CHAPTER 8

Transformations

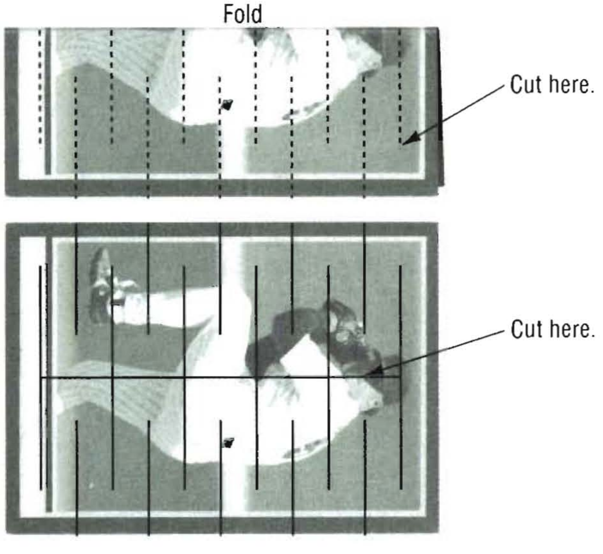
Can you pass your body through a baseball card? You can if you change the card's shape with some scissors.

Fold the card lengthwise.

Starting at one end of the card, make the first cut from the fold, the second from the open side, and so on. Never cut all the way across from either side. Make as many cuts as possible. Begin the last cut from the fold.

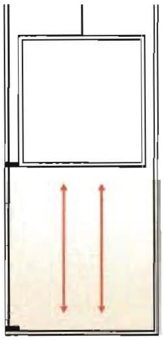
Unfold the card and cut along the fold as shown, stopping before the ends.

Now stretch the card into a large circle and pass your body through it.



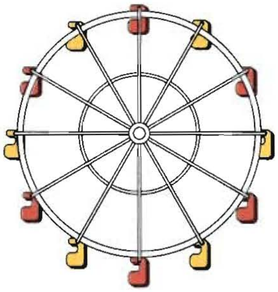
Activity ❶ Slides, Flips, and Turns

An elevator slides up and down in the elevator shaft. The elevator on the second floor can be called a slide image of the elevator on the ground floor.



The reflection of a mountain in a still lake is a flip image of the mountain.

When a chair on a Ferris wheel moves from the bottom to the top, the chair at the top can be called a turn image of the chair at the bottom.



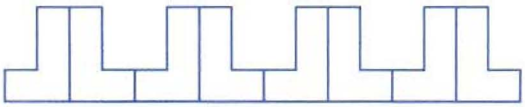
Find 3 more examples of each of the following and sketch the original figure and its image.

1. slides **2.** flips **3.** turns

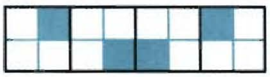
Activity ❷ Patterns

1. Identify each pattern as a slide, flip, or turn.

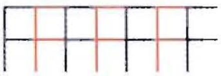
a)



b)



c)

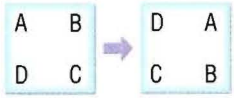


2. Draw a slide pattern, a flip pattern, and a turn pattern.

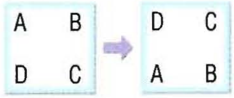
Activity ❸ The Square

Write the slide, flip, or turn that gives each image.

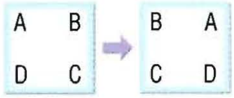
1.



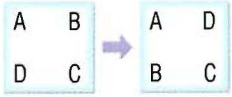
2.



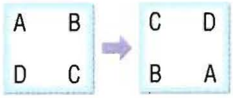
3.



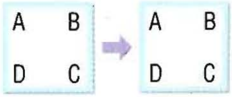
4.



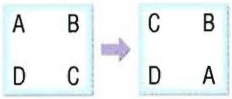
5.



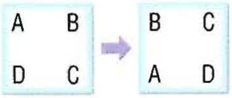
6.



7.



8.

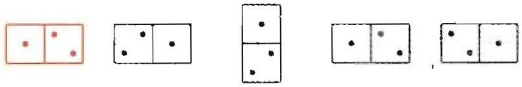


Activity ❹

Identify each figure as a slide, flip, or turn of the red figure.

a) b) c) d)

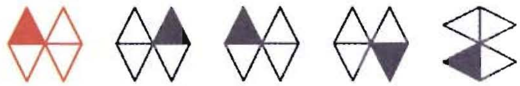
1.



2.

3.

4.

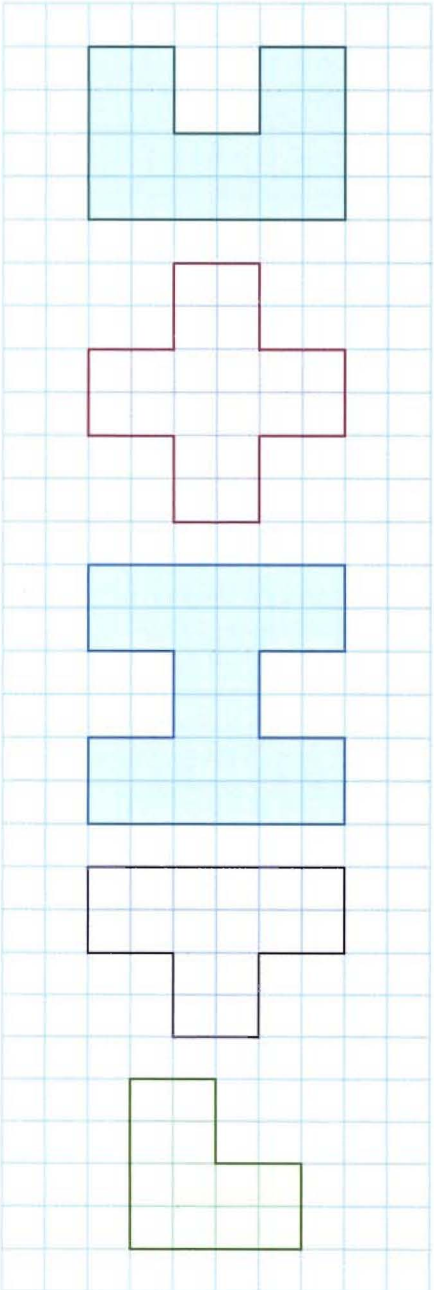


5.

Activity 5 Tile Patterns

In mathematics, tiling the plane means to cover an area with shapes so that there are no gaps and the shapes do not overlap.

Select 1 or more of these shapes and tile the plane to make an interesting design.



Mental Math

Calculate.

1. 220×0.1	**2.** 2000×0.01
3. $300 \div 100$	**4.** 4500×0.001
5. $0.2 \div 10$	**6.** $600 \div 0.01$
7. 0.003×1000	**8.** $0.01 \div 0.01$

Add.

9. \$3.99 + \$4.99	**10.** \$3.50 + \$4.25
11. \$3.98 + \$4.98	**12.** \$7.99 + \$2.95
13. \$5.90 + \$2.95	**14.** \$0.99 + \$2.45
15. \$1.95 + \$2.95	**16.** \$4.60 + \$3.60

Simplify.

17. $-7 - (-3)$	**18.** $-12 \div (-6)$
19. $-4 \times (-5)$	**20.** $-6 - 4 - 2$
21. $-6 + 3 - 8$	**22.** $20 \div (-5)$
23. $9 - 8 - 7$	**24.** $5 \times (-2) \times (-1)$

Calculate.

25. 2 at \$2.99	**26.** 4 at \$2.98
27. 5 at \$0.99	**28.** 3 at \$19.98
29. 4 at \$3.95	**30.** 6 at \$4.98
31. 3 at \$2.95	**32.** 5 at \$1.98

Calculate.

33. $\frac{5}{6} - \frac{2}{3}$	**34.** $\frac{3}{4} + \frac{1}{8}$	**35.** $\frac{2}{5} \div \frac{2}{3}$
36. $\frac{1}{2} \times \frac{3}{5}$	**37.** $\frac{1}{3}$ of 72	**38.** $\frac{1}{4}$ of 84

Simplify.

39. $6^2 - 24 + 3$	**40.** $100 - 8^2 - 6^2$
41. $500 \div 10 + 47$	**42.** $200 - 12^2$
43. $3^3 + 40 + 10$	**44.** $2^2 + 3^2 + 4^2$
45. $4 \times 50 + 23 - 1$	**46.** $7 + 20 \times 3 + 2$
47. $3^2 \times 100 + 47$	**48.** $29 + 29 + 29$
49. $19 \times 3 - 4$	**50.** $10^2 + 7^2 - 4$

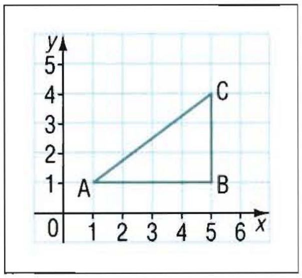
8.1 Translations

A **translation**, or slide, is a motion that is described by length and direction.

A helicopter can go straight up and down, or back and forth, or some combination of these two motions. If a helicopter goes down 30 m, then goes west 60 m, we say that the helicopter is "translated down 30 m," then "translated west 60 m." The description of each translation includes the direction and the distance.

Activity: Draw a Translation Image

Plot △ABC with coordinates A(1, 1), B(5, 1), and C(5, 4). Translate each point 6 units to the right and 4 units up. Label the new points A′, B′, and C′. Join A′, B′, and C′ to form a triangle.



Inquire

1. What are the coordinates of the vertices of △A′B′C′?

2. △A′B′C′ is called the **translation image** of △ABC. How do the lengths of the sides of △A′B′C′ compare with the lengths of the sides of △ABC?

3. Are the two triangles congruent? Explain.

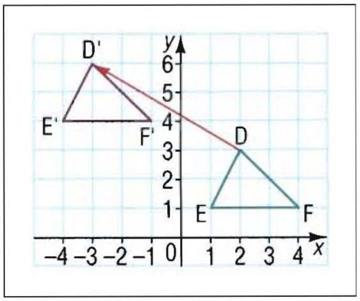
4. What is true about the measures of the angles after a translation?

5. What number do you add to the x-coordinate of each vertex of △ABC to give the x-coordinate of each vertex of △A′B′C′?

6. What number do you add to the y-coordinate of each vertex of △ABC to give the y-coordinate of each vertex of △A′B′C′?

7. Describe a translation in your own words.

△DEF has been translated 5 units to the left and 3 units up (5L, 3U). △D′E′F′ is the image of △DEF. Notice that D–E–F and D′–E′–F′ read in the same direction; in this case it is counterclockwise (ccw). We say that:



"△DEF and △D′E′F′ have the same **sense**."

The translation arrow shows the distance the triangle is translated and the direction of the translation.

The translation can be described mathematically as the ordered pair [−5, 3] or as the following **mapping**.

$(x, y) \rightarrow (x - 5, y + 3)$

The lengths of line segments and the sizes of angles do not change in a translation. The original figure and its image have the same sense.

Example

$\triangle RST$ has vertices R(–3, –2), S(1, –3), and T(0, 4). Use the mapping $(x,y) \rightarrow (x-2,y+4)$ to draw the translation image of $\triangle RST$.

