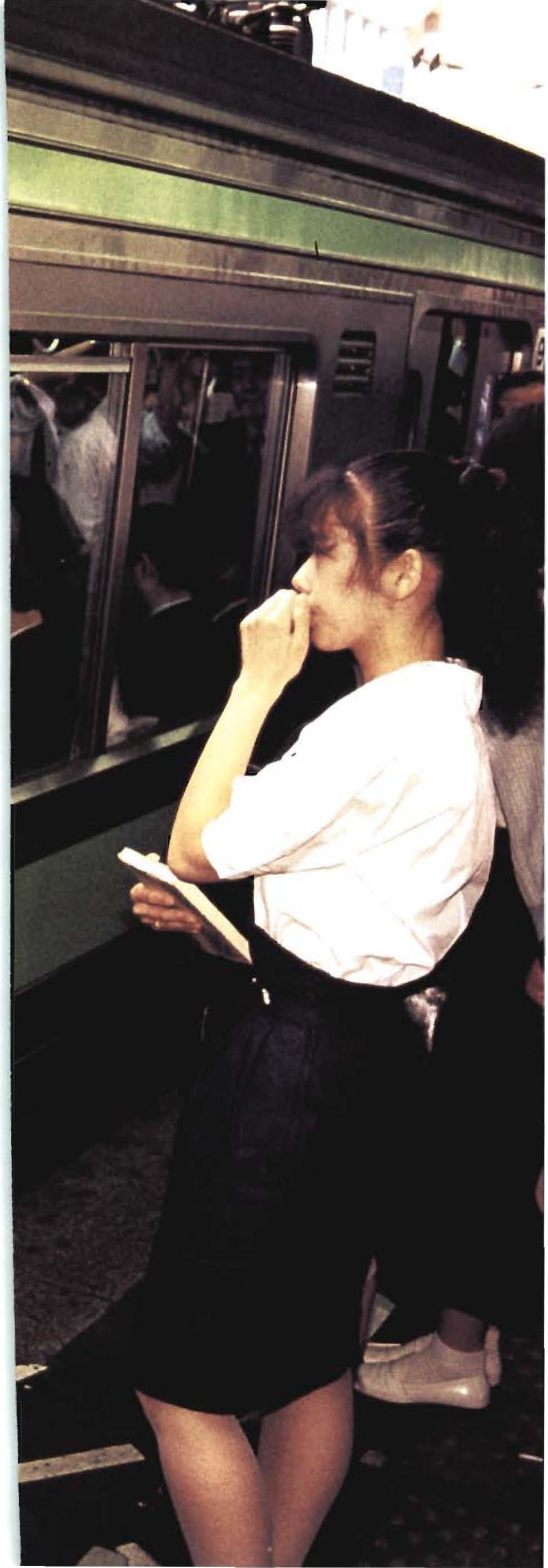


CHAPTER 7

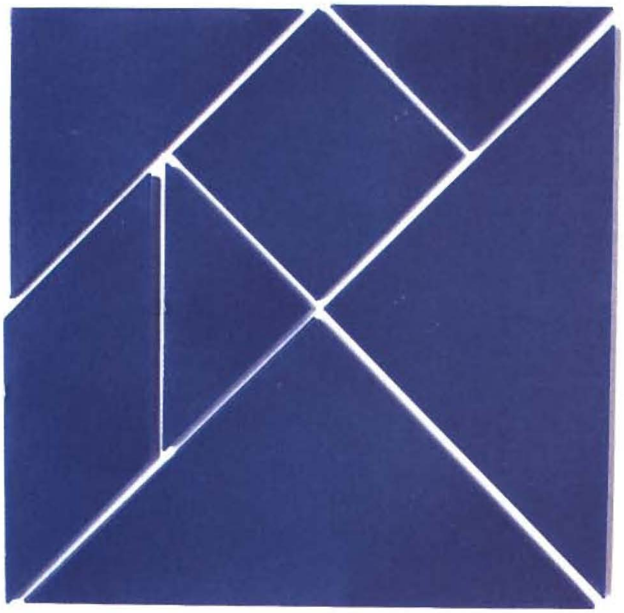
Shape and Space

The average number of people per square metre on a Tokyo train filled to capacity is officially 4. In fact, the average number of people per square metre on Tokyo trains in rush hour is 10. Can 10 people in your class stand on 1 m^2 of floor?

Estimate the area of your classroom floor in square metres. About how many people are there per square metre in your classroom?

If your classroom were as crowded as a Tokyo train in rush hour, about how many people would be in the room?

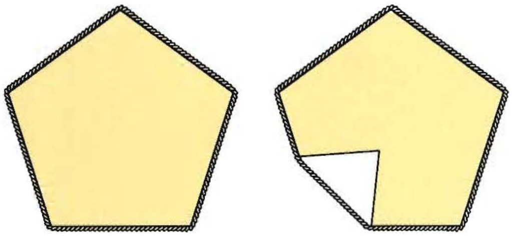
Tangrams



A tangram is a 7-piece puzzle that comes from China. The origin of the word "tangram" is itself interesting. The ending of the word, *gram*, refers to something drawn, such as a diagram. The beginning of the word, *tan*, dates back to the Tang dynasty, A.D. 618-906, the greatest dynasty in Chinese history. The 7 tangram pieces are shown assembled as a square.

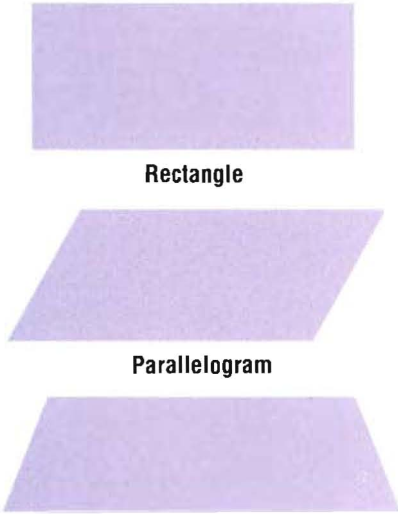
Activity ❶ Convex and Concave Figures

Polygons are closed geometric figures with line segments as sides. You are already familiar with triangles (3 sides), quadrilaterals (4 sides), pentagons (5 sides), and hexagons (6 sides).



Convex Polygon   Concave Polygon

Polygons can be convex or concave. If you wrap a piece of string around the perimeter of a convex polygon and pull the string tight, the string makes contact with every point of the polygon. On the other hand, if you wrap a piece of string around a **concave polygon**, the string does not make contact with every point of the polygon. A concave polygon bends in at some point or points, leaving gaps.



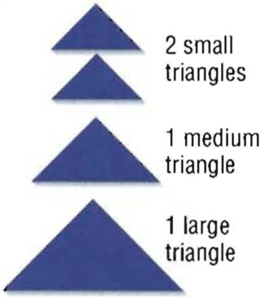
Rectangle

Parallelogram

Trapezoid

Using all 7 tangram pieces, it is possible to construct 9 different convex quadrilaterals without holes in the middle. One of them is the square shown at the top of this page. The others are 2 rectangles, 2 parallelograms, and 4 trapezoids. Use your 7 tangram pieces to construct these 8 quadrilaterals. Sketch your solutions on grid paper.

Activity ❷ Quadrilaterals



1. From a tangram, select the 4 triangles shown. Use the 4 triangles to make the following quadrilaterals. Sketch your solutions on grid paper.

a) parallelogram **b)** square
c) rectangle **d)** trapezoid

2. Are any of these quadrilaterals regular polygons? Explain.

Activity ❸ More Convex Figures

1. There are 7 other convex polygons without holes in the middle that can be constructed. Use the 7 tangram pieces to construct these polygons.

a) 1 triangle **b)** 3 pentagons **c)** 3 hexagons

2. Are any of these polygons regular polygons? Explain.

Activity ❹ Area with Tangrams

Let the area of the smallest tangram piece be 1 square unit. Calculate the areas of the other tangram pieces, and record the areas in a chart.

Mental Math

Calculate.

1. 3.14×10 **2.** 3.14×100 **3.** 3.14×1000 **4.** $3.14 \times 10\,000$

5. $3.14 \div 10$ **6.** $3.14 \div 100$ **7.** $3.14 \div 1000$ **8.** $3.14 \div 10\,000$

Calculate.

9. $5 \times 25 \div 1000$ **10.** 12×8 **11.** 14×7 **12.** 2.5×6

13. $8 \times 5 \times 3$ **14.** $10 \times 7 \times 9$ **15.** $4(7 + 6)$ **16.** $15.5 \div 5$

Estimate.

17. $(3.14)(1.2)$ **18.** $(3.14)(1.8)$ **19.** $(3.14)(5.8)$ **20.** $(3.14)(0.9)$

Estimate, then calculate.

21. $\frac{1}{2}(6.2 + 4.8)$ **22.** $\frac{1}{2}(2.2 + 5.8)$ **23.** $\frac{1}{2}(3.9 + 8.1)$ **24.** $\frac{1}{2}(10.2 + 9.8)$

Estimate.

25. $(3.14)(2.2)^2$ **26.** $(3.14)(3.3)^2$ **27.** $(3.14)(5.4)^2$ **28.** $(3.14)(4.9)^2$

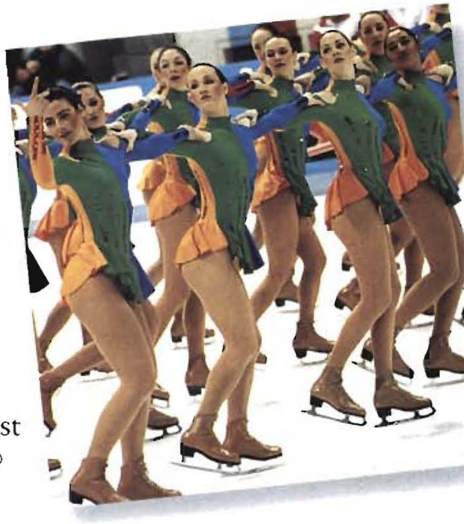
Estimate.

29. $(3.14)(2.1)(3.2)$ **30.** $(3.14)(5.2)(2.4)$ **31.** $(3.14)(5.6)(5.1)$ **32.** $(3.14)(7.2)(10.4)$

Estimate.

33. $(3.14)(2.4)^3$ **34.** $(3.14)(4.3)^3$ **35.** $(3.14)(9.8)^3$ **36.** $(3.14)(5.1)^3$

7.1 Areas of Rectangles, Squares, and Circles



Activity: Use the Information

Precision figure skating teams perform routines set to music. Teams may be made up of 20 or 24 skaters.

Several times in a routine, the full team lines up across the ice. When the team rotates around the centre spot on the ice, the skaters cover a complete circle. When the skaters sweep the ice from one end or one side to the other, they cover a rectangle.

Inquire

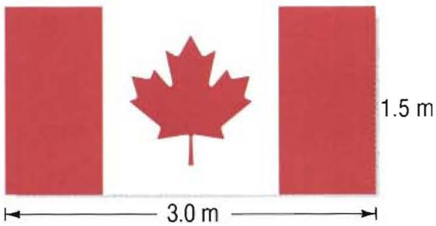
1. On an ice surface 30 m by 56 m, what is the area of the largest rectangle the skaters could cover? the area of the largest square? the area of the largest circle?

2. If the skaters split into 2 groups to practise, they could divide the ice into 2 congruent rectangles in 2 ways. Use diagrams to show the 2 ways and calculate the area that each group would have.

3. a) Which of the 2 ways of dividing the ice surface would allow each group to cover the largest possible circle? Explain.
b) Find the area of the largest possible circle in part a).

Example 1

The Canadian flag has 2 red rectangles, with a square in the middle. Calculate the area of 1 red rectangle.



Solution

The formula for the area of the rectangular flag is $A = lw$.

$$A = lw$$
$$= (3.0)(1.5)$$
$$= 4.5$$

The formula for the area of the white square is $A = s^2$.

$$A = s^2$$
$$= (1.5)^2$$
$$= 2.25$$

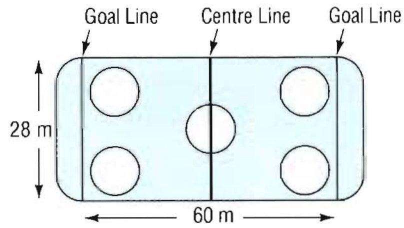
Area of 2 red rectangles is $4.5 - 2.25 = 2.25$

Area of 1 red rectangle is $\frac{2.25}{2} = 1.125$

The area of 1 red rectangle is 1.125 m^2.

Example 2

There are 5 circles marked on a hockey rink. Each circle has a radius of 4.5 m. For the rink shown, calculate the area of the ice that lies outside the circles and between the goal lines. Round your answer to the nearest tenth of a square metre.



Solution

The ice between the goal lines is rectangular.

$$A = lw$$
$$= 60 \times 28$$
$$= 1680$$

EST: $60 \times 30 = 1800$

The area of each circle is given by the formula

$$A = \pi r^2$$
$$= 3.14(4.5)^2$$
$$= 63.585$$

EST: $3 \times 5 \times 5 = 75$

For 5 circles, $A = 5 \times 63.585$
$= 317.925$

EST: $5 \times 60 = 300$

Area of rectangle less total area of circles $= 1680 - 317.925$
$= 1362.075$

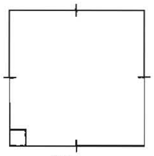
EST: $1700 - 300 = 1400$

The area of the ice outside the circles and between the goal lines is 1362.1 m^2, to the nearest tenth of a square metre.

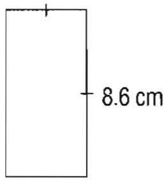
Practice

Determine each area.

1.

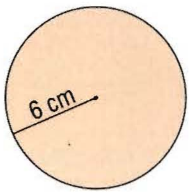


2.

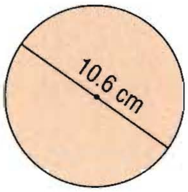


Calculate the area of each circle, to the nearest square centimetre.

3.



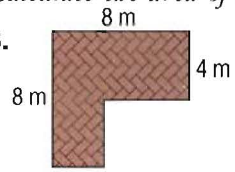
4.



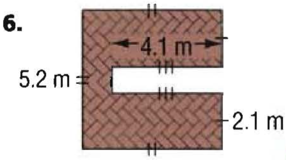
Problems and Applications

Calculate the area of each patio.

5.

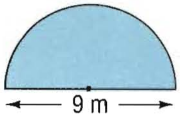


6.

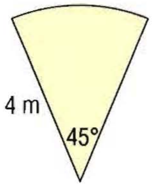


7. Calculate each area to the nearest square unit.

a)

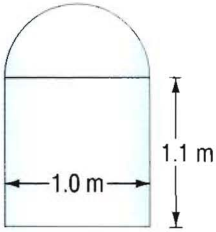


b)



8. a) Calculate the area of the glass in this window to the nearest tenth of a square metre.

b) A protective storm window for this window costs \$19.99/m^2. What is the cost of the storm window?

