

CHAPTER 2

Patterns and Equations

Crime writers have used codes many times. "The Shadow" was a crime fighter in the 1930s. He was created by Walter B. Gibson. Here is a cipher used by "The Shadow."

A		B		C		D	
E		F		G		H	
I		J		K		L	
M		N		O		P	
Q		R		S		T	
U		V		W		X	
Y		Z					

Extra Symbols: 1 2 3 4

The extra symbols tell the decoder how far the letter symbols have been rotated to the right in the message in order to disguise them. The symbol means they have been rotated 90° to the right.

Decode this message.

Sir Arthur Conan Doyle used a code in the Sherlock Holmes story "The Adventure of the Dancing Men."

Research how he used the code.

GETTING STARTED

Activity ① Patterns in Tables



The patterns in the tables are found in the columns. Copy and complete the tables. Write the rules for completing each table.

1.

1	4	7	9	12	9		
3	3	4	5	4		7	9
4	7	11			16	20	
3	12	28					72

2.

1	4	6	9	11			
2	8	12			14		30
4	16	24				20	
7	28	42					

3.

1	4	6	9	3			
30	77	91				99	96
6	11	13			8		12
5	7	7	1	5	2	9	

4.

15	14	12			36	40
3	2	3		3	4	
5	7	4	6	9		20
2	5	1	2			

5.

3	5	6	4		7	5
24	45	42		16	28	25
35	59	55				
8	9	7	5	8		

6.

2	3	5	4			10
6	9	15		18		
7	10	16			25	
13	19	31				

Activity ② Stamp Patterns

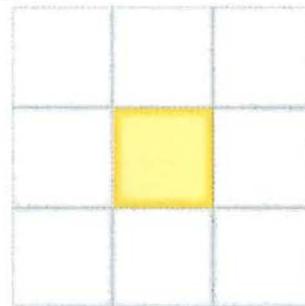
Two stamps can be attached to form two rectangular shapes.



1. In how many ways can 12 stamps be attached to form rectangular shapes? Sketch your solutions.
2. How many rectangular shapes can be made with 10 attached stamps?
3. How many rectangular shapes can be made with 24 attached stamps?
4. Without drawing the stamps, how can you determine how many rectangular shapes can be made with 100 attached stamps?

Activity ③ Tile Patterns

A floor is being tiled using the following pattern. Each yellow tile is surrounded by white tiles.

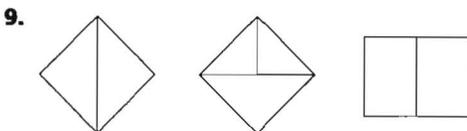
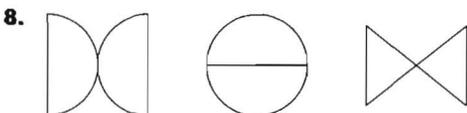
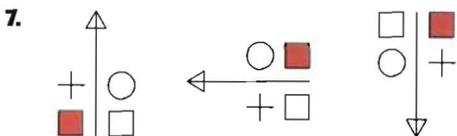
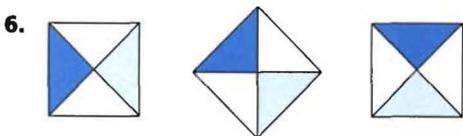
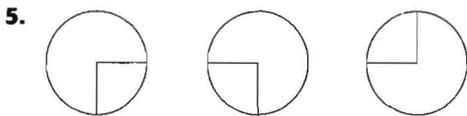
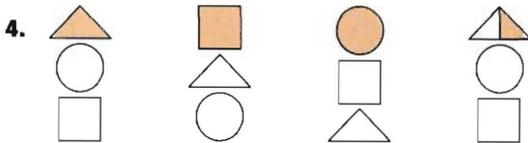
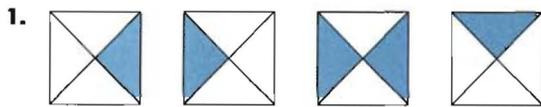


1. How many tiles of each colour do you need to tile each of the following areas?

a) 3 tiles by 3 tiles	b) 5 tiles by 3 tiles
c) 5 tiles by 5 tiles	d) 7 tiles by 3 tiles
e) 7 tiles by 5 tiles	f) 7 tiles by 7 tiles

Activity 4 Patterns in Drawings

Determine the pattern and then sketch the next figure in the sequence.



Mental Math

Simplify.

- | | |
|--------------------|---------------------|
| 1. $10 + 20 - 30$ | 2. $10 - 10 + 10$ |
| 3. $-10 + 2 + 2$ | 4. $15 - 5 - 4$ |
| 5. $-15 + 2 + 1$ | 6. $-10 - 10 - 10$ |
| 7. $-10 - 10 - 20$ | 8. $-15 + 5 - 15$ |
| 9. $-50 + 25 + 5$ | 10. $100 - 80 - 10$ |

Simplify.

- | | | |
|--------------|--------------|--------------|
| 11. $(2)^3$ | 12. $(-3)^2$ | 13. $(-1)^5$ |
| 14. $(-1)^8$ | 15. -5^2 | 16. -5^0 |
| 17. $(4)^2$ | 18. $(-2)^2$ | 19. -10^2 |
| 20. $(3)^3$ | 21. -1^3 | 22. -1^6 |
| 23. $(-8)^0$ | 24. $(-3)^3$ | 25. -3^2 |

Multiply.

- | | |
|---------------------------------|----------------------------------|
| 26. $-2 \times (-3) \times 5$ | 27. $-2 \times (-2) \times (-5)$ |
| 28. $5 \times (-5) \times 2$ | 29. $5 \times 4 \times (-3)$ |
| 30. $-4 \times 3 \times 2$ | 31. $6 \times (-5) \times (-2)$ |
| 32. $10 \times 10 \times (-10)$ | 33. $-10 \times (-10) \times 2$ |

Divide.

- | | | |
|----------------------|----------------------|----------------------|
| 34. $\frac{-18}{9}$ | 35. $\frac{21}{-7}$ | 36. $\frac{-42}{6}$ |
| 37. $\frac{-14}{-7}$ | 38. $\frac{-20}{-5}$ | 39. $\frac{-24}{-6}$ |
| 40. $\frac{21}{3}$ | 41. $\frac{-49}{7}$ | 42. $\frac{-54}{-6}$ |
| 43. $\frac{-33}{11}$ | 44. $\frac{44}{-4}$ | 45. $\frac{-50}{-5}$ |

Simplify. State each answer in exponential form.

- | | | |
|-------------------------|-------------------------|-------------------------|
| 46. 3×3^0 | 47. 2×2^2 | 48. $3^5 \times 3^{-2}$ |
| 49. $6^4 \times 6^{-2}$ | 50. $4^{-2} \times 4^3$ | 51. $5^{-3} \times 5^5$ |
| 52. $3^7 \div 3^5$ | 53. $4^{10} \div 4^8$ | 54. $2^8 \div 2^7$ |
| 55. $2 \div 2^{-2}$ | 56. $3 \div 3^{-1}$ | 57. $4 \div 4^{-1}$ |
| 58. $\frac{5^5}{5^3}$ | 59. $\frac{6^3}{6^2}$ | 60. $\frac{2}{2^{-2}}$ |

2.1 Look for a Pattern

Patterns are everywhere, and people often use them in their work. Scientists look for patterns in their research findings. Air traffic controllers use traffic patterns.

Using patterns to *predict results* is a very useful problem solving tool. Store managers and hotel managers use patterns to predict and plan for their busy times. Meteorologists use patterns to help predict the weather.

A group of towns has a phone network to call volunteer firefighters in the event of a forest fire. The fire chief phones 2 firefighters and gives them information about the fire. In the second round of calls, these 2 firefighters each phone 2 more. In the third round of calls, each person called in the second round phones 2 more people. How many firefighters are called in the ninth round of calls?

1. What information are you given?
2. What are you asked to find?
3. Do you need an exact or approximate answer?

Make a table. Write the numbers of people called in the first few rounds of calls and look for a pattern.

Round of Calls	1	2	3	4	5	6	7	8	9
Number of People Called	2	4	8	16					
	or 2^1	or 2^2	or 2^3	or 2^4					

The number of calls in a round equals 2 raised to an exponent that equals the round number.

From this pattern, we predict that 2^5 people are called in the fifth round, 2^6 people in the sixth round, and so on.

In the ninth round of calls, 2^9 or 512 people are called.

Does the answer seem reasonable?

Can you think of another way to solve the problem?

Understand the Problem

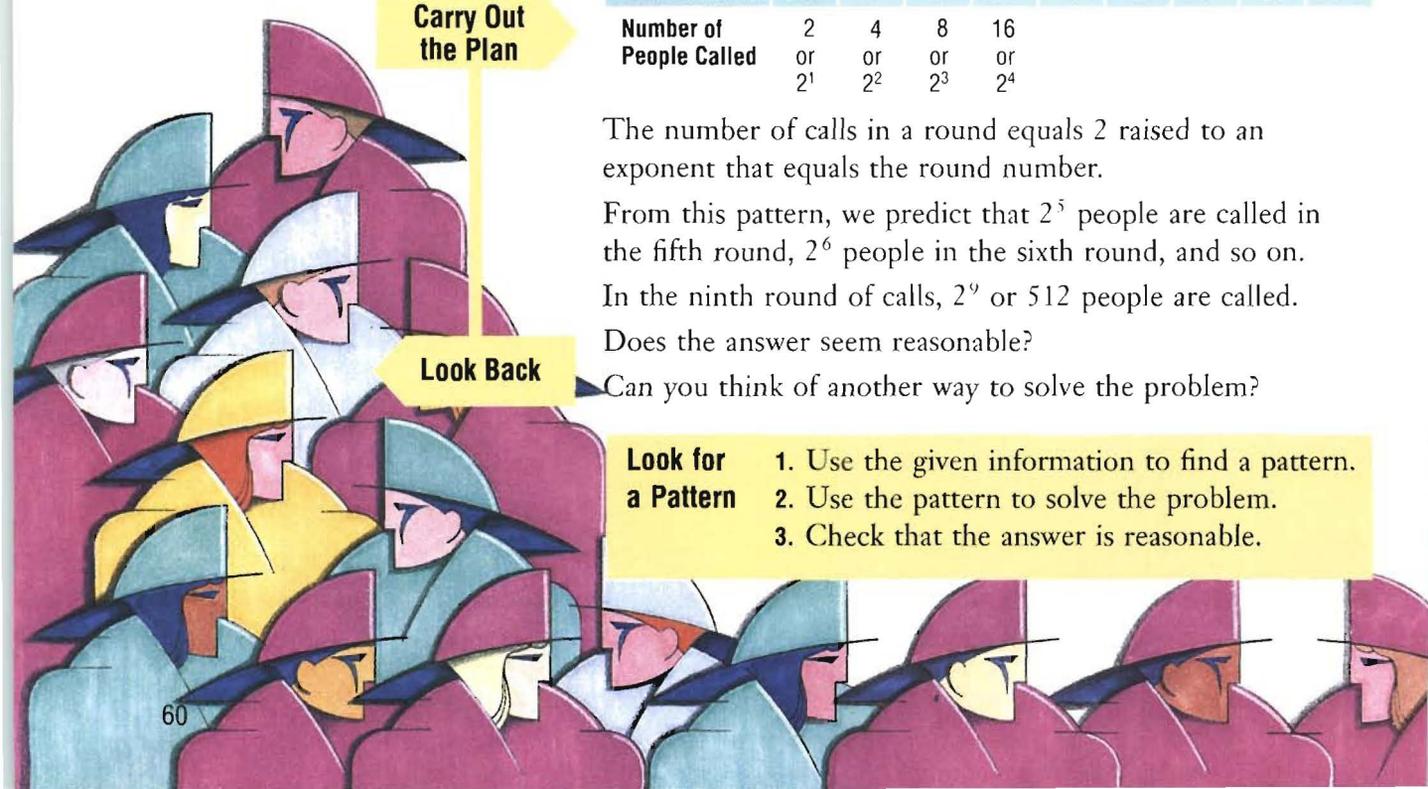
Think of a Plan

Carry Out the Plan

Look Back

Look for a Pattern

1. Use the given information to find a pattern.
2. Use the pattern to solve the problem.
3. Check that the answer is reasonable.



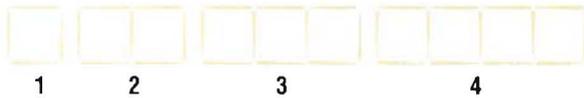
Problems and Applications

In questions 1–5, determine the pattern in each set of numbers. Then, write the next 2 numbers.

1. 5, 8, 11, 14, ■, ■
2. 14, 12, 10, 8, ■, ■
3. 10, 12, 9, 11, 8, 10, 7, ■, ■
4. 5, 10, 20, 40, ■, ■
5. 256, 128, 64, 32, ■, ■

6. If the pattern continues, how many toothpicks are needed for

- a) the 5th diagram?
- b) the 6th diagram?
- c) the 50th diagram?



7. The numbers 1, 8, 27, and 64 are the first 4 cubes.

- a) Find the sum of the first 2 cubes.
- b) Find the sum of the first 3 cubes.
- c) Find the sum of the first 4 cubes.
- d) Describe the pattern in the sums.
- e) Use the pattern to find the sum of the first 9 cubes.

8. Find the pattern and complete the chart.

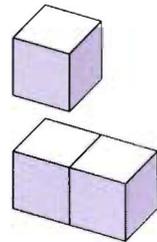
$$\begin{array}{r}
 1 \times 1 = \blacksquare \\
 11 \times 11 = \blacksquare \\
 111 \times 111 = \blacksquare \\
 1111 \times 1111 = \blacksquare \\
 11\ 111 \times 11\ 111 = \blacksquare \\
 111\ 111 \times 111\ 111 = \blacksquare
 \end{array}$$

9. A computer company donated money to a university scholarship fund for 10 years. In the first year, the company gave \$1 000 000. Every year after that, it gave \$100 000 more than in the previous year. How much did the university receive in the tenth year?

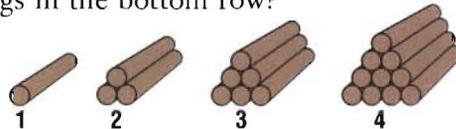
10. Six people entered a chess tournament. Each player played until losing a game. If there were no draws, how many games were played?

11. There are 10 rooms on each floor of a hotel. There are 9 floors. The rooms are numbered from 10 to 19 on the first floor, from 20 to 29 on the second floor, and so on. You have been hired to put new brass numbers on each room door. For example, you will put one brass “1” and one brass “0” on the door of the first room. To have enough brass numbers, how many will you need to buy for each digit from 0 to 9?

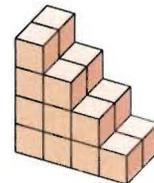
12. If you place a block on a table, 5 faces are showing. If you add another block as shown, 8 faces are showing. How many faces are showing when there are 21 blocks in a row?



13. How many logs will be in a pile with 9 logs in the bottom row?



14. The staircase has 4 steps and is made up of 20 cubes. How many cubes are needed for a staircase with 12 steps?



15. Eight people entered a tennis tournament. Each person played everyone else once. How many games were played?

16. A triangle has no diagonals. A quadrilateral has 2 diagonals. A pentagon has 5 diagonals. How many diagonals does a 12-sided figure have?

17. Write a problem you can solve by finding a pattern. Have a classmate solve your problem.



2.2 Evaluating Expressions

At the 1993 Juno Awards, the theme song from the movie *Beauty and the Beast* was voted best single of the year. It was sung by Quebec's Celine Dion and an American, Peabo Bryson.

Activity: Discover the Relationship

The prices of tickets for a recent Celine Dion concert are shown in the table. Copy and complete the table.

Number of Tickets Purchased	Main Floor	Balcony
1	\$32.50	\$28.50
2		
3		
4		
5		

Inquire

- If n is the number of seats on the main floor, write an algebraic expression for the total cost of the main-floor tickets.
- If m is the number of seats in the balcony, write an expression for the total cost of the balcony tickets.
- Use the expressions from 1 and 2 to calculate the cost of tickets for everyone in your class to
 - sit on the main floor.
 - sit in the balcony.

A letter that represents one or more numbers is called a **variable**. Expressions such as $x + 2y + 10$, which include numbers and variables, are called **algebraic expressions**.

The expression $x + 2y + 10$ has 3 **terms**. The number 10 is called a **constant**. The letters x and y are variables. The **coefficient** of the variable x is 1. The coefficient of the variable y is 2.

Algebraic expressions can have many different values, depending on the value assigned to each variable. When we assign a specific value to a variable in an algebraic expression, the process is known as **substitution**.

Example 1

Evaluate for $x = 4$.