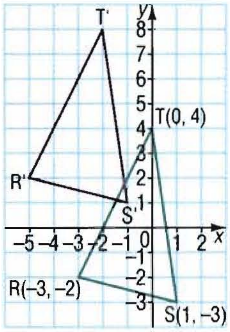
Solution

Plot $\triangle RST$ and find the coordinates of the vertices of its image.

$$(x,y) \rightarrow (x-2,y+4)$$

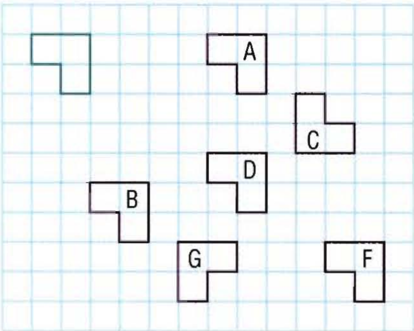
For R: $(-3,-2) \rightarrow (-3-2,-2+4)$ so R′ is (–5, 2)
For S: $(1,-3) \rightarrow (1-2,-3+4)$ so S′ is (–1, 1)
For T: $(0,4) \rightarrow (0-2,4+4)$ so T′ is (–2, 8)

Plot R′, S′, and T′ and join them.
$\triangle R'S'T'$ is the translation image of $\triangle RST$.



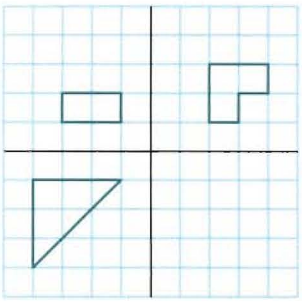
Practice

1. Which of the lettered figures are translation images of the green figure? Give reasons for your answers.

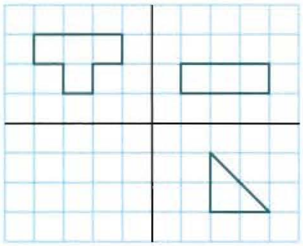


2. Plot each set of figures on a grid. Graph the image of each figure under the given translation.

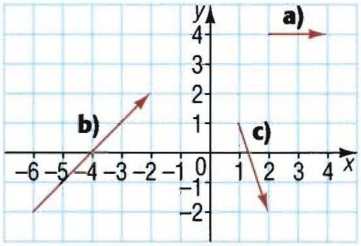
a) 4 units right and 3 units up



b) 2 units left and 5 units down



3. State the translation described by each arrow. Write your answers in the form $[x,y]$.



Describe each translation in words.

4. $(x,y) \rightarrow (x+3,y+2)$

5. $(x,y) \rightarrow (x-1,y+4)$

6. $(x,y) \rightarrow (x-2,y-3)$

7. $(x,y) \rightarrow (x+5,y-1)$

8. $(x,y) \rightarrow (x,y+6)$ **9.** $(x,y) \rightarrow (x-3,y)$

10. $[2,-3]$ **11.** $[-3,5]$ **12.** $[-4,2]$ **13.** $[1,-6]$

CONTINUED

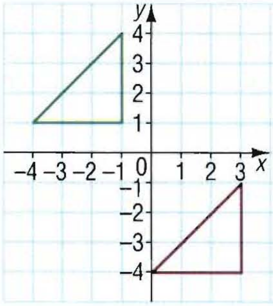
Draw an arrow on grid paper to show each translation.

14. [2, −3] **15.** [−1, −5] **16.** [1, −5] **17.** [5, 0]

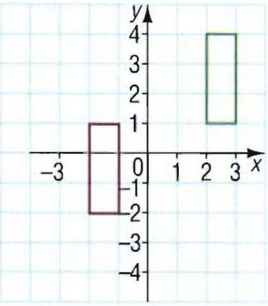
18. [3, −2] **19.** [−2, 3] **20.** [0, −5] **21.** [5, −4]

Name the translation that maps each green figure onto its purple image.

22.



23.



Find the coordinates of each point after the translation.

	Point	Translation
24.	(2, 3)	[3, 4]
25.	(−3, −1)	[2, −2]
26.	(5, −2)	[−1, 5]
27.	(−5, 7)	[−2, −6]

Express each translation as an ordered pair.

28. A(3, 5) to A′(7, 9)

29. C(2, 1) to C′(3, 0)

30. X(−2, −4) to X′(−3, −5)

31. M(0, −1) to M′(4, −6)

32. T(−1, −3) to T′(−6, 5)

33. P(0, 4) to P′(−1, −2)

Draw each triangle on grid paper. Then, draw the translation image.

34. A(3, 2), B(−1, 4), C(−3, −5)

$(x, y) \rightarrow (x - 4, y + 2)$

35. D(−2, 0), E(4, −1), F(2, −5)

$(x, y) \rightarrow (x + 3, y - 3)$

36. R(0, 0), S(−4, 0), T(−3, −5)

$(x, y) \rightarrow (x + 2, y + 3)$

Problems and Applications

37. The translation [2, 3] translates △ABC so that the coordinates of the vertices of the image, △A′B′C′, are A′(3, 4), B′(0, 7), and C′(−2, −1). What are the coordinates of A, B, and C?

38. a) △RST has vertices R(3, 0), S(−1, 5), and T(−3, 1). Draw △RST on grid paper.

b) Determine the coordinates of the vertices of △R′S′T′ under the translation $(x, y) \rightarrow (x + 4, y + 2)$. Draw △R′S′T′.

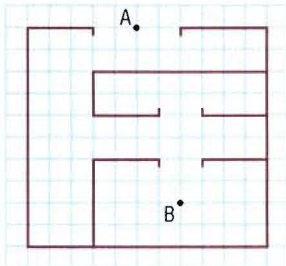
c) Apply the translation $(x, y) \rightarrow (x + 1, y - 5)$ to △R′S′T′ and find the coordinates of the vertices of the image, △R″S″T″. Draw △R″S″T″.

d) Determine the translation that maps △RST onto △R″S″T″.

39. a) Draw the line $y = 2x + 1$ on grid paper.

b) Draw the image of the line after the translation $(x, y) \rightarrow (x + 2, y + 5)$.

40. What translations are needed to get a robot at point A to move an object from point B to point A?



41. a) Draw △XYZ so that it lies entirely in the first quadrant. Name the coordinates of X, Y, and Z.

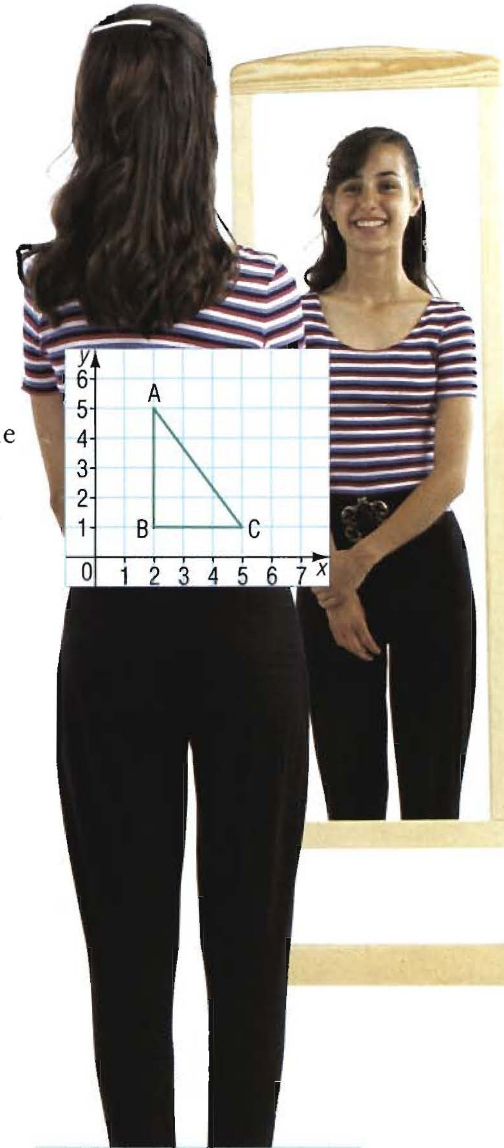
b) Name a translation that gives an image, △X′Y′Z′, that lies entirely in the fourth quadrant.

c) Name a translation of △X′Y′Z′ that gives an image, △X″Y″Z″, that lies entirely in the second quadrant.

d) Give a classmate the coordinates of X, Y, and Z, and the names of both translations. Have your classmate find the coordinates of X″, Y″, and Z″.

8.2 Reflections

When you stand in front of a mirror, you see an image of yourself, your reflection. If you move closer to the mirror, the image moves closer. If you move back, the image moves back. The image is always the same distance from the mirror as you are. In mathematics, a **reflection** is a transformation in which a figure is reflected or flipped over a **mirror line** or **reflection line**.



Activity: Draw a Reflection Image

Plot △ABC with coordinates A(2, 5), B(2, 1), and C(5, 1). Reflect point A in the *y*-axis to locate point A′ so that A and A′ are the same perpendicular distance from the *y*-axis. Reflect points B and C in the *y*-axis to locate B′ and C′. Join A′, B′, and C′ to form a triangle.

Inquire

1. What are the coordinates of A′, B′, and C′?

2. △A′B′C′ is the **reflection image** of △ABC. How do the side lengths in △A′B′C′ compare with the side lengths in △ABC?

3. Are the two triangles congruent? Explain.

4. What is true about the measures of the angles after a reflection?

5. Do △ABC and △A′B′C′ have the same sense? Explain.

6. Describe a reflection in your own words.

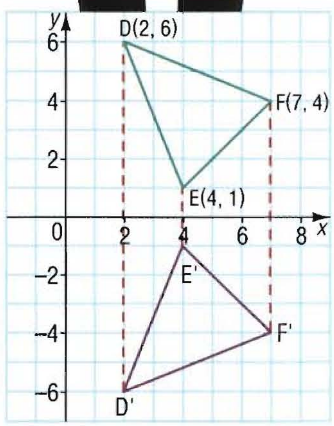
Example

△DEF has vertices D(2, 6), E(4, 1), and F(7, 4). Draw the image of △DEF after a reflection in the *x*-axis.

Solution

Draw △DEF.
Locate D′ so that the perpendicular distance from D′ to the *x*-axis equals the perpendicular distance from D to the *x*-axis. The coordinates of D′ are (2, –6). Locate E′ and F′ in the same way.

D(2, 6) → D′(2, –6)
E(4, 1) → E′(4, –1)
F(7, 4) → F ′(7, –4)



Join points D′, E′, and F′. △D′E′F′ is the image of △DEF after a reflection in the *x*-axis.

The lengths of line segments and the sizes of angles do not change in a reflection. The sense of a reflection image is the reverse of the sense of the original figure.

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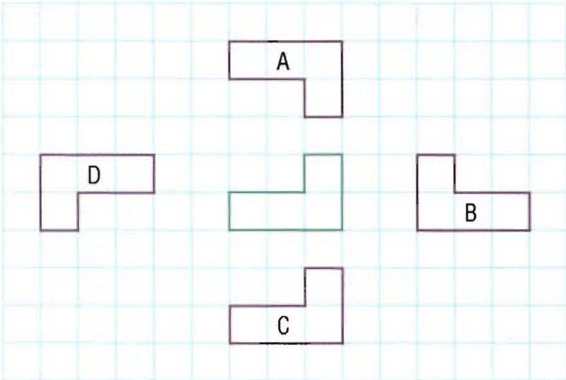
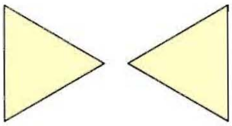
Practice

1. Which of the lettered figures are reflections of the green figure? Give reasons for your answer.

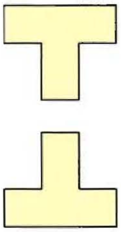


Trace each diagram. Use a Mira or paper folding to locate the reflection line.