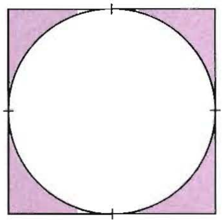


9. A rectangular lot measures 66 m by 107 m. A sidewalk surrounds the lot. The sidewalk is 2 m wide. What is the area of the sidewalk?

10. You want to plant a 16-m^2 rectangular garden and build the shortest possible fence around it. What shape should you make the garden?

11. A gardener wants to rope off a rectangular area of grass. The gardener has 10 m of rope to block off 2 sides of the area and will use a wall of the garage and a wall of the house for the other 2 sides. What is the largest area that can be roped off?

12. The area of the circle is 153.86 m^2. Work with a partner to calculate the area of the shaded region. Use $\pi = 3.14$.

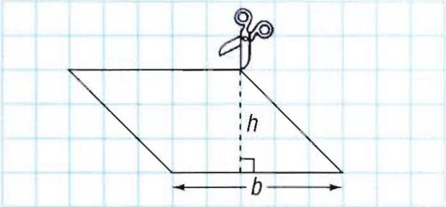


7.2 Areas of Parallelograms, Triangles, and Trapezoids

Some of the most distinctive architecture is composed of parallelograms, triangles, and trapezoids. This photograph of the ceiling in the Hockey Hall of Fame is an example.

Activity: Determine the Formula

Copy the parallelogram onto grid paper. Cut a right triangle from the parallelogram along the height, h. Then, fit the right triangle onto the opposite end of the remainder of the parallelogram so that the two pieces form a rectangle.



1. What is the area of the rectangle in square units?

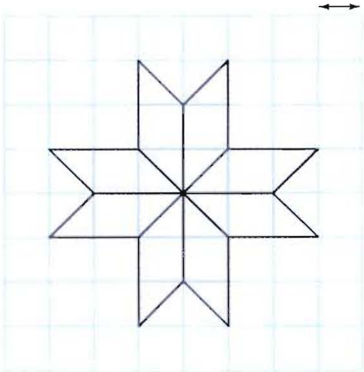
2. Write a formula for the area of the rectangle in terms of the variables b and h.

3. What is the area of the parallelogram in square units?

4. Write a formula for the area of the parallelogram in terms of b and h.

Example 1

Many Canadian heritage quilts are made from simple geometric shapes such as identical parallelograms. If the side of each square on the grid is 5 cm, what is the area of each parallelogram in this section of a quilt?



Solution

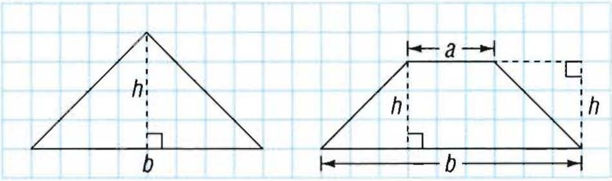
The formula for the area of a parallelogram is $A = bh$. The base of each parallelogram in this quilt is 10 cm and each height is 5 cm.

$$\begin{aligned} A &= bh \\ &= (10)(5) \\ &= 50 \end{aligned}$$

The area of each parallelogram in the quilt is 50 cm^2.

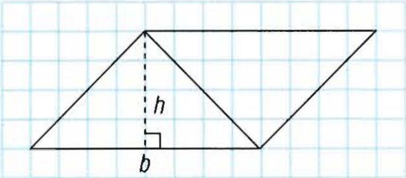
Activity: Determine the Formula

Copy this triangle and trapezoid onto grid paper.



Inquire

1. Along one side of the triangle, draw another identical triangle to form a parallelogram similar to the one shown.

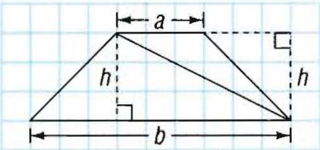


2. Write a formula for the area of the parallelogram in terms of b and h.

3. How is the area of the triangle related to the area of the parallelogram?

4. Write a formula for the area of the triangle in terms of b and h.

5. In the trapezoid, draw a diagonal to form 2 triangles as shown. Then, write the area of each triangle in terms of its base and height.



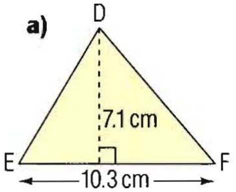
6. What is the sum of the areas of both triangles in terms of a, b, and h?

7. Factor the sum you found in question 6, and write a formula for the area of a trapezoid in terms of a, b, and h.

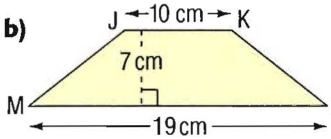
Example 2

Calculate the area of each figure to the nearest tenth of a square centimetre.

a)



b)



Solution

a) The formula for the **area of a triangle** is $A = \frac{1}{2}bh$.

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(10.3)(7.1)$$

EST $(0.5)(10)(7) = 35$

$$= \frac{1}{2}(73.13)$$
$$= 36.565$$

The area of $\triangle$DEF is about 36.6 cm^2.

b) The formula for the **area of a trapezoid** is $A = \frac{1}{2}(a + b)h$.

$$A = \frac{1}{2}(a + b)h$$
$$= \frac{1}{2}(10 + 19)(7)$$
$$= \frac{1}{2}(29)(7)$$

EST $(0.5)(30)(7) = 105$

$$= \frac{1}{2}(203)$$
$$= 101.5$$

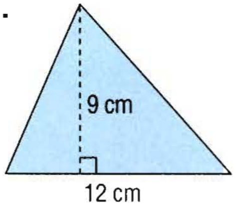
The area of trapezoid JKLM is 101.5 cm^2.

CONTINUED

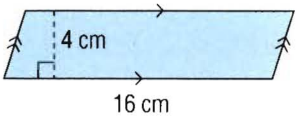
Practice

Calculate the area of each figure.

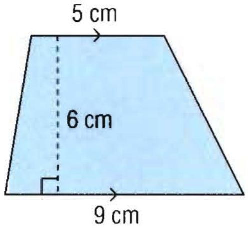
1.



2.



3.



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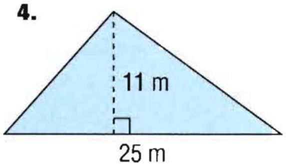
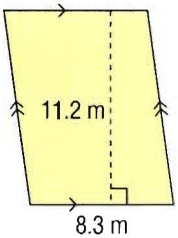
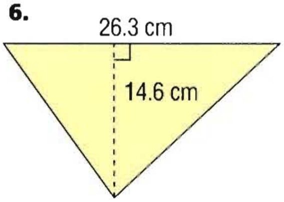


Estimate each area. Then, calculate it to the nearest square centimetre or square metre.

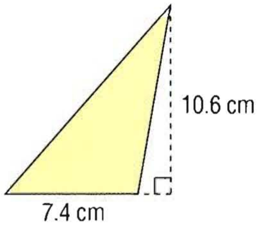
5.



6.



7.



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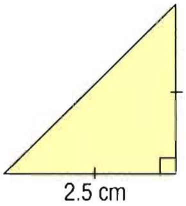
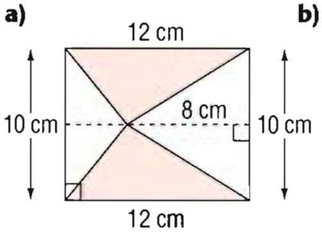


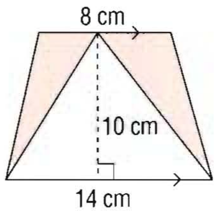
Problems and Applications

9. Calculate the area of the shaded region in each diagram.

a)

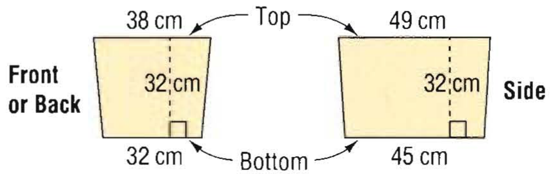


b)



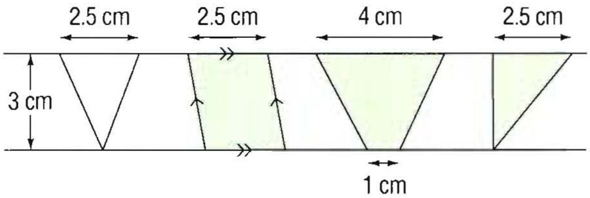
10. The area of a parallelogram is 8400 cm^2. Its height is 60 cm. What length is its base?

11. Many Canadian cities use the Blue Box for recycling. The sides, the front, and the back of the Blue Box are trapezoids. What is the area of a side and a front or back?

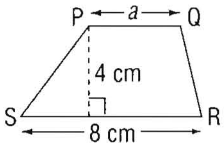


12. The area of a triangle is 14.4 cm^2, and the length of its base is 6.4 cm. What is its height?

13. Without calculating any areas, decide how the areas of the triangles, parallelogram, and trapezoid compare. Explain.



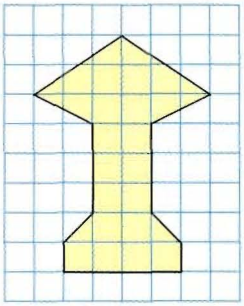
14. The area of trapezoid PQRS is 26 cm^2. Work with a partner to calculate a when b = 8 cm and h = 4 cm.



WORD POWER

Change the word SLOW to the word FAST by changing 1 letter at a time. Each time you change a letter, you must form a real word. The best solution requires the fewest steps. Compare your list of words with a classmate's.

Areas of Irregular Figures



Look at the figure of the arrow. It has been drawn on a grid with each square equal to 1 square unit.

The area of the arrow can be estimated by counting the number of whole and part squares that it covers.

The number of whole squares is 16. The number of part squares is 14.

$$\text{Total Area} = (\text{Number of whole squares}) + \frac{1}{2}(\text{Number of part squares})$$
$$= 16 + 0.5(14)$$
$$= 16 + 7$$
$$= 23$$

The total area of the arrow is about 23 square units.

Why do you think you calculate one-half of the part squares?

Activity

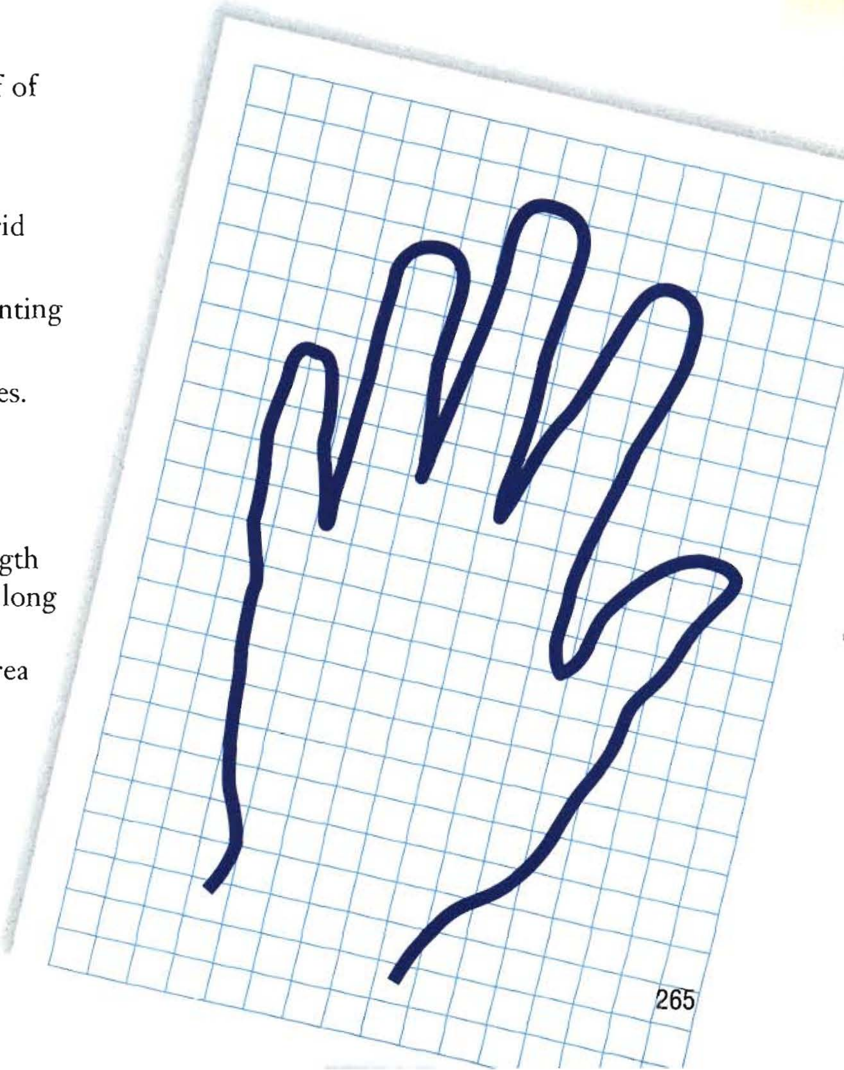
1. Trace your hand on a sheet of 1-cm grid paper.

2. Estimate the area of your hand by counting whole squares and part squares.

3. Measure your arm length in centimetres.

4. Collect a class set of arm lengths in centimetres and hand areas in square centimetres.

5. Graph the data on a grid with arm length along the horizontal axis and hand area along the vertical axis. Can you make a general conclusion about arm length and hand area from the data? If you can, what is it?



Volumes

Volume is the amount of space occupied by an object.

Activity ❶ Volume of a Prism

1. Cut the top from an empty 250-mL milk carton to form a prism.

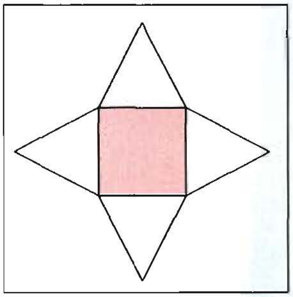
2. What type of prism have you made?

3. Fill the prism with layers of 1-cm cubes.

4. How many 1-cm cubes just fill the prism?

5. What is the volume of the prism?

Activity ❷ Volume of a Pyramid



1. Draw the net of a pyramid that has the same base area and height as the prism you made in Activity 1.

2. Cut out the net, tape it, and place the pyramid inside the prism.

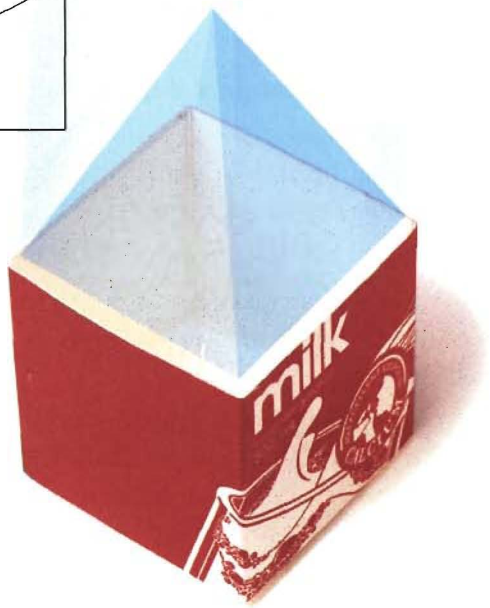
3. Estimate how many times greater the volume of the prism is than the volume of the pyramid.