- $2x + 1$
- $-7x$
- $x - 11.5$

Solution

$$\begin{array}{ll} \mathbf{a)} & 2x + 1 = 2(\mathbf{4}) + 1 \\ & = 8 + 1 \\ \mathbf{b)} & -7x = -7(\mathbf{4}) \\ & = -28 \\ & = 9 \end{array}$$

$$\mathbf{c)} \quad x - 11.5 = \mathbf{4} - 11.5 \\ = -7.5$$



Example 2

Evaluate for $x = 3$, $y = -4$, and $z = 5$.

a) $5x + y - 2z$

b) $9x^{-2} - 5y^2 + 2z$

Solution

$$\begin{aligned}\text{a) } 5x + y - 2z &= 5(3) + (-4) - 2(5) \\ &= 15 - 4 - 10 \\ &= 1\end{aligned}$$

$$\begin{aligned}\text{b) } 9x^{-2} - 5y^2 + 2z &= 9(3)^{-2} - 5(-4)^2 + 2(5) \\ &= 9 \times \frac{1}{9} - 5 \times 16 + 10 \\ &= 1 - 80 + 10 \\ &= -69\end{aligned}$$

Practice

Name the variable in each expression.

1. $5a + 8$

2. $7c - 3$

3. $\frac{1}{4} + 3q$

4. $4b - 5$

5. $-k - 0.1$

6. $6t^2 + 1$

Name each variable and constant.

7. $-2c + 10$

8. $a + 3b^2 - 6$

9. $x^3 - 11y^2 - 3$

10. $xy + 4z^5 + 0.5$

State the number of terms and the coefficient of each variable in each expression.

11. $2a + 3b$

12. $7s + 9t - 5$

13. $1 - 5c + \frac{1}{4}d$

14. $6jkl + 11mn$

15. Evaluate $5x$ for each value of x .

a) 2

b) -5

c) 0

d) -4

16. Evaluate $3y + 1$ for each value of y .

a) 0

b) 1

c) 5

d) -3

17. Evaluate $2z - 1$ for each value of z .

a) -1

b) 2

c) -5

d) 10

18. Evaluate for $x = -2$.

a) $x + 3$

b) $3x + 2$

c) $2x - 6$

d) $8 - 2x$

19. Evaluate for $x = 2$, $y = 3$, and $z = -4$.

a) $x + y + z$

b) xyz

c) $x + 2z$

d) $xy - z$

20. Evaluate for $a = 1.5$, $b = 2.5$, and $c = 3.5$.

a) $a + b + c$

b) $ab - 0.75$

c) $5a - 3b$

d) $1.2a + 3.2b$

e) $5.65 - ab$

f) $2.4b - a$

g) $ac - 2b$

h) $ab - bc - ca$

Problems and Applications

21. a) In a basketball game, team A scored $7x + 46y + 15z$ points and team B scored $2x + 27y + 31z$ points. In these expressions, x is the value of a 3-point field goal, y is the value of a 2-point field goal, and z is the value of a 1-point free throw. Which team won?

b) At the end of the 1991–1992 season, the Chicago Bulls' Michael Jordan had the highest all-time scoring average in the NBA at 32.3 points per game. His total points were described by the expression $206x + 6881y + 4620z$. The variables have the same values as in part a). How many points had Michael Jordan scored?

22. An apple contains about 290 kJ of energy, a banana 360 kJ, and a grapefruit 210 kJ.

a) Write an expression for the energy in a mixture of apples, bananas, and grapefruit.

b) Calculate the energy in a fruit salad made from 2 apples, 3 bananas, and $1\frac{1}{2}$ grapefruit.

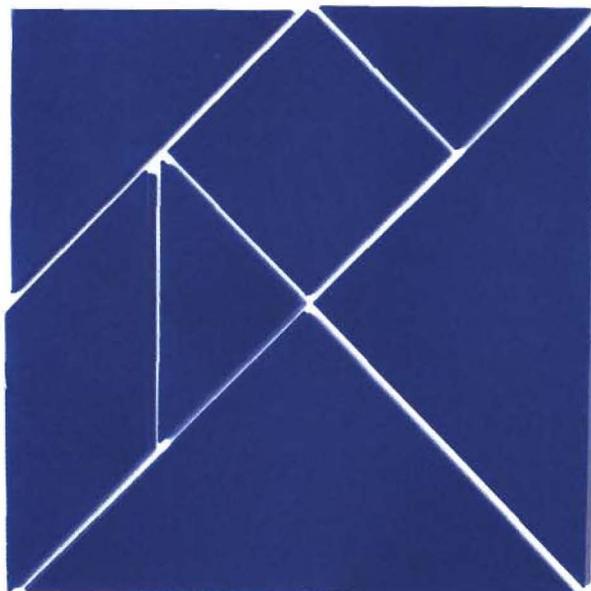
2.3 Like Terms

Activity: Use a Tangram

The tangram square is made up of 7 pieces. Copy the square into your notebook. Label the pieces with letters starting at A. Label pieces with equal areas with the same letter.

Inquire

1. How many different areas make the square?
2. Which areas are identical or *like*?
3. Which areas are not identical or *unlike*?
4. Write 2 different expressions that give the area of the tangram square.
5. Which of the 2 expressions in question 4 is simpler?



Like terms in algebra have the same variables raised to the same exponents. For example, the terms a , $3a$, and $7a$ are like terms. So are $2x^2$, $3x^2$, and $7x^2$.

Unlike terms in algebra have different variables, or the same variable but different exponents. For example, the terms $7b$, $3b^2$, $10q$, $5t$, $3x$, $4x^3$, and $17y$ are unlike terms.

Example 1

Simplify.

- a) $x + 2x$ b) $3b - b$
c) $2y + y - 4$

Solution

Combine like terms.

- a) $x + 2x = 3x$ b) $3b - b = 2b$
c) $2y + y - 4 = 3y - 4$

Example 2

Simplify.

- a) $5x - 6y - 2x + 2y + 1$
b) $-11 + a - 2c + 16 + 3a - b - 5c$

Solution

- a) $5x - 6y - 2x + 2y + 1$
 $= 5x - 2x - 6y + 2y + 1$
 $= 3x - 4y + 1$
b) $-11 + a - 2c + 16 + 3a - b - 5c$
 $= a + 3a - b - 2c - 5c - 11 + 16$
 $= 4a - b - 7c + 5$

Practice

Simplify.

1. $3x + 5x$
2. $11t - t$
3. $-10b + 3b$
4. $-12y - y$
5. $11m + 10m$
6. $15p - 9p$

Simplify.

7. $6r + 4r - r$
8. $2t + 3t - t$
9. $9p - p - 6p$
10. $11a - 10a - 5a$
11. $-5y - 2y + 9y$
12. $-8q - 9q + 10q$

Simplify.

13. $3t + 5 - 2t$
14. $2x - 5x - 7$
15. $2a - b + 3a$
16. $-4x + y - 6x$
17. $8y - 2z + 7y$
18. $-9p - 10q + 8p$

Simplify.

19. $5c + d - 2c - d$
20. $7p - q + p - 2q$
21. $3j + k - 5j - 2k$
22. $8a - 2b - 6a - 3b$
23. $-r + s + 2r - 2s$
24. $-5x + 5y + 5x - y$

Simplify.

25. $5x + 10 + 5y - 3x + 1$
 26. $3a - 4b - 5 + a - b$
 27. $-8 - 3z + 9x - 11 - 6z$
 28. $8r - 11 - 18q + 5p + 7q$
 29. $-4w + 7c - 8x - 9w - 3c$
 30. $13j - 18d - 5d + 2j + 2c$
- Simplify.
31. $-p + 5q - 8r - q + 3p + 9r + 1$
 32. $-2z + 3y - 10x - 4y + 4z - 3x$
 33. $5q - 9s - 8r + 8q - 7r + 13s$
 34. $-10 - a + 6c - 11 - 9c + 8a - d$

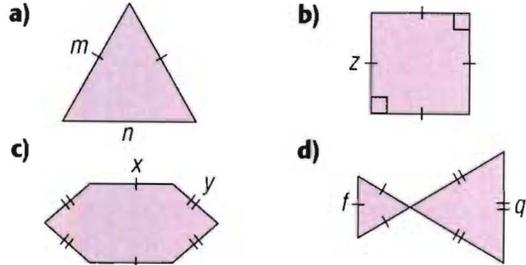
Problems and Applications

35. Simplify, then evaluate.

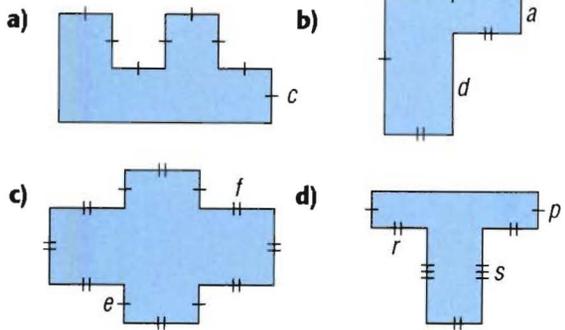
- a) $2a + 8a$, when $a = 2$

- b) $7t - 3t$, when $t = 3$
 c) $-6k - 8k$, when $k = -2$
 d) $-6y - 9y - y$, when $y = -3$
 e) $3 + 4x - 2x$, when $x = 5$
 f) $-5p - 3p + 1 - 6$, when $p = -1$

36. Write an expression for each perimeter in 2 different ways.



37. Write and simplify an expression for each perimeter.

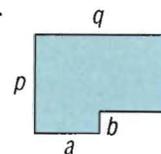


38. What is wrong with this sign?

SMITHVILLE	
Population	460
Established	1830
Elevation	750
Sum	3040



39. Work with a partner to calculate the perimeter of this figure. Describe the simplest method you can find.



Words to Symbols

Activity ①

The longest parade in the world runs from March to July. The “floats” have masses between 500 000 t and 10 000 000 t. They are up to 10 000 years old. The parade consists of hundreds of icebergs that pass Newfoundland’s east coast.

The part of an iceberg above the water is called the tip. The height, t , of the tip is $\frac{1}{7}$ of the total height of an iceberg. So, an expression for the total height of an iceberg is $7t$. Write an expression in terms of t for the height of the part of an iceberg that is under water.

Activity ②

Match the words with the symbols.

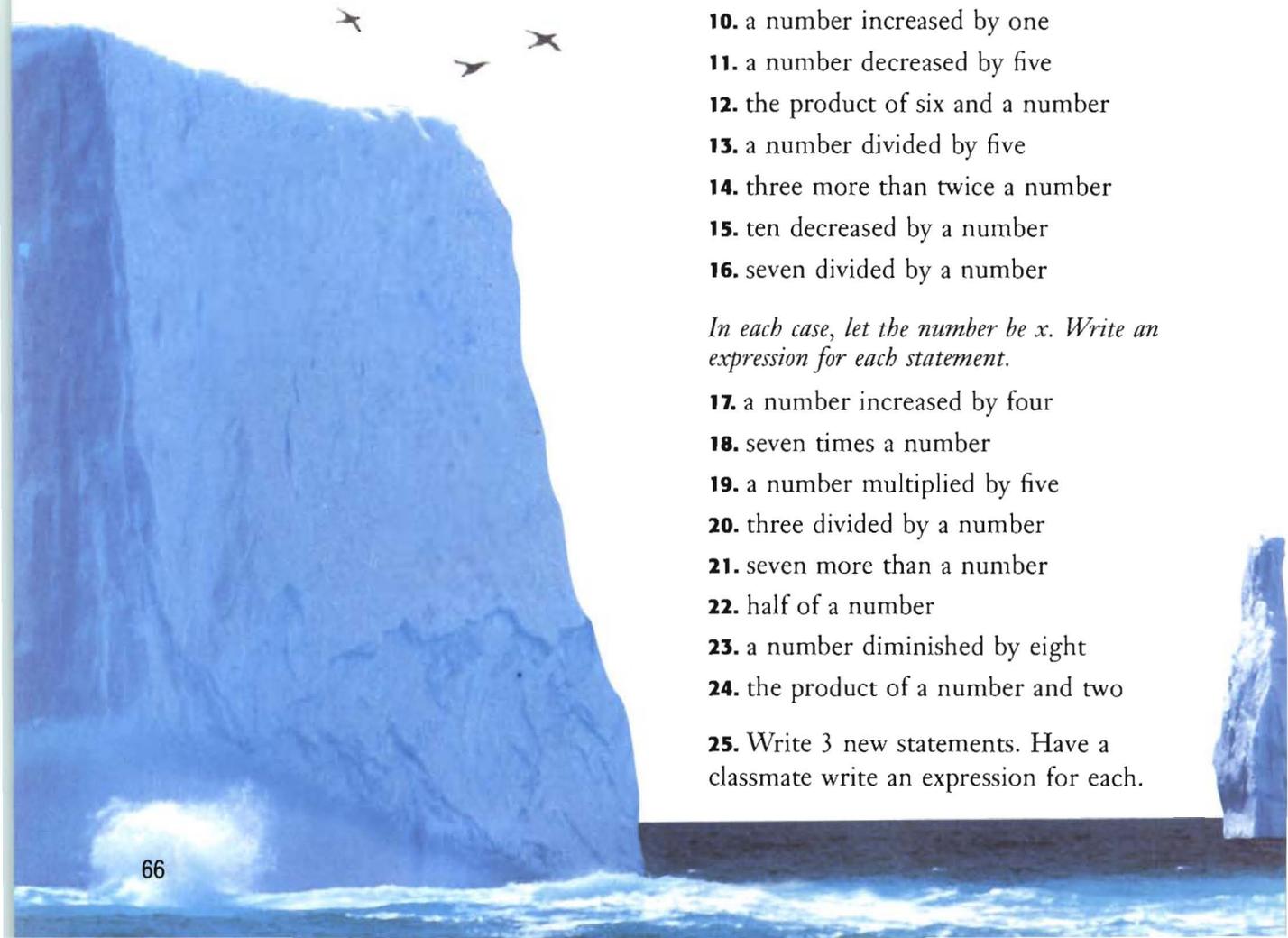
- | | |
|-------------------------------|------------|
| 1. a number increased by five | $5x$ |
| 2. a number decreased by one | $w + 3$ |
| 3. a number divided by nine | $n - 1$ |
| 4. two less than a number | $b + 5$ |
| 5. three more than a number | $6 - t$ |
| 6. five times a number | $m - 2$ |
| 7. six decreased by a number | $2 + y$ |
| 8. two increased by a number | $q \div 9$ |

If n represents a number, write an algebraic expression using numbers and symbols for each of the following statements.

- three times a number
- a number increased by one
- a number decreased by five
- the product of six and a number
- a number divided by five
- three more than twice a number
- ten decreased by a number
- seven divided by a number

In each case, let the number be x . Write an expression for each statement.

- a number increased by four
- seven times a number
- a number multiplied by five
- three divided by a number
- seven more than a number
- half of a number
- a number diminished by eight
- the product of a number and two
- Write 3 new statements. Have a classmate write an expression for each.



Activity 3

Copy and complete the tables.

1. The first number is 23 more than the second.

First Number	45	56	x	$3m$	$4t + 21$
Second Number		18	y		$6r - 5$

2. Francine is 4 years younger than Terry.

Francine's Age	19	m	$4t$	
Terry's Age	16	x	$5y$	$3w + 8$

3. There are 6 times as many cars as motorcycles.

Number of Cars	72	y		
Number of Motorcycles	13	x	$m + 7$	$3t + 8$

Activity 4

1. The length of a rectangle is 5 cm more than its width.

- Which dimension is smaller?
 - Let the smaller dimension be x . Then, write the larger dimension in terms of x .
 - Draw and label a diagram of the rectangle.
 - Write an expression for the perimeter of the rectangle in terms of your variable.
2. The length of a rectangle is 4 m more than its width. Write an expression for its area.
3. The side length of a square is x metres. What is its perimeter?
4. a) Copy and complete the table for the dimensions of a rectangle. Its length is 3 m more than its width.

Width (m)	10	15	x	$10x$	$4x + 1$
Length (m)					

- Write expressions for the perimeter and area of each rectangle. Compare your expressions with a classmate's.
5. Repeat question 4 for the following table. The length of the rectangle is now 7 m more than the width.

Width (m)	10	x	$3t$	$5z + 4$
Length (m)	24	y	$4n$	

2.4 Writing Equations

The Canadian Coast Guard patrols Canada's rivers and coastline. The Coast Guard is mainly involved in search-and-rescue operations.

Activity: Study the Information

A Coast Guard boat that travels at a speed of 15 km/h in still water was patrolling up a river with a current of 6 km/h. The boat's actual speed, s kilometres per hour relative to the river bank, was given by the equation $s = 15 - 6$.

When it patrolled down the river, the boat's actual speed, s kilometres per hour relative to the river bank, was given by the equation $s = 15 + 6$.

Inquire

- Let the speed of the boat in still water be x kilometres per hour. Write an equation to find x if the boat patrols up a river where
 - the current is 6 km/h and the boat's actual speed is 11 km/h.
 - the current is 4 km/h and the boat's actual speed is 10 km/h.
- Write an equation to find x if the boat patrols down a river where
 - the current is 5 km/h and the actual speed is 17 km/h.
 - the current is 7 km/h and the actual speed is 20 km/h.