2.



3.



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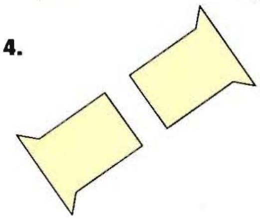
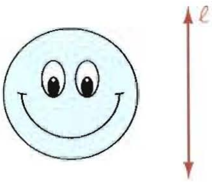
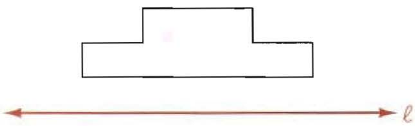


In each diagram, the reflection line is l. Trace each diagram and use a Mira or paper folding to draw the reflection image.

5.



6.



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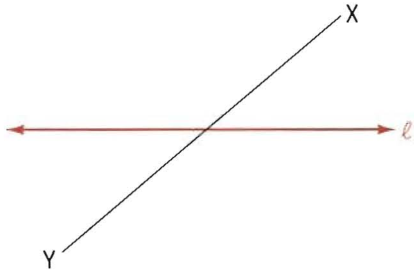
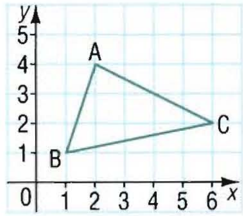


Copy each figure onto a grid and draw its reflection image in the x-axis.

8.



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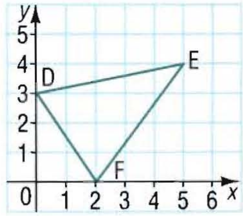
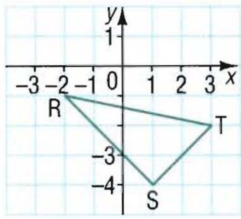
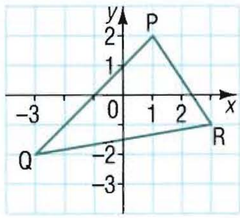


10.

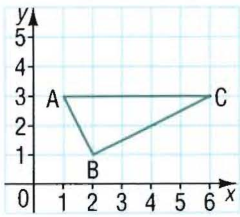


11.

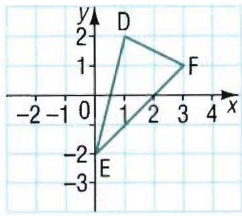


Draw each triangle on a grid. Draw the image after a reflection in the y-axis.

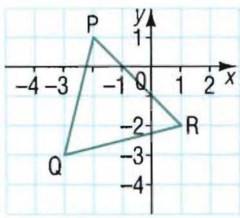
12.



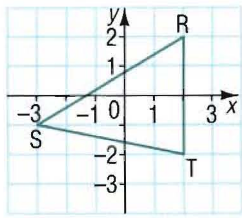
13.



14.



15.



16. Find the coordinates of the image of each point after a reflection in each axis.

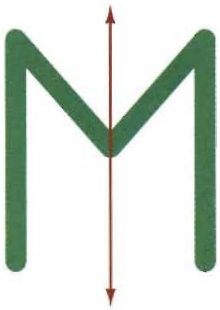
		Reflection Line	
	Point	x-axis	y-axis
a)	(1, 4)		
b)	(2, 3)		
c)	(−1, −2)		
d)	(−3, −2)		
e)	(−3, 2)		
f)	(4, 0)		

17. $\triangle ABC$ has vertices A(1, 1), B(5, 2), and C(3, 6). Draw the image of $\triangle ABC$ after a reflection in the y-axis.

18. $\triangle RST$ has vertices R(2, 5), S(−2, 4), and T(−1, −2). Draw the image of $\triangle RST$ after a reflection in the x-axis.

Problems and Applications

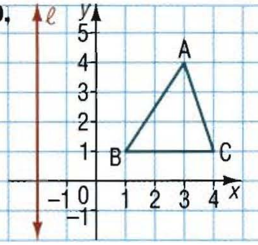
19. The letter M is a reflection of itself through the vertical red reflection line.



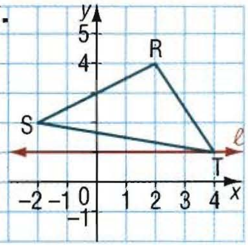
a) What other letters are reflections of themselves through vertical reflection lines?
b) What letters are reflections of themselves through horizontal reflection lines?
c) What letters are reflections of themselves through horizontal and vertical reflection lines?
d) Which of the ten digits are reflections of themselves?
e) Are you a reflection of yourself through a vertical reflection line? Explain.

Draw the triangles on a grid. Draw the image after a reflection in the given reflection line.

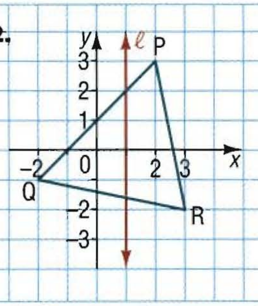
20.



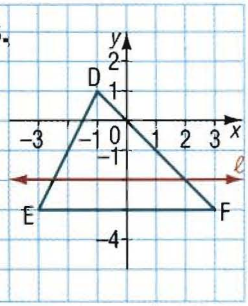
21.



22.



23.



24. The coordinates of the vertices of $\triangle DEF$ are D(1, 5), E(−2, −2), and F(5, 1). Draw $\triangle DEF$ and its image after a reflection in the line $y = -1$.

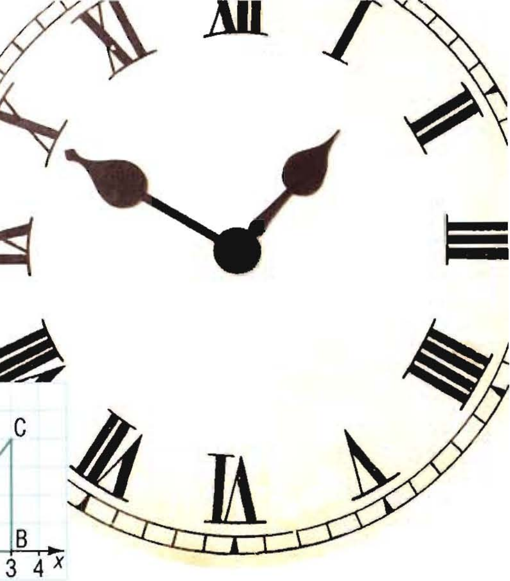
25. a) $\triangle PQR$ has vertices P(−5, 6), Q(−7, 2), and R(−1, 1). Draw $\triangle PQR$ on grid paper.
b) Reflect $\triangle PQR$ in the y-axis and draw the image $\triangle P'Q'R'$.
c) Reflect $\triangle P'Q'R'$ in the x-axis and draw the image $\triangle P''Q''R''$.

26. Which coordinate of a point does not change when the point is reflected
a) in the x-axis? **b)** in the y-axis?

27. a) Draw a graph of the line $x + y = 4$.
b) Reflect the line in the x-axis.
c) Reflect the line in the y-axis.

28. $\triangle A''B''C''$ has vertices A″(3, 4), B″(2, 1), and C″(1, 3). $\triangle A''B''C''$ is an image obtained from the translation $(x, y) \rightarrow (x + 1, y - 2)$ on $\triangle ABC$, followed by a reflection in the y-axis. Work with a classmate to find the coordinates of the vertices of the original triangle, $\triangle ABC$.

8.3 Rotations

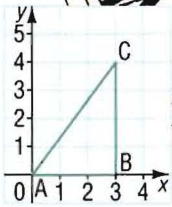


The hands of this antique clock turn or rotate about a point. This point is the centre of the clock face.

In mathematics, a **rotation** is a transformation in which a figure is turned or rotated about a point.

Activity: Draw a Rotation Image

Plot △ABC with coordinates A(0, 0), B(3, 0), and C(3, 4). Use tracing paper to rotate △ABC 90° counterclockwise (ccw) about the origin. Draw the **rotation image** of △ABC and name it △A′B′C′.



Inquire

1. What are the coordinates of A′, B′, and C′?

2. How do the lengths of the sides of △ABC compare with the lengths of the sides of △A′B′C′?

3. Are the two triangles congruent? Explain.

4. What is true about the measures of the angles after a rotation?

5. Do △ABC and △A′B′C′ have the same sense? Explain.

6. A 90° rotation is called a $\frac{1}{4}$ turn.

a) What is a 180° rotation called?

b) What is a 270° rotation called?

7. Describe a rotation in your own words.

To rotate an object and find its image, you need to know the amount and direction of the rotation. You also need to know the **centre of rotation** or **turn centre**.

Example

△RST has vertices R(−5, 0), S(−1, 0), and T(−2, −3). Draw the image of △RST after a rotation of 180° clockwise (cw) about the origin.

Solution

Draw △RST. Find the rotation image of each vertex.

R(−5, 0) → R′(5, 0)

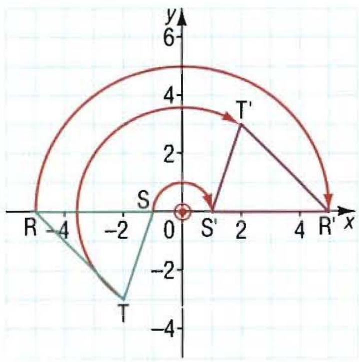
S(−1, 0) → S′(1, 0)

T(−2, −3) → T′(2, 3)

Join points R′, S′, and T′.

△R′S′T′ is the rotation image of △RST.

⊙ marks the turn centre

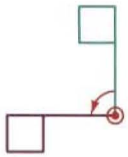


The lengths of line segments and the sizes of angles do not change in a rotation. The original figure and its image have the same sense.

Practice

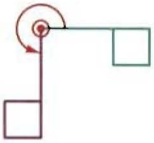
The green flag has been rotated about the origin. The purple flag is the image. Name the rotation.

1.



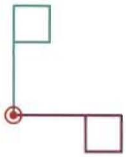
2.

3.



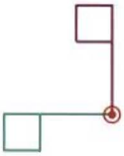
The green flag has been rotated about the origin. The purple flag is the image. Name 2 rotations for each case.

4.



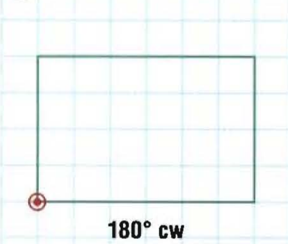
5.

6.

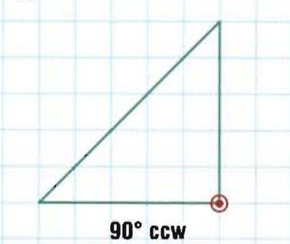


Copy each figure onto grid paper and draw the image after the given rotation about the red turn centre.