4. Cut along 3 sides of the pyramid's base. Then, fill the pyramid with sand and empty it into the prism. Repeat until the prism is full.
a) How many pyramids full of sand does it take to fill the prism?
b) How close was your estimate in step 3?

5. From your results, what fraction of the volume of the prism is the volume of the pyramid?

6. Construct the net of a triangular prism and the net of a triangular pyramid, so that the prism and the pyramid have congruent bases and equal heights.

7. Cut out and tape the prism and the pyramid. Estimate how many times greater the volume of the prism is than the volume of the pyramid.



8. Check your estimate by using the pyramid to fill the prism with sand.

9. Are your results from step 8 consistent with the fraction you found in step 5?

Activity ❸ Volume of a Cylinder

1. Obtain an empty cylindrical tin can. Make sure that there are no sharp edges where the lid was removed.

2. a) Cover the bottom of the can as completely as possible with a layer of 1-cm cubes. Add more layers to fill the can as completely as possible.
b) Use the number of cubes in the can to estimate its volume.
c) Is your estimated volume an underestimate or an overestimate? Explain.

3. Empty the can, then fill it with water. Use a graduated cylinder to measure the volume of water in the can.

4. What is the volume of the can in cubic centimetres? Explain.

5. Compare the volume you stated in step 4 with your estimate in step 2.

Activity ❹ Volume of a Cone

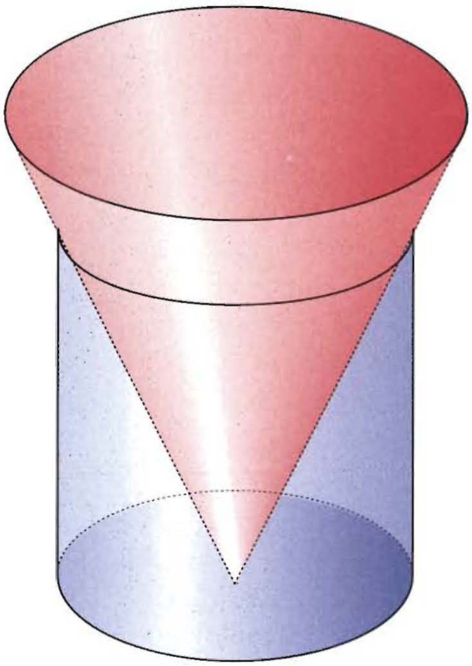
1. Use a piece of construction paper to form a cone whose tip just touches the bottom of the tin can you used in Activity 3. The curved surface of the cone should touch the inside circumference of the top of the can.

2. Tape your cone and cut it so that its height is the same as the height of the cylinder.

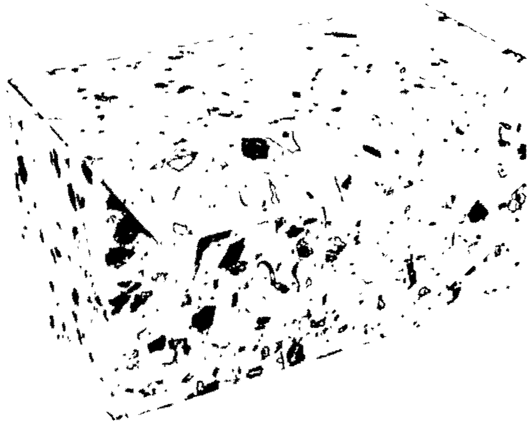
3. Estimate how many times greater the volume of the cylinder is than the volume of the cone.

4. Fill the cone with sand and empty it into the cylinder. Repeat until the cylinder is full.
a) How many cones full of sand does it take to fill the cylinder?
b) How close was your estimate in step 3?

5. From your results, what fraction of the volume of the cylinder is the volume of the cone?



7.3 Surface Area and Volume of a Prism



Activity: Solve the Problem

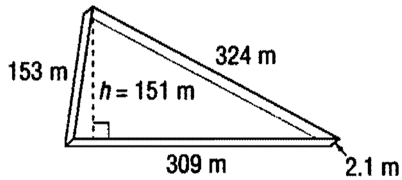
Mark has to wrap a gift for his friend. The gift box measures 40 cm by 20 cm by 20 cm. Mark has 16 000 cm^2 of wrapping paper. Mark wants to know if he has enough paper.

Inquire

1. What type of prism is the box?
2. What types of figures are the faces of the box?
3. Calculate the area of each face.
4. What is the sum of all the areas?
5. Does Mark have enough paper? What assumptions did you make?

Example 1

Fort York was built next to Lake Ontario during the War of 1812. The 2.1 m high parapet that surrounds it forms a triangle with sides of 309 m, 153 m, and 324 m, as shown. The shape of the Fort resembles a triangular prism, but without a top. Find the interior surface area of the Fort to the nearest square metre.



Solution

The surface area of a prism is the sum of the areas of the faces. In this case, the faces are the triangular base of the Fort and the 3 rectangular walls. The area of the triangular base is given by

$$\begin{aligned} A &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2}(309 \times 151) \\ &= \tfrac{1}{2}(46\ 659) \\ &= 23\ 329.5 \end{aligned}$$

EST: $\frac{1}{2}(300)(150) = 22\ 500$

C 0 . 5 × 309 × 151 = 23329.5

The area of the 3 rectangular faces is

$$\begin{aligned} &2.1(309) + 2.1(153) + 2.1(324) \\ &= 648.9 + 321.3 + 680.4 \\ &= 1650.6 \end{aligned}$$

EST: $2(300) + 2(200) + 2(300) = 1600$

C 2 . 1 × (309 + 153 + 324) = 1650.6

The surface area is

$$23\ 329.5 + 1650.6 = 24\ 980.1$$

EST: $23\ 000 + 2000 = 25\ 000$

The interior surface area of Fort York is 24 980 m^2 to the nearest square metre.

Activity: Investigate Volume

Build this rectangular prism with interlocking cubes.

Inquire

1. How many cubes represent the amount of space occupied by the prism?

2. Let the side of 1 cube be 1 unit of length.

a) What is the height of the prism?

b) What is the area of the base of the prism in square units?

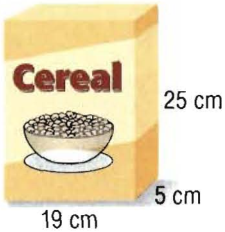
3. Multiply the area of the base by the height, and compare the result with the amount of space you calculated in question 1.

4. The amount of space occupied by a prism is called the **volume**. Write a formula to calculate the volume of this prism. Use the variables V, B, and h, where B is the area of the base and h is the height.

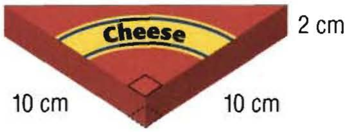
Example 2

Calculate each volume.

a)



b)



Solution

The volume of a prism is the area of its base times its height.

a) The cereal box is a rectangular prism.

$V = B \times h$

$= [(19)(5)](25)$ EST $(20)(5)(25) = 2500$

$= 2375$

The volume of the box is 2375 cm^3.

b) The cheese box is a triangular prism. The base and height of the triangle are 10 cm each.

$V = B \times h$

$= [\frac{1}{2}(10)(10)](2)$

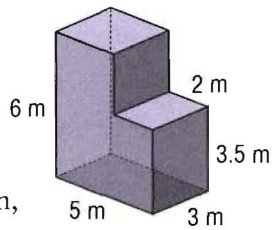
$= (50)(2)$

$= 100$

The volume of the box is 100 cm^3.

Example 3

A **composite solid** is made up of 2 or more prisms joined together. Find the volume of this composite solid.



Solution

The solid is composed of a smaller prism, where $l = 3$ m, $w = 2$ m, and $h = 3.5$ m, and a larger prism, where $l = 3$ m, $w = 3$ m, and $h = 6$ m. The volume of the smaller prism is (3)(2)(3.5) or 21 m^3. The volume of the larger prism is (3)(3)(6) or 54 m^3.

Total volume $= 21 + 54$

$= 75$

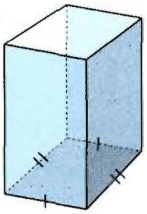
The volume of the composite solid is 75 m^3.

CONTINUED

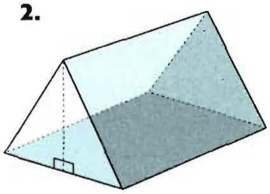
Practice

Name each prism.

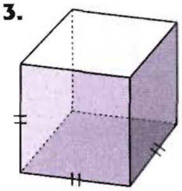
1.



2.



3.



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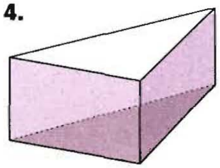
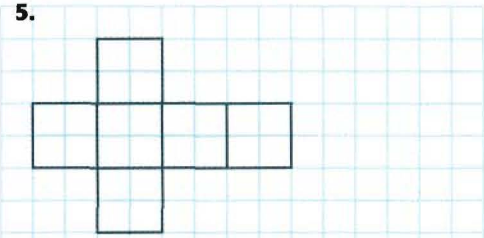
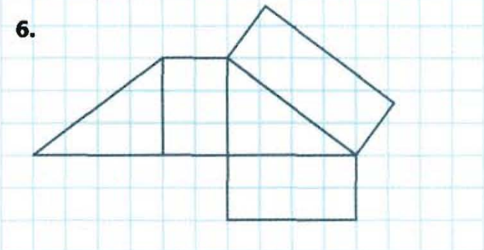


Which prism can be formed from each net?

5.

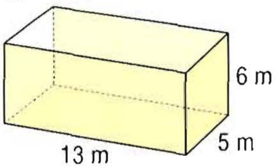


6.



Estimate the surface area of each prism. Then, calculate it to the nearest square centimetre or square metre.

7.



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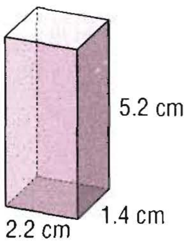
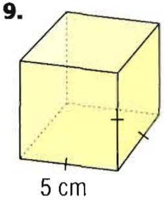


Calculate each surface area.

9.



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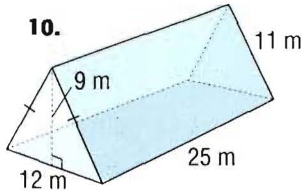
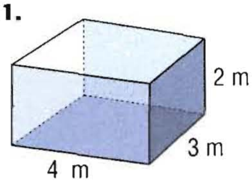


Estimate, then calculate the volume of each rectangular prism.

11.



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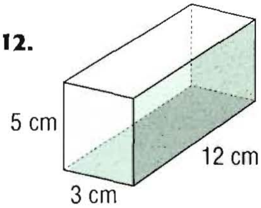

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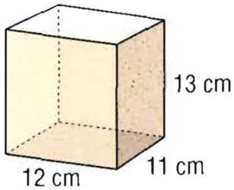

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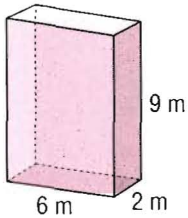
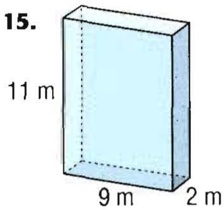


Calculate the surface area and volume of each prism to the nearest square or cubic unit.

15.



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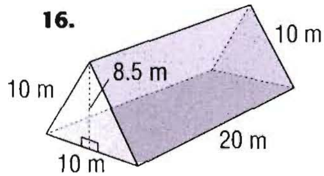

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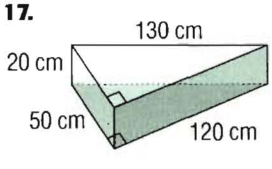

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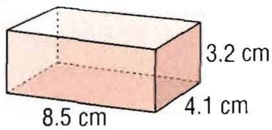

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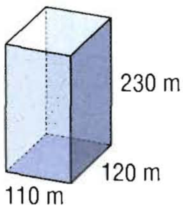
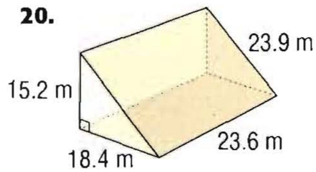


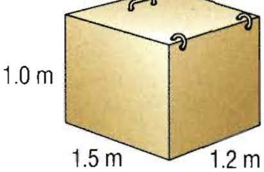
20.



Problems and Applications

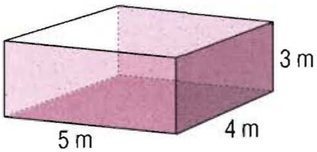
21. A covered garbage bin is to be built so that it measures 1.5 m by 1.2 m by 1.0 m.

a) How much plywood will it take to build the garbage bin?



b) How many cubic metres of garbage will it hold?

22. a) Calculate the surface area of this room.



b) One 4-L can of paint will cover 36 m^2. If you want to give the ceiling and walls of the room 2 coats of paint, how many 4-L cans will you need? What assumptions have you made?

23. a) The dimensions of the base of a composter are 1 m by 1 m. Its height is 0.65 m. It is a prism with a top, a bottom, and 4 sides. Calculate its surface area.

b) The cost of material to build this composter is $9.98/m^2. What is the total cost of the material?

c) If a town has set aside $1 250 000 for the materials to build these composters, how many composters can be built?

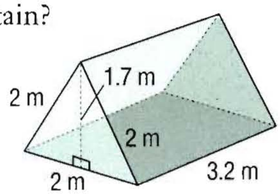
24. The surface area of a cube is 216 cm^2. What are the dimensions of this cube?

25. a) A prism has a height of 10 cm. Find its surface area if the dimensions of the base are 8 cm by 2 cm.

b) Draw and label a diagram of the prism on dot paper or centimetre grid paper.

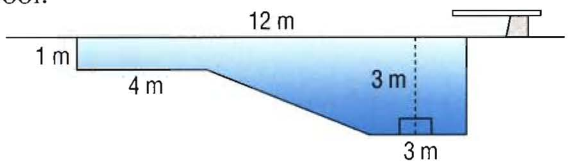
c) What is the name of the prism?