Example 1

It costs 4 times as much to go to the movie theatre as it does to rent a movie. Together, both ways of seeing a movie cost \$10. Write an equation to find each cost.

Example 2

Lake Ontario is 77 km shorter than Lake Erie. The lengths of the 2 lakes total 699 km. Write an equation to find the length of Lake Ontario.

Solution

Let the smaller cost, a movie rental, be represented by x . Then, the cost of going to a movie theatre is $4x$. Together, both ways cost \$10. The total cost of both ways equals the sum of x and $4x$.

$$\begin{aligned}x + 4x &= 10 \\5x &= 10\end{aligned}$$

Solution

Let the length of Lake Ontario be represented by x . Then, the length of Lake Erie is $x + 77$. The sum of x and $x + 77$ is equal to 699.

$$\begin{aligned}x + (x + 77) &= 699 \\2x + 77 &= 699\end{aligned}$$



Practice

Match each sentence with the correct equation.

1. Three times a number is equal to eighteen. $x - 4 = 10$

2. A number decreased by six is four. $3x = 18$

3. Kim's age in four years will be eighteen. $\frac{m}{4} = 18$

4. Shuji's age four years ago was ten. $y - 6 = 4$

5. A number divided by four is eighteen. $x + 4 = 18$

Write an equation for each sentence.

6. Four times a number is twenty.

7. A number divided by two equals five.

8. Six more than a number is fifteen.

9. A number increased by five is twelve.

10. A number decreased by six is ten.

11. Four less than a number is seven.

12. The square of a number is twenty-five.

13. Ten decreased by a number is two.

14. A number multiplied by three is nine.

15. A number divided by five is ten.

16. A number decreased by six is negative eight.

Problems and Applications

Write an equation for each statement in questions 17–22.

17. There are 16 more white keys than black keys on a full-sized piano keyboard. There are 88 keys on a piano.

18. Brad has \$12 more than Pietro. Together they have \$84.

19. Mike is seven years older than Carol. The sum of their ages is 29.

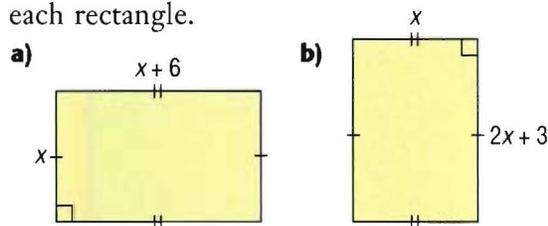
20. The area of the Pacific Ocean is twice the area of the Atlantic. The sum of their areas is 250 000 000 km².

21. The population of Japan is about 4.5 times the population of Canada. The sum of the populations is about 150 000 000.



22. Niagara Falls has 2 parts. The American Falls are 2 m higher than the Horseshoe Falls. Their average height is 58 m.

23. The perimeter of each rectangle is 36 cm. Write an equation to find the dimensions of each rectangle.



24. It costs \$30 to rent a VCR plus \$15 a day. How much will it cost to rent a VCR for 2 days, 10 days, x days?



25. Compact discs cost \$15 for the first one and \$14 for each additional disc. Gina buys m compact discs and spends n dollars. Write an equation that shows the relationship between m and n .

WORD POWER

Lewis Carroll invented a word game called “doublets.” The object is to go from one word to another by changing one letter at a time.

Change the word WARM to the word COLD by changing one letter at a time. You must form a real word each time you change a letter. The best solution has the fewest steps.

2.5 Solving Equations

Many horses take part in sports and shows in Canada. In the RCMP Musical Ride, 32 horses and their riders perform complex movements to music.

Activity: Solve the Problem

Suppose that you own a horse. The monthly cost to board your horse at a public stable is \$100. You have \$225 a month to spend, and you want to take riding lessons that cost \$25 each. Use the equation $25n + 100 = 225$ to find the number of riding lessons you can take per month.

Inquire

1. How did you solve the equation $25n + 100 = 225$?
2. What is the value of x in each of the following equations?
a) $x + 4 = 9$ b) $x - 7 = 11$ c) $3x = 27$ d) $4x + 11 = 31$

An **equation** is a statement that two expressions or numbers are equal.

This equation is true. $5 + 3 = 8$

This equation is false. $7 - 4 = 2$

This equation is neither true nor false. $x + 5 = 9$

The equation $x + 5 = 9$ is called an **open sentence**, because it is not possible to say whether it is true or false. It is true if x is replaced by 4. Other replacements make the statement false. A value of the variable that makes the statement true is called the **solution** or **root** of the equation.

This value is said to *satisfy* the equation.

Example

Solve.

- a) $x + 5 = 2$
- b) $7x + 13 = 55$

Solution

a) Simple equations like $x + 5 = 2$ can be solved mentally. This process is called **solving by inspection**. The solution is $x = -3$.

b) Use **systematic trial**. Substitute different values for the variable until you find the solution.

Substitute 3 for x . $7x + 13 = 7(3) + 13$
 $= 21 + 13$
 $= 34$ When $x = 3$, $7x + 13$ is less than 55.

Substitute 7 for x . $7x + 13 = 7(7) + 13$
 $= 49 + 13$
 $= 62$ When $x = 7$, $7x + 13$ is greater than 55.

Substitute 6 for x . $7x + 13 = 7(6) + 13$
 $= 42 + 13$
 $= 55$ The solution is $x = 6$.



Practice

Solve by inspection.

- | | |
|-----------------|------------------|
| 1. $x + 3 = 7$ | 2. $x + 5 = 4$ |
| 3. $x + 5 = 5$ | 4. $x + 5 = 3$ |
| 5. $2 + m = 9$ | 6. $n + 1 = 6$ |
| 7. $p + 10 = 3$ | 8. $4 = 7 + s$ |
| 9. $y + 8 = 4$ | 10. $z + 8 = 12$ |

Solve by inspection.

- | | |
|-------------------|-------------------|
| 11. $x - 5 = -7$ | 12. $x - 3 = 7$ |
| 13. $4 = -10 + y$ | 14. $y + 5 = -3$ |
| 15. $z - 1 = 6$ | 16. $t - 12 = 10$ |
| 17. $n + 5 = -2$ | 18. $4 - x = 0$ |
| 19. $m + 3 = 0$ | 20. $2 = x + 7$ |

Is the number in brackets the correct solution to the given equation?

- | | | | |
|---------------------------------|------|-----------------------------------|------|
| 21. $x + 7 = 10$ | (3) | 22. $x - 5 = 0$ | (-5) |
| 23. $3x = -18$ | (-6) | 24. $5x = 20$ | (-4) |
| 25. $\frac{x}{5} = \frac{4}{5}$ | (4) | 26. $\frac{2x}{7} = -\frac{6}{7}$ | (-3) |
| 27. $2x + 1 = 7$ | (-3) | 28. $10 = -2 + 3x$ | (4) |

Solve by systematic trial.

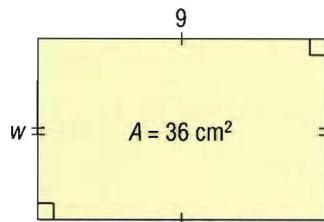
- | | |
|--------------------|---------------------|
| 29. $6x + 17 = 65$ | 30. $73 - 7x = 45$ |
| 31. $14 + 9x = 86$ | 32. $35 = 83 - 8t$ |
| 33. $8k - 37 = 59$ | 34. $5b + 41 = 76$ |
| 35. $4 - 3s = -11$ | 36. $2m - 15 = -11$ |
| 37. $-15 = 7 - x$ | 38. $21 = 5b + 31$ |

Problems and Applications

Solve by inspection.

- | | |
|---------------------|----------------------|
| 39. $x + 2.5 = 3.5$ | 40. $t - 4.3 = 1.7$ |
| 41. $6.6 - m = 2.6$ | 42. $2x = 4.8$ |
| 43. $0.5t = 16$ | 44. $7.1 + x = 10.2$ |
| 45. $5 = m - 3.7$ | 46. $12 = x + 4.6$ |
| 47. $9 - x = 8.2$ | 48. $1.2 + x = -5.3$ |

49. Solve the equation $9w = 36$ to find the width of the rectangle.



50. Point Pelee National Park has an area of 15 km^2 , which is 7 km^2 less than the area of Prince Edward Island National Park. Solve the equation $x - 7 = 15$ to find the area of Prince Edward Island National Park.

51. London, Ontario, has thunderstorms on 36 days a year, the highest number in Canada. Prince Rupert, B.C., has the fewest days of thunderstorms. If you multiply the number for Prince Rupert by 10 and add 6, you get the number for London. Solve the equation $10d + 6 = 36$ to find the number of days of thunderstorms in Prince Rupert per year.

52. Use x , 2, and 6 to write equations with the following solutions. Check your answers by substitution.

- | | | |
|------|-------|------------------|
| a) 3 | b) -4 | c) 4 |
| d) 8 | e) 12 | f) $\frac{1}{3}$ |

53. Find 2 solutions to each equation.

- | | |
|---------------|-------------------|
| a) $x^2 = 16$ | b) $t^2 - 6 = 19$ |
|---------------|-------------------|

54. Write an equation that can be solved by inspection and in which the solution is $x = -4$. Have a classmate solve it.

55. Write an equation that can be solved by systematic trial and in which the solution is $x = 8$. Have a classmate solve it.

56. Write an equation that can be solved by systematic trial and in which the solution is $x = -15$. Have a classmate solve it.

Algebra Tiles

Each red tile represents $+1$.

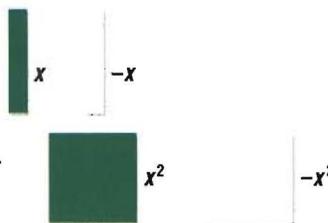
Each white tile represents -1 .

Each long green tile represents $+x$ or x .

Each long white tile represents $-x$.

Each square green tile represents $+x^2$ or x^2 .

Each square white tile represents $-x^2$.



Activity 1 Representing Variables with Tiles

1. Write the expression represented by each group of tiles and then evaluate each expression for $x = 2$ and $x = -3$.

a)



b)



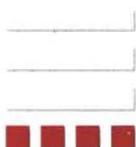
c)



d)



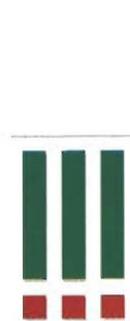
e)



f)



g)



h)



2. Use tiles to model the following expressions. Sketch your solutions.

a) $4x$

b) $2x^2$

c) $-4x$

d) $3x^2 - 2x$

e) $2x - x^2$

f) $-3x^2 + 4x + 3$

g) $-2x^2 - 2x - 1$

h) $3x^2 - 2x + 4$

Activity 2 Representing Zero with Variables

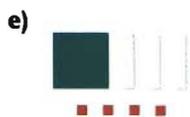
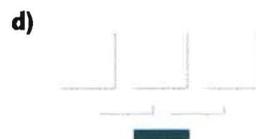
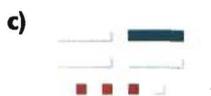
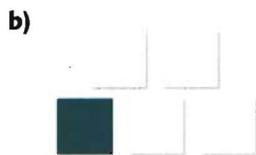
Each pair represents zero.



1. Copy and complete the table. The first row has been done for you.

Tile Display	Simplified Form	Expression	Substituting $x = 3$ and $x = -2$
		$2x$	When $x = 3$, $2x = 6$ When $x = -2$, $2x = -4$

2. What is the smallest number of tiles you can add to each group to make zero? Explain.



3. Represent each integer using exactly 10 tiles. Use only tiles that represent +1 and -1.

a) +8

b) -6

c) -4

d) -8

e) +2

f) -2

2.6 Using Addition to Solve Equations

One year, Canada won 2 gold medals at the Winter Olympics. This number was 4 less than the number of gold medals won by Canada at the Summer Olympics that same year.

If we let the number of gold medals Canada won at the Summer Olympics be x , we can represent the above information with the equation $x - 4 = 2$.

Activity: Use Algebra Tiles

Write the equation represented by the algebra tiles.



Inquire

1. How many red 1-tiles must be added to the left side so that only the value of the x -tile remains?
2. What must you do to the right side to keep the equation balanced?
3. What is the value of x ?
4. How many gold medals did Canada win at the Summer Olympics?
5. Model the following equations using algebra tiles. Then, solve for x . Compare your answers with a classmate's.

- a) $x - 2 = 5$ b) $x - 1 = 2$
 c) $x - 4 = 2$ d) $x - 5 = -4$

6. Write a rule for solving equations by addition.

To solve an equation, isolate the variable on one side of the equation.

Example 1

Solve the equation $x - 2 = 6$.

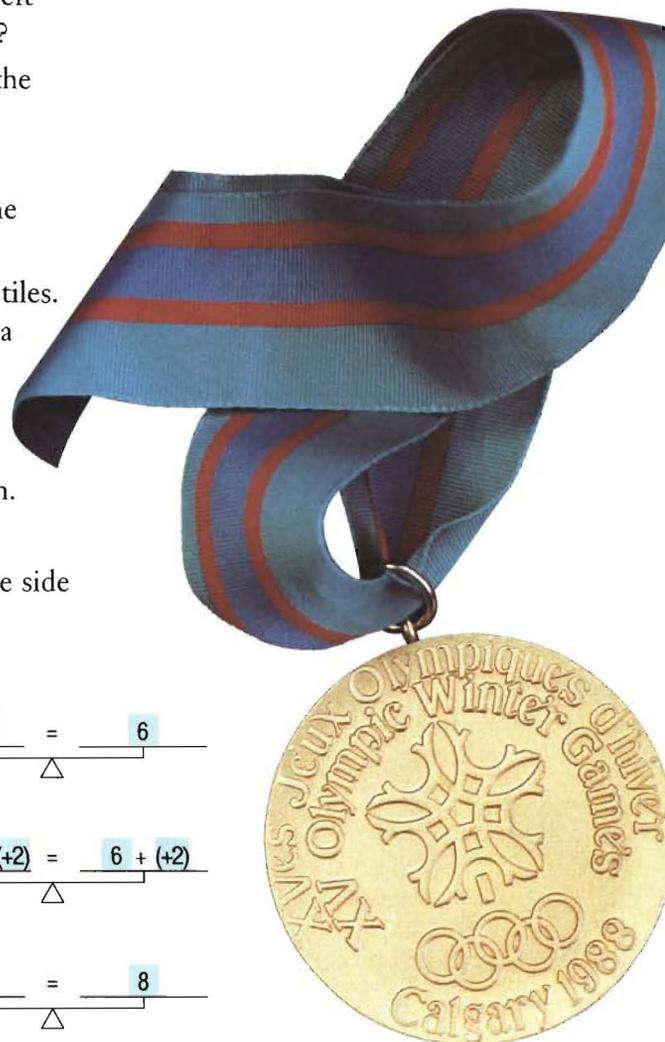
Solution

Add 2 to both sides of the equation.

$$\begin{aligned} x - 2 &= 6 \\ x - 2 + 2 &= 6 + 2 \\ x &= 8 \end{aligned}$$

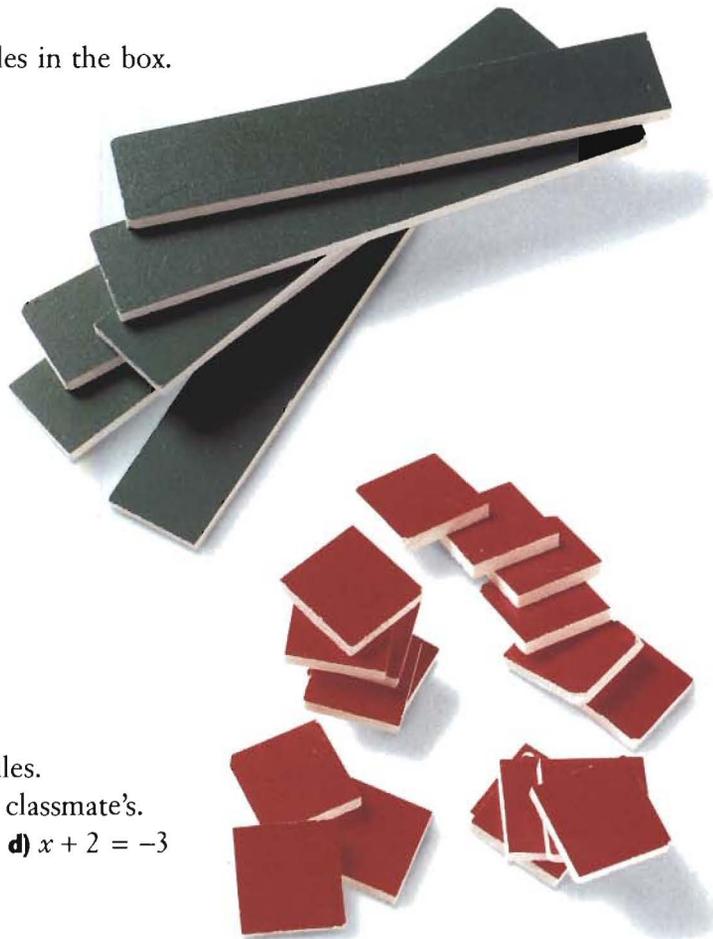
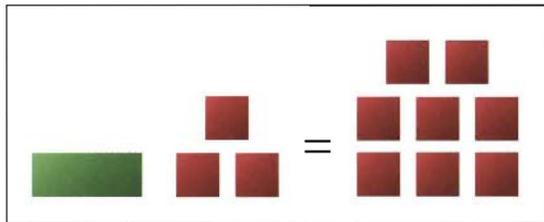
The solution is $x = 8$.

$$\begin{aligned} \frac{x - 2}{\triangle} &= \frac{6}{\triangle} \\ \frac{x - 2 + (+2)}{\triangle} &= \frac{6 + (+2)}{\triangle} \\ \frac{x}{\triangle} &= \frac{8}{\triangle} \end{aligned}$$



Activity: Use Algebra Tiles

Write the equation represented by the algebra tiles in the box.