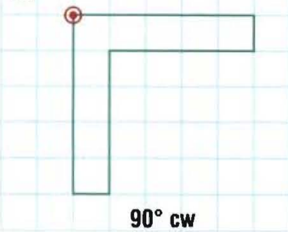
7.



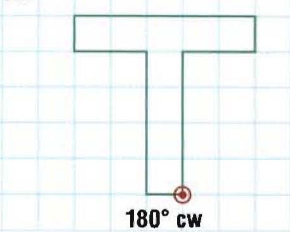
8.



9.

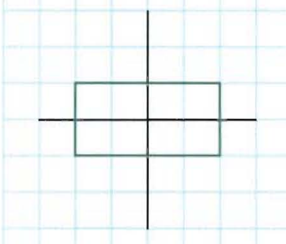


10.

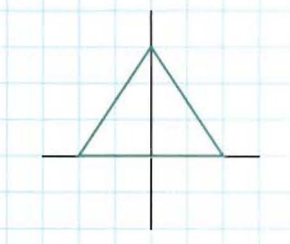


Draw the image of each figure after a 90° clockwise rotation about the origin.

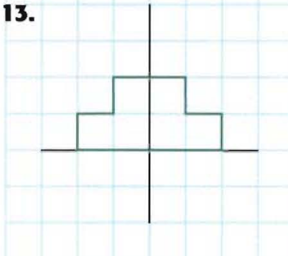
11.



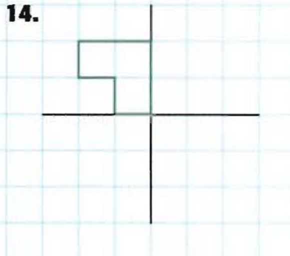
12.



13.

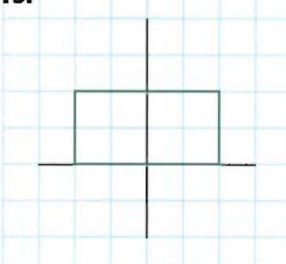


14.

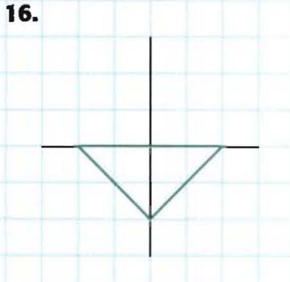


Draw the image of each figure after a 180° counterclockwise rotation about the origin.

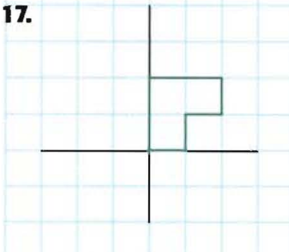
15.



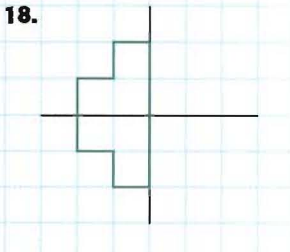
16.



17.



18.



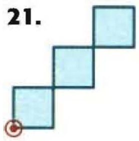
19. △DEF has vertices D(0, 0), E(−4, 2), and F(0, 6). Draw the image of △DEF after a 90° counterclockwise rotation about the origin.

20. △ABC has vertices A(1, 0), B(3, 4), and C(3, 0). Draw the image of △ABC after a 90° clockwise rotation about the origin.

CONTINUED

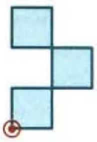
Copy each figure and find the image after the given rotation about the red turn centre.

21.



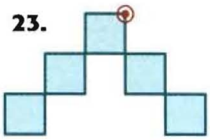
90° cw

22.



90° ccw

23.



180° ccw

Copy and complete the table with the image of each rotation about the origin.

		Rotation		
	Point	90° ccw	180° cw	270° ccw
24.	(0, 6)			
25.	(–4, 0)			
26.	(0, –3)			
27.	(3, 4)			
28.	(–4, 3)			

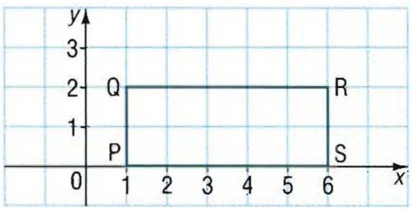
Problems and Applications

29. Rectangle ABCD has vertices A(1, 0), B(1, 2), C(6, 2), and D(6, 0). Draw the image of the rectangle after a 180° counterclockwise rotation about the origin.

30. Parallelogram ABCD has vertices A(–1, 0), B(–2, 2), C(–6, 2), and D(–5, 0). Draw the image of the parallelogram after a 90° clockwise rotation about the origin.

31. Copy the figure onto grid paper and rotate it 90° clockwise about

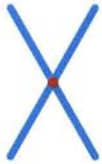
a) point P **b)** the origin



32. The parallelogram PQRS has vertices P(1, 0), Q(3, 3), R(8, 3), and S(6, 0). Draw the image of the parallelogram after a 90° clockwise rotation about S. What sort of figure is the image?

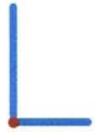
33. Rotate each letter by a $\frac{1}{2}$ turn clockwise about the indicated point. Which ones make readable letters when rotated?

a)

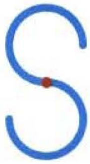


b)

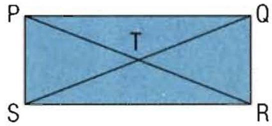
c)



d)



34. This figure is to be rotated so that its image lies exactly on top of the original figure. Which point(s) named in the figure could be the turn centre if



a) a 180° rotation is used?

b) a 360° rotation is used?

35. Plot the line $x + y = 2$ on a grid and rotate it 90° clockwise about the origin. At what angle do the line and its image intersect?

36. a) Plot the line $y = x - 3$ on a grid. Rotate the line 180° about the origin. How are the line and its image related?

b) Repeat part a) for the line $y = x$.

c) Explain why your findings in parts a) and b) are different.

37. a) Name another clockwise rotation that will give the same image of a figure as a 90° clockwise rotation about the origin.

b) Name a counterclockwise rotation that will give the same image of the figure as in part a).

38. The stars in the sky appear to rotate 360° about a point every day. What is that point?

39. List 3 examples of real objects that rotate about a turn centre. Compare your list with a classmate's. Choose objects that have not been mentioned in this section.

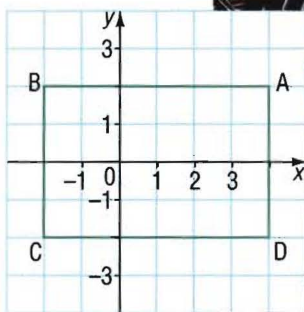
8.4 Dilatations

Lewis Carroll's *Alice's Adventures in Wonderland* and *Through the Looking Glass* are considered the most famous children's books written in English. In both books, the story is a dream in which Alice changes size and encounters fantastic creatures.

A transformation that changes the size of an object is called a **dilatation**. Dilatations are called **enlargements** or **reductions**, depending on the way in which the size is changed.

Activity: Draw a Dilatation Image

Plot rectangle ABCD with coordinates A(4, 2), B(–2, 2), C(–2, –2), and D(4, –2). Multiply the coordinates of A by 2 to give its image A′(8, 4). Multiply the coordinates of the other vertices by 2 to give B′, C′, and D′. Join A′, B′, C′, and D′ to make rectangle A′B′C′D′, which is the dilatation image of rectangle ABCD.



Inquire

1. How do the side lengths of rectangle A′B′C′D′ compare with the side lengths of rectangle ABCD?

2. How do the ratios $\frac{AB}{A'B'}$, $\frac{BC}{B'C'}$, $\frac{CD}{C'D'}$, and $\frac{AD}{A'D'}$ compare?

3. How do the angles of rectangle A′B′C′D′ compare with the angles of rectangle ABCD?

4. Calculate the area of each rectangle. How do the areas compare?

5. Do rectangle ABCD and its image, rectangle A′B′C′D′, have the same sense?

6. a) Draw a line from the origin, (0, 0), through A and extend it. Does it pass through A′?

b) Does a straight line pass through the origin, B, and B′? the origin, C, and C′? the origin, D, and D′?

7. We say that rectangle ABCD has been enlarged by a dilatation with **centre** (0, 0) and a **scale factor** of 2. Describe a dilatation of this type in your own words.

8. Repeat this activity by dividing each coordinate of rectangle ABCD by 2 to give the coordinates of the image rectangle A″B″C″D″. How do the lengths of the sides, the sizes of the angles, and the area compare with those of rectangle ABCD?

The dilatation with centre (0, 0) and scale factor k can be described mathematically as a mapping.

$(x, y) \rightarrow (kx, ky)$

When $k > 1$, the mapping gives an enlargement.

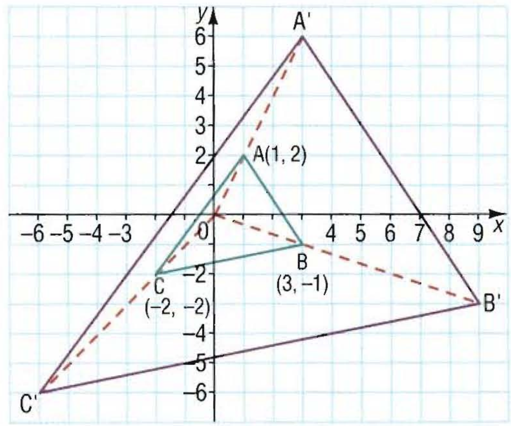
When $k < 1$, the mapping gives a reduction.

The image and the original figure are **similar**.

They have the same shape, but not the same size.

Example

△ABC has vertices A(1, 2), B(3, −1), and C(−2, −2).
Find the image of △ABC under the mapping $(x, y) \rightarrow (3x, 3y)$.



Solution

Draw △ABC.

For $(x, y) \rightarrow (3x, 3y)$

$A(1, 2) \rightarrow A'(3, 6)$

$B(3, -1) \rightarrow B'(9, -3)$

$C(-2, -2) \rightarrow C'(-6, -6)$

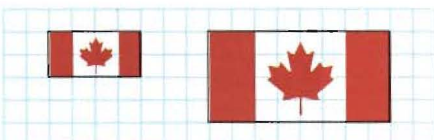
Plot A′, B′, and C′ and join them.

△A′B′C′ is the image of △ABC.

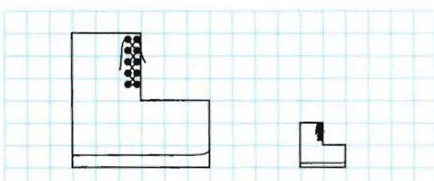
Practice

A figure is shown with its image to the right. What is the scale factor?

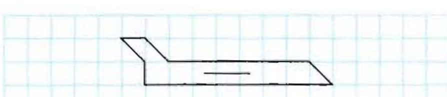
1.



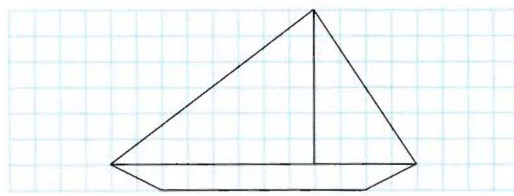
2.



3. Copy the figure onto grid paper and enlarge it by a scale factor of 2.



4. Copy the figure onto grid paper and reduce it by a scale factor of $\frac{1}{2}$.



Draw the dilatation image of each line segment under the given mapping.