26. How many cubic metres of air does the tent contain?



27. Canada's Anik E1 is a domestic communications satellite. Launched in September, 1991, it is a rectangular prism with l = 23 m, w = 8.5 m, and h = 4.3 m. Calculate the surface area and volume of Anik E1 to the nearest square or cubic unit.

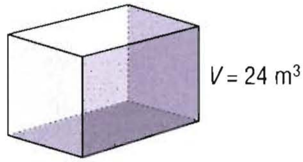
28. The diagram shows the side view of a pool.



a) The pool is 5 m wide. Calculate its volume.

b) A pump can drain water from the pool at 0.3 m^3/min. How long does it take to drain the pool?

29. A garden storage shed is to be built in the shape of a rectangular prism before the roof is added. The volume of the shed before the roof is put on is 24 m^3. What are the most appropriate dimensions for the rectangular prism?

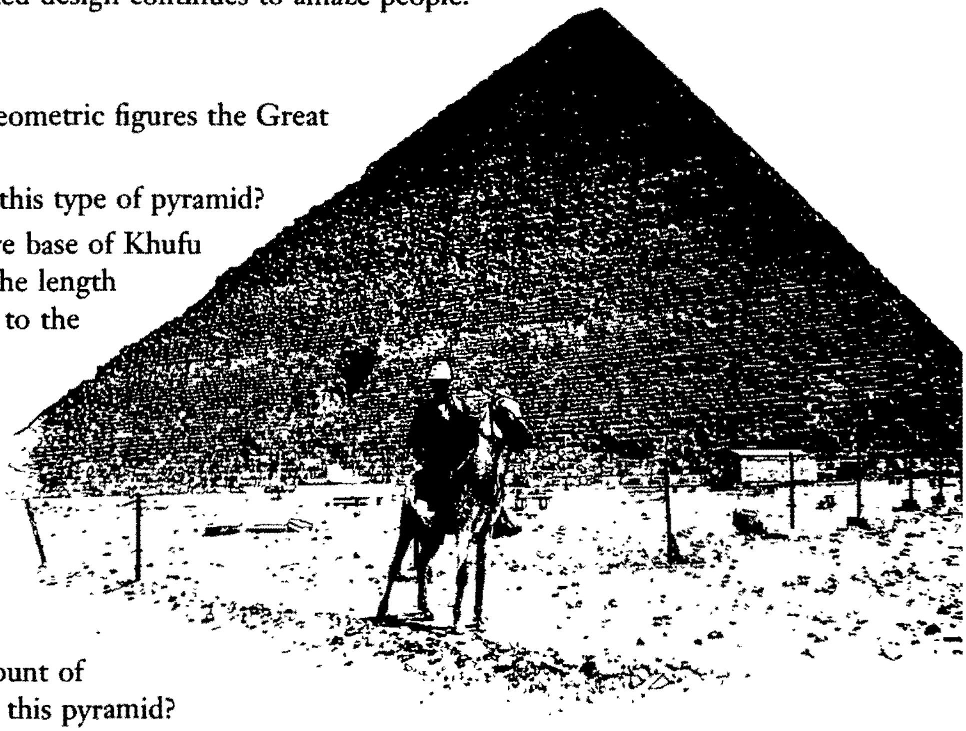


30. Work with a partner to calculate the surface area and volume of the interior of your classroom.

31. Write a problem that requires the calculation of the surface area and volume of a prism. Have another classmate solve your problem.

7.4 Surface Area and Volume of a Pyramid

The Great Pyramid of Khufu is shown in this photograph. It was built by an ancient Egyptian civilization around 2500 B.C. In spite of its age, its sophisticated design continues to amaze people.



Inquire

1. Name the different geometric figures the Great Pyramid is made up of.

2. What is the name of this type of pyramid?

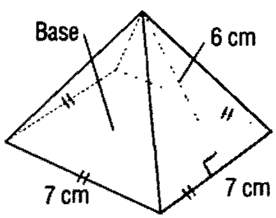
3. The area of the square base of Khufu is 53 058 m^2. What is the length of each side of the base to the nearest metre?

4. The **slant height** of the pyramid, or the height of each triangular face, is 187 m. What is the surface area of each face?

5. What is the total amount of exposed surface area on this pyramid?

Example 1

Calculate the surface area of this square-based pyramid.



Solution

The **surface area of a pyramid** is the sum of the area of the base and the areas of the triangular faces.

The base of the pyramid is a square.

Base Area $= (7)(7)$

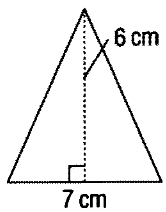
$= 49$

The 4 triangular faces are identical.

Face Area $= 4[\frac{1}{2}(7)(6)]$

$= 4(21)$

$= 84$



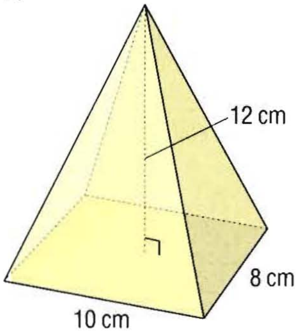
Surface Area $= 49 + 84$

$= 133$

The surface area of the pyramid is 133 cm^2.

Example 2

Calculate the volume of this pyramid.



Solution

The **volume of a pyramid** is one-third the product of the area of the base and the height.

$$V = \frac{1}{3}(\text{Area of Base})(\text{Height})$$
$$= \frac{1}{3}[(lw)(h)]$$
$$= \frac{1}{3}[(10)(8)(12)]$$
$$= \frac{1}{3}[(80)(12)]$$
$$= \frac{960}{3}$$
$$= 320$$

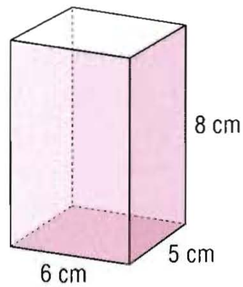
EST $\frac{(10)(10)(10)}{3} \doteq 333$

The volume of the pyramid is 320 cm^3.

Example 3

a) What is the volume of this prism?

b) What is the maximum volume of a pyramid that can fit upright inside this prism?



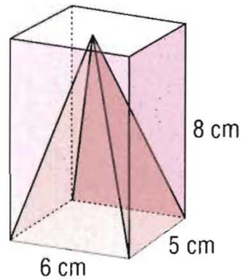
Solution

a) The volume of the prism is $V = B \times h$

$$= [(6)(5)](8)$$
$$= 240$$

The volume of the prism is 240 cm^3.

b) The biggest pyramid that can fit upright inside this prism has the same base and height as the prism.



The volume of the pyramid is $V = \frac{1}{3} \times B \times h$

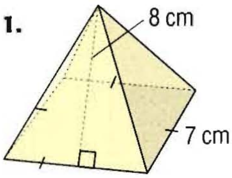
$$= \frac{1}{3}(240)$$
$$= 80$$

The maximum volume of the pyramid is 80 cm^3.

CONTINUED

Practice

Calculate each surface area.

1.


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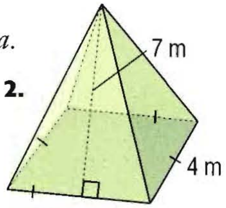
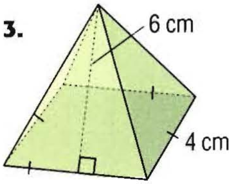


3.


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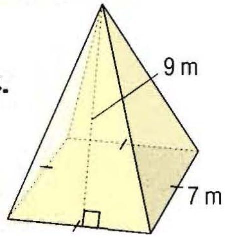
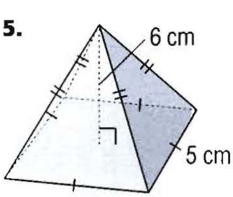


Estimate, then calculate each volume to the nearest cubic unit.

5.


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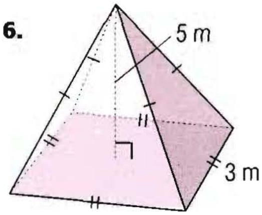

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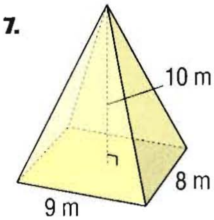

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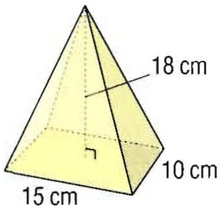


Problems and Applications

9. The rectangular base of a pyramid measures 10 cm × 9 cm. The height of the pyramid is 12 cm. Calculate its volume.

10. The length of one side of the base of the Great Pyramid is 230 m, and the height of the pyramid is 146 m. What is its volume?

11. What happens to the volume of a square pyramid under these conditions?

a) The base is unchanged and the height is doubled.

b) The base is unchanged and the height is tripled.

c) The height is unchanged and the area of the base is doubled.

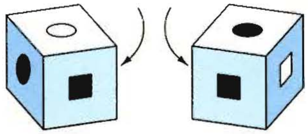
12. The slant height of a pyramid is about 120 m and each side of its base measures about 180 m. Calculate the surface area to the nearest thousand square metres.

13. A pyramid sits inside a cube so that the base of the pyramid is a face of the cube. What is the maximum volume of the pyramid if the side length of the cube is 24 cm?

14. A pyramid and a prism have congruent square bases and equal volumes. How do their heights compare? Explain.

LOGIC POWER

The cubes are identical. Each face has a ○, □, ■, or ● on it.



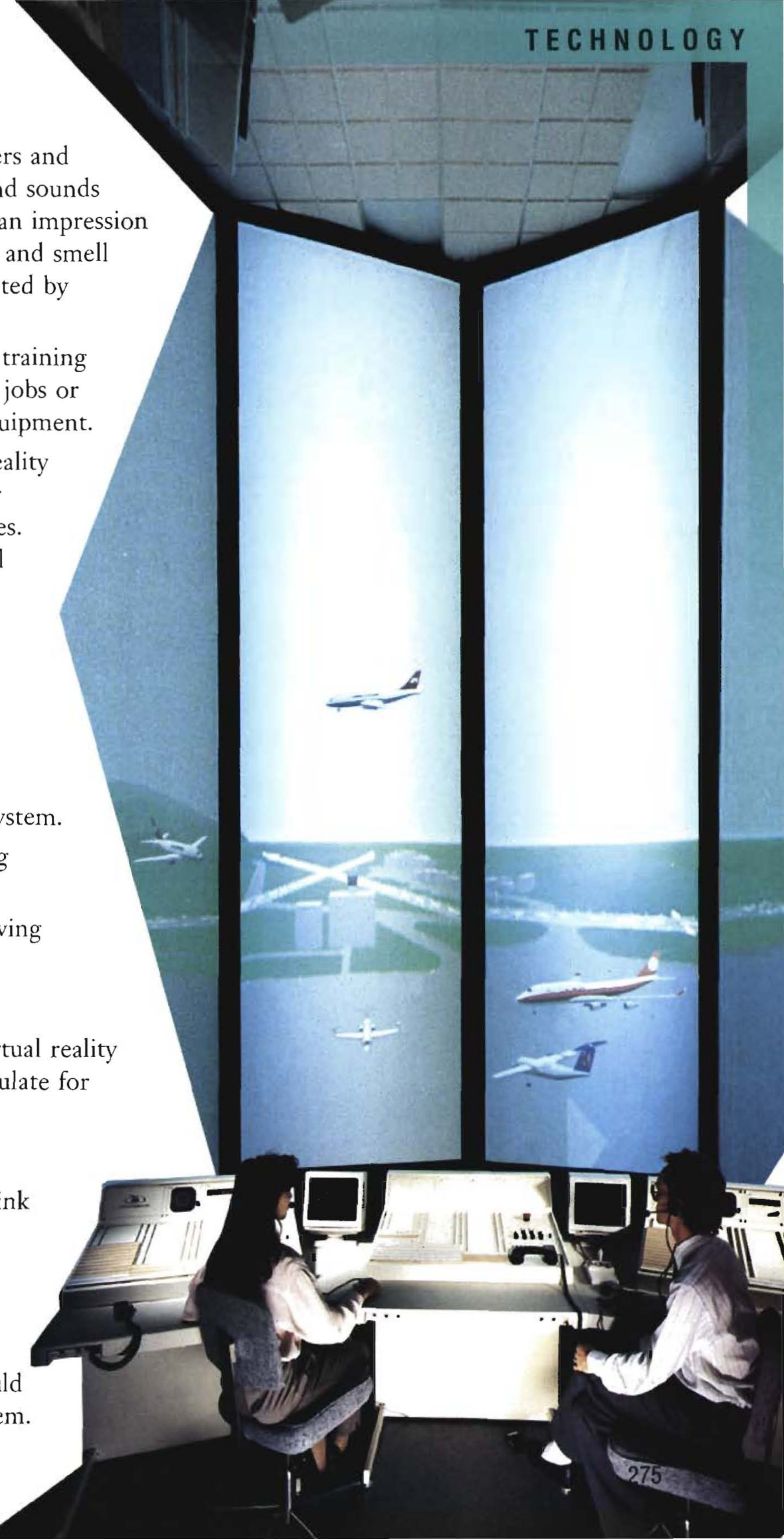
What symbol is on each of the faces indicated by the arrows?

Virtual Reality

Virtual reality is the use of computers and graphics software to create sights and sounds from real-world data. The result is an impression of the real world. Soon, even touch and smell will be included in the "world" created by virtual reality.

Virtual reality is seen as a powerful training tool for people who take hazardous jobs or who must learn to use expensive equipment.

A Canadian company uses virtual reality in the systems it designs to train air traffic controllers in several countries. The controllers practise on a virtual reality system before tackling the real job at an airport.



Activity ❶

Virtual reality may be used to train doctors.

1. List 5 skills that trainee doctors might practise on a virtual reality system.

2. Describe the advantages of having doctors train in this way.

3. Describe any disadvantages of having doctors train in this way.

Activity ❷

Firefighters may be trained on a virtual reality system. What could the system simulate for a firefighter?

Activity ❸

Choose a situation in which you think a virtual system could be used for training. Describe what your system would do.

Activity ❹

Describe a computer game that could be played with a virtual reality system.

7.5 Surface Area and Volume of a Cylinder and a Cone

Nearly perfect cylindrical shapes can be found in nature. An example is this part of the trunk of one of Canada's biggest trees. It is a Douglas fir that grows near Port Renfrew on Vancouver Island.

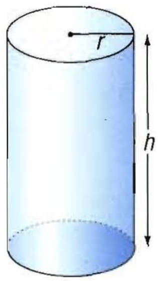
Activity: Develop a Formula

Join the shorter edges of a sheet of paper to form a cylinder. Do not crease the paper.

Inquire

1. a) Name the shapes of the top and bottom of your cylinder.
b) Write a formula for the combined area of these shapes.

2. a) What name is given to the distance around the top of the cylinder?
b) Write a formula for this distance in terms of r.