Inquire

- How many white 1-tiles must be added to the left side so that only the value of the x -tile remains?
- What must you do to the right side to keep the equation balanced?
- What is the value of x ?
- Model the following equations using algebra tiles. Then, solve for x . Compare your answers with a classmate's.
 - $x + 1 = 5$
 - $x + 3 = 2$
 - $x + 4 = -2$
 - $x + 2 = -3$

Example 2

Solve and check $x + 7 = -2$.

Solution

To isolate the variable, add -7 to both sides of the equation.

$$\begin{aligned} x + 7 &= -2 \\ x + 7 + (-7) &= -2 + (-7) \\ x &= -9 \end{aligned}$$

Substitute $x = -9$ into the left side of the equation. The solution is correct if the value of the left side equals the value of the right side.

Check:

L.S.	$= x + 7$	R.S.	$= -2$
	$= -9 + 7$		
	$= -2$		

The solution is $x = -9$.

$$\frac{x + 7}{\triangle} = \frac{-2}{\triangle}$$

$$\frac{x + 7 + (-7)}{\triangle} = \frac{-2 + (-7)}{\triangle}$$

$$\frac{x}{\triangle} = \frac{-9}{\triangle}$$

Practice

Write and solve the equations shown by the tiles.

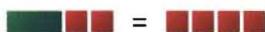
1.



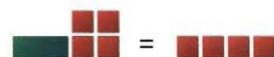
2.



3.



4.



What number would you add to both sides to solve each equation?

- | | |
|------------------|-------------------|
| 5. $x - 3 = 11$ | 6. $x - 1 = 5$ |
| 7. $n - 7 = -8$ | 8. $-8 = -4 + m$ |
| 9. $3 = y - 10$ | 10. $z - 5 = -11$ |
| 11. $x + 6 = 13$ | 12. $8 = x + 1$ |
| 13. $y + 2 = -7$ | 14. $m + 5 = -5$ |
| 15. $x + 3 = 9$ | 16. $-10 = 7 + z$ |

Solve.

- | | |
|-------------------|-------------------|
| 17. $m - 5 = -4$ | 18. $2 = -3 + n$ |
| 19. $p - 7 = -3$ | 20. $r + 7 = -9$ |
| 21. $-3 = s + 5$ | 22. $t - 8 = 10$ |
| 23. $2 + x = -4$ | 24. $7 = x - 5$ |
| 25. $x + 4 = -8$ | 26. $-11 = t - 1$ |
| 27. $13 = -2 + x$ | 28. $y - 5 = -8$ |
| 29. $12 + q = 10$ | 30. $d - 7 = -7$ |

Problems and Applications

Solve and check.

- | | |
|----------------------|------------------------|
| 31. $x + 1.5 = 3.5$ | 32. $m - 3.2 = 4.8$ |
| 33. $4.6 = t - 1.4$ | 34. $5.7 = r + 3.6$ |
| 35. $4.3 + m = 1.3$ | 36. $y - 2.4 = -1.4$ |
| 37. $9 = 8.2 + x$ | 38. $3.7 + t = 6.4$ |
| 39. $-1.4 + q = 4.4$ | 40. $10.3 = x - 5.4$ |
| 41. $-8.8 = w - 1.1$ | 42. $x + 15.7 = -18.1$ |

In questions 43–45, find the equation that represents the problem and solve it.

43. Three more than a number, x , is eight. What is the number?

- | | |
|----------------|----------------|
| a) $x - 3 = 8$ | b) $x + 8 = 3$ |
| c) $x + 3 = 8$ | d) $x - 8 = 3$ |

44. How many boxes are left to unload if there are 195 boxes in the shipment and 72 boxes have been unloaded?

- | | |
|-------------------|-------------------|
| a) $x + 72 = 195$ | b) $x + 195 = 72$ |
| c) $x - 72 = 195$ | d) $x - 195 = 72$ |

45. In a triangle, the sum of the sides is 9.7 cm. The lengths of two of the sides are 3.2 cm and 4.5 cm. What is the length of the unknown side?

- | |
|--------------------------|
| a) $x - 4.5 = 9.7 - 3.2$ |
| b) $9.7 - x = 4.5 - 3.2$ |
| c) $4.5 + x - 3.2 = 9.7$ |
| d) $x + 3.2 + 4.5 = 9.7$ |

46. The number of moons around Uranus is 5 more than the number of rings. Uranus has 15 moons. Solve the equation $r + 5 = 15$ to find the number of rings around Uranus.

47. A nurse walks about 6.3 km a day, which is 0.8 km less than a letter carrier walks. Solve the equation $x - 0.8 = 6.3$ to find out how far a letter carrier walks in a day.



48. What is the result if you add 0 to both sides of an equation?



49. Find 2 solutions for each equation.

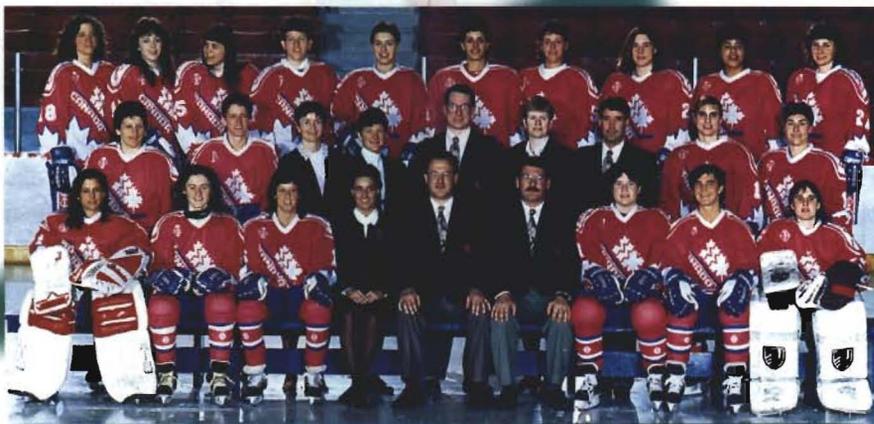
- | | |
|----------------------|------------------------|
| a) $x^2 - 1.8 = 7.2$ | b) $82.2 = t^2 + 18.2$ |
|----------------------|------------------------|



50. Write 1 equation that can be solved by adding a positive number and 1 equation that can be solved by adding a negative number. Each equation should have $x = -3$ as its solution. Have a classmate solve your equations.

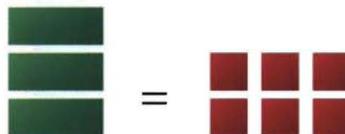
2.7 Using Division and Multiplication to Solve Equations

In 1992, Canada won the Women's World Hockey Championships. In the semi-final game against Finland, Canada scored 6 goals, 3 times as many as Finland. If we let x be the number of goals Finland scored, we can represent the above information with the equation $3x = 6$.



Activity: Use Algebra Tiles

Write the equation represented by the algebra tiles.



Inquire

- How many pairs of 1-tiles are shown?
- How many x -tiles are shown?
- How many 1-tiles does an x -tile represent?
- Divide both sides of the equation $3x = 6$ by 3. Compare your result with your answer to question 3.
- How many goals did Finland score against Canada?
- Represent the following equations using algebra tiles. Then, solve for x .

a) $2x = 10$ b) $3x = -6$ c) $-2x = 4$ d) $-4x = -4$

7. Write a rule for solving equations by division.

Example 1

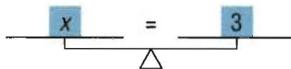
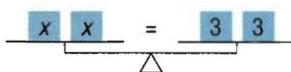
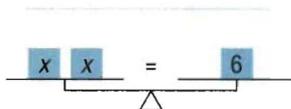
Solve the equation $2x = 6$.

Solution

Divide both sides of the equation by 2.

$$\begin{aligned} 2x &= 6 \\ \frac{2x}{2} &= \frac{6}{2} \\ x &= 3 \end{aligned}$$

The solution is $x = 3$.



Activity 2 Interpret the Diagram

The algebra tiles represent the equation $\frac{x}{2} = 1$.

What part of an x -tile is shown on the left side?



$$\frac{x}{2} = 1$$

Inquire

1. How many 1-tiles are shown on the right side?
2. How many 1-tiles does a whole x -tile represent?
3. Multiply both sides of the equation by 2.

What is the value of an x -tile? Compare this result with your answer to question 2.



4. Write a rule for solving equations by multiplication.

Example 2

Solve the equation

$$\frac{x}{2} = -5.$$

Example 3

Solve the equation

$$\frac{12}{x} = -4.$$

Example 4

Solve and check

$$-2x = 18.4.$$

Solution

Multiply both sides of the equation by 2.

$$\begin{aligned}\frac{x}{2} &= -5 \\ 2 \times \frac{x}{2} &= 2 \times (-5) \\ x &= -10\end{aligned}$$

The solution is $x = -10$.

Solution

Multiply both sides of the equation by x .

$$\begin{aligned}x \times \frac{12}{x} &= x \times (-4) \\ 12 &= -4x\end{aligned}$$

Divide both sides of the equation by -4 .

$$\begin{aligned}\frac{12}{-4} &= \frac{-4x}{-4} \\ -3 &= x\end{aligned}$$

The solution is $x = -3$.

Solution

Divide both sides of the equation by -2 .

$$\begin{aligned}-2x &= 18.4 \\ \frac{-2x}{-2} &= \frac{18.4}{-2} \\ x &= -9.2\end{aligned}$$

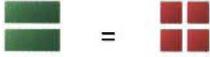
$$\begin{aligned}\text{Check: L.S.} &= -2x & \text{R.S.} &= 18.4 \\ &= -2(-9.2) \\ &= 18.4\end{aligned}$$

The solution is $x = -9.2$.

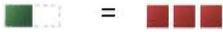
Practice

Write and solve the equation represented by the tiles.

1.



2.



By what number would you divide both sides to solve each equation?

3. $6x = 12$ 4. $2x = -8$ 5. $7z = -14$

6. $11r = 22$ 7. $6n = -18$ 8. $8y = 64$

By what number would you multiply both sides to solve each equation?

9. $\frac{x}{3} = 9$ 10. $\frac{x}{2} = -4$ 11. $\frac{y}{7} = 4$

12. $\frac{m}{5} = -6$ 13. $\frac{t}{4} = -1$ 14. $\frac{x}{6} = 0$

Solve.

15. $3x = 15$ 16. $2y = -12$ 17. $-8x = 16$

18. $-15 = 5p$ 19. $-6t = -24$ 20. $12 = 3s$

21. $2a = 24$ 22. $-25 = 5j$ 23. $-7x = -21$

Solve.

24. $\frac{x}{2} = 2$ 25. $\frac{y}{2} = -4$ 26. $\frac{x}{3} = 7$

27. $-4 = \frac{p}{3}$ 28. $\frac{8}{m} = 4$ 29. $-3 = \frac{6}{n}$

30. $\frac{10}{s} = 5$ 31. $-2 = \frac{12}{t}$ 32. $\frac{16}{u} = 4$

Solve and check.

33. $2x = 8.4$ 34. $3m = -6.3$

35. $\frac{x}{5} = 2.4$ 36. $\frac{3}{x} = -1.5$

37. $-4x = -2.8$ 38. $1.6x = 4.8$

39. $4 = \frac{m}{1.5}$ 40. $-8.4 = 0.2x$

Problems and Applications

In questions 41 and 42, find the equation that represents the problem and solve it.

41. Six dollars is to be divided equally among three people. How much money does each person receive?

a) $3x = 6$ b) $6x = 3$

c) $\frac{x}{3} = 6$ d) $\frac{x}{6} = 3$

42. One-third of the sum of the side lengths of an equilateral triangle is 4.2 cm. What is the sum?

a) $3x = 4.2$ b) $\frac{1}{3}x = 4.2$

c) $\frac{1}{3}x + 4.2 = 0$ d) $x = \frac{1}{3}(4.2)$

43. The average annual snowfall in Vancouver is about one-sixth of the average value for St. John's. Vancouver averages 60 cm of snow a year. Solve the equation $\frac{s}{6} = 60$ to find the average annual snowfall in St. John's.

44. The mass of a white-tailed deer is 1.4 times the mass of a cougar. White-tailed deer average 98 kg in mass. Solve the equation $1.4x = 98$ to find the mass of a cougar.

 45. What is the result if you multiply both sides of an equation by 0?

 46. Describe the solutions to these equations.

a) $3x = 0$ b) $0y = 2$

 47. Find 2 solutions to each equation.

a) $2x^2 = 72$ b) $\frac{m^2}{4} = 1$

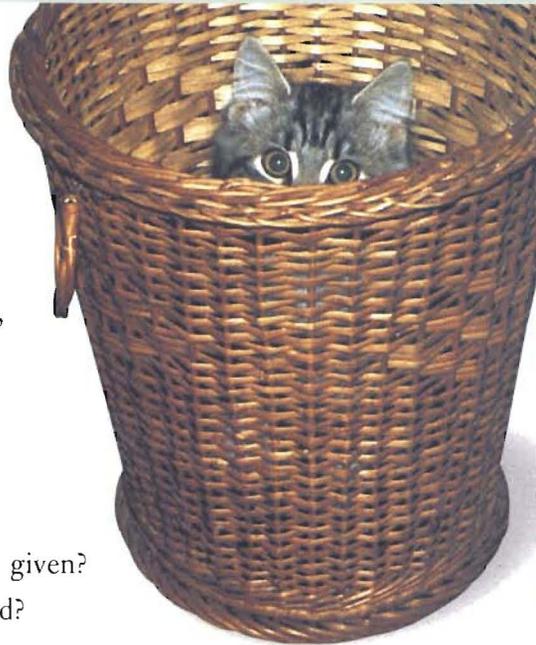
 48. Write one equation that can be solved by division and one that can be solved by multiplication. Each equation should have $x = 7$ as its solution. Have a classmate solve your equations.

2.8 Use Logic

Problems that can be solved using logic do not require any special skills in mathematics. These problems sharpen your deductive thinking skills.

A cat, dog, monkey, and elephant are named Trixie, Wags, Snow, and Boots. Use the clues to name each animal.

- Trixie is a friend of the cat and the elephant.
- The dog and Boots enjoy popcorn.
- Snow and Boots play golf with the monkey.
- The elephant does not play outdoor sports.



Understand the Problem

1. What information are you given?
2. What are you asked to find?

Think of a Plan

Make a table and fill in the facts from the clues.

Trixie is a friend of the cat and the elephant. So, Trixie is not the cat or the elephant.

	C	D	M	E
Trixie	n			n
Wags				
Snow				
Boots				

Carry Out the Plan

The dog and Boots enjoy popcorn. So, the dog is not Boots.

	C	D	M	E
Trixie	n			n
Wags				
Snow				
Boots		n		

Snow and Boots play golf with the monkey. So, Snow and Boots are not the monkey.

	C	D	M	E
Trixie	n			n
Wags				
Snow			n	
Boots		n	n	

The elephant does not play outdoor sports. So, the elephant does not play golf and is not Snow or Boots. The table now shows that the Elephant must be Wags, and the cat must be Boots. We can now complete the table.

	C	D	M	E
Trixie	n			n
Wags				
Snow			n	n
Boots		n	n	n

	C	D	M	E
Trixie	n	n	y	n
Wags	n	n	n	y
Snow	n	y	n	n
Boots	y	n	n	n

Trixie is the monkey, Wags is the elephant, Snow is the dog, and Boots is the cat.

Look Back

Check that the answer agrees with the given facts.

Use Logic

1. Organize the information.
2. Draw conclusions from the information.
3. Check that your answer is reasonable.



Problems and Applications

1. Maria, Paula, and Shelly attend the same school. One is in grade 9, one is in grade 10, and one is in grade 11. Use the clues to find which grade each is in.

