



CHAPTER 3

Using Equations to Solve Problems

The following are the times for a typical space shuttle launch.

At $t - 3.8$ s, the shuttle's 3 main engines are ignited.

At $t + 2.88$ s, the twin boosters are ignited, and the hold-down bolts are released.

At $t + 2$ min 12 s, the boosters burn out and are jettisoned.

At $t + 8$ min 32 s, just before the shuttle reaches orbital velocity, the engines are shut down.

The huge external tank is jettisoned at $t + 8$ min 50 s.

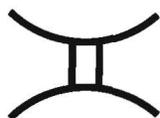
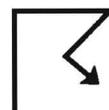
The shuttle's own orbit-adjust engines put the shuttle into orbit.

If t is 08:00, at what time does each of the above events take place? For how long is the shuttle bolted to the lift-off platform while the engines are on? Why?

Activity 1 A Scavenger Hunt

When you solve problems in mathematics, you often replace words with symbols.

1. The following are some symbols that have been used to replace words. State what each symbol means and, if possible, give an example of where you have seen it.



2. Design an international symbol of your own. Ask your classmates to tell you what they think it means.

Activity 2 Equations

Solve.

1. $x - 4 = -6$

2. $y - 5 = -8$

3. $8 = m + 3$

4. $n + 5 = 8$

5. $s - 7 = 7$

6. $t + 7 = 7$

7. $x + 5 = 11$

8. $-9 = -3 + y$

9. $q + 9 = -1$

Solve.

10. $3y = 12$

11. $5x = -20$

12. $-m = 6$

13. $-n = -3$

14. $18 = 6s$

15. $-45 = 9t$

16. $\frac{x}{3} = 5$

17. $\frac{y}{4} = 1$

18. $\frac{y}{2} = -3$

Solve and check.

19. $2x + 7 = -13$

20. $1 = -5 + 3y$

21. $m + 6 = 4m + 18$

22. $4n - 5 = 2n + 1$

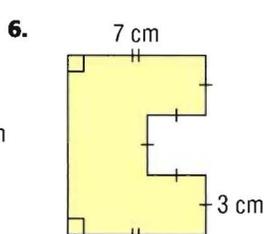
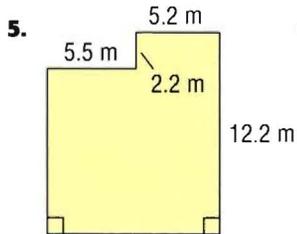
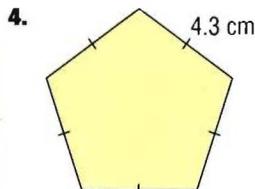
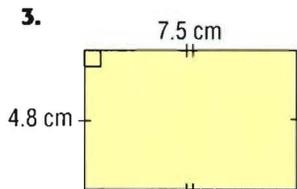
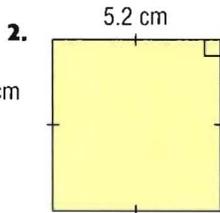
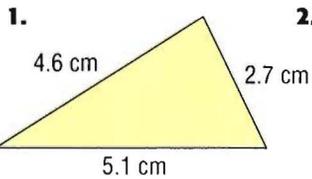
23. $6 = 2(x - 1)$

24. $3(y + 2) + 1 = y + 9$

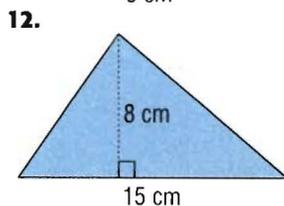
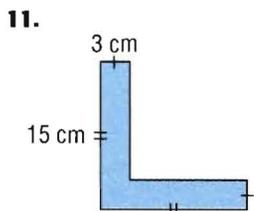
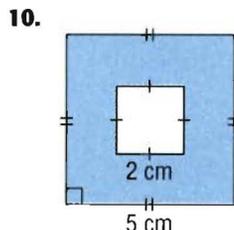
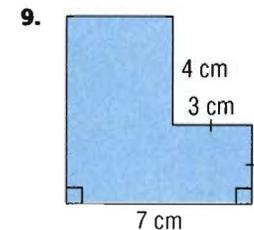
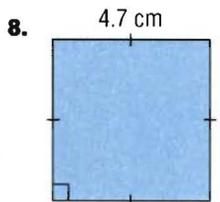
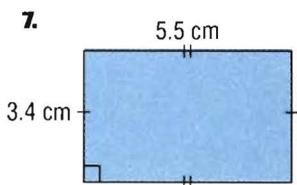
25. $2(m + 3) + 4 = 3(m - 7) + 19$

Activity 3 Perimeter and Area

Calculate each perimeter.



Calculate each area.



Mental Math

Add.