3. a) What was the original shape of the sheet of paper?
b) How does the length of this shape compare with the distance around the top of the cylinder?
c) What is the length of this shape in terms of r?

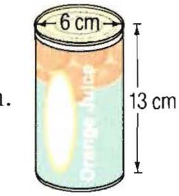
4. How does the height, h, of the cylinder compare with the width of the sheet of paper?

5. Write the formula for the area of the sheet of paper in terms of r and h.

6. Use the results of questions 1 b) and 5 to write a formula for the surface area of a cylinder that has a top and a bottom.

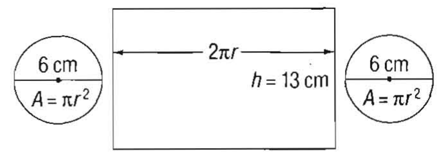
Example 1

The cylindrical can of juice has a diameter of 6 cm and a height of 13 cm. Find the surface area to the nearest tenth of a square centimetre.



Solution

The area of the curved surface is the area of a rectangle. The top and bottom are circles. The formula for the **surface area of a closed cylinder**, a cylinder with a top and bottom, is $2\pi r^2 + 2\pi rh$.



$$\begin{aligned}\text{Surface Area} &= 2\pi r^2 + 2\pi rh \\ &= 2(3.14)(3)^2 + 2(3.14)(3)(13) \\ &= 56.52 + 244.92 \\ &= 301.44\end{aligned}$$

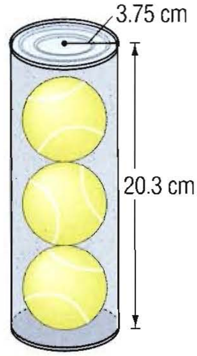
EST

$$\begin{aligned}&(2)(3)(10) + (2)(3)(3)(10) \\ &= 60 + 180 \\ &= 240\end{aligned}$$

The surface area of the juice can is 301.4 cm^2 to the nearest tenth of a square centimetre.

Example 2

Find the volume of the can of tennis balls to the nearest cubic centimetre.



Solution

The **volume of a cylinder** is calculated by multiplying the area of the base by the height.

V = (Area of Base)(Height)

$= (\pi r^2)(h)$

$= (3.14)(3.75)^2(20.3)$

$\doteq 896.4$

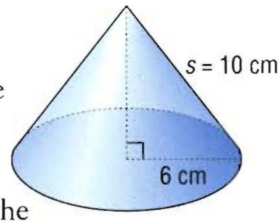
EST: $(3)(4)^2(20) = 960$

C π × 3 . 75 x² × 20 . 3 = 896.82653

The volume of the can of tennis balls is about 896 cm^3.

Example 3

Calculate the surface area of this cone to the nearest tenth of a square centimetre.



Solution

The **surface area of a cone** is the sum of the circular base area and the curved surface area. The **slant height**, s, is the side length of the cone.

Surface Area of a Cone = (Base Area) + (Curved Surface Area)

$= \pi r^2 + \pi rs$

$= (3.14)(6)^2 + (3.14)(6)(10)$

$= 113.04 + 188.4$

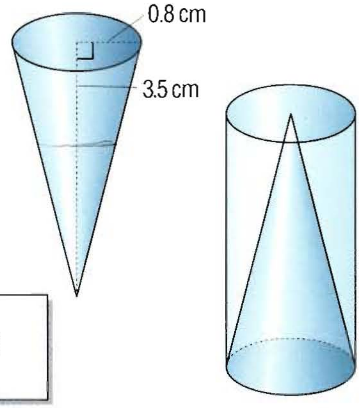
$= 301.44$

EST: $(3)(40) + (3)(6)(10)$
$= 120 + 180$
$= 300$

The surface area of the cone is about 301.4 cm^2.

Example 4

Find the volume of this cone to the nearest tenth of a cubic centimetre.



Solution

The **volume of a cone** is equal to one-third the volume of a cylinder with the same base and height.

Volume of a Cone $= \frac{1}{3}$(Base Area)(Height)

$= \frac{1}{3}(\pi r^2)(h)$

$= \frac{(3.14)(0.8)^2(3.5)}{3}$

$= \frac{7.0336}{3}$

$\doteq 2.3$

EST: $\frac{(3)(1^2)(3)}{3} = 3$

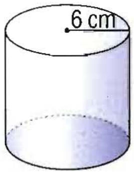
C π × . 8 x² × 3 . 5 ÷ 3 = 2.3457225

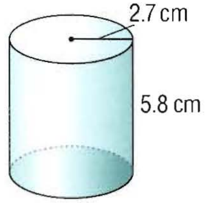
The volume of the cone is about 2.3 cm^3.

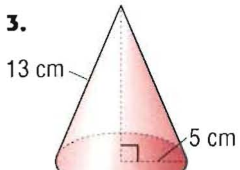
CONTINUED

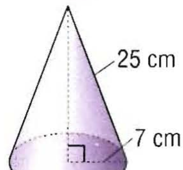
Practice

Calculate each surface area to the nearest square centimetre.

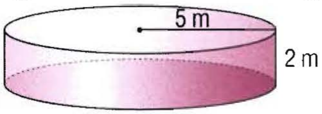
1. 

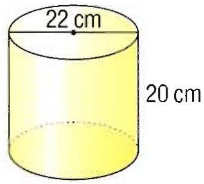
2. 

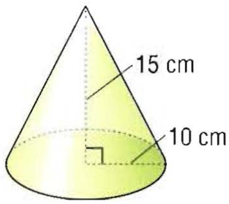
3. 

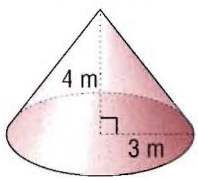
4. 

Estimate, then calculate each volume.

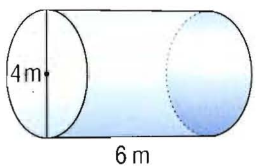
5. 

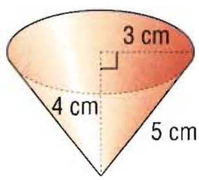
6. 

7. 

8. 

Calculate the surface area and volume of the following.

9. 

10. 

Problems and Applications

11. A paper cup at a water dispenser has a conical shape. The radius of the cup is 3 cm, and its height is 6 cm.

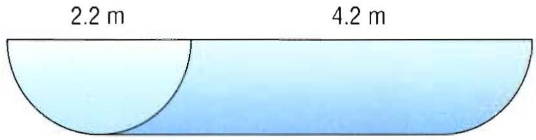
a) Find the slant height of the cup to the nearest tenth of a centimetre.

b) Calculate how much paper, to the nearest square centimetre, is required to make the cup.

12. The Canadarm used on the space shuttle is a cylinder that is 15.2 m long with a diameter of 38 cm. What is its surface area to the nearest tenth of a square unit?

13. A cone-shaped container has a diameter of 20 cm and a height of 20 cm. How many litres of water will the cone hold to the nearest litre?

14. A hobby club runs remote-controlled boats in a tank that is shaped as shown.



What volume of water can this tank hold to the nearest cubic metre?

15. Paper towels are sold in packages of 2 rolls. Each roll is a cylinder with a height of 30 cm and an outer diameter of 12 cm.

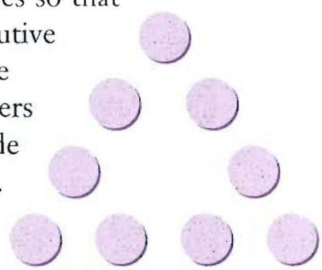
a) Design a shipping carton that will contain 12 packages of paper towels.

b) The inner diameter of each roll is 4 cm. How much wasted space is there in the carton?

16. Write a problem that requires the calculation of the surface area and volume of a cone. Have a classmate solve your problem.

NUMBER POWER

Place the digits from 1 to 9 in the circles so that the consecutive sums of the four numbers on each side differ by 1.



Programming Formulas in BASIC

Work with a partner to examine the following computer programs, and describe what each line of the program does or asks you to do.

1.
```
NEW
10 PRINT"SURFACE AREA OF A"
20 PRINT"RECTANGULAR PRISM"
30 INPUT"LENGTH IS";L
40 INPUT"WIDTH IS";W
50 INPUT"HEIGHT IS";H
60 A=2*L*W+2*L*H+2*W*H
70 PRINT"SURFACE AREA IS";A
80 END
```

2.
```
NEW
10 PRINT"VOLUME OF A"
20 PRINT"RECTANGULAR PRISM"
30 INPUT"LENGTH IS";L
40 INPUT"WIDTH IS";W
50 INPUT"HEIGHT IS";H
60 V=L*W*H
70 PRINT"VOLUME IS";V
80 END
```

3.
```
NEW
10 PRINT"SURFACE AREA OF A"
20 PRINT"SQUARE PYRAMID"
30 INPUT"LENGTH IS";L
40 INPUT"SLANT HEIGHT IS";S
50 A=4*1/2*L*S + L^2
60 PRINT"AREA IS";A
70 END
```

4.
```
NEW
10 PRINT"VOLUME OF A"
20 INPUT"SQUARE PYRAMID"
30 INPUT"LENGTH IS";L
40 INPUT"HEIGHT IS";H
50 V=1/3*L^2*H
60 PRINT"VOLUME IS";V
70 END
```

5.
```
NEW
10 PRINT"SURFACE AREA"
20 PRINT"OF A CYLINDER"
30 INPUT"RADIUS IS";R
40 INPUT"HEIGHT IS";H
50 A=2*3.14*R^2+2*3.14*R*H
60 PRINT"THE SURFACE AREA IS";A
70 END
```

6.
```
NEW
10 PRINT"VOLUME OF A CYLINDER"
20 INPUT"RADIUS IS";R
30 INPUT"HEIGHT IS";H
40 V=3.14*R^2*H
50 PRINT"VOLUME IS";V
60 END
```

7.
```
NEW
10 PRINT"SURFACE AREA OF A CONE"
20 INPUT"RADIUS IS";R
30 INPUT"SLANT HEIGHT IS";S
40 A=3.14*R^2+3.14*R*S
50 PRINT"THE SURFACE AREA IS";A
60 END
```

8.
```
NEW
10 PRINT"VOLUME OF A CONE"
20 INPUT"RADIUS OF BASE IS";R
30 INPUT"HEIGHT IS";H
40 V=1/3*3.14*R^2*H
50 PRINT"VOLUME IS";V
60 END
```

Estimate, then use a program to calculate the surface area and volume of each of the following.

9. rectangular prism with L = 3.5 cm, W = 4.5 cm, H = 6.2 cm

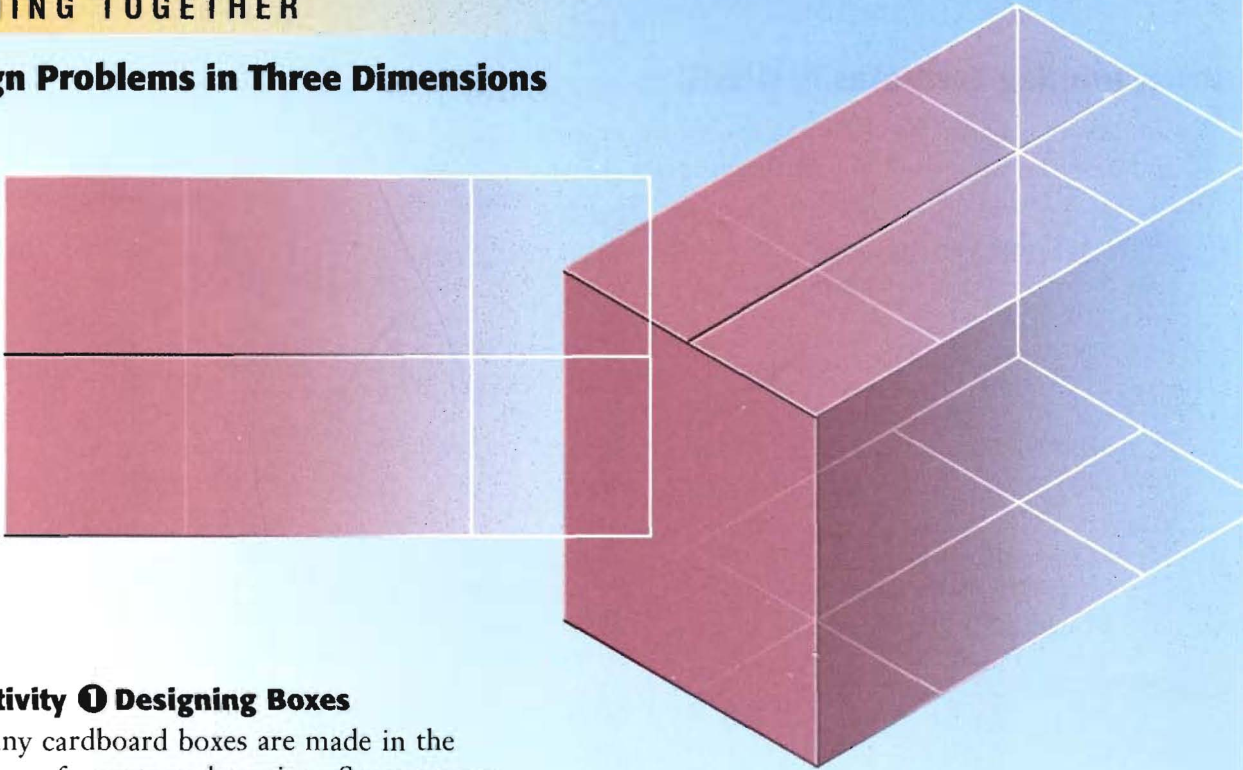
10. square pyramid with L = 1 m, H = 1.2 m, S = 1.3 m

11. cylinder with R = 2 cm, H = 3.5 cm

12. cone with R = 3 m, H = 4 m, S = 5 m

13. What advantages are there in using a computer program to perform these types of computations?

Design Problems in Three Dimensions



Activity ❶ Designing Boxes

Many cardboard boxes are made in the shape of a rectangular prism. Suppose you are an industrial designer who must decide what dimensions to use for a box to hold 12 objects, each the size of a one-centimetre cube.

1. Use 12 one-centimetre cubes to make a rectangular prism. Record its dimensions.

2. Repeat step 1 three times, making a different rectangular prism each time.

3. Copy and complete the table.

Prism	Dimensions (cm)	Volume (cm^3)	Surface Area (cm^2)
1			
2			
3			
4			

4. The most cost-effective, or cheapest, container of a given volume is the one that uses the least material. To package 12 cm^3 of product in a box with sides that are whole numbers of centimetres, what dimensions would give the most cost-effective box? Explain.

5. a) For each prism, calculate the rate $\frac{\text{volume}}{\text{surface area}}$, to 2 decimal places.

b) What happens to this rate as a box becomes more cost-effective?