- Maria and the grade 9 student eat lunch with Shelly.
- Maria is not in grade 10.

2. Four students named Al, Bjorn, Carl, and Don each have a favourite sport. The sports are swimming, running, bowling, and golf. Use the clues to match each student with his sport.

- The runner met Bjorn and the golfer for lunch.
- Neither Bjorn's sport nor Carl's sport requires a ball.
- Don is in the same math class as the golfer's sister.

3. Susan, Irina, Traci, and Debbie are a pilot, a dentist, a doctor, and a writer. Use the clues to match each person with her profession.

- Debbie is a friend of the doctor.
- Susan and the writer sail with Traci.
- Neither Susan nor Irina has patients.

4. Four people are running a marathon. Manuel is 30 m behind Margaret. Coreen is 20 m behind Tom. Margaret is 75 m ahead of Coreen. How far ahead of Tom is Manuel?

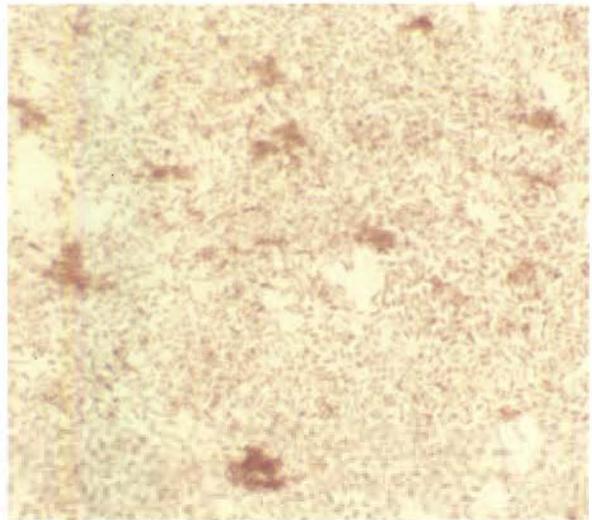
5. Sonya had \$1.19 in change. None of the coins was a dollar. Greg asked her for change for a dollar, but Sonya could not make change. What coins did she have?

6. The odometer on Anitha's car showed 25952 km. The number 25952 is called a **palindrome**. It reads the same forward and backward. Anitha drove for 2 h, and the odometer showed the next palindrome. How far did she drive in the 2 h?

7. A building has 5 doors. In how many ways can you enter the building by one door and leave by another?

8. A photographer wants to take a picture of the curling team. In how many ways can the 4 curlers be arranged side by side?

9. In a bacterial culture, the number of bacteria doubled every minute. At midnight, the glass jar was full and contained 1 000 000 bacteria. At what time was the jar half full?



10. Evans, Thompson, Smith, and DiMaggio are a teacher, plumber, artist, and banker. Use the clues to match each person with a profession.

- Thompson met the artist when he hired her to paint a picture of his horse.
- The plumber and DiMaggio are friends who have never had any business dealings.
- Neither Evans nor the plumber has ever met Smith.
- DiMaggio asked the banker for a loan.



11. Make your own logic puzzle like those in questions 2 and 3. First, set up a chart and mark the answers you want. Then, write the clues. Test your puzzle before asking a classmate to solve it.

Equations and Number Tiles

Prepare a set of 10 number tiles with the numbers from 0 to 9 written on them.



Activity 1

1. Use each of the tiles only once to solve the equations. The x -value for each equation is shown in brackets. Three of the tiles have been placed for you.

$$x - \blacksquare = 2 \quad (x = 5)$$

$$x - \blacksquare = \blacksquare - 3 \quad (x = 5)$$

$$0 + x - \blacksquare = \blacksquare + 1 \quad (x = 10)$$

$$x + \blacksquare - 8 = 9 + \blacksquare \quad (x = 11)$$

Activity 2

Use each of the tiles only once to solve the equations. The x -value for each equation is shown in brackets. Three of the tiles have been placed for you.

$$x + \blacksquare = 7 \quad (x = 4)$$

$$x + \blacksquare = \blacksquare + 2 \quad (x = 3)$$

$$\blacksquare + x + 6 = \blacksquare + 3 \quad (x = 5)$$

$$x - 2 + \blacksquare = \blacksquare - 0 \quad (x = 3)$$



Activity 3

Use each of the tiles only once to solve the equations. The x -value for each equation is shown in brackets. Three of the tiles have been placed for you.

$$\blacksquare x = 8 \quad (x = 2)$$

$$\blacksquare x + \blacksquare = 8 \quad (x = 1)$$

$$\blacksquare + 2x = \blacksquare \quad (x = 3)$$

$$0 + \blacksquare x = \blacksquare + 9 + 1 \quad (x = 3)$$

Activity 4

Use each of the tiles only once to solve the equations. The x -value for each equation is shown in brackets. Two of the tiles have been placed for you.

$$\frac{x}{\blacksquare} = 2 \quad (x = 12)$$

$$\frac{x}{\blacksquare} + \blacksquare = 5 \quad (x = 5)$$

$$\blacksquare + \frac{x}{2} = \blacksquare + 2 \quad (x = 6)$$

$$\frac{x}{\blacksquare} + \blacksquare = 7 - \blacksquare \quad (x = 18)$$

Activity 5

Create a set of equations similar to those in Activities 1, 2, 3, and 4. Have a classmate solve your equations.



2.9 Work Backward

Working backward can be a very useful problem solving technique. To solve some crimes, a detective will start at the scene of the crime and retrace the criminal's steps, or work backward, until the crime is solved.

The departure time for the Orient Express train from Calais to Paris is 21:20. Sam Orwell, a private detective, plans to board the train 35 min before departure. Orwell's hotel, the Rampant Lion, is a 25-min cab ride from the train station.

Before leaving the hotel, Orwell needs 30 min to eat dinner. After dinner, he needs 10 min to pack and 15 min to phone Interpol from his room. On the way to the train station, Orwell needs to stop for 15 min at the ambassador's residence to get a copy of a set of fingerprints. To reach the train on time, when should Orwell

- a) leave his hotel for the train? b) go to dinner?

1. What information are you given?
2. What are you asked to find?
3. Do you need an exact or approximate answer?

Start at the departure time of the train and work backward.

Train departure time	21:20
Time to arrive before departure	35 min
Orwell's arrival at the train	20:45
Time for cab ride and ambassador	$25 + 15 = 40$ min
When to leave hotel for the train	20:05
Time for dinner, packing, and phone call	$30 + 10 + 15 = 55$ min
When to go to dinner	19:10

- a) Orwell should leave the hotel at 20:05.
 b) Orwell should go to dinner at 19:10.

Is the answer reasonable?
 Can you think of another way to solve the problem?

Work Backward

1. Start with what you know.
2. Work backward to get an answer.
3. Check that the answer is reasonable.

Understand the Problem

Think of a Plan

Carry Out the Plan

Look Back

Problems and Applications

1. Clara's plane leaves at 08:15. She has to be at the airport 40 min early. The cab ride to the airport is about 55 min. Clara wants to spend 30 min at her office on her way to the airport. When should she leave home?
2. Michiko went to the county fair. She spent half her money on rides. She spent half of what was left on food. Of the remainder, she spent half at the arcade. She had \$6 left. How much money did she start with?
3. Sandra bought a gold chain for \$150. The jeweller added one-half of his cost price to get the price of \$150. What did the jeweller pay for the chain?
4. The exam starts at school at 09:00. You want to get there 15 min early. It takes 20 min to 30 min to get to school, depending on the buses. You plan to study for 1 h at home. It will take you 1 h to shower, dress, and eat breakfast. For what time should you set your alarm?
5. Jennifer wants a jacket that costs \$230, including tax. She has saved \$70. If she saves the rest in equal amounts over the next 4 months, how much will she save each month?
6. Ahmed's plane leaves at 12:30. Ahmed can drive 120 km to the airport at an average speed of 80 km/h. He needs to be at the airport 45 min before departure. When should he leave home?
7. Tanya saved for a band trip costing \$725. She worked part-time at a convenience store and earned \$7.50/h. Then, her aunt gave her \$125 toward the trip. For how many hours did Tanya have to work to afford the trip?
8. Dino multiplied his age by 3. Then he subtracted 3, divided by 7, and added 12. The result was 18. How old was Dino?

9. The train left Sutton for Bent Creek. At Brownsville, 35 people got on and 27 got off. Next came Liberty, where 24 people got on and 11 got off. At Long Lake, 55 people got on and 42 people got off. At Bent Creek, 105 people got off and 10 got on. There were 203 people on the train when it left Bent Creek. How many people were on the train when it left Sutton?

10. Mike works for an art gallery. An art dealer offers him a painting for \$72 000. The dealer says the painting has increased in value by one-half of the previous year's value in each of the past 2 years.

- a) What was its value 2 years ago?
- b) If the painting's value continues to increase at this rate, what will it be 3 years from now?

11. The Canadian population was about 27 300 000 in 1991. It had increased by about 3 000 000 since 1981. The 1981 figure was about 7 times the figure at Confederation in 1867. What was the Canadian population at Confederation to the nearest 100 000?

12. Write a problem that can be solved by working backward. Have a classmate solve your problem.



WORD POWER

In an anagram, the letters can be in any order except the right one. The word TEN has 5 anagrams.

NET NTE ETN ENT TNE

1. How many anagrams are there for

- a) FOUR? b) NINE?



2. Why are your answers different, even though FOUR and NINE both contain 4 letters?

Knowledge-Based and Expert Systems

Knowledge-based systems and **expert systems** are specialized types of computer software. Knowledge-based systems use facts and rules you might find in a manual or textbook. An example of a knowledge-based system is a chess-playing program. Expert systems provide you with the knowledge of human experts in a particular field. The programs are based on a combination of textbook knowledge and the decision-making processes supplied by experts. One example is the medical consultation system called CADUCEUS. This system is like many expert systems in that it is interactive. The user works with the system to solve a problem through a series of questions and answers, which eventually leads to a solution.

Activity 1 Knowledge-Based Systems

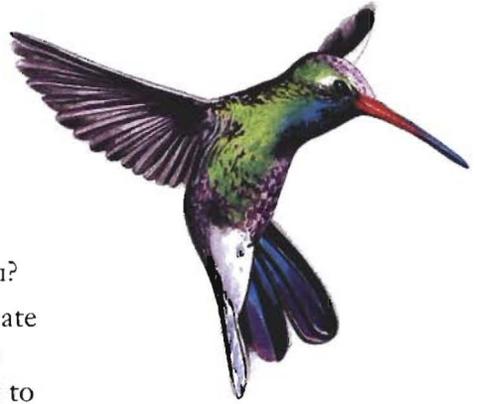
1. Work with your classmates to list the knowledge-based systems that you know. You are probably familiar with more than you think.
2. Describe what each knowledge-based system is used for.





Activity 2 Expert Systems

1. Describe what you would expect to find in an expert system called *Bird Species Identification*.
2. Use your research skills to find 5 areas in which expert systems have already been developed. Write a brief description of what each system does.
3. Predict 3 areas in which you think expert systems will be developed in the future. Describe how you think the systems will be used.
4. In what area would you like to see an expert system developed? What would you want the program to tell you?
5. There are many areas in which it is very difficult to create expert systems. A system that tells you how to predict the weather is one example. Why is an expert system difficult to create in this area? In what other areas do you think it would be difficult to have an expert system?



Example 1

Solve the equation

$$5x + 4 = -16.$$

SolutionAdd -4 to both sides:

$$5x + 4 = -16$$

$$5x + 4 + (-4) = -16 + (-4)$$

Divide both sides by 5:

$$5x = -20$$

$$\frac{5x}{5} = \frac{-20}{5}$$

$$x = -4$$

The solution is $x = -4$.**Example 2**

Solve and check

$$7.2 = 3x - 4.2.$$

Solution

Add 4.2 to both sides:

$$7.2 = 3x - 4.2$$

$$7.2 + 4.2 = 3x - 4.2 + 4.2$$

Divide both sides by 3:

$$11.4 = 3x$$

$$\frac{11.4}{3} = \frac{3x}{3}$$

$$3.8 = x$$

Substitute $x = 3.8$.

Check: L.S. = 7.2

R.S. = $3(3.8) - 4.2$
 $= 11.4 - 4.2$
 $= 7.2$

The solution is $x = 3.8$.**Example 3**

Solve and check

$$\frac{9x}{2} - 6 = 21.$$

Solution

Add 6 to both sides:

$$\frac{9x}{2} - 6 = 21$$

$$\frac{9x}{2} - 6 + 6 = 21 + 6$$

Multiply both sides by 2:

$$\frac{9x}{2} = 27$$

$$2 \times \frac{9x}{2} = 2 \times 27$$

Divide both sides by 9:

$$9x = 54$$

$$\frac{9x}{9} = \frac{54}{9}$$

$$x = 6$$

Substitute $x = 6$.**Check:**

L.S. = $\frac{9x}{2} - 6$

R.S. = 21

$$= \frac{9(6)}{2} - 6$$

$$= \frac{54}{2} - 6$$

$$= 27 - 6$$

$$= 21$$

The solution is $x = 6$.

Example 4

Solve the equation
 $7x - 9x - 6 = 21 - 5$.

Solution

Simplify both sides of the equation.
Then, apply the rules for solving equations.

$$7x - 9x - 6 = 21 - 5$$

$$-2x - 6 = 16$$

Add 6 to both sides: $-2x - 6 + 6 = 16 + 6$

$$-2x = 22$$

Divide both sides by -2 :

$$\frac{-2x}{-2} = \frac{22}{-2}$$

$$x = -11$$

The solution is $x = -11$.

Example 5

Solve $3x - 2 = 5$.
Round your answer to the
nearest tenth.

Solution

$$3x - 2 = 5$$

Add 2 to both sides: $3x - 2 + 2 = 5 + 2$

$$3x = 7$$

Divide both sides by 3:

$$\frac{3x}{3} = \frac{7}{3}$$

$$x \doteq 2.33$$

The solution is $x = 2.3$ to the nearest tenth.

Practice

Draw flow charts to show the solution steps for each equation.

1. $5x + 2 = 22$

2. $2x + 5 = 25$

Solve.

3. $3x = 11 + 1$

4. $2y - 5 = 9$

5. $5n + 2n = -14$

6. $3m + 7 = 19$

7. $4x + 2x = -18$

8. $4y - 9y = 35$

9. $5w + w = 7 + 23$

10. $6n + 3n = -18$

Solve.

11. $3y - 5y = 4$

12. $3t + 7t = -30$

13. $3x + 2x = -20$

14. $5 + 11 = 4y$

15. $15 = 2n + 3n$

16. $5 + n = -15$

17. $3 + m = -7$

18. $4 + x = 20$

Solve and check.

19. $\frac{x}{3} = 4 + 2$

20. $\frac{y}{2} = 6 - 3$

21. $\frac{n}{3} + 3 = 5$

22. $\frac{m}{5} - 5 = -9$

23. $4 + \frac{x}{3} = -7$

24. $\frac{y}{2} + 5 = 3$

25. $\frac{x}{4} - \frac{1}{4} = -\frac{3}{4}$

26. $\frac{y}{2} + \frac{1}{4} = \frac{3}{4}$

Solve and check.

27. $4x + 3x + 7 = 21$

28. $2y - 5y - 5 = 13$

29. $4t + 7t = 15 - 4$

30. $6s + 2s + s = 18$

31. $4t - 7t = 8 + 4$

32. $3y = 10 - 6 - 7$

33. $4 + 3t = -6 - 2$

34. $x + 2x = -15 - 6$

Solve and check.

35. $5n + 3n - 2n = 17 - 2 + 9$

36. $2m + 5m + m = -10 - 5 - 1$

37. $3x + 5x + 1 = 11 - 2$

38. $4 + 3t - 5t = 12$

39. $2s + 3s - 5 = 18 + 12$

40. $4y - 7 + 2y = -24 - 1$

Solve.

41. $2x + 1.4 = 7.8$

42. $5t - 2.1 = 8.9$

43. $1.4m - 3.6 = 3.4$

44. $9.2 = 1.5t + 1.7$

45. $6 - 1.2x = 8.4$

46. $9.3 + 2.5k = 1.3$

47. $2 - 1.8r = 11$

48. $4.2 + 0.5y = 8.1$

Solve. Round your answer to the nearest tenth.

49. $7x + 5 = -3$

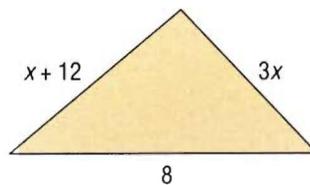
50. $2.4n - 3.8 = 1.1$

51. $3 = 3y + 8$

52. $4 = -6z + 14$

Problems and Applications

53. The equation $(x + 12) + 3x + 8 = 40$ represents the perimeter of the triangle. Find the lengths of the sides.



54. Pierre Trudeau was Prime Minister of Canada for 15 years. This was 3 years longer than twice the number of years that John Diefenbaker was Prime Minister. Solve the equation $2y + 3 = 15$ to find out how long John Diefenbaker was Prime Minister.