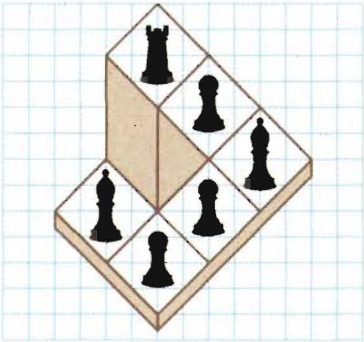
	Line Segment	Mapping
5.	A(3, 2), B(1, 4)	$(x, y) \rightarrow (2x, 2y)$
6.	C(6, 4), D(−2, 2)	$(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$
7.	E(−1, −1), F(1, 2)	$(x, y) \rightarrow (3x, 3y)$
8.	G(9, 3), H(−6, 0)	$(x, y) \rightarrow (\frac{1}{3}x, \frac{1}{3}y)$

9. △RST has vertices R(2, 3), S(−1, 4), and T(−3, −2). Find the image of △RST under the mapping $(x, y) \rightarrow (3x, 3y)$.

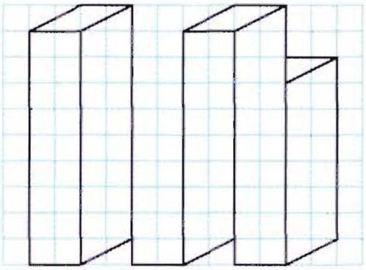
10. Quadrilateral DEFG has vertices D(6, 4), E(–2, 6), F(–4, –4), and G(4, –6). Find the image of quadrilateral DEFG under the mapping $(x,y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$.

Problems and Applications

11. Copy this piece of an impossible chessboard onto grid paper and enlarge it by a scale factor of 2.



12. Copy this disappearing block puzzle onto grid paper and reduce it by a scale factor of $\frac{1}{2}$.



13. △PQR has vertices P(–2, 2), Q(–2, –2), and R(2, –2).

a) Draw △PQR on grid paper.
b) Calculate the area of △PQR.
c) Find the image of △PQR, △P′Q′R′, under the mapping $(x,y) \rightarrow (3x, 3y)$.
d) Calculate the area of △P′Q′R′.
e) Find the image of △PQR, △P″Q″R″, under the mapping $(x,y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$.
f) Calculate the area of △P″Q″R″.
g) Write the following ratios.
area △P′Q′R′ : area △PQR
area △P″Q″R″ : area △PQR
h) How is each ratio related to the scale factor that produced the image?

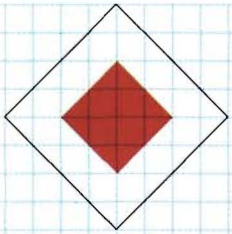
14. a) State the types of dilatations you see in:

- a map of a province
- a movie screen
- a television screen
- a blueprint of a house

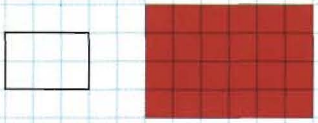
b) Find 2 other examples of enlargements or reductions in real-world objects. Estimate the scale factors involved.

In questions 15–17, copy the figure and its red image onto grid paper. Find the dilatation centre and state the scale factor.

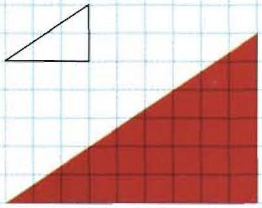
15.



16.



17.



18. What would the image of a triangle look like under this mapping?
$(x,y) \rightarrow (1x, 1y)$

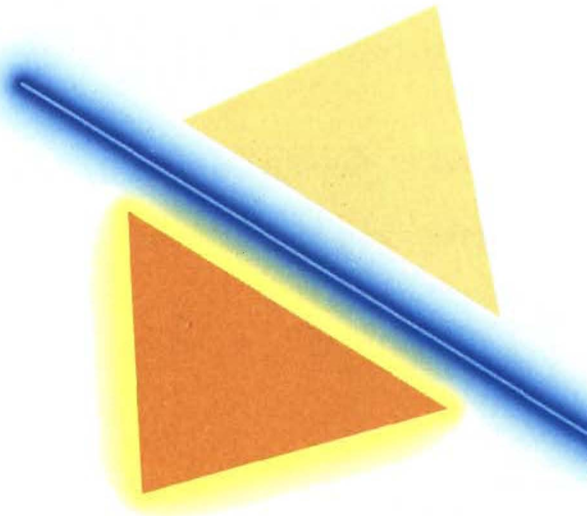
19. △ABC has vertices A(4, 2), B(1, –2), and C(8, –3).

a) Draw △ABC on grid paper.
b) Find the image of △ABC for a dilatation with centre (4, 0) and a scale factor of 2.

20. Use a figure of your choice and investigate these mappings. Describe your findings.

a) $(x,y) \rightarrow (2x, 4y)$
b) $(x,y) \rightarrow (-2x, -2y)$

Transformations with Geometry Software



Complete the following activities with a geometry software package. If you do not have suitable software, use grid paper.

Activity ❶ Reflections and Coordinates

1. Draw △ABC with vertices A(2, 3), B(4, 1), and C(1, 2).

2. Reflect △ABC in the x-axis and determine the coordinates of the vertices of its image.

3. Reflect △ABC in the y-axis and determine the coordinates of the vertices of its image.

4. Copy and complete the table for reflections of △ABC and the other triangles in each axis.

		Coordinates after Reflection	
Triangle	**Original Coordinates**	**in x-axis**	**in y-axis**
△ABC	A(2, 3), B(4, 1), C(1, 2)		
△DEF	D(−1, 4), E(−4, 3), F(−2, 1)		
△PQR	P(−2, −3), Q(−1, −4), R(−3, −5)		
△XYZ	X(3, −1), Y(2, −4), Z(4, −3)		

5. Write a rule for finding the coordinates of the point (x, y) after a reflection in

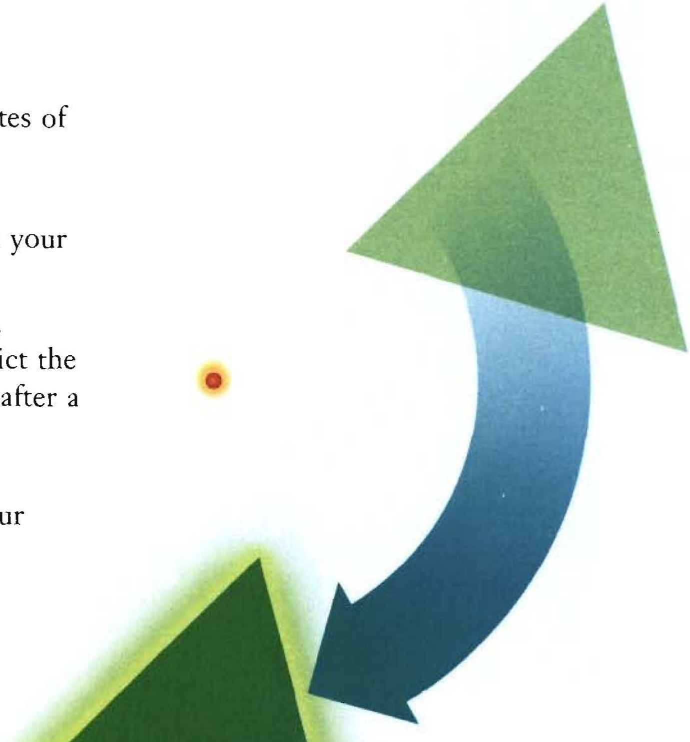
a) the x-axis **b)** the y-axis

6. Are any x- and y-values exceptions to your rule? Explain.

7. Rectangle ABCD has vertices A(0, 3), B(−4, 3), C(−4, −2), and D(0, −2). Predict the coordinates of the vertices of its image after a reflection in

a) the x-axis **b)** the y-axis

8. Carry out the reflections to check your predictions.



Activity ❷ Combined Reflections

1. For each triangle in Activity 1, carry out a reflection in the *x*-axis, then reflect the image in the *y*-axis. Determine the coordinates of the final image. Tabulate your findings.

Triangle	Original Coordinates	Coordinates after Combined Reflection in Both Axes
△ABC	A(2, 3), B(4, 1), C(1, 2)	
△DEF	D(−1, 4), E(−4, 3), F(−2, 1)	
△PQR	P(−2, −3), Q(−1, −4), R(−3, −5)	
△XYZ	X(3, −1), Y(2, −4), Z(4, −3)	

2. Write a rule for finding the coordinates of the point (x, y) after a combined reflection in both axes.

3. Are any x- and y-values exceptions to your rule? Explain.

4. Is your rule affected by the order in which the two reflections are carried out? Explain.

5. Quadrilateral WXYZ has vertices W(1, 3), X(−5, 0), Y(−3, −2), and Z(0, −2). Predict the coordinates of the vertices of its image after a combined reflection in both axes. Then, carry out the combined reflection to check your predictions.

Activity ❸ 180° Rotations

1. For each of the triangles in Activity 1, carry out a 180° rotation, clockwise or counterclockwise, about the origin. Determine the coordinates of the final image. Tabulate your findings.

Triangle	Original Coordinates	Coordinates after 180° Rotation about the Origin
△ABC	A(2, 3), B(4, 1), C(1, 2)	
△DEF	D(−1, 4), E(−4, 3), F(−2, 1)	
△PQR	P(−2, −3), Q(−1, −4), R(−3, −5)	
△XYZ	X(3, −1), Y(2, −4), Z(4, −3)	

2. How do the results of a 180° rotation about the origin compare with the results of a combined reflection in both axes?

3. Quadrilateral PQRS has vertices P(0, 0), Q(3, 4), R(−2, 5), and S(−4, −1). Predict the coordinates of the vertices of its image after a 180° rotation about the origin. Then, carry out the rotation to check your predictions.

Activity ❹ Using the Results

1. Without carrying out the transformations, predict the coordinates of the vertices of the following images.

a) △KLM with vertices K(1, −4), L(−2, −5), M(3, 2), following a reflection in the *y*-axis

b) △FGH with vertices F(4, 0), G(3, −3), H(−1, 5), following a reflection in the *x*-axis

c) △STU with vertices S(3, −5), T(2, 2), U(−4, −1), following a combined reflection in both axes

d) △QRS with vertices Q(−2, −2), R(−4, 3), S(0, 0), following a 180° rotation about the origin

2. State the coordinates of the vertices of a rectangle whose image after a combined reflection in both axes lies exactly on the original rectangle. Compare your rectangle with that of a classmate.

8.5 Symmetry

A **line of symmetry** is a mirror line that reflects an object onto itself. Line symmetry is also called **reflectional symmetry** or **mirror symmetry**. Insects, flowers, and many other natural objects have lines of symmetry.

Describe where the line of symmetry is on the ladybug.

Activity: Draw Lines of Symmetry

When water drops are sprinkled onto a dry, hot skillet, the drops dance across the skillet. The drops take many different shapes. A few of them are shown. Trace the water drops and draw all their lines of symmetry. You may want to use a Mira.

