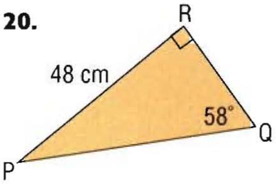
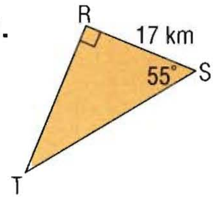


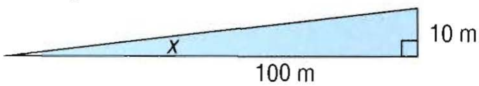
20.



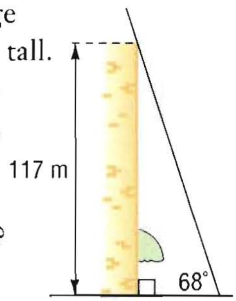
21.



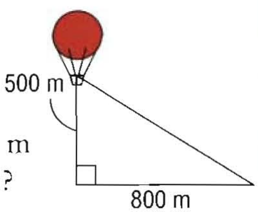
22. When a road has a 10% gradient, it means that the road rises 10 m for every 100 m of horizontal distance travelled. What is the angle of inclination of the road, to the nearest degree?



23. The Commodity Exchange Tower in Winnipeg is 117 m tall. When the sun's rays make an angle of 68° with the ground, what is the length of the building's shadow on level ground, to the nearest metre?



24. If you were in a hot air balloon 500 m above Selkirk, Manitoba, at what angle of depression would you look at a point on the ground 800 m horizontally from the balloon?



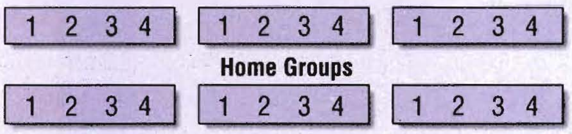
Group Decision Making
Advertising in the Media

In this activity, your group will do one of the following.

- write and act out a 30-s television commercial
- write and act out a 30-s radio commercial
- design a full-page newspaper advertisement

1. Meet as a class and choose 6 or 7 products you would like to advertise. Decide as a class which group will advertise which product.

2. In your home group, decide which type of advertising best suits your product. List the reasons for your decision.



3. Prepare your advertisement in your home group.

4. Make a presentation to the class. Include a description of how you used math in preparing the advertisement.

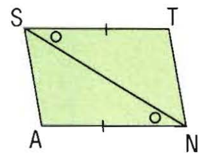
5. In your home group, discuss your group work. Decide what worked well and what you would do differently next time.

6. Meet as a class to compare your opinions on your group work.

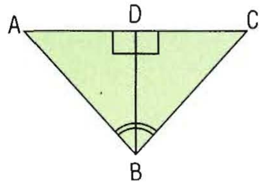
Chapter Check

Which pairs of triangles are congruent? Explain.

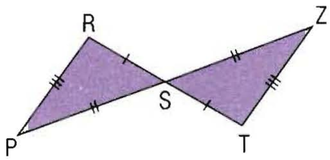
1.



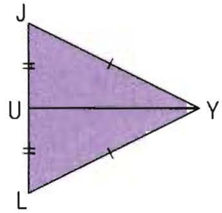
2.



3.

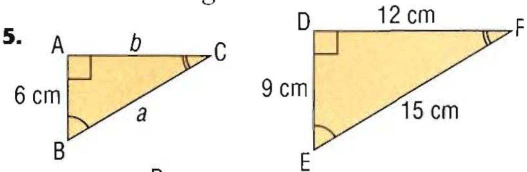


4.

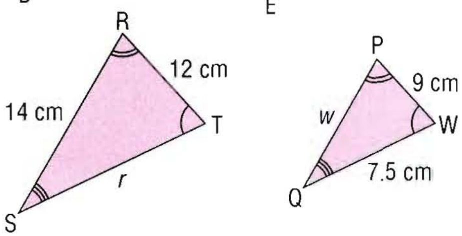


The pairs of triangles are similar. Find the unknown side lengths.

5.

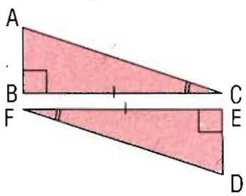


6.

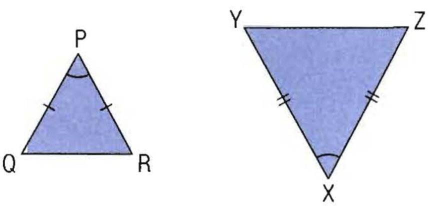


Are the triangles similar? Are the triangles congruent? Explain.