6. Many boxes do not have whole-number dimensions. Suppose you had to design a box to hold 8 m^3 of product. Possible dimensions of the box would include 8 m × 1 m × 1 m, 2 m × 2 m × 2 m, 1.6 m × 2 m × 2.5 m, and 1 m × 1.25 m × 6.4 m. For these sets of dimensions, and any others you choose, complete a table like the one in step 3.

7. Use the rate $\frac{\text{volume}}{\text{surface area}}$ to choose the most cost-effective box. What shape is this box?

8. Why are many products not packed in the shape of box you found in step 7?

Activity ❷ Boxes Within Boxes

Many shipping boxes hold smaller boxes.

1. What is the maximum number of 6 cm × 6 cm × 6 cm boxes you can pack in shipping boxes with the following dimensions?

a) 24 cm × 24 cm × 24 cm

b) 48 cm × 48 cm × 48 cm

2. Of the two shipping boxes in step 1, which is the more cost-effective?

3. a) Do shipping boxes become more cost-effective as they get larger or smaller?

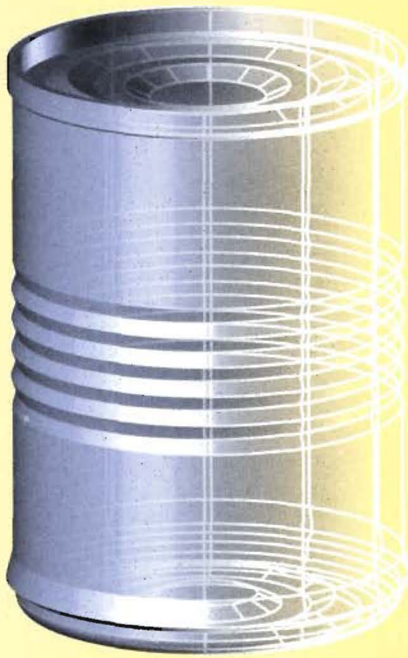
b) What other factors do you think are used to decide the size of a shipping box?

4. What is the maximum number of boxes measuring 5 cm × 4 cm × 3 cm that can be packed into a shipping box that is 24 cm × 20 cm × 18 cm?

5. What is the maximum number of boxes measuring 10 cm × 6 cm × 5 cm that can be packed into a shipping box that is 30 cm × 20 cm × 11 cm?

Activity ❸ Designing Cans

Industrial designers must also decide what height and radius to use for a cylindrical container.



1. The table shows the radii and heights of 4 possible cylindrical cans. Copy and complete the table. Round each volume and surface area to the nearest whole number.

Can	Radius (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
1	1	36		
2	2	9		
3	3	4		
4	6	1		

2. Which are the two most cost-effective cans? Explain.

3. a) What general shape of cylinder is the most cost-effective choice for a can?

b) Why do you think cylindrical cans are not all made with this general shape?

4. Think about food products that come in cylindrical cans. Give possible reasons why each shape of cylinder is used for each can.

Activity ❹ Variations in Cans

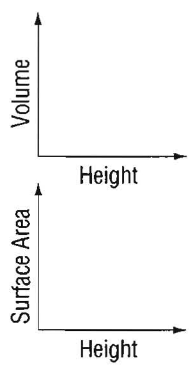
1. Make up the dimensions of 4 cans with the same radius but different heights. Calculate the volume and surface area of each can. Record the results in a table similar to the one in Activity 3.

2. Use the table to draw separate graphs.

a) volume versus height

b) surface area versus height

3. How are the two graphs similar? How are they different?



Vanishing Points and Perspective

Many artists from the 14th to 16th century, a period known as the Renaissance, studied mathematics and engineering. The artists used geometry to help them show the three-dimensional world on a two-dimensional surface. They used the concepts of perspective and vanishing points to help create great works of art.

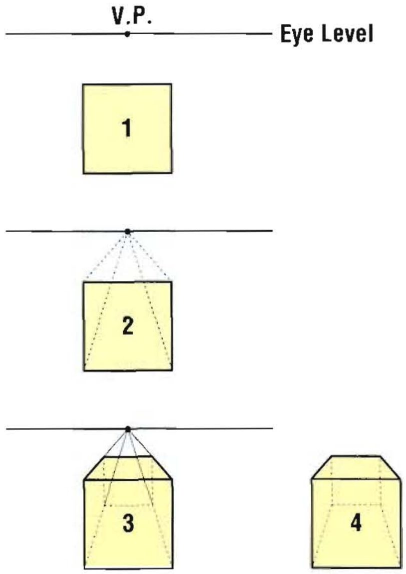
Activity ❶ One Vanishing Point

1. a) To sketch a cube using 1 vanishing point, draw a square for the front face of the cube.
b) Draw a horizontal line parallel to the horizontal edges of the square. This line is called the **eye level.**
c) Mark a vanishing point above the top edge of the square on the eye level line.

2. Draw a line from each corner of the square to the vanishing point. These lines are called **vanishing lines.**

3. Complete the sketch of the cube using the vanishing lines as edges of the cube.

4. Erase the eye level line and any unnecessary parts of the vanishing lines. The viewer sees the cube from above and in front.



Activity ❷ Perspectives I

1. Sketch a cube where the eye level line is below the cube and the vanishing point is to the right of the cube. How does the viewer see the cube?

2. Sketch a cube where the eye level line is through the centre of the cube and the vanishing point is to the right of the cube. How does the viewer see the cube?

3. Sketch a cube where the eye level line is below the cube and the vanishing point is directly below the cube. How does the viewer see the cube?

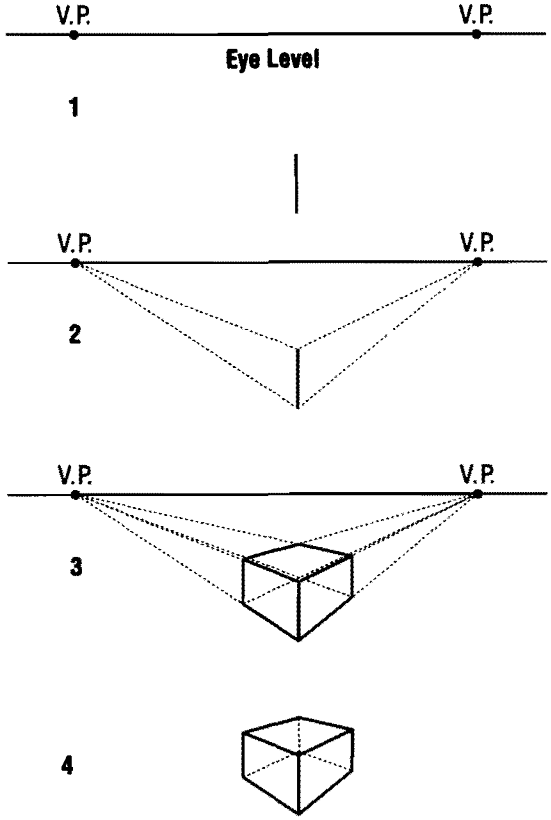
Activity ❸ Two Vanishing Points

1. a) To sketch a cube using 2 vanishing points, draw the front edge of the cube.
b) Draw an eye level line above the front edge of the cube, and mark 2 vanishing points on the line.

2. Draw vanishing lines from each end of the front edge of the cube to the vanishing points.

3. a) To form the remaining front vertical edges, draw vertical line segments, parallel to the front edge. Draw these lines to intersect the vanishing lines.
b) Draw more vanishing lines to these new edges to determine the back edges of the cube.

4. Erase the eye level line and any unnecessary parts of the vanishing lines.



Activity ❹ Perspectives II

1. Use 2 vanishing points to sketch a cube with the eye level line above the cube.

2. Use 2 vanishing points to sketch a cube with the eye level line passing through the cube.

Activity ❺ Sketching Letters in Perspective

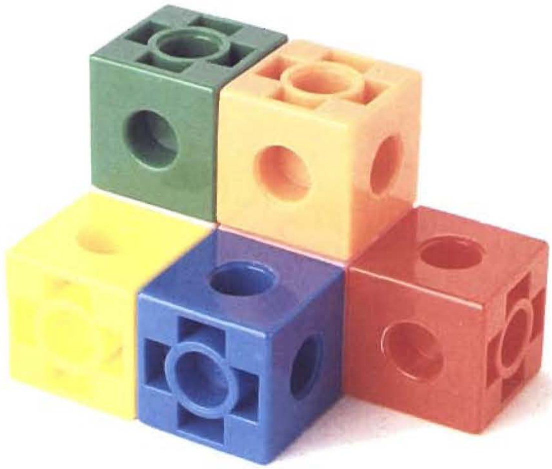
1. Sketch the letter M using 1 vanishing point.

2. Sketch the letter T using 2 vanishing points.

3. Sketch your initials using 1 or 2 vanishing points.

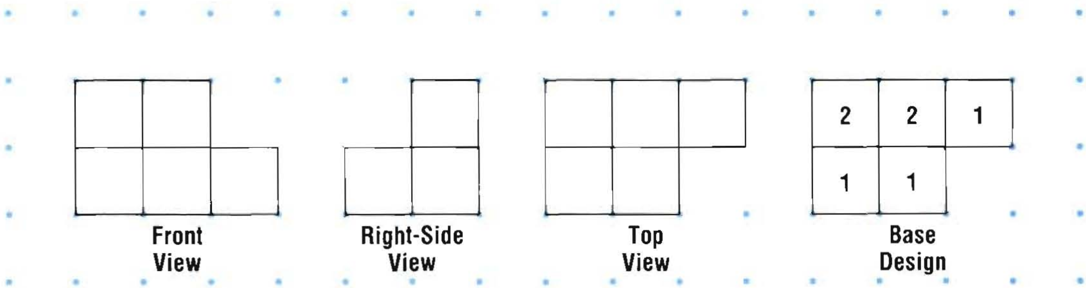
Different Views

Knowing how to represent 3-dimensional objects in 2 dimensions is an important skill. Taking 2-dimensional drawings of an object and building the 3-dimensional figure is also useful.



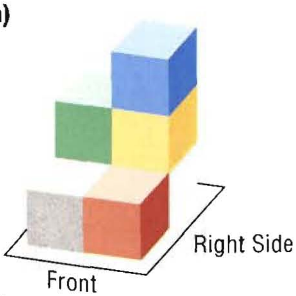
Activity ❶ Front, Side, and Top Views

This 3-dimensional shape was built using 7 cubes. The front view, right-side view, top view, and base design are shown. The base design is the top view with numbers giving the number of cubes in each position.

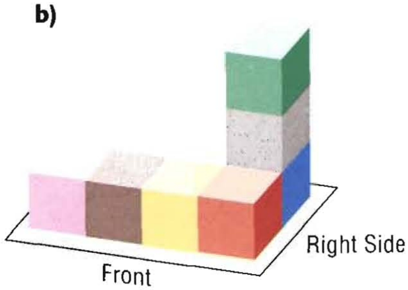


On grid paper, sketch the front view, right-side view, top view, and base design for each of the following shapes.

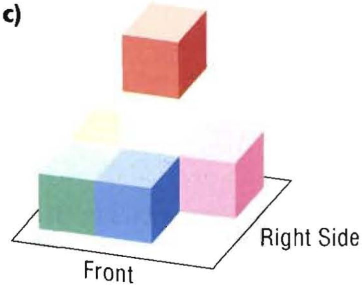
a)



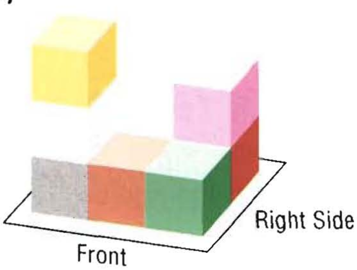
b)



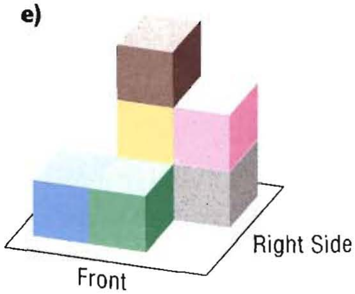
c)



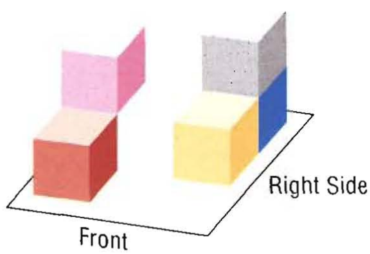
d)



e)

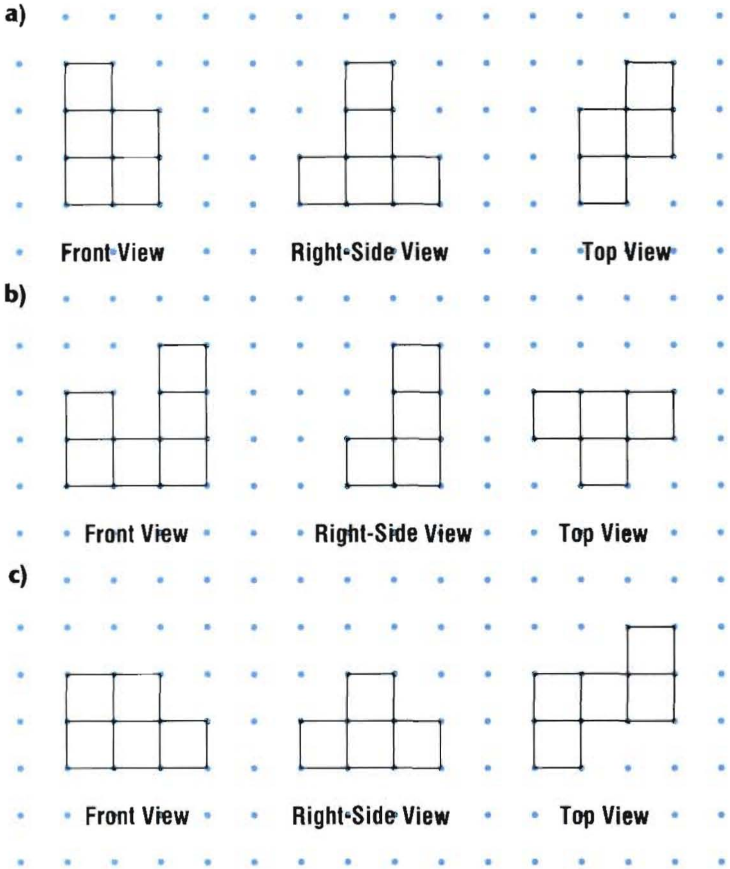


f)



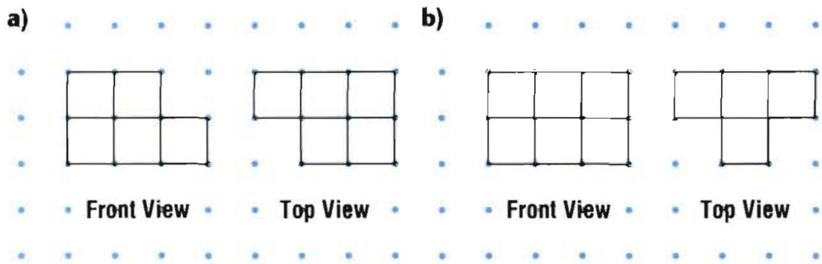
Activity ❷ Build the Model

Use these 2-dimensional drawings of 3 views to build the 3-dimensional shape. Draw the base design on grid paper.