Inquire

1. How many lines of symmetry does each water drop have?

2. How many lines of symmetry does a square have?

3. How many lines of symmetry does an isosceles triangle have? an equilateral triangle?

4. List some objects in the classroom that have several lines of symmetry.

A figure that can be mapped onto itself with a turn of less than one complete rotation has **rotational symmetry** or **turn symmetry**. The number of times the figure matches with itself in a turn of 360° is the **order** of rotational symmetry. Rotational symmetry of order 2 is also called **point symmetry**.

Example

Determine the order of turn symmetry of water drop 2.

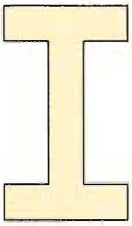
Solution

Trace the water drop and put the tracing on top of the original. Put a pencil point or pen point on the centre and turn the tracing. Count the number of times the tracing matches the original in a 360° turn.
The tracing matches 3 times.
The water drop has turn symmetry of order 3.

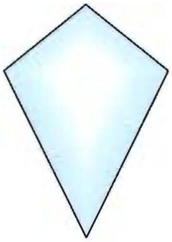
Practice

How many lines of symmetry does each figure have?

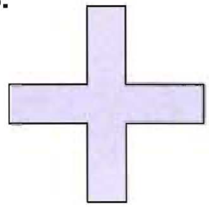
1.



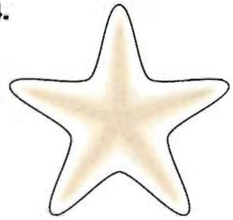
2.



3.



4.

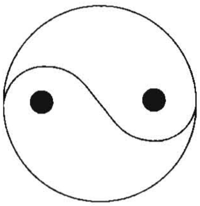


What is the order of turn symmetry for each figure?

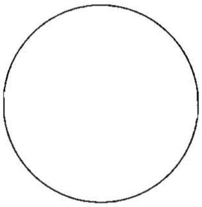
5.

6.

7.



8.



Problems and Applications

9. Print the capital letters of the alphabet.

a) Which letters have a vertical line of symmetry?

b) Which letters have a horizontal line of symmetry?

c) Which letters have both a vertical and a horizontal line of symmetry?

d) Which of the letters have point symmetry?

Trace each figure. Then, add enough parts so that it is symmetric about the red line.

10.

11.

Trace each figure. Then, add parts to each of the other 3 quadrants so that the figure has

a) *1 line of symmetry*

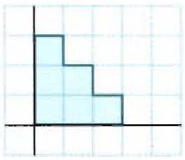
b) *2 lines of symmetry*

c) *no lines of symmetry*

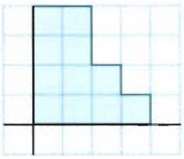
d) *rotational symmetry of order 4*

e) *rotational symmetry of order 2*

12.



13.



14. A double six set of dominoes uses seven patterns from "blank" to six. The set starts with the double blank domino and goes to the double six domino.

In a double six domino set, how many dominoes have

a) no lines of symmetry?

b) 1 line of symmetry?

c) 2 lines of symmetry?

d) rotational symmetry of order 2?

15. Many company logos have lines of symmetry and rotational symmetry. Find examples of company logos with symmetry. Copy them into your notebook and record the type of symmetry they have. Why do logo designers use symmetry?

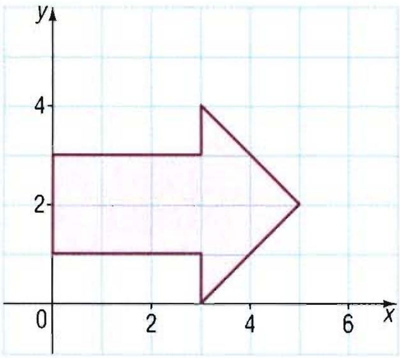
16. Work with a classmate to find flags of the world that have lines of symmetry. Sketch the flags.

Distortions on a Grid

When you stand in front of a mirror in a fun house, your image is distorted. When you distort a figure in mathematics, you may stretch, shrink, and turn it in many directions.

Activity 1

1. Draw the arrow on grid paper.



Find the image of the arrow under the following mappings.

a) $(x,y) \rightarrow (2x,y)$

b) $(x,y) \rightarrow (x,2y)$

Describe how each image has been distorted from the original arrow.

2. A kite ABCD has vertices A(0, 0), B(2, 4), C(0, 6), and D(–2, 4). Draw the kite on grid paper, then draw the images of the kite under the following mappings.

a) $(x,y) \rightarrow (3x,y)$

b) $(x,y) \rightarrow (x,2y)$

c) $(x,y) \rightarrow \left(x,\frac{y}{2}\right)$

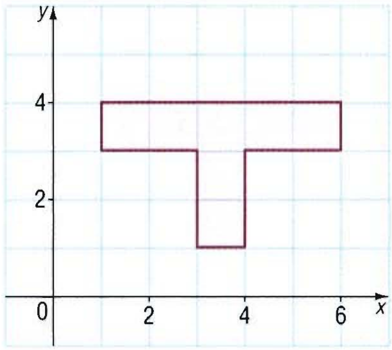
d) $(x,y) \rightarrow (2x,2y)$

Does each distortion give an image that is also a kite?

3. Draw a square with vertices (3, 3), (–3, 3), (–3, –3), and (3, –3). If you draw the square under each of the mappings you used for the kite in question 2, is the image always a square?

Activity 2

Draw the T on grid paper.



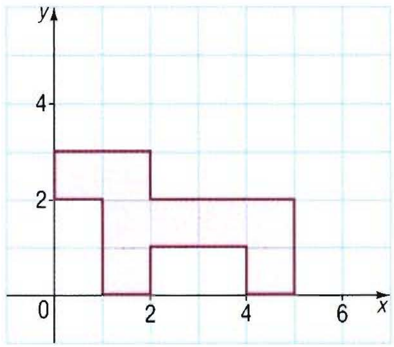
Draw the images of the T under the following mappings.

a) $(x,y) \rightarrow (-x,2y)$ **b)** $(x,y) \rightarrow (2x,-y)$

Describe how each image has been distorted from the original T-shape.

Activity 3

Draw the figure on grid paper.



Draw the images of the figure under the following mappings.

a) $(x,y) \rightarrow \left(\frac{x}{2},3y\right)$ **b)** $(x,y) \rightarrow \left(3x,\frac{y}{2}\right)$

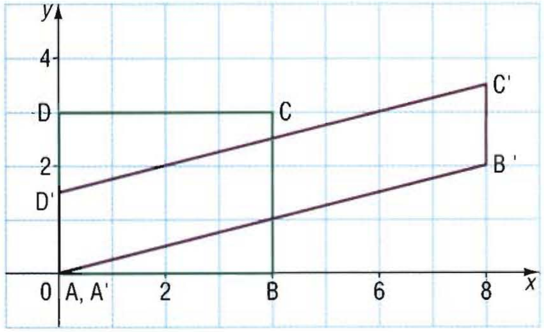
Describe how each image has been distorted from the original figure.

Activity ❹

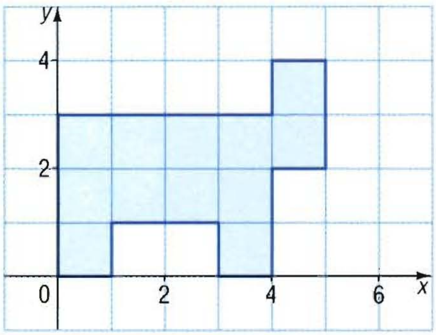
The rectangle ABCD has been transformed under the mapping $(x,y) \rightarrow \left(2x, \frac{x+y}{2}\right)$.

$$(x,y) \rightarrow \left(2x, \frac{x+y}{2}\right)$$

$$A(0,0) \rightarrow A'(0,0)$$
$$B(4,0) \rightarrow B'(8,2)$$
$$C(4,3) \rightarrow C'(8,3.5)$$
$$D(0,3) \rightarrow D'(0,1.5)$$



1. Draw the following figure on grid paper.



Draw the image of the figure under this mapping.

$$(x,y) \rightarrow \left(2x, \frac{x+y}{2}\right)$$

2. Draw the image of the same figure under this mapping.

$$(x,y) \rightarrow \left(\frac{x+y}{2}, 2y\right)$$

3. Redraw the figure away from the axes and apply the mappings from steps 1 and 2 of this activity to the figure.

Activity ❺

1. With any combinations of the mappings you have used, write your own mapping that will distort a figure in an unusual way.

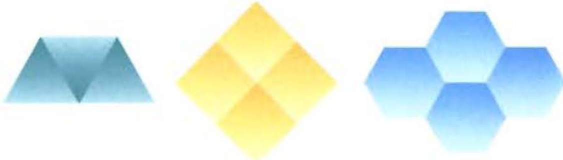
2. Have a classmate use your mapping to distort a figure.

Escher Drawings

Islamic artists use tessellations in much of their work. A **tessellation** or tiling is an arrangement of shapes that completely covers the plane without overlapping or leaving gaps. The Dutch artist M.C. Escher became fascinated with tile patterns he found in Spain. Escher used translations, rotations, and reflections to make tessellations of animals and humans. This Escher tessellation is based on translations in parallelograms.

Activity ❶ Geometric Figures that Tessellate

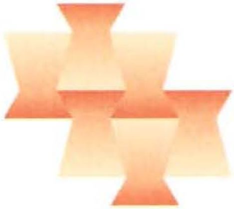
Only 3 regular polygons will tile the plane: an equilateral triangle, a square, and a regular hexagon. However, many irregular figures will tile the plane.



1. Use grid paper to show how an irregular triangle tiles the plane.

2. Draw an example of an irregular quadrilateral that tiles the plane.

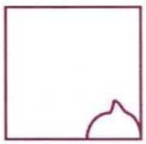
3. The diagram shows one example of an irregular hexagon that tiles the plane. Draw another example.



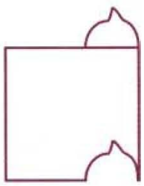
Activity ❷ Tessellations Using Translations

1. You can alter the opposite sides of a square and use translations to tile the plane.

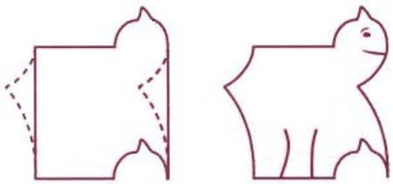
a) Make an alteration to one side.



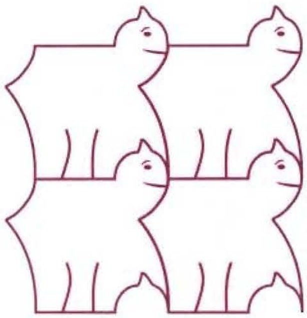
b) Translate the alteration to the opposite side.



c) Alter the other sides in a similar way.



d) Use the new figure to tile the plane.