55. a) Solve the equation $2x + 5 = 11$, using the division rule first.

b) Is the result the same when you use the addition rule first?



c) Is one method better than the other? Explain.



56. Write an equation in which the solution requires at least 2 steps and is $x = -15$. Have a classmate solve your equation.

NUMBER POWER

Use each of the numbers from 1 to 9 only once to make each statement true.

(■ - ■) ÷ ■ = 1

■ + ■ - ■ = 1

■ - ■ × ■ = 1

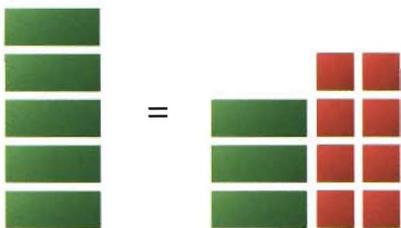
2.11 Solving Equations with Variables on Both Sides

At the 1992 Summer Olympics, Canadians won medals in 19 events, including the demonstration sport of taekwondo. Marcia King won a silver medal for Canada in this event.

Canadians were more successful in rowing than in any other sport. If we let x be the number of rowing events in which Canadians won medals, we can write the equation $x + 9 = 19 - x$. This is an example of an equation with a variable on both sides.

Activity: Use Algebra Tiles

Write the equation represented by the algebra tiles.



Inquire



1. Solve the equation for x . Describe your method.
2. Solve the equation $x + 7 = 2x + 3$ by isolating the variable on the right side of the equation.
3. Solve the equation $x + 7 = 2x + 3$ by isolating the variable on the left side of the equation.
4. Compare your results in questions 2 and 3.
5. Write the steps for solving equations with variables on both sides.
6. Solve the equation $x + 9 = 19 - x$ to find the number of rowing events in which Canadians won medals at the 1992 Summer Olympics.



Example

Solve the equation $5x - 8x = x + 8$.

Solution

Simplify before solving.

$$5x - 8x = x + 8$$

$$-3x = x + 8$$

Add $-x$ to both sides: $-3x + (-x) = x + 8 + (-x)$

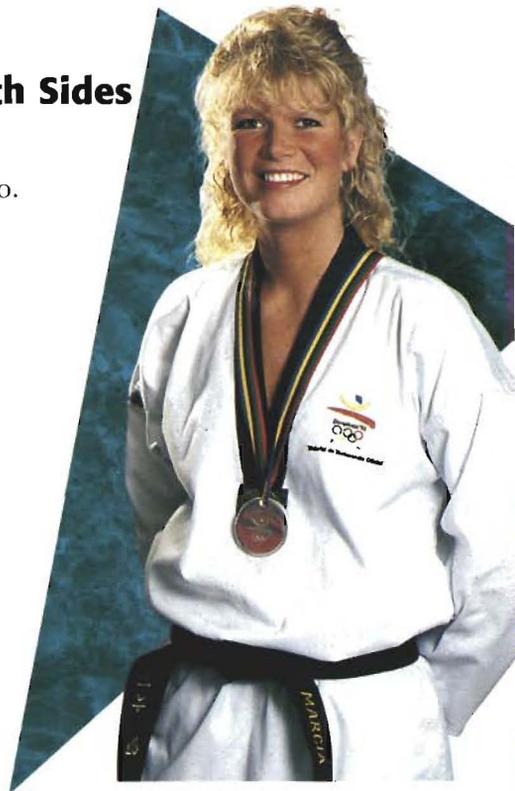
$$-4x = 8$$

Divide both sides by -4 :

$$\frac{-4x}{-4} = \frac{8}{-4}$$

$$x = -2$$

The solution is $x = -2$.



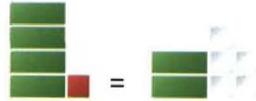
Practice

Write and solve the equations shown by the tiles.

1.



2.



Solve.

3. $5x + 1 = 4x + 7$ 4. $6y + 2 = 7y - 4$
5. $3m + 2 = m + 4$ 6. $7n + 6 = 4n - 6$
7. $9t + 4 = 5t - 8$ 8. $4s - 2 = 2s + 10$
9. $8x - 10 = -2x - 20$ 10. $3y - 5 = -2y + 20$

Solve.

11. $2x = -30 + 5x$ 12. $7y = 20 + 3y$
13. $3t = -12 + 7t$ 14. $12s = -36 + 6s$
15. $2a = 0.35 - 5a$ 16. $4n = 3.6 - 2n$

Solve.

17. $2x + 3x = 8x - 3$ 18. $5x - 4x = -x + 6$
19. $3y + y = 2y - 8$ 20. $5a - 3a = 6 - a$
21. $6s + 4s = 20 + 5s$ 22. $3t + 3t = -9 + 9t$
23. $6r + 4r = 16 + 2r$ 24. $7x - 4x = x - 10$

Solve.

25. $5x - 3x = 4x - 2$ 26. $7x - 4x = x + 6$
27. $3y + 5y = 5y - 6$ 28. $4t - 7t = -7 - 2$
29. $3s - 5s = 3 + 7$ 30. $6j - 7j = 3j + 12$
31. $5a - 2a = 5a - 8$ 32. $8y = 5y + y + 14$

Solve.

33. $15x - 7x = -52 - 5x$
34. $21y = -205 + 75 + 47y$
35. $7t + 6t = 14 - 53$
36. $m = 26.8 - 6.8 + 3m$
37. $2x = -8.4 - 3.6 - 4x$
38. $7y = 0.3 + 2.7 + 4y$
39. $3n = 10.1 + 9.9 - 2n$

Solve.

40. $2x + 10 = 9 + 11 - 3x$
41. $11s + 25 = 54 + 25 + 2s$
42. $65 + n = 85 - 16 + 3n$

Problems and Applications

43. Bill has x baseball cards. Abbas has $4x$ baseball cards. The number of cards Abbas has is equal to the number of cards Bill has plus 60. Solve the equation $x + 60 = 4x$ to find the number of cards Bill has.

44. Marlene scored y ringette goals. Lynn scored $3y$ goals. The number of goals that Marlene scored equals the number of goals Lynn scored minus 12. Solve the equation $y = 3y - 12$ to find the number of goals Marlene scored.

45. Solve for x . Assume the letters a , b , c , and y represent non-zero integers.

- a) $x + a = b + c + y$ b) $a + (x - b) = c - y$
c) $ax + b = c - y$ d) $a(x - b) + y = c$
e) $b + y = cx - a$ f) $\frac{x}{a} + b - y = -c$

46. The average wind speed in Bonavista, Newfoundland, is twice the average wind speed in Whitehorse, Yukon Territory. The difference between the wind speeds is 14 km/h. Solve the equation $2x - 14 = x$ to find the average wind speed in Whitehorse.

47. The number of states in India is 1 more than 4 times the number of states in Australia. There are 19 more states in India than in Australia. Solve the equation $x + 19 = 4x + 1$ to find the number of states in Australia.

-  48. Write an equation with a variable on both sides and a solution of $x = -2$. Have a classmate solve your equation.

2.12 The Distributive Property

Activity: Use Algebra Tiles

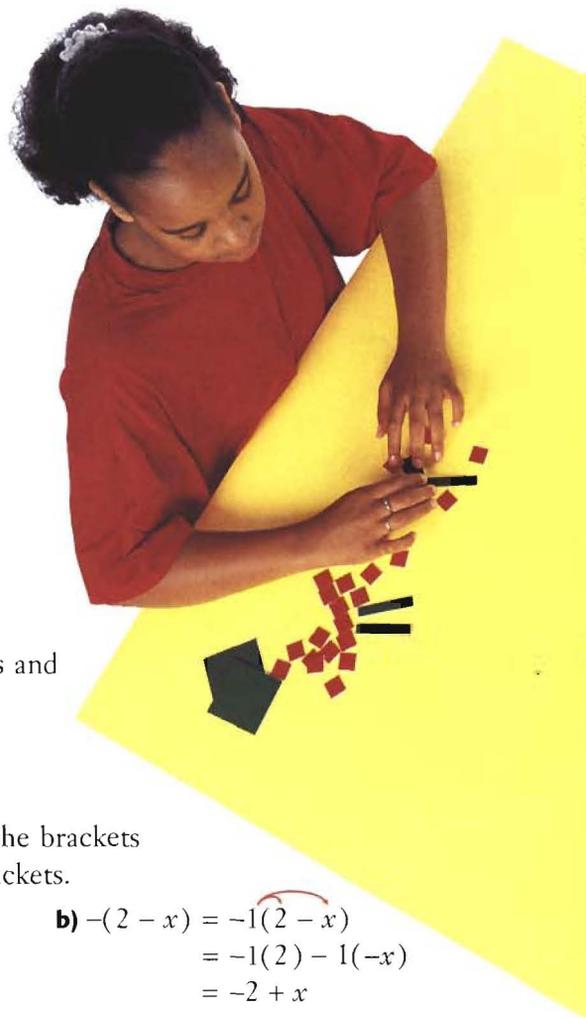
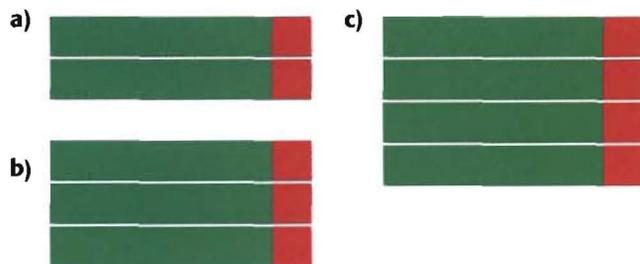
An x -tile has an area of x square units.

A 1-tile has an area of 1 square unit.

How many units long are the sides of a 1-tile?

How many units long is each side of an x -tile?

Use x -tiles and 1-tiles to model these rectangles.



Inquire

1. What is the area of each rectangle?
2. What are the length and width of each rectangle?
3. Describe a method for multiplying the length by the width to give the area of each rectangle.

To **expand** an expression means to remove the brackets and simplify. To do this, we use the **distributive property**.

$$a(b + c) = ab + ac$$

Example 1

Expand using the distributive property.

- a) $3(x + 5)$
 b) $-(2 - x)$

Solution

Multiply each term inside the brackets by the term outside the brackets.

$$\begin{aligned} \text{a) } 3(x + 5) &= 3(x + 5) \\ &= 3(x) + 3(5) \\ &= 3x + 15 \end{aligned} \qquad \begin{aligned} \text{b) } -(2 - x) &= -1(2 - x) \\ &= -1(2) - 1(-x) \\ &= -2 + x \end{aligned}$$

Example 2

Expand.

- a) $4(3x - 2y + 7)$
 b) $-2(x + 3) - (x - 5)$

Solution

$$\begin{aligned} \text{a) } 4(3x - 2y + 7) &= 4(3x - 2y + 7) \\ &= 12x - 8y + 28 \end{aligned} \qquad \begin{aligned} \text{b) } -2(x + 3) - (x - 5) &= -2(x + 3) - 1(x - 5) \\ &= -2x - 6 - x + 5 \\ &= -2x - x - 6 + 5 \\ &= -3x - 1 \end{aligned}$$

Practice

Expand.

1. $5(x + 1)$ 2. $3(x - 2)$ 3. $4(x + 2)$
4. $2(x - 3)$ 5. $7(x - 1)$ 6. $5(x + 3)$
7. $2(x + 6)$ 8. $4(x - 5)$ 9. $7(x + 3)$
10. $3(x - 4)$ 11. $10(x + 2)$ 12. $9(x - 3)$

Expand.

13. $2(3x + 2)$ 14. $3(3x + 1)$
15. $5(2x + 1)$ 16. $4(2x + 3)$
17. $6(2x - 1)$ 18. $5(3x - 2)$
19. $7(2x + 1)$ 20. $6(3x + 2)$

Expand.

21. $-3(x + 2)$ 22. $-4(2x + 1)$
23. $-2(5x - 2)$ 24. $-3(3x - 2)$
25. $-5(2x - 1)$ 26. $-2(5x - 3)$
27. $-4(x + 5)$ 28. $-(2x - 1)$

Expand.

29. $-2(3x - 2y)$ 30. $-3(5x + 3y)$
31. $-4(5x + 2y)$ 32. $-(2x + y)$
33. $-5(-x + y)$ 34. $-7(x - 3y)$
35. $-2(3x + 7)$ 36. $-4(2x - y)$

Expand.

37. $-4(2x + 3y + tz)$ 38. $3(5x - 2y + 2)$
39. $6(-x + 3y + 4)$ 40. $-(2x - 3y + 5)$

Expand and simplify.

41. $4(y - 3) + 2(y + 4)$
42. $2(5x - 1) - (3x + 2)$
43. $4(x + 7) - (3x + 2)$
44. $3(2x - 3) - 3(2x - 5)$
45. $3(2x - 5) + 3(2x + 1)$
46. $4(-x + 3) - 2(3x + 1)$
47. $-2(x - 3y + 2) - 3(x - 2y)$

Problems and Applications

Expand and simplify.

48. $3(x^2 + 4x - 3) + 2(x^2 + 5)$
49. $2(3x^2 + 7x - 3) + 4(x - 3)$
50. $5(y^2 + 3y - 4) + 2(y^2 - 4y + 1)$
51. $-(2x + 3y - 5) + 5(2x - 3y + 6)$
52. $-(2x + 3) + 3(x - 1) - 4(x - 2)$
53. $(2x + 1) - 4(2x - 3) + 2(3x - 1)$

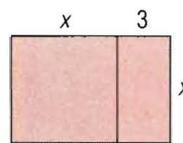
Expand and simplify.

54. $4(4x - 3y + 2) - (2x + 5y - 4)$
55. $-(3x - 2y + 7) + 4(x + y - 2)$
56. $2(x^2 + y^2) - 3x^2 + 4y^2 + 7$
57. $4(x^2 + 2x - 3) - 2(x^2 + 4y - 1)$
58. $5(2y^2 + 3y - 2) - 2(y^2 - 4y + 1)$

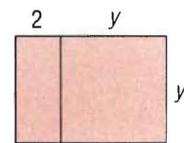
For each large rectangle, state

- a) the length and width.
b) the area in expanded form.

59.

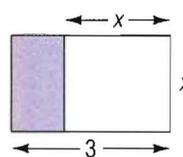


60.

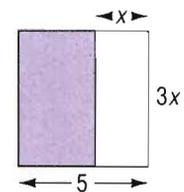


Write the area of the shaded rectangle in expanded form.

61.



62.



63. The money you deposit in a bank is called the principal, P . After your money earns interest, the new principal is given by the expression $P(1 + rt)$, where r is the interest rate and t is the length of time your money was deposited.

- a) Expand the expression $P(1 + rt)$.
b) Evaluate the expression for $P = \$300$, $r = 0.05$, and $t = 2$ years.

2.13 Solving Equations with Brackets

Activity: Solve the Equation

Canada became a country in 1867. If we take the number of provinces in Canada at that time, add 1, and double the sum, the result is Canada's present number of provinces. Thus, we can find the number of provinces in 1867 by solving the equation $2(x + 1) = 10$.

Inquire

- Expand to remove the brackets in the equation $2(x + 1) = 10$.
- Solve for x to find the number of provinces in 1867.
- Solve these equations.
 - $3(x - 1) = 12$
 - $-2(x + 1) = 6$
 - $2(x + 3) = -4$
 - $-(x + 1) = -7$
- Write a rule for solving equations with brackets.
- Research and list Canada's provinces in 1867.

Example 1

Solve the equation $2(x + 2) = -6$ and check.

Solution

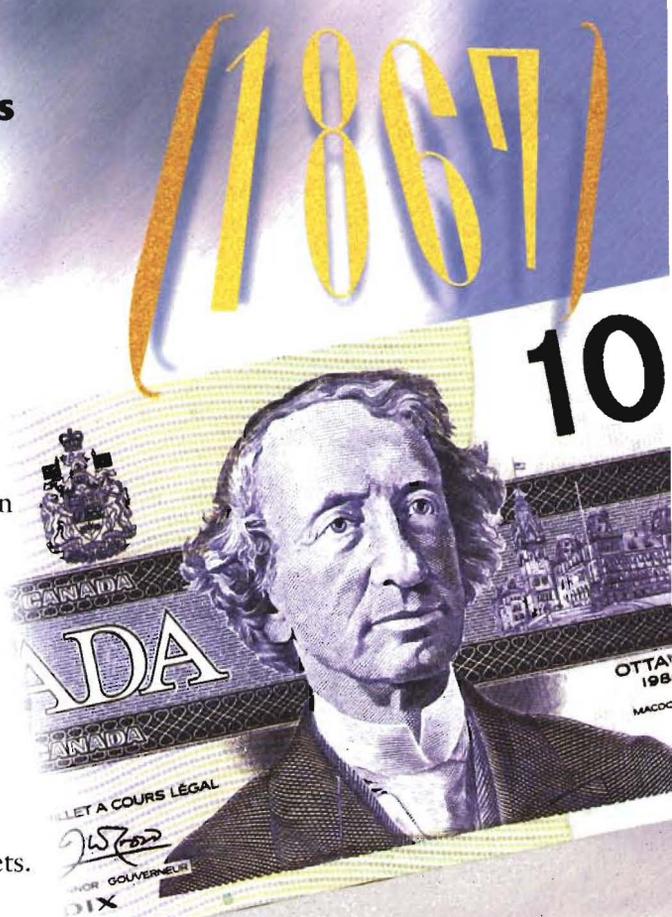
Expand then solve.

$$\begin{aligned}
 2(x + 2) &= -6 \\
 2x + 4 &= -6 \\
 \text{Add } -4 \text{ to both sides: } & \quad 2x + 4 + (-4) = -6 + (-4) \\
 & \quad 2x = -10 \\
 \text{Divide both sides by } 2: & \quad \frac{2x}{2} = \frac{-10}{2} \\
 & \quad x = -5
 \end{aligned}$$

Substitute $x = -5$.