1. $18 + 2$ 2. $18 + 12$ 3. $25 + 5$
 4. $25 + 25$ 5. $46 + 5$ 6. $46 + 15$
 7. $54 + 6$ 8. $54 + 16$ 9. $54 + 26$

Subtract.

10. $36 - 6$ 11. $36 - 16$ 12. $54 - 4$
 13. $54 - 24$ 14. $65 - 6$ 15. $65 - 16$
 16. $71 - 2$ 17. $71 - 12$ 18. $71 - 22$

Multiply.

19. $(25)(4)$ 20. $(25)(40)$ 21. $(4)(5)$
 22. $(400)(5)$ 23. $(5)(8)$ 24. $(5)(800)$
 25. $(4)(9)$ 26. $(40)(9)$ 27. $(400)(9)$

Divide.

28. $28 \div 4$ 29. $280 \div 4$ 30. $35 \div 7$
 31. $3500 \div 7$ 32. $2000 \div 5$ 33. $2000 \div 50$

Simplify.

34. $(3)(2) + 5$ 35. $4 + (2)(5)$
 36. $5(2 + 6)$ 37. $(8 - 3) \times 4$
 38. $5 + 9 - 2$ 39. $24 - (8 - 2)$
 40. $27 - 9 - 3$ 41. $30 - (10 + 5)$

Simplify.

42. $4 + 6 + 5 + 7$ 43. $7 - 3 + 4 - 5$
 44. $(2)(6) - 3 + 4$ 45. $12 - 3 + 6 - 5$
 46. $24 - (4)(2)(1)$ 47. $20 - 5 - (4)(3)$
 48. $(6 + 4)(3 - 1)$ 49. $5 + 7 - (7 - 4)$

Simplify.

50. $\frac{4}{8} + \frac{1}{8}$ 51. $\frac{3}{9} + \frac{1}{9}$ 52. $\frac{5}{10} + \frac{2}{10}$
 53. $\frac{5}{10} - \frac{1}{5}$ 54. $\frac{5}{6} - \frac{1}{6}$ 55. $\frac{3}{10} - \frac{1}{5}$

3.1 Solving Problems Using Equations

The ground speed of a jet depends on its air speed and the speed of the headwind or tailwind.

Activity: Study the Information

The equation for the ground speed of a jet, y , flying at 450 km/h with a headwind, x , is:

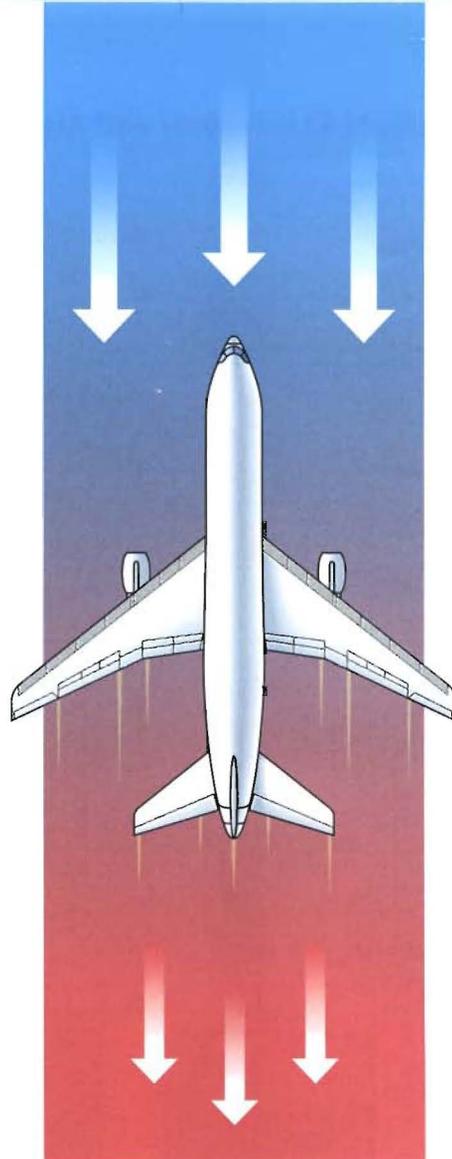
$$y = 450 - x$$

If there is a tailwind, the equation is:

$$y = 450 + x$$

Inquire

1. What is the ground speed when there is
 - a) a headwind of 80 km/h?
 - b) a tailwind of 120 km/h?
2. What is the headwind if the ground speed is 390 km/h?
3. What is the tailwind if the ground speed is 515 km/h?
4. Write the equation to find the ground speed when the headwind is 60 km/h. Solve the equation.
5. Write the equation to find the ground speed when the tailwind is 100 km/h. Solve the equation.
6. Write the equation to find the headwind when the ground speed is 400 km/h. Solve the equation.
7. Write the equation to find the tailwind when the ground speed is 475 km/h. Solve the equation.