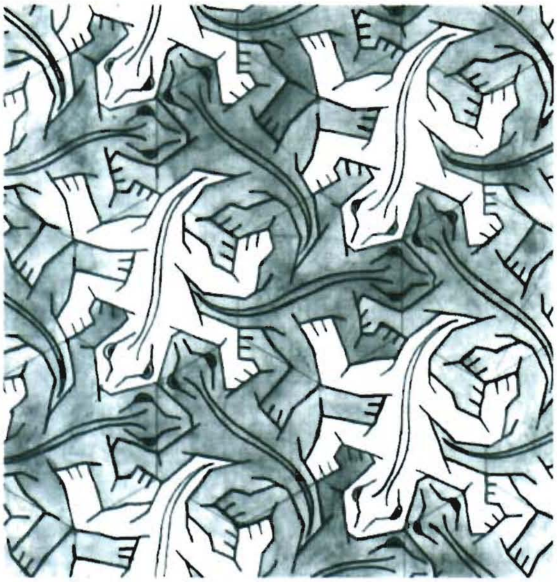


2. Alter the opposite sides of a parallelogram and use translations to tile the plane.

3. Alter the opposite sides of a hexagon and use translations to tile the plane.

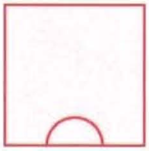
Activity ❸ Tessellations Using Rotations

This Escher tessellation is based on rotations in regular hexagons.

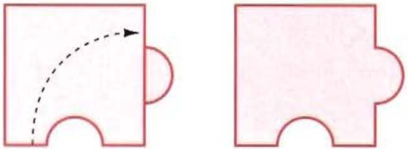


1. You can alter the sides of a square and use rotations to tile the plane.

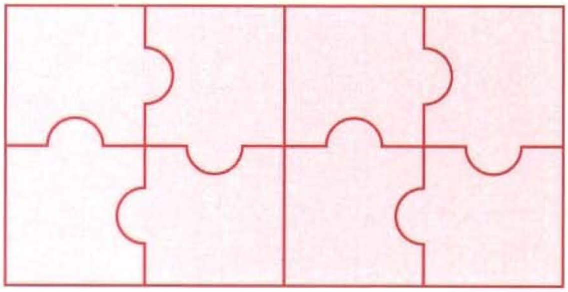
a) Make an alteration to one side.



b) Rotate the alteration.



c) Use the new figure to tile the plane.



2. Alter the sides of a rhombus and use rotations to tile the plane.

3. Alter the sides of a triangle and use rotations to tile the plane.

Activity ❹ Impossible Figures

Escher also used impossible geometric figures, such as the one to the right, in his art.

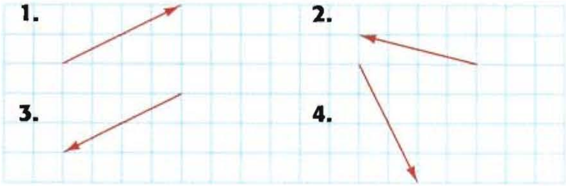
1. What are impossible figures?

2. Research other examples of Escher's art that include these figures.

Review

State each translation arrow as an ordered pair.

1. **2.** **3.** **4.**



Describe each translation in words.

5. $(x,y) \rightarrow (x+2, y+3)$

6. $(x,y) \rightarrow (x-3, y-1)$

7. $[-4, 5]$

8. $[0, 6]$

Express each translation as an ordered pair.

9. $A(5,4) \rightarrow A'(6,2)$

10. $B(3,-1) \rightarrow B'(-1,5)$

11. $C(0,-3) \rightarrow C'(2,-6)$

12. △ABC has vertices A(3, 2), B(4, –1), and C(–2, 1). Find the coordinates of the image of △ABC under this mapping.
$(x,y) \rightarrow (x+4, y+3)$

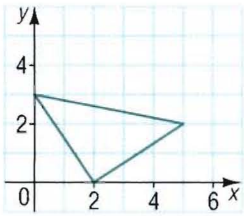
13. △RIK has vertices R(–2, 3), I(3, 0), and K(2, –3). Find the image of △RIK under the mapping $(x,y) \rightarrow (x-3, y-2)$.

Find the coordinates of the image after a reflection in each axis.

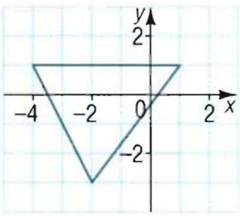
	Point	x-axis	y-axis
14.	(4, 5)		
15.	(5, –2)		
16.	(–3, 6)		
17.	(–3, –2)		

Copy each figure onto a grid and draw its reflection image in the x-axis.

18.

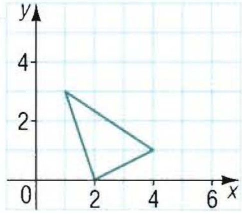


19.

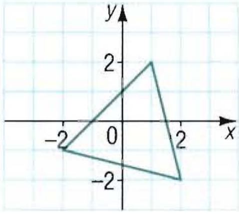


Copy each figure onto a grid and draw its reflection image in the y-axis.

20.



21.

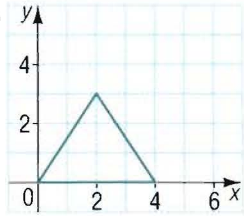


22. △DEF has vertices D(2, 3), E(5, 1), and F(4, 6). Find the coordinates of the image of △DEF after a reflection in the x-axis.

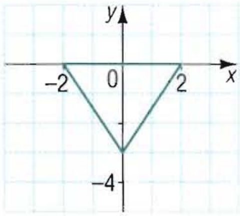
23. △RST has vertices R(3, –1), S(–3, 2), and T(–4, –3). Find the coordinates of the image of △RST after a reflection in the y-axis.

Draw the image of each figure after a 90° counterclockwise rotation about the origin.

24.

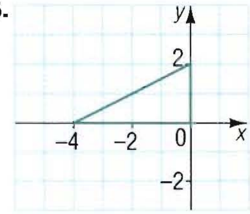


25.

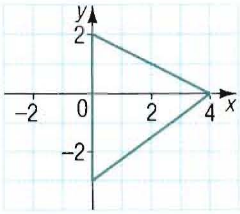


Draw the image of each figure after a 180° clockwise rotation about the origin.

26.



27.



28. △DEF has vertices D(3, 0), E(3, –4), and F(0, –4). Draw the image of △DEF after a 180° counterclockwise rotation about the origin.

29. △ABC has vertices A(3, 0), B(3, 5), and C(6, 0). Draw the image of △ABC after a 90° clockwise rotation about the origin.

30. △PQR has vertices P(2, 1), Q(2, −2), and R(−1, −1). △PQR undergoes the translation [1, 2], followed by the translation [−3, 1], to give the image △P″Q″R″.
a) Find the coordinates of P″, Q″, and R″.
b) Name the single translation that maps △PQR onto △P″Q″R″.

31. △XYZ has vertices X(−2, 2), Y(1, 4), and Z(2, 1). △XYZ is reflected in the y-axis and then in the x-axis to give the image △X″Y″Z″.
a) Find the coordinates of X″, Y″, and Z″.
b) Name a single transformation that maps △XYZ onto △X″Y″Z″.

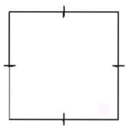
Draw the dilatation image of each line segment under the given mapping.

	Line Segment	Mapping
32.	A(2, 3), B(5, 0)	$(x, y) \rightarrow (2x, 2y)$
33.	C(−1, 2), D(3, −3)	$(x, y) \rightarrow (3x, 3y)$
34.	E(4, 6), F(−6, −2)	$(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$
35.	G(−6, 0), H(0, 6)	$(x, y) \rightarrow (\frac{1}{3}x, \frac{1}{3}y)$

36. △PQR has vertices P(3, 2), Q(3, −1), and R(2, −2). Find the image of △PQR under the mapping $(x,y) \rightarrow (2x, 2y)$.

37. Quadrilateral ABCD has vertices A(−6, 2), B(−4, −6), C(2, −8), and D(6, 10). Find the image of quadrilateral ABCD under the mapping $(x,y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$.

How many lines of symmetry does each figure have?

38.


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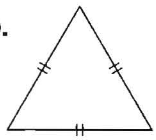

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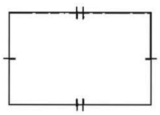
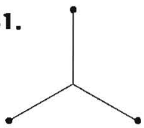
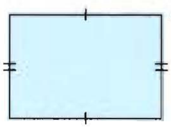


What is the order of turn symmetry for each figure?

41.


42.


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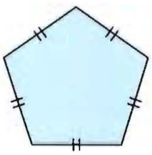


Group Decision Making
Math in the Workplace

1. Decide as a class what local businesses, trades, or professions you would like to investigate to see how math is used. Your choices might include a jewellery store, clothing store, gas station, dentist's office, doctor's office, and a professional sports team. List your choices on the chalkboard.

2. Form pairs. Each pair chooses which workplace to research.

3. In your pair, research how math is used in your chosen workplace. You may have to visit the workplace, interview people, and use the library.

4. Prepare a visual presentation for the class to show how math is used. Presentations should include a description of how you conducted the research.

5. Write 2 problems based on your research. Be creative.

6. Make a presentation to the class. Include the math problems.

7. As a class, evaluate the visual presentations and the math problems. Check that the math problems can be solved.

Chapter Check

1. $\triangle ABC$ has vertices A(−2, 0), B(0, −3), and C(3, −2). Find the coordinates of the image of $\triangle ABC$ under the mapping $(x, y) \rightarrow (x + 5, y + 4)$.

2. $\triangle PQR$ has vertices P(2, −1), Q(0, 2), and R(−3, −2). Draw the image of $\triangle PQR$ under the mapping $(x, y) \rightarrow (x - 4, y + 3)$.

3. $\triangle RST$ has vertices R(−1, 1), S(−3, 4), and T(−6, 3). Draw the image of $\triangle RST$ after a reflection in the x-axis.

4. $\triangle DEF$ has vertices D(2, 4), E(−4, 1), and F(−1, −3). Draw the image of $\triangle DEF$ after a reflection in the y-axis.

5. $\triangle JKL$ has vertices J(−4, 0), K(−3, 5), and L(0, −3). Draw the image of $\triangle JKL$ after a 180° clockwise rotation about the origin.

6. $\triangle GHI$ has vertices G(4, 0), H(4, −3), and I(0, −2). Draw the image of $\triangle GHI$ after a 90° counterclockwise rotation about the origin.

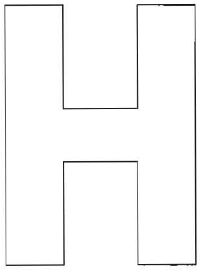
7. $\triangle STU$ has vertices S(0, 1), T(−3, 3), and U(4, −3). Draw the image of $\triangle STU$ under the mapping $(x, y) \rightarrow (2x, 2y)$.

8. Quadrilateral PQRS has vertices P(−2, 2), Q(2, 4), R(3, 1), and S(1, −1).
a) Draw quadrilateral PQRS on grid paper.
b) Quadrilateral P′Q′R′S′ is the image of quadrilateral PQRS under the mapping $(x, y) \rightarrow (3x, 3y)$. Find the coordinates of P′, Q′, R′, and S′, and draw quadrilateral P′Q′R′S′.

9. $\triangle XYZ$ has vertices X(2, 1), Y(−2, 1), and Z(−1, 3). $\triangle XYZ$ is reflected in the x-axis and then in the y-axis to give the image $\triangle X''Y''Z''$. Find the coordinates of X″, Y″, and Z″.

How many lines of symmetry does each figure have?

10.