Activity ❸ Build the Model

The front and top views of models built with 7 cubes are shown.

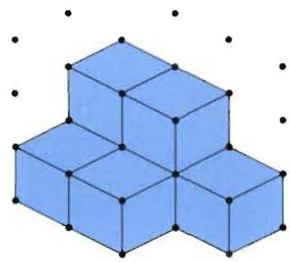


Build the model. On grid paper, draw the right-side view and the base design. Is there more than 1 solution? Compare with a classmate's solution.

CONTINUED

Activity ❹ Perspective Drawings

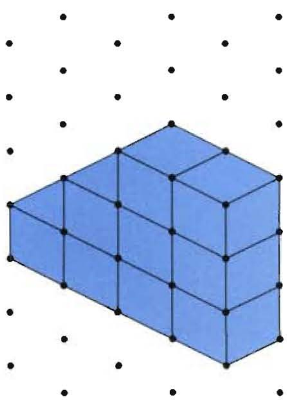
Instead of drawing two-dimensional views of a three-dimensional shape, you can draw a perspective view of the whole shape on isometric dot paper. For example, the shape built from interlocking cubes at the top of page 284 looks like this when it is represented on isometric dot paper.



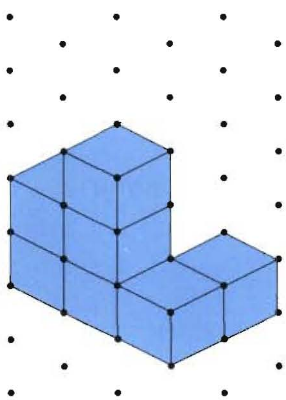
1. Use isometric dot paper to represent each of the three-dimensional shapes in parts a) to f) of Activity 1 on page 284.

2. Use the following perspective drawings to draw the front view, right-side view, and top view of each shape.

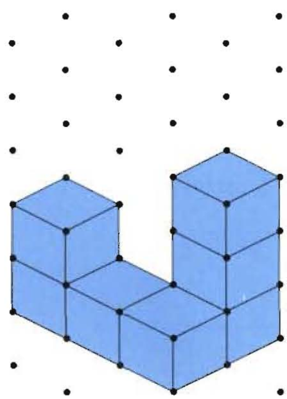
a)



b)



c)

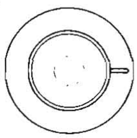


3. Use the views shown in parts a), b), and c) of Activity 2 on page 285 to complete a perspective drawing of each object on isometric dot paper.

Activity ❺ Views of Everyday Objects

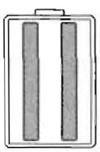
1. The objects are viewed from above. Name each object.

a)



b)

c)

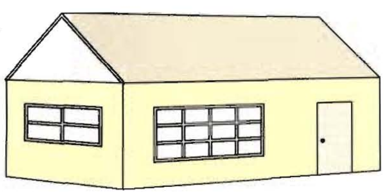


2. Sketch the house as it would look from each view.

a) front

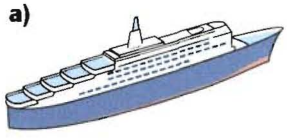
b) side

c) top

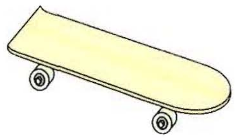


3. Draw the front view, side view, and top view of each object.

a)



b)



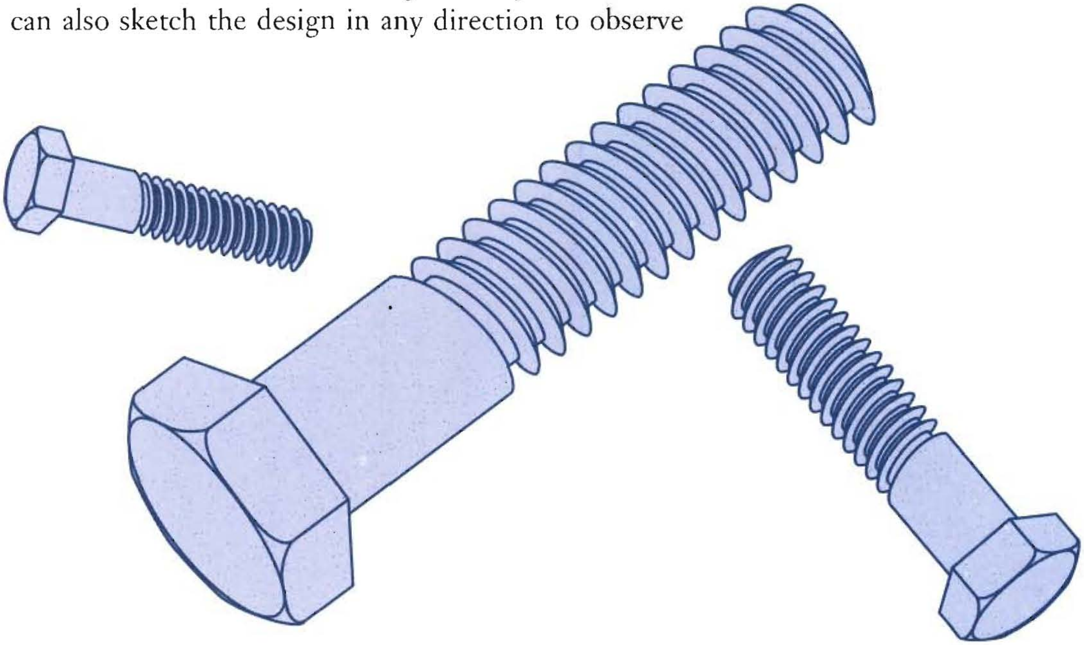
c)

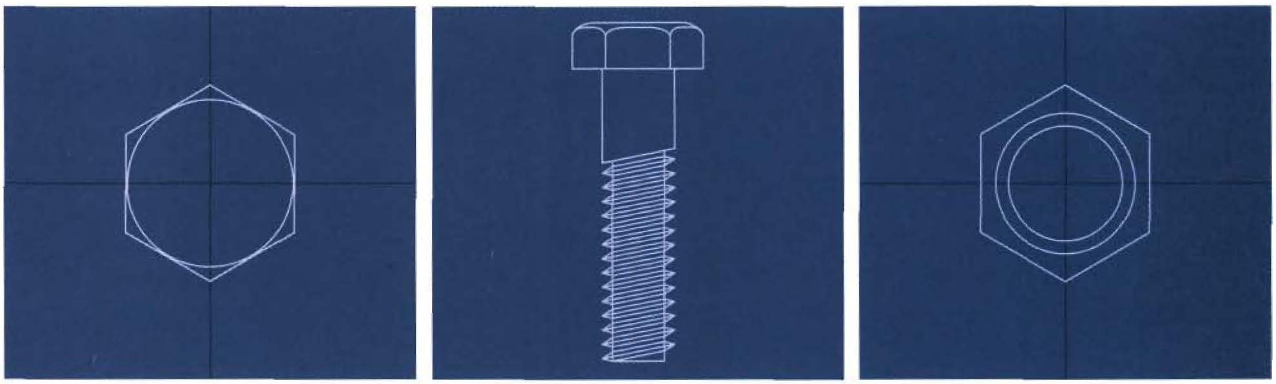
d)

Computer-Assisted Design

The diagram shows a computer-generated design for a simple threaded bolt.

The computer allows us to rotate the design to inspect different views. We can also sketch the design in any direction to observe the effect.





Top View Side View Bottom View

Activity ❶

1. What are the advantages of computer-assisted design over drawing by hand?

2. Describe any advantages that drafting has over computer-assisted design.

3. Compare your opinions with your classmates'.

Activity ❷

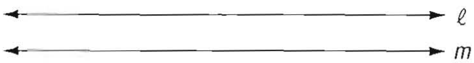
1. Research the uses of computer-assisted design. What mathematics does the designer need to know?

2. Describe any uses that you did not expect.

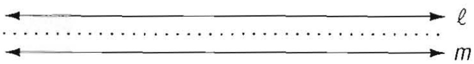
3. Describe your findings to your group.

Locus

A **locus** in a plane is the set of all points in the plane that satisfy a given condition. Suppose a locus is defined as the set of all points in the plane that are equidistant from parallel lines l and m.



We can identify examples of points that satisfy the condition.



All points in the plane that satisfy the condition lie on a third line that is parallel to lines l and m and midway between them.

If lines l and m are 2 cm apart, we can describe the locus as the line parallel to lines l and m and 1 cm from each of them.

Equidistant from means *the same distance from.*

Activity ❶ Identifying a Locus

1. For each condition, sketch the locus in the plane. Describe the locus fully in words.

a) all points that are 3 cm from point A
b) all points equidistant from the endpoints of line segment AB
c) all points equidistant from the arms of ∠DEF
d) all points that are 4 cm from line n
e) all points equidistant from all 4 sides of square ABCD

2. State whether it is possible to sketch the complete locus. Explain.

a) all points 4 cm or less from point K
b) all points more than 4 cm from point K

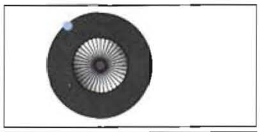
Activity ❷ Applications

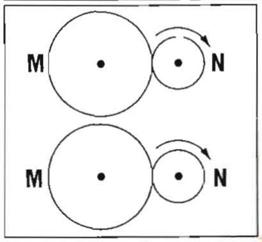
1. Sketch and describe each locus in the plane.

a) all points equidistant from the goal lines on a hockey rink
b) all points on a dart board that triple the score when a dart lands on them
c) all points equidistant from the edges of a city street
d) all points that can receive a signal from a radio transmitter with a range of 80 km
e) all points that can be wiped by a car's windshield wiper

Activity ❸ Problem Solving

In questions 1 to 3, sketch and describe each locus in the plane.



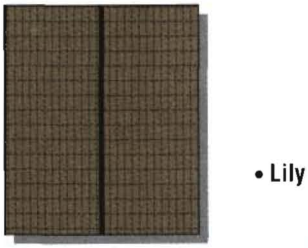


1. all points traced by a point on the outside of a car tire as the car is driven down the highway

2. all points traced by a point on the outside of a small wheel, N, rolling clockwise around a large drive wheel, M

3. all points traced by the centre of the smaller wheel in question 2

4. Lily and Katerina are playing hide and seek. Lily is hiding behind a garden shed. The diagram shows her location from above. For each of the following conditions, sketch the locus of all points in the plane. If it is not possible to sketch the complete locus, explain why.



a) all points from which Katerina would not be able to see Lily

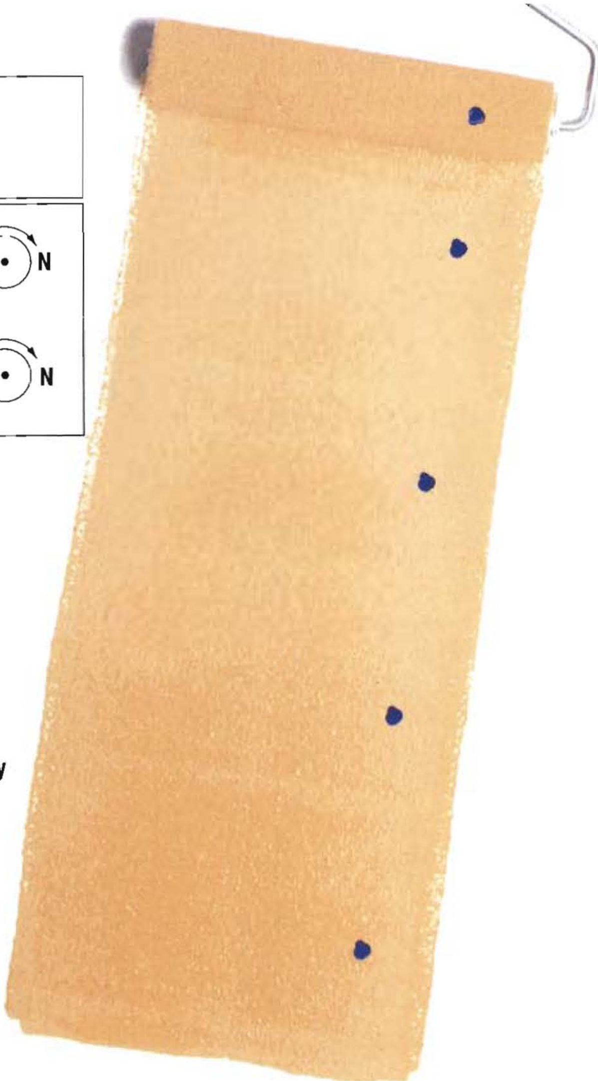
b) all points from which Katerina would be able to see Lily

5. A circular fountain has a radius of 2 m. A circular flower bed surrounds the fountain. There is no gap between the flower bed and the fountain. The outside circumference of the flower bed is the locus of points in the plane that are 3 m from the centre of the fountain. Sketch the flower bed and calculate its area.

6. A horse grazing in a rectangular field is tethered to the fence at the midpoint of the longer side. The field is 50 m by 40 m, and the tether is 20 m long.

a) Sketch the locus that shows the area in which the horse can graze.

b) Calculate the area of grass that the horse cannot reach.



Activity ❹ Using Geometry Software

Some geometry software programs will draw a locus.

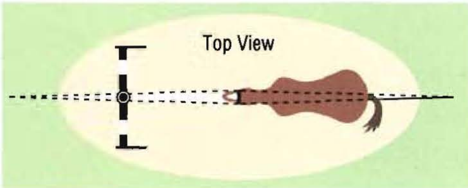
1. Explore a suitable program to discover how to draw each locus described in question 1 of Activity 1.

2. Use the program to draw a locus of your own. Describe your locus to a classmate and challenge your classmate to draw it with the software.

Sight

Even though a horse's eyes are twice as large as a human's, the horse cannot see an area directly in front of it. This area is called the **blind spot**. When horses, such as Canada's Big Ben, compete in jumping competitions they actually jump the barriers blind.

Peripheral vision in humans and animals is the range or angle through which vision is possible. The angle of peripheral vision for a horse is 170° on one side and 170° on the other side. A horse cannot see an area within an angle of 10° directly in front of it or 10° directly behind it.