7.

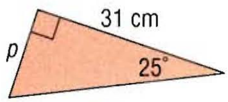


8.

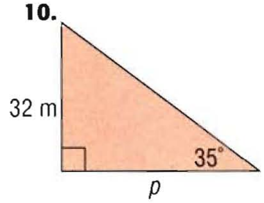


Use the tangent ratio to find p to the nearest tenth of a unit.

9.

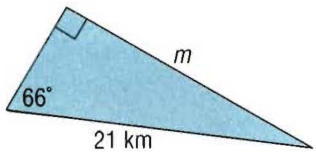


10.

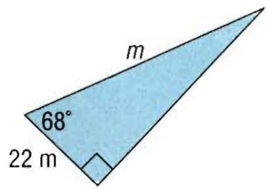


Use the sine ratio to find m to the nearest unit.

11.

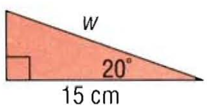


12.

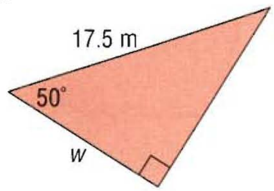


Use the cosine ratio to find w to the nearest tenth of a unit.

13.

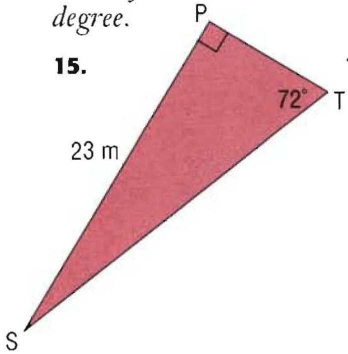


14.

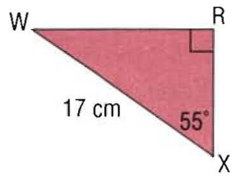


Solve the triangles for the unknown sides and angles. Round each side length to the nearest tenth of a unit and each angle to the nearest degree.

15.

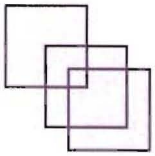


16.

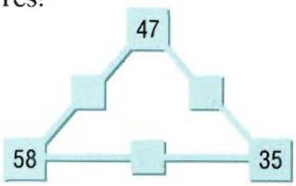


Using the Strategies

1. How many squares are in this diagram?



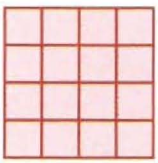
2. The number in each large square is found by adding the numbers in the 2 small squares connected to it. What are the numbers in the small squares?



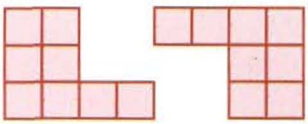
3. The Beach bus leaves every 20 min and the Sand bus leaves every 45 min. If they leave together at noon, when is the next time that they will leave together?

4. The amount of a purchase is $12.43. How can the exact amount be paid without using a $10.00 bill, but using the smallest number of bills and coins?

5. This is a 4-by-4 square.



One way to separate it into 2 congruent shapes, made up of smaller squares, is shown.

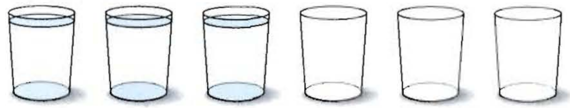


Find at least 5 other ways.

6. Six consecutive even numbers are written on a piece of paper. If the sum of the first 3 numbers is 60, what is the sum of the last 3?

7. Which of these pizzas is the best buy? Why?

8. Six water glasses are in a row. The first 3 are full, and the last 3 are empty. How can 1 glass be moved so that full glasses and empty glasses alternate?



9. What is the minimum number of mass comparisons an inspector needs to make to find 1 counterfeit coin in a collection of 40 coins. Assume that the counterfeit coin is lighter than the others.

10. You fill a glass half full of water from the tap. Then, you add enough ice cubes to the water to fill the glass. Sketch a graph of temperature versus time from the moment you add the ice to the water until the moment when the ice has all melted.

DATA BANK

1. A bathtub can hold 142 L of water up to the overflow. How many bathtubs can the world's largest reservoir fill when it is full?

2. Which 2 provinces are closest to each other in surface area? Which province is larger and by how much?