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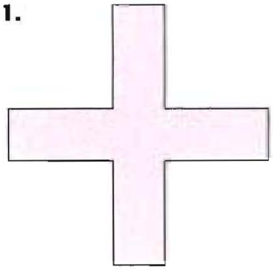
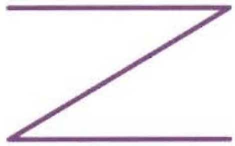


What is the order of turn symmetry for each figure?

12.



13.

This will be a wonderful day for self reflection.

Using the Strategies

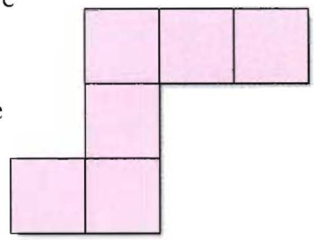
1. How many different-sized squares can be constructed on a 5-by-5 geoboard?

2. Find 2 multiples of 13 that are also multiples of 17.

3. Find 3 consecutive integers whose product is 1716.

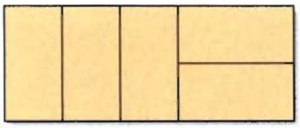
4. When you write the whole numbers in words, "one," "two," "three," and so on, what is the first word that has the letters in alphabetical order?

5. The squares are exactly the same size. The total area of the figure is 384 cm^2. What is the perimeter of the figure?



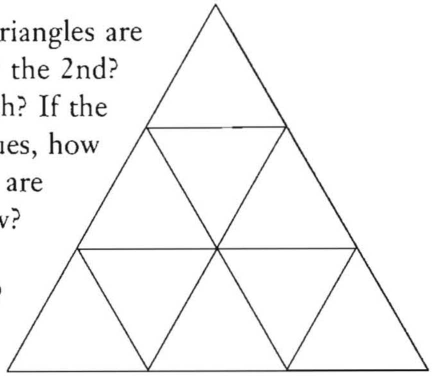
6. On a "prime day," both the month and the day are prime. March 13 is a prime day because March is month 3, which is prime, and so is 13. How many prime days are there this year?

7. The diagram shows one way of covering a 5 cm by 2 cm rectangle with 5 rectangles measuring 2 cm by 1 cm.



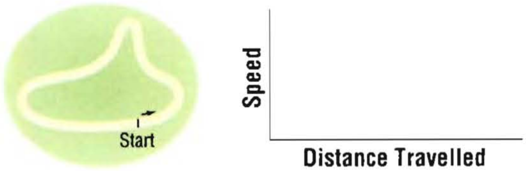
In how many other ways can you cover this rectangle with the smaller rectangles?

8. How many triangles are in the 1st row? the 2nd? the 3rd? the 4th? If the pattern continues, how many triangles are in the 10th row? the 20th row? the 100th row? the nth row?



9. You spend \$2.75 in a store and receive \$7.25 change from \$10.00. Notice that the arrangement of the digits in the amount you spent is a rearrangement of the digits in your change. Find 4 other pairs of amounts spent and change from \$10.00 that share this property.

10. A car is racing around this track.



Sketch a graph of speed versus distance travelled for 1 lap of the track.

DATA BANK

1. Write the ratio of the speed of the strongest strong breeze to the speed of the strongest moderate breeze. Express your answer in lowest terms.

2. The longest covered bridge is in Hartland, New Brunswick. The longest cable suspension bridge is under construction in Akashi Kaikyo, Japan. How many times longer is the Japanese bridge than the bridge in New Brunswick?

Chapter 5

Find the GCF of each pair.

1. 42, 56 **2.** $10t^2$, $2t$ **3.** $14xy$, $35x^2y^2$

Factor.

4. $3x - 21$ **5.** $5x^2y + 15xy^2$

Expand.

6. $(4x)(5y)$ **7.** $-2x(3 - x)$ **8.** $3x(2x - 3)$

Divide.

9. $45x \div 9$ **10.** $\frac{-32xy}{8y}$ **11.** $\frac{4x - 8x^2 + 12x^3}{4x}$

Expand.

12. $(x - 3)(x + 1)$ **13.** $(2x - 1)(3x - 2)$

Factor.

14. $x^2 - 7x - 8$ **15.** $a^2 + 10a - 11$

16. $b^2 - 2b + 1$ **17.** $y^2 - 10y - 56$

Factor fully.

18. $2x^2 - 20x + 50$ **19.** $3x^2 + 3x - 36$

20. $4a^2 - 4a - 80$ **21.** $5a^2 - 25a + 30$

Expand.

22. $(x + 7)(x - 7)$ **23.** $(3 - t)(3 + t)$

24. $(5p - 1)(5p + 1)$ **25.** $(6 - 9y)(6 + 9y)$

Factor.

26. $x^2 - 4$ **27.** $4p^2 - 49$ **28.** $8x^2 - 72$

Simplify.

29. $\frac{25x^2y}{5} \times \frac{2x^4y^2}{5x^3y^3}$ **30.** $\frac{2 + x}{2} + \frac{5 - x}{5}$

Expand.

31. $(2x - 1)^2$ **32.** $(3 - 2y)^2$ **33.** $(p - 3q)^2$

Factor.

34. $m^2 + 10m + 25$ **35.** $4w^2 - 12w + 9$

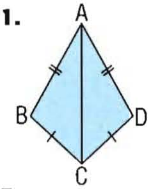
Expand and simplify.

36. $(y + 3)(2y^2 + 7y - 4y)$

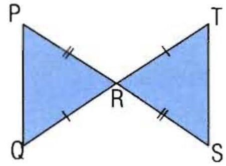
Chapter 6

Explain why the pairs of triangles are congruent.

1.



2.



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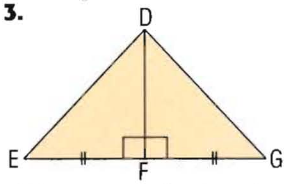

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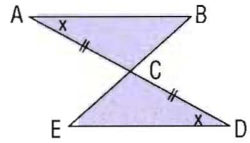
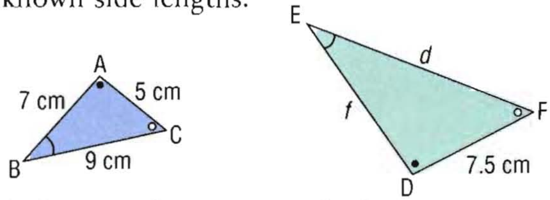
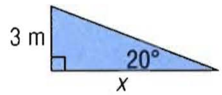


5. This pair of triangles is similar. Find the unknown side lengths.

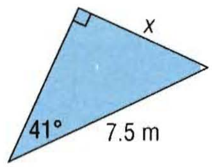


Calculate x to the nearest tenth of a metre.

6.



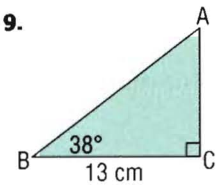
7.



8. In △PQR, ∠P is 90°, ∠Q is 74° and QR is 23.5 cm in length. What is the length of side QP? Round your answer to the nearest tenth of a centimetre.

Solve each triangle.

9.



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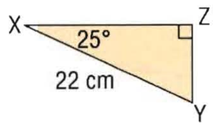
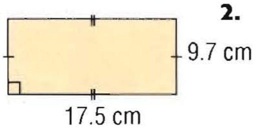


11. A plane is 800 m above the ground. A person standing on the ground looks up through an angle of elevation of 23° to look directly at the plane. What is the horizontal distance of the person from the plane?

Chapter 7

Estimate, then calculate the area of each figure.

1.



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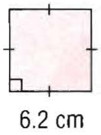

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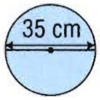

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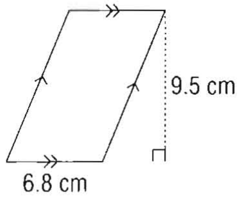

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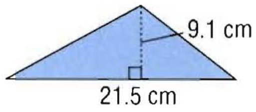

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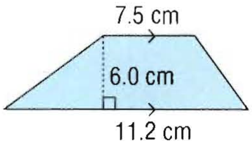

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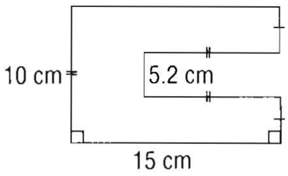
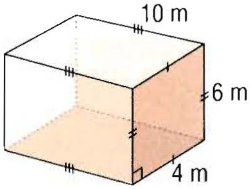


Calculate the volume and surface area of each solid.

8.



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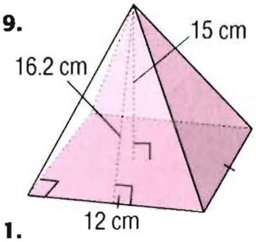
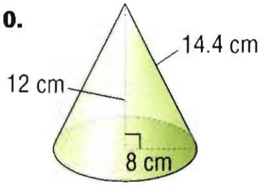
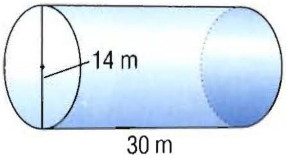


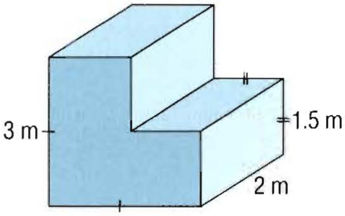
10.



11.



12. Calculate the volume of the composite solid.



13. How many boxes 1 m by 1 m by 0.5 m will fit into a storage container 3 m by 4 m by 10 m?

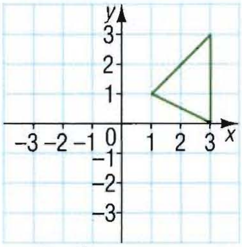
Chapter 8

1. Draw the triangle on grid paper. Then, draw its translation image.
W(−1, 4), Y(−3, 1), Z(−1, 1)
$(x, y) \rightarrow (x + 2, y - 3)$

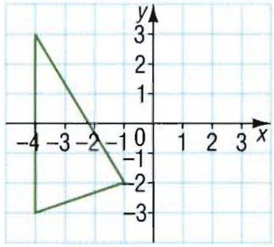
2. $\triangle K'L'M'$ is the image of $\triangle KLM$ under the translation [−3, 2]. The coordinates of the vertices of $\triangle K'L'M'$ are K′(2, 1), L′(2, −3), and M′(7, 1). What are the coordinates of the vertices of $\triangle KLM$?

Draw each triangle on a grid. Draw its image after a reflection in
a) *the x-axis.* **b)** *the y-axis.*

3.



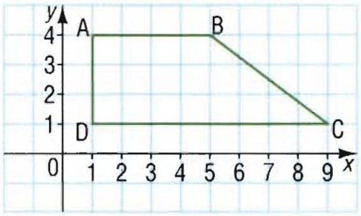
4.



Copy the figure onto grid paper. Draw the image after the given rotation about the given turn centre.

5. 90° turn clockwise about the turn centre A

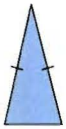
6. 180° turn counterclockwise about the origin



7. Square ABCD has vertices A(3, 3), B(3, −3), C(−3, −3) and D(−3, 3). Draw the image of square ABCD under each mapping.
a) $(x, y) \rightarrow (2x, 2y)$ **b)** $(x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)$

Copy each figure.
a) *Draw the lines of symmetry.*
b) *State the order of turn symmetry.*

8.



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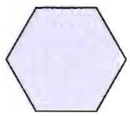


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SURF STARS