Check:	L.S.	$= 2(x + 2)$	R.S.	$= -6$
		$= 2(-5 + 2)$		
		$= 2(-3)$		
		$= -6$		

The solution is $x = -5$.



Example 2

Solve the equation $2(x - 3) - 5 = 13 - 4x$ and check.

Solution

To follow the order of operations, first expand to remove the brackets.

$$\begin{aligned}2(x - 3) - 5 &= 13 - 4x \\2x - 6 - 5 &= 13 - 4x \\2x - 11 &= 13 - 4x \\2x - 11 + 11 &= 13 - 4x + 11 \\2x &= 24 - 4x \\2x + 4x &= 24 - 4x + 4x \\6x &= 24 \\\frac{6x}{6} &= \frac{24}{6} \\x &= 4\end{aligned}$$

Substitute $x = 4$.

Check:	L.S.	$= 2(x - 3) - 5$	R.S.	$= 13 - 4x$
		$= 2(4 - 3) - 5$		$= 13 - 4(4)$
		$= 2(1) - 5$		$= 13 - 16$
		$= 2 - 5$		$= -3$
		$= -3$		

The solution is $x = 4$.

Example 3

Solve and check.

$$3(2x - 5) - (x + 3) = 2(x + 1) + 4$$

Solution

$$\begin{aligned}3(2x - 5) - (x + 3) &= 2(x + 1) + 4 \\6x - 15 - x - 3 &= 2x + 2 + 4 \\6x - x - 15 - 3 &= 2x + 6 \\5x - 18 &= 2x + 6 \\5x - 2x &= 6 + 18 \\3x &= 24 \\x &= 8\end{aligned}$$

Check:	L.S.	$= 3(2x - 5) - (x + 3)$	R.S.	$= 2(x + 1) + 4$
		$= 3[2(8) - 5] - (8 + 3)$		$= 2(8 + 1) + 4$
		$= 3(16 - 5) - (11)$		$= 2(9) + 4$
		$= 3(11) - 11$		$= 18 + 4$
		$= 33 - 11$		$= 22$
		$= 22$		

The solution is $x = 8$.



Practice

Solve.

- $2(x + 1) = 4$
- $2(x - 3) = 2$
- $3(x + 1) = 6$
- $2(x + 3) = -6$
- $3(x + 2) = -9$
- $2(x + 5) = -4$

Solve and check.

- $2(3x + 4) = 14$
- $14 = 2(3x - 2)$
- $3(x + 5) = 18$
- $3(2x + 3) = -3$
- $-24 = 4(x + 3)$
- $5(2x + 3) = -15$

Solve.

- $2(x + 3) - 3 = 8 - 3x$
- $3(x + 1) + 10 = 8 - 2x$
- $8 - 3x = 4(x - 3) + 6$
- $5(2x - 3) + 6 = -35 - 3x$

Solve.

- $5(2x - 3) = 2(-3x - 2) + 5$
- $2(5x - 6) = (3x - 2) + 4$
- $3(5y + 4) = 5(2y - 3) + 22$
- $2(8n + 7) = -50 + 2(-5n + 6)$
- $5(2x - 3) = 2(3x + 7) + 11$

Solve and check.

- $2(x - 3) + (x + 3) = 6x$
- $5(x + 4) - (x + 2) = 8x + 2$
- $4(m - 2) - (m + 3) = m - 1$
- $4(n - 7) - 2(n + 3) = -15n$
- $4(y + 2) - 5(y + 1) = y - 1$

Solve.

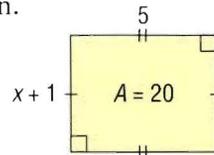
- $3(2x + 1) - (x - 2) = 2(3x + 4)$
- $12(2s - 1) - 4(-2s - 1) = 2(s + 11)$
- $2(x - 8) - (x - 4) = 3(x + 5) + 3$
- $7(2x - 1) - 2(5x - 6) = 2(4x - 5) + 7$
- $3(4n - 1) = 4(-n + 9) - 7$

Problems and Applications

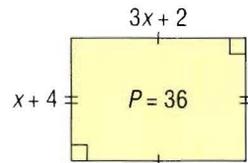
32. The equation $5(x + 1) = 20$ represents the area of the following rectangle.

a) Solve the equation.

b) Find the rectangle's width.



33. The equation $2(3x + 2) + 2(x + 4) = 36$ represents the perimeter of the rectangle shown below.



a) Solve the equation.

b) Calculate the rectangle's dimensions.

34. The Pacific Ocean accounts for 46% of the area of the water on the Earth's surface. If we take the percent that the Atlantic Ocean accounts for, subtract 1, and double the difference, the result is the percent that the Pacific Ocean accounts for. Solve the equation $2(x - 1) = 46$ to find the percent that the Atlantic Ocean accounts for.

- 35.** Write an equation that has 1 bracketed expression on its left side and 1 bracketed expression plus a number on its right side. The equation should have a solution of $x = -2$. Have a classmate solve your equation.

WORD POWER

Change the word BARN to the word DOOR by changing one letter at a time. You must form a real word each time you change a letter. The best solution has the fewest steps.

Equivalent Forms of Algebraic Equations

An algebraic equation has an infinite number of equivalent forms. For example, dividing each term in the equation $4x + 2 = 6$ by 2 gives the equivalent form $2x + 1 = 3$. Because the forms are equivalent, the solution to both forms is the same, $x = 1$.

Activity 1 Equivalent Whole-Number Forms

1. a) What whole numbers can be divided into each of the 3 terms of the equation $10x + 20 = 30$ to produce equivalent whole-number equations?

b) Write the equivalent whole-number equations you found by division.

c) Which equation from part b) is the least complex? Explain.

d) How could you write more whole-number equations equivalent to $10x + 20 = 30$?

2. a) Write three whole-number equations equivalent to $6x + 15 = 3$.

b) State the least complex whole-number equation that is equivalent to $6x + 15 = 3$.

c) Solve the equation you wrote in part b).

d) Without solving the equation $6x + 15 = 3$, state what the solution is. Check your answer by substitution.

3. Reduce each equation to its least complex equivalent whole-number form.

a) $8x - 4 = 12$

b) $100x + 300 = 1000$

c) $50x - 125 = 575$

d) $35x - 75 = 350$



Activity 2 Equations with Decimal Coefficients

1. a) Multiply each of the three terms in the equation $1.5x + 0.7 = 3.7$ by 10.

b) What type of equation is the resulting equation?

c) What is its solution?

2. a) State the smallest whole number by which you can multiply each term in the equation $0.01p - 0.2 = 0.25$ to produce an equivalent whole-number equation.

b) What is the resulting equation?

c) What is its solution?

3. Can equivalent whole-number equations always be written for any equation written in decimal form? Explain.

Activity 3 Equations with Fractional Coefficients

1. What is the coefficient of the variable in each of these equations?

a) $\frac{1}{2}x + \frac{1}{5} = 5$ b) $\frac{3}{2}x + \frac{1}{3} = -4$

c) $\frac{x}{7} + 8 = -3$ d) $\frac{5x}{4} - 11 = 13$

2. a) What is the coefficient of x in the equation $\frac{2x}{5} + \frac{1}{5} = 1$?

b) Why can we write $\frac{2x}{5} + \frac{1}{5}$ in the equivalent form $\frac{2x+1}{5}$?

c) The equation $2x + 1 = 5$ is the least complex whole-number form for the equation in part a). Explain how this equation was produced.

3. Write two other whole-number equations equivalent to the equation $2x + 1 = 5$.

4. Are these equations related? Explain.

$\frac{x}{3} + \frac{1}{2} = 1$ $2x + 3 = 6$ $34x + 51 = 102$

5. How are these equations related?

$\frac{x}{2} + \frac{3}{7} = 2$ $\frac{5x}{2} + \frac{15}{7} = 10$ $7x + 6 = 28$



Activity 4 Alternate Equivalent Rational Forms

1. Is the expression $\frac{1}{3}(x+1)$ an equivalent form of the expression $\frac{x+1}{3}$? Explain.

2. Write the expression $\frac{x+1}{3}$ as two separate expressions with a common denominator.

3. Which of the following expressions is equivalent to $\frac{3x-4}{5}$?

a) $3x - 4 \div 5$ b) $\frac{3x}{5} - \frac{4}{5}$ c) $5(3x - 4)$

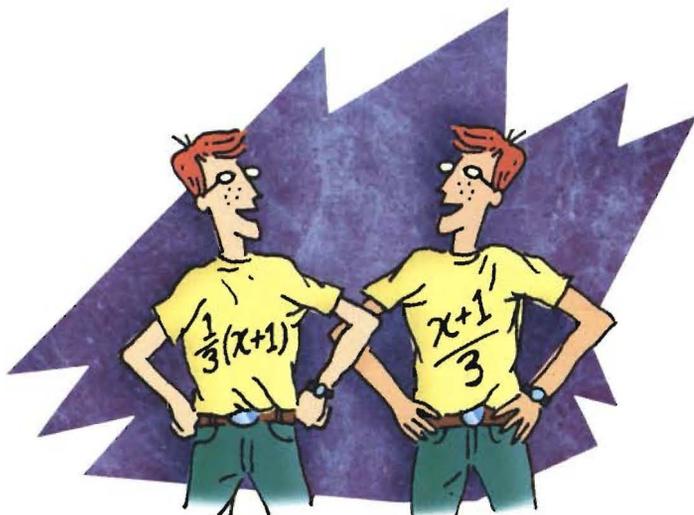
4. Explain how $\frac{x+3}{2} = 5$, $\frac{3x+9}{2} = 15$, and $2x + 6 = 20$ are related.

5. a) The equations $\frac{x}{2} + \frac{1}{5} = 5$, $\frac{3x}{2} + \frac{3}{5} = 15$, and $5x + 2 = 50$ are equivalent forms with the same solution. If you were asked to find the solution, which form would you solve? Explain.

b) Solve the equation.

6. Explain how $C = \pi d$ and $\pi = \frac{C}{d}$ are related.

7. If speed is distance divided by time, explain why time is distance divided by speed.



2.14 Solving Equations with Fractions and Decimals

In 1990, a survey of Canadian households found that the percent with camcorders was 2.1% less than one-half the percent with compact disc players. The percent of households with camcorders was 5.6%.

We can write the equation $0.5x - 2.1 = 5.6$, where x is the percent of households with compact disc players.

Activity: Solve the Equations

Solve these equations.

a) $1.2x - 0.4 = 2$

b) $0.25x - 5 = 2.75$

c) $3.14n + 2 = 12.99$

d) $2.3t - 1.47 = 6.85 + 0.7t$

Inquire

1. Multiply the terms in the equations a) to d) by a power of 10 to produce an equation with whole-number coefficients.

2. Solve each equation you wrote in question 1.

 3. Compare the 2 solution methods you used. Which method do you prefer? Explain.

4. Solve the equation $0.5x - 2.1 = 5.6$ to find the percent of Canadian households with compact disc players in 1990.

5. Research the percent of Canadian households with compact disc players today. Compare your findings with your answer to question 4.

Example 1

Solve the equation $5.84 - 2.2y = 18.6$.

Solution 1

The number 5.84 has the greatest number of decimal places. To obtain whole-number coefficients, multiply each term by 100.

$$5.84 - 2.2y = 18.6$$

$$100 \times (5.84 - 2.2y) = 100 \times 18.6$$

$$100 \times 5.84 - 100 \times 2.2y = 100 \times 18.6$$

$$584 - 220y = 1860$$

$$-220y = 1276$$

$$y = -5.8$$

 $1000 \div (-200) = -5$

The solution is $y = -5.8$.

Solution 2

Keep the decimals and solve with a calculator.

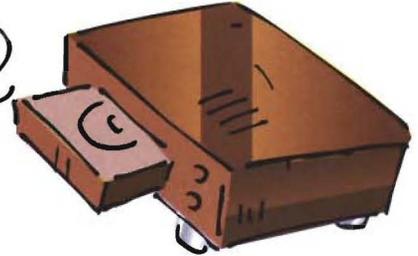
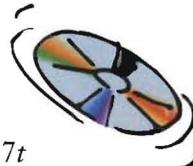
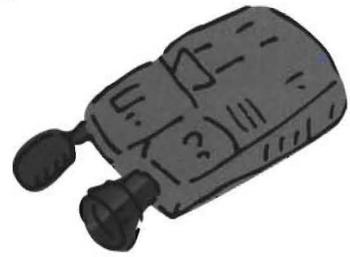
$$5.84 - 2.2y = 18.6$$

$$-2.2y = 18.6 - 5.84$$

$$-2.2y = 12.76$$

$$y = -5.8$$

 $10 \div (-2) = -5$



Example 2

Solve $\frac{x}{3} - \frac{3x}{2} = \frac{1}{6} - x$.

Solution

To eliminate the fractions, multiply each term in the equation by the lowest common denominator, 6.

$$\begin{aligned}\frac{x}{3} - \frac{3x}{2} &= \frac{1}{6} - x \\ 6 \times \left(\frac{x}{3}\right) - 6 \times \left(\frac{3x}{2}\right) &= 6 \times \left(\frac{1}{6}\right) - 6 \times x \\ 2x - 9x &= 1 - 6x \\ -7x &= 1 - 6x \\ -7x + 6x &= 1 \\ -x &= 1 \\ x &= -1\end{aligned}$$

The solution is $x = -1$.

Example 3

Solve and check
 $\frac{x-2}{3} + \frac{x+1}{2} = 4$.

Solution

Multiply all terms by the lowest common denominator, 6.

$$\begin{aligned}\frac{x-2}{3} + \frac{x+1}{2} &= 4 \\ 6 \times \frac{(x-2)}{3} + 6 \times \frac{(x+1)}{2} &= 6 \times 4 \\ 2(x-2) + 3(x+1) &= 24 \\ 2x - 4 + 3x + 3 &= 24 \\ 5x - 1 &= 24 \\ 5x &= 25 \\ x &= 5\end{aligned}$$

The division bar is a grouping symbol. It acts like a bracket.

Check: L.S. = $\frac{x-2}{3} + \frac{x+1}{2}$ R.S. = 4

$$\begin{aligned}&= \frac{5-2}{3} + \frac{5+1}{2} \\ &= \frac{3}{3} + \frac{6}{2} \\ &= 1 + 3 \\ &= 4\end{aligned}$$

The solution is $x = 5$.

Practice

Solve.

- $x + 0.2 = 0.8$
- $s + 1.2 = 1.5$
- $m + 5.2 = 2.2$

- $y - 0.5 = 1.2$
- $t + 4.5 = -3.0$
- $-2.3 = 4.6 + n$

Solve.

- $0.3x = 0.3$
- $1.2m = -3.6$
- $4.8 = 2.0n$
- $-5 = 0.5y$
- $0.3n = 1.2$
- $1.1s = -4.4$

Solve and check.

13. $3x - 0.4 = 0.8$ 14. $3.64 = 2m + 4.7$

15. $4y + 0.88 = 5.24$ 16. $2.5 = 3.7 + 0.2x$

17. $0.3r + 0.54 = -3$ 18. $1.2s + 3.6 = 4.8$

Solve.

19. $\frac{x}{4} = \frac{1}{2}$ 20. $\frac{y}{12} = \frac{1}{3}$ 21. $\frac{8}{10} = \frac{n}{5}$

22. $\frac{m}{6} = -\frac{1}{3}$ 23. $-\frac{1}{9} = \frac{y}{27}$ 24. $\frac{x}{8} = -\frac{1}{4}$

Solve and check.

25. $\frac{y}{2} = \frac{y}{3} - 1$ 26. $\frac{y}{4} = \frac{y}{5} + 1$

27. $\frac{5n}{2} = \frac{4n}{3} - \frac{7}{6}$ 28. $\frac{p}{2} - \frac{3p}{4} = \frac{3}{4} - p$

29. $\frac{n}{3} + 2 = \frac{n}{5} + 4$ 30. $\frac{(x+1)}{3} = \frac{(x-1)}{5}$

31. $\frac{(3-y)}{5} = \frac{(-2-3y)}{4}$ 32. $\frac{(2x-3)}{2} = \frac{(-x-1)}{4}$

Solve and check.