Activity ❶ The Blind Spot

1. To locate your blind spot, mark 2 black dots 5 cm apart on a blank piece of paper as shown.

•   •

2. Hold the paper in front of you with your arm extended.

3. Cover your left eye with your hand and look at the left dot with your right eye.

4. Continue looking at the left dot and move the paper toward you slowly until the right dot disappears. The point at which this happens is called the blind spot for your right eye.

5. Have a classmate measure the distance from your eye to the paper.

6. Repeat the experiment to find the blind spot for your left eye by covering your right eye and looking at the right dot.

7. Are the blind spot distances the same for both eyes?

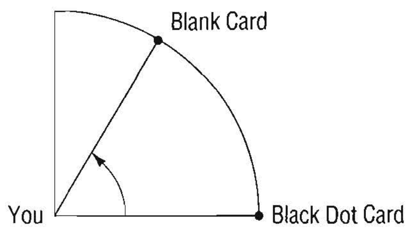
Activity ❷ Peripheral Vision

1. To determine your angle of peripheral vision, you will need a blank index card and an index card with a black dot drawn on it that has a diameter of 1 cm.

2. Use chalk to draw a circle with a radius of 1 m on a large piece of paper placed on the floor.

3. Stand on the centre of the circle and have 1 classmate stand in front of you at the edge of the circle holding the card with the black dot on it. Have another classmate hold the blank card beside the card with the black dot on it. The blank card should be to your left of the card with the dot on it.

4. Stare at the black dot while the classmate with the blank card moves slowly to your left, along the edge of the circle. Say, "Stop" when you can no longer see the blank card.



5. Have another classmate draw a line from you to the person holding the card with the dot and a line from you to the person holding the blank card.

6. Measure and record the angle between the 2 lines drawn in step 5.

7. Repeat the procedure with the blank card moving to your right.

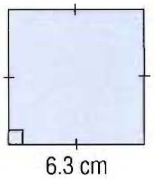
8. Add the 2 angles you have found and record them. The sum of these 2 angles is your angle of peripheral vision.

9. Find the average angle of peripheral vision for your class.

Review

Estimate, then calculate each area.

1.



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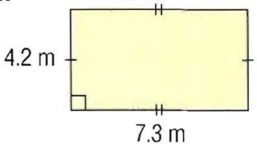

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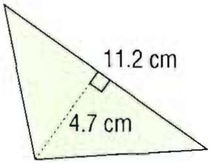

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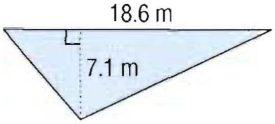

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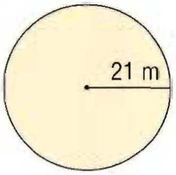

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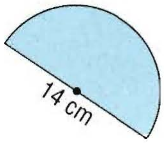

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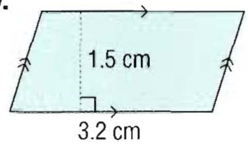

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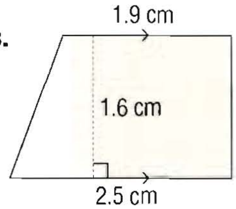

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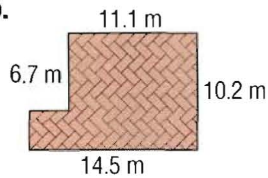

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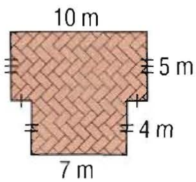
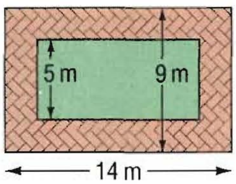
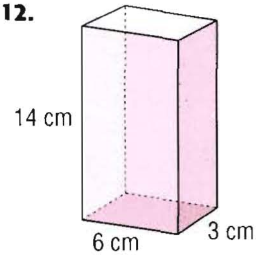


11. a) Calculate the area of the stonework around this rectangular garden.
b) What is the total cost of landscape fabric needed to cover the garden if the fabric costs $0.99/m^2? Allow an extra 10% for overlapping.



Calculate the surface area.

12.



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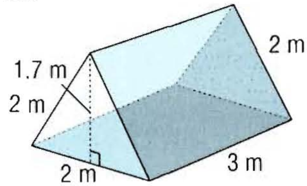

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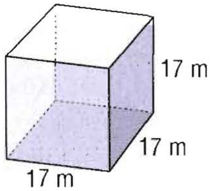

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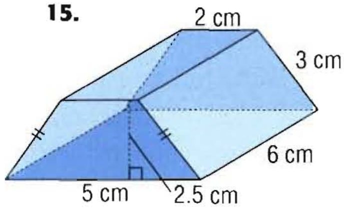
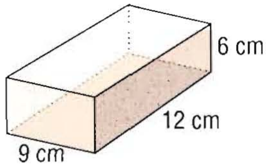


Calculate the volume.

16.



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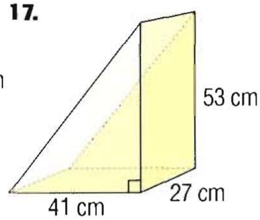
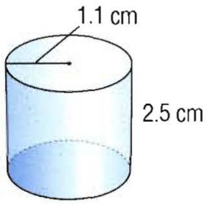


Calculate the surface area.

18.



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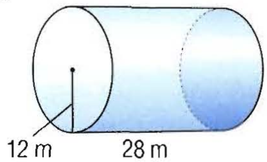
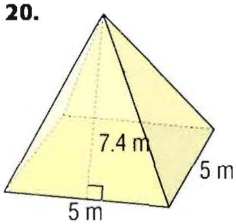


20.



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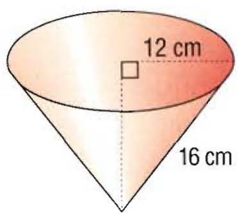
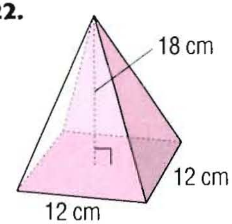


Calculate the volume to the nearest unit.

22.



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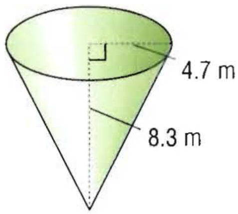

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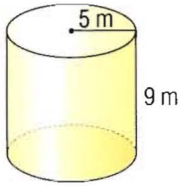

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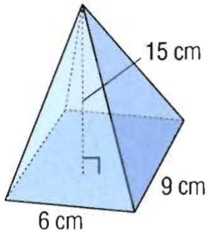
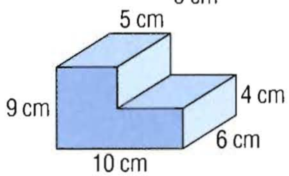


26. Calculate the volume of the composite solid.



27. A foam container has the shape of a rectangular prism. Its inside dimensions are 12 cm by 10 cm by 15 cm. Its outside dimensions are 14 cm by 12 cm by 17 cm.

a) Find the outside surface area.

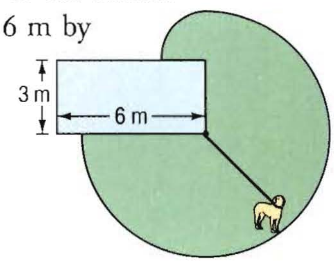
b) Find the inside surface area.

c) Find the volume of foam in the container.

28. A 4 m by 4 m square is removed from the centre of a 15 m by 30 m rectangle. What is the area of the remaining part?

29. A pizza with a diameter of 45 cm has a circular ring of tomato sauce with a radius of 20 cm. Find the area of the bare crust around the edge of the pizza, to the nearest square centimetre.

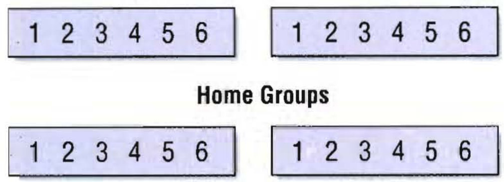
30. Suneel tied his dog to the corner of a shed. The shed is 6 m by 3 m. The rope is 5 m long. Calculate the area within the dog's reach to the nearest square metre.



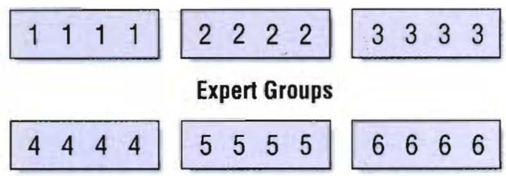
Group Decision Making
Researching Construction Careers

1. Meet as a class to choose 6 careers in the construction industry. Your choices might include a plumber, an electrician, a crane operator, an architect, or a building contractor.

2. Go to home groups. Assign one career to each home group member.



3. Form an expert group of 4 with students who have the same career as you to research. Decide on the questions you want to answer about your career. One of the questions should be: "How is math used in this career?" Research your assigned career in your expert group.



4. Return to your home group and share your findings about the career you researched.

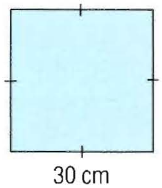
5. In your home group, prepare a report on the 6 careers.

6. Meet as a class to discuss your group work. Decide what went well and what you would do differently next time.

Chapter Check

Calculate the area.

1.



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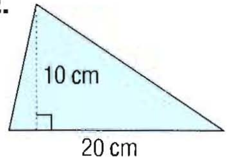

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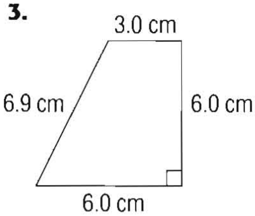

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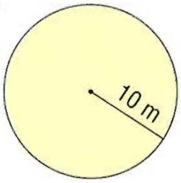
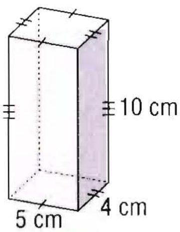


Calculate the surface area.

5.



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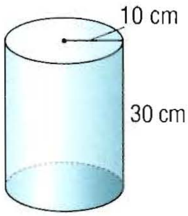
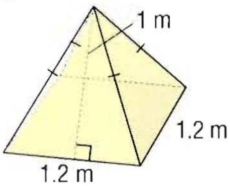


7.



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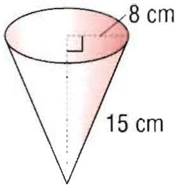
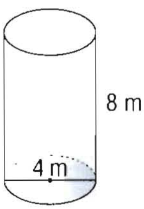


Calculate the volume to the nearest unit.

9.



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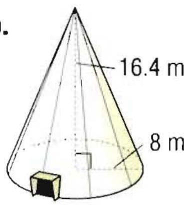

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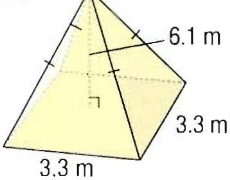

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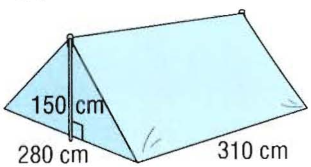
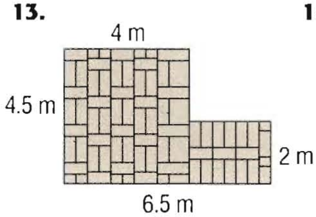


Determine the area of each patio.

13.



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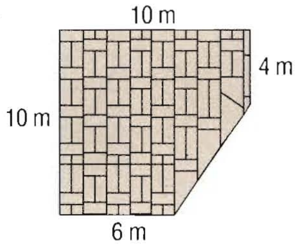
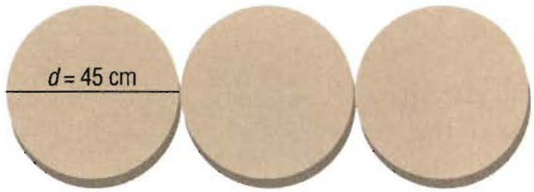


15. Circular patio stones are placed as shown to make a walkway that is 7.2 m long.



a) How many patio stones are used?

b) The patio stones do not cover the entire width of the walkway, which is filled with gravel. If the walkway is 0.6 m wide, what area is covered with gravel?

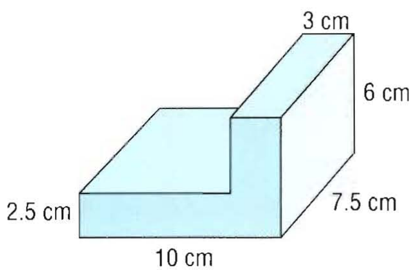
16. A juice box measures 10 cm × 6 cm × 3 cm.

a) What is its surface area?

b) What is its volume?

17. A block is a rectangular prism of volume 600 cm^3. What is the height of the block if its length is 20 cm and its width is 10 cm?

18. Determine the volume of the composite solid.

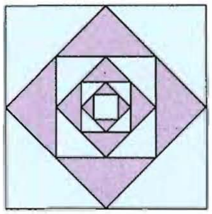


Using the Strategies

1. What is the sum of these numbers?
a) first 3 odd numbers
b) first 4 odd numbers
c) first 5 odd numbers
d) first 6 odd numbers
e) first 25 odd numbers
f) first 105 odd numbers

2. If a year has 2 months in a row with Friday the 13th, what months are they?

3. The area of the large square is 64 cm^2.



Each smaller square is formed by joining the midpoints of the sides of the next larger square. What is the area of the smallest square?

4. How many pieces of string each 8.5 cm long can be cut from a spool of string 400 cm long? What assumptions have you made?

5. The number 2601 is a 4-digit number that is a perfect square because $51^2 = 2601$. What is the smallest 4-digit number that is a perfect square and that has all even digits?

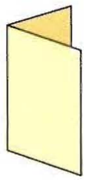
6. The average of two numbers is 21. When a third number is included, the average of the three numbers is 23. What is the third number?

7. Justine, Chris, and Meelang plan to travel together in a van. They will all sit in the front seat. In how many different arrangements can they sit in each of the following situations?
a) all three can drive
b) only Chris and Meelang can drive
c) only Justine can drive

8. List the different ways you can make change for a dollar using only quarters and nickels.

9. Starting at the letter A, how many different pathways can you follow to spell ANGLE?

10. A square piece of paper is folded in half as shown. The perimeter of each new rectangle formed is 24 cm. What is the perimeter of the original square?



11. The ship will leave the harbour at 08:30. You have to be on board 20 min before departure. It takes 25 min to drive to the ship from your hotel. You must allow 15 min to check out of the hotel. It will take you 20 min to pack. You need half an hour to eat breakfast and at least 45 min to shower and dress. For what time should you place your wake-up call?

DATA BANK

1. How many times will Mercury orbit the sun in the time it takes Jupiter to orbit the sun once?

2. Use the DATA BANK to make up your own problem. Have a classmate solve your problem. Check your classmate's solution.