33. $\frac{1-x}{4} - \frac{x}{2} = 7$

34. $\frac{x-1}{3} + \frac{x+2}{6} = 7$

35. $\frac{x+1}{2} - \frac{x-7}{6} = 3$

36. $\frac{2n+10}{4} - \frac{n}{3} = 1$

37. $\frac{4x+5}{3} - \frac{3x}{2} = -x$

38. $4 = \frac{x+1}{3} + \frac{x+5}{5}$

39. $\frac{x+1}{3} + \frac{2-3x}{2} = -1$

40. $\frac{x+1}{3} + \frac{x-2}{7} = 1$

Problems and Applications

41. On the average, the mass of a raccoon is 1 kg more than $\frac{1}{4}$ of the mass of a coyote.

The mass of a raccoon is 9.5 kg. To find the mass of a coyote, solve the equation

$$\frac{m}{4} + 1 = 9.5.$$

42. On the average, a Canadian adult spends 35% or 0.35 of the time sleeping. Solve the equation $9t - 0.01 = 0.35$ to find the fraction of the time spent shopping.

PATTERN POWER

1. Take a 20 cm by 20 cm square piece of paper and cut out a 1-by-1 square from each corner. Fold up the sides to make a box. Calculate the volume of the box.

2. Cut out a 2-by-2 square from each corner and calculate the volume of the new box.

3. Repeat the procedure for a 3-by-3 square, a 4-by-4 square, and so on. Copy and complete the table.

Dimensions of Corners (cm)	Volume of Box (cm ³)
1 × 1	
2 × 2	
3 × 3	
...	

4. What are the dimensions of the box with the greatest volume?

5. What was the size of the square you removed from each corner to give the box with the greatest volume?

6. To get the box with the greatest volume, what size of square must you remove from each corner of each of the following squares?

a) 14 cm by 14 cm

b) 25 cm by 25 cm

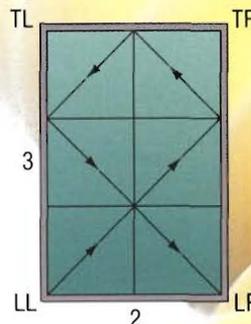
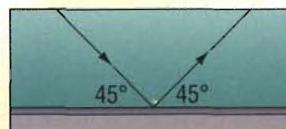
Rolling Balls and Inductive Reasoning

Inductive reasoning is the process of drawing conclusions from a number of specific cases. In the following activities, you will consider a number of cases in which a ball rebounds from the sides of a table.

When a ball moves on a flat table with raised sides, the angle at which the ball strikes a side equals the angle at which the ball rebounds.

In the following activities, assume that:

- a) The corners of the table are named top right (TR), top left (TL), lower right (LR), and lower left (LL).
- b) The ball always starts from the lower left (LL) corner at an angle of 45° to the sides.
- c) The ball keeps rolling until it hits a corner. Then, the ball stops.
- d) The table dimensions are stated as height first and then width. A 3-by-2 table is shown here.



Activity 1

The diagram shows the path of a ball on a 3-by-2 table. The ball stops at the lower right (LR) corner.

1. Draw tables with the following dimensions on grid paper. For each grid, draw the path of a ball that starts at the LL corner.

a)

b)

c)

d)

e)

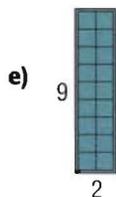
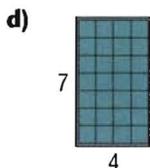
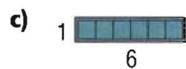
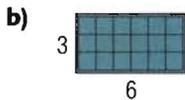
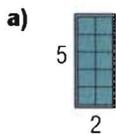
f)

- 2. In which corner does the ball stop on each table?
- 3. What are the dimensions of the table that gives
 - a) the most complicated path?
 - b) the simplest path?
- 4. On which table does the ball cross every square?
- 5. Will a ball ever stop at the LL corner? Explain.



Activity 2

1. Draw tables with the following dimensions and draw the path of the ball.



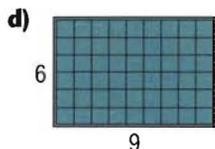
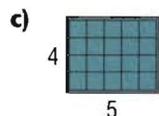
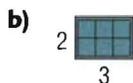
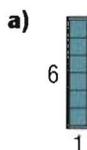
2. What kind of number is the height of each table?

3. What kind of number is the width of each table?

4. Where does the ball always stop?

Activity 3

1. Draw each table and the path of the ball.



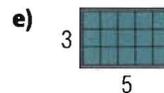
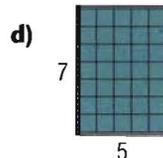
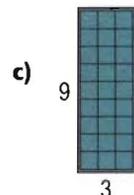
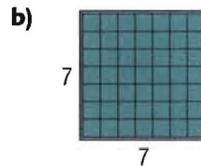
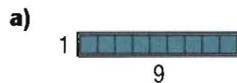
2. What kind of number is the height of each table?

3. What kind of number is the width of each table?

4. Where does the ball always stop?

Activity 4

1. Draw tables with the following dimensions and draw the path of the ball.



2. What kind of number is the height of each table?

3. What kind of number is the width of each table?

4. Where does the ball always stop?

Activity 5



1. Use inductive reasoning to write a rule that states where the ball stops if the dimensions are

a) odd-by-even

b) even-by-odd

c) odd-by-odd

2. State where the ball stops on tables with the following dimensions.

a) 32-by-13

b) 37-by-37

c) 29-by-28

d) 101-by-101

e) 244-by-133

f) 79-by-88



Review

Evaluate $3x + 2y$ for the following values of the variables.

- | | |
|--------------------|---------------------|
| 1. $x = 1, y = 3$ | 2. $x = 4, y = 2$ |
| 3. $x = 7, y = 1$ | 4. $x = 3, y = 3$ |
| 5. $x = -4, y = 0$ | 6. $x = -2, y = -1$ |
| 7. $x = 3, y = -5$ | 8. $x = -1, y = -3$ |

Evaluate each expression for $a = 2.5$, $b = -3.5$, and $c = 5.5$.

9. $a + 2b - 3c$ 10. $2a - b + 4c$

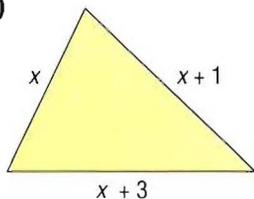
Simplify.

11. $7x + 10 + 7y - 4x + 1$
 12. $2a - 3b - 5 + a - b$
 13. $-q + 4q - 9r - a + 2r + 1$

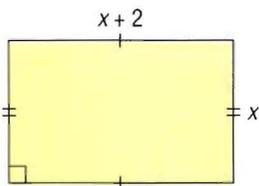
In questions 14–18, write an equation for each statement.

14. Eight more than a number is twenty.
 15. A number multiplied by six is seventy-two.
 16. Hadib is eight years older than Cam. The sum of their ages is twenty-four.
 17. Sonya has ten dollars less than Paula. Together they have ninety dollars.
 18. Carl has three times as many stamps as Tim. Together they have 2400 stamps.
 19. The perimeter of each figure is 64 m. Write an equation to find the dimensions of each figure.

a)



b)



Solve.

- | | |
|------------------|------------------|
| 20. $3x = 12$ | 21. $y + 7 = 12$ |
| 22. $2 = -4 + s$ | 23. $3 + z = 7$ |
| 24. $x + 1 = -2$ | 25. $5m = 40$ |
| 26. $3n = -21$ | 27. $-24 = 12x$ |
| 28. $5p = 3.0$ | 29. $7z = -2.8$ |

Solve.

- | | |
|------------------------|------------------------|
| 30. $\frac{x}{2} = 5$ | 31. $\frac{y}{2} = -5$ |
| 32. $\frac{b}{3} = -3$ | 33. $\frac{z}{4} = 6$ |

Solve.

- | | |
|----------------------------|----------------------------|
| 34. $\frac{x}{3} + 2 = 4$ | 35. $\frac{y}{2} - 3 = 2$ |
| 36. $4 + \frac{b}{3} = -1$ | 37. $2 - \frac{z}{4} = -1$ |

Solve and check.

- | | |
|---------------------|----------------------|
| 38. $2x - 3 = 9$ | 39. $3y - 2 = 13$ |
| 40. $5y - 7 = 3$ | 41. $-22 = 7y - 8$ |
| 42. $3x + 2x = -25$ | 43. $4x + 5x = -81$ |
| 44. $3 + 2q = 21$ | 45. $2.9 + 6x = 1.1$ |

Solve.

- | | |
|----------------------|------------------------|
| 46. $3x - 17 = 13$ | 47. $56 = 5y + 11$ |
| 48. $2x + 3.1 = 1.1$ | 49. $5m + 25 = -65$ |
| 50. $3y + 10 = 82$ | 51. $0.3x + 2.6 = 4.4$ |
| 52. $-39 = 25 + 4a$ | 53. $-54 = 6z - 18$ |
| 54. $-33 + 7t = -61$ | 55. $8q - 42 = -26$ |

Solve.

- | |
|------------------------------------|
| 56. $6a - 3a + 2a = 10 - 2 + 7$ |
| 57. $2p + 3p - p = 9 - 15 - 6$ |
| 58. $10j - 5j - 8j = -6 + 15 + 12$ |
| 59. $12k - 8k - 6k = -18 + 20 - 6$ |

Solve and check.

60. $3x - 4 = 8 - x$ 61. $6x - 6 = 4x - 10$
62. $7 - x = -7 + 6x$ 63. $4 + x = 36 - 7x$
64. $2 + 3j = 5j - 24$ 65. $-3 + 2r = -30 - 7r$

Solve.

66. $2x + 15 = -12 - 13 - 3x$
67. $b - 17 = 15 + 3b + 18$
68. $21 + 4q = q + 13 + 17$

Solve and check.

69. $4(y - 4) + 3(y + 7) = -30$
70. $-38 = 5(2y - 1) + y$
71. $6(3x - 4) - (1 - 3x) = -67$

Solve.

72. $5y + 0.4 - 3y = -6$
73. $1.6m - 2.4 + 0.1m = 0.1m + 0.8$
74. $8.4x - 0.8 = 4.4x - 4$
75. $12.2x + 0.3 = 20.7 + 5.4x$

Solve.

76. $\frac{y}{2} + \frac{y}{3} = \frac{5}{6}$ 77. $\frac{5y}{4} - \frac{y}{2} = \frac{-3}{4}$
78. $\frac{2y + 1}{3} = -5$ 79. $\frac{x - 2}{6} = \frac{x + 2}{2}$
80. $\frac{x + 2}{4} = \frac{2x - 5}{2}$ 81. $\frac{x - 2}{2} - \frac{x}{6} = -2$

Solve and check.

82. $\frac{2x - 1}{3} - \frac{x + 2}{4} = -5$
83. $\frac{3y + 5}{3} - \frac{y - 3}{6} = -2$
84. $\frac{2y + 1}{3} - \frac{y + 4}{5} = 7$

85. The number of Members of Parliament (MPs) elected from Ontario is 99. This number is 5 less than 4 times the number elected from Alberta. Solve the equation $4n - 5 = 99$ to find the number of MPs elected from Alberta.

Group Decision Making

Designing a Rube Goldberg Invention



The cartoonist Rube Goldberg is famous for his impractical inventions, which are shown as one-scene cartoons. This cartoon is like a Rube Goldberg cartoon. It shows a different way of pouring water into a sink.

Sources of power for Goldberg inventions might include:

- the sun melting a block of ice that releases a rope that was frozen into it
- the rays of the sun being focused through a magnifying glass
- a cat chasing a dog

1. Work in home groups. Decide on a Rube Goldberg invention designed to do something in an original way. Some tasks might include sharpening a pencil, travelling to school, or cleaning the chalkboard.
2. As a group, design a flow chart to show the sequence of events in your invention before you start to draw the cartoon.
3. In your group, draw and discuss rough sketches of the cartoon. Draw the final version and present it to the class.
4. As a class, evaluate the cartoons for originality and humour.

Chapter Check

1. Evaluate $ab + bc - ca$ for $a = 2$, $b = 3$, and $c = 4$.

Collect like terms and simplify.

2. $3x - 2y - 3x + 3y + 2$
3. $-15 + 2a - 3c + 10 + 4a - b - 7c$
4. Burt is four years older than Anna and the sum of their ages is 36. Write the equation to find their ages.

Solve and check.

5. $x - 12 = -4$ 6. $x + 15 = 12$
7. $x + 2 = 8$ 8. $x - 10 = -19$
9. $5 = x - 3$ 10. $15 - x = -5$

Solve.

11. $6x = -54$ 12. $4x = 44$
13. $\frac{x}{5} = 7$ 14. $\frac{x}{3} = 4$

Solve.

15. $5x + 8 = 3x + 2$
16. $5x + 12 = 7x + 16$
17. $6z + 8 = 9z - 7$
18. $2y - 6 = 10 + 11y + 2$

Solve.

19. $3(2x - 4) = 9x + 3$
20. $5(2x - 1) + 9 = 2(x - 2)$

Solve.

21. $10x - 0.4 = 2x - 3.6$
22. $7.6x - 0.7 = 5.6x - 6.5$

Solve.

23. $\frac{2x}{3} = -6$ 24. $\frac{y}{3} + \frac{1}{3} = -\frac{2}{3}$
25. $\frac{y}{3} - \frac{5y}{6} = -\frac{1}{2}$

Solve.

26. $\frac{3y + 1}{2} = 5$ 27. $\frac{x + 2}{2} = \frac{x - 1}{5}$
28. $\frac{x + 2}{2} + \frac{x - 2}{5} = 2$

29. The average Canadian spends 23% or 0.23 of the time on leisure activities. This fraction is 0.01 less than 4 times the fraction of the time spent eating. Solve the equation $4t - 0.01 = 0.23$ to find the fraction of the time spent eating.

Using the Strategies

1. Determine the pattern and write the next 3 rows.

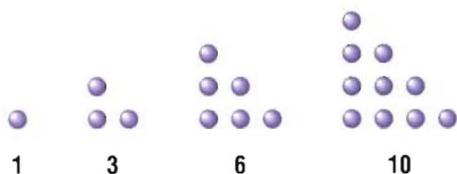
1	1	1							
1	2	3	2	1					
1	3	6	7	6	3	1			
1	4	10	16	19	16	10	4	1	

2. The perimeter of an isosceles triangle is 8 cm. The length of each side is a whole number. How long is the shortest side?
3. The number 72 can be factored as follows.

$$\begin{aligned}
 72 &= 36 \times 2 \\
 &= 18 \times 2 \times 2 \\
 &= 9 \times 2 \times 2 \times 2 \\
 &= 3 \times 3 \times 2 \times 2 \times 2
 \end{aligned}$$

Use this technique to find the age of each person in a group of teenagers if the product of their ages is 661 500.

4. The first 4 triangular numbers are shown.



What are the next 3 triangular numbers?

5. There are 20 students in the gym. The teacher wants them in 3 groups, with an even number of students in each group. In how many ways can the teacher put them into groups?
6. Your train leaves at 08:15. The bus trip to the train station takes 25 min. The bus stop is a 5-min walk from your place. You should get to the train station to buy your ticket 15 min before the train leaves. It will take you 55 min to get dressed, eat breakfast, and pack. For what time should you set your alarm clock?

7. Determine the pattern and predict the next 2 lines.

$$101 \times 101 = \blacksquare$$

$$202 \times 202 = \blacksquare$$

$$303 \times 303 = \blacksquare$$

8. The number 8 is a perfect cube.
 $2^3 = 2 \times 2 \times 2 = 8$ or $2^3 = 8$
 The number 27 is also a perfect cube.
 $3^3 = 3 \times 3 \times 3 = 27$ or $3^3 = 27$

We can write 8 as the sum of 2 consecutive odd numbers.

$$3 + 5 = 8$$

We can write 27 as the sum of 3 consecutive odd numbers.

$$7 + 9 + 11 = 27$$

- a) The next perfect cube is 64 because $4^3 = 64$. Write 64 as the sum of 4 consecutive odd numbers.
- b) The next perfect cube is 125. Write 125 as the sum of 5 consecutive odd numbers.
- c) The next perfect cube is 216. Write 216 as the sum of 6 consecutive odd numbers.
-  d) How does the square of the number that is cubed fit into the sum of the consecutive odd numbers?
- e) Use this pattern to write 7^3 as the sum of 7 consecutive odd numbers.

9. Sketch a graph to show the length of time you spend watching TV versus the day of the week.

DATA BANK

- How much higher above sea level is Mexico City than Calgary?
- Express Venezuela's annual gold production as a percent of Canada's annual gold production.