Example 1

A compact disc player costs \$75 more than a tape deck. Together, they cost \$725. How much does each cost?

Solution

Let the cost of the tape deck be x dollars.

Then, the cost of the compact disc player is $(x + 75)$ dollars.

The sum of the costs is \$725.

$$x + (x + 75) = 725$$

$$2x + 75 = 725$$

$$2x = 725 - 75$$

$$2x = 650$$

$$x = 325$$

The cost of the tape deck is \$325.

The cost of the compact disc player is $\$325 + \75 or \$400.

Check: $\$325 + \$400 = \$725$

Example 2

The sides of a triangle are 3 consecutive whole numbers of centimetres. The perimeter of the triangle is 48 cm. How long is each side?

Solution

Let x represent the length of the shortest side in centimetres. Then, the lengths of the other 2 sides are $x + 1$ and $x + 2$. The sum of the 3 sides is 48 cm.

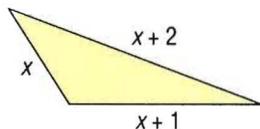
$$x + (x + 1) + (x + 2) = 48$$

$$x + x + x + 1 + 2 = 48$$

$$3x + 3 = 48$$

$$3x = 45$$

$$x = 15$$



If $x = 15$, $x + 1 = 16$, and $x + 2 = 17$.

The lengths of the sides are 15 cm, 16 cm, and 17 cm.

Check: The numbers 15, 16, and 17 are consecutive whole numbers, and $P = 15 + 16 + 17$ or 48 cm.

Example 3

A parking meter contains \$27.05 in quarters and dimes. There are 146 coins. How many quarters are there?

Solution

Let x represent the number of quarters.

Then, $(146 - x)$ is the number of dimes.

If there are x quarters, then the value of the quarters is $25x$ cents.

If there are $(146 - x)$ dimes, the value of the dimes is $10(146 - x)$ cents.

The total value of the coins is \$27.05 or 2705 cents.

$$10(146 - x) + 25x = 2705$$

$$1460 - 10x + 25x = 2705$$

$$1460 + 15x = 2705$$

$$15x = 1245$$

$$x = 83$$

The number of quarters is 83.

The number of dimes is $146 - 83$ or 63.

Check: The value of the coins in dollars is $(0.25)(83) + (0.10)(63)$ or \$27.05.



Example 4

One number is 2 times another number. If you subtract 10 from each number, the sum is 40. What are the numbers?

Solution

Let the smaller number be x . Then, the larger number is $2x$.

Subtract 10 from each number to give $x - 10$ and $2x - 10$.

The sum equals 40.

$$(x - 10) + (2x - 10) = 40$$

$$x + 2x - 10 - 10 = 40$$

$$3x - 20 = 40$$

$$3x = 60$$

$$x = 20$$

If $x = 20$, $2x = 40$. The numbers are 20 and 40.

Check: $20 - 10 = 10$ $40 - 10 = 30$ $10 + 30 = 40$



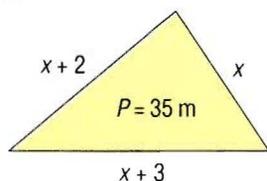
Practice

Each statement has two unknowns. Represent both in terms of x .

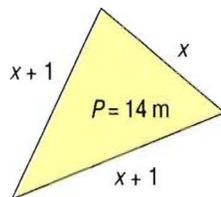
- The sum of two numbers is 35.
- There are 50 nickels and dimes.
- There are 125 quarters and dimes.
- The length and width of a rectangle total 36 cm.
- There is a total of 32 males and females in the class.
- The cafeteria sold 758 hamburgers and hot dogs.
- Jim and Janice sold 468 kg of cheese.
- The parking meter had 246 coins in quarters and dimes.

Find the length of each side.

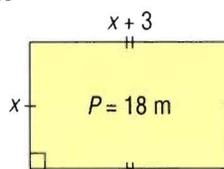
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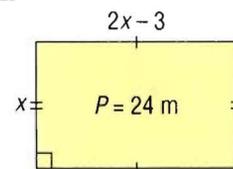
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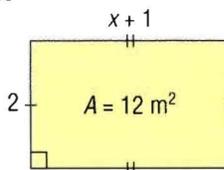
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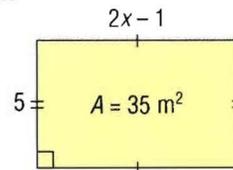
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13.



14.

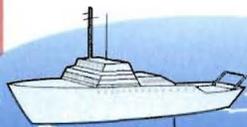


Problems and Applications

- The sum of 2 numbers is 46. One number is 12 more than the other. What are the numbers?
- The Mackenzie river is 1183 km longer than the St. Lawrence. The sum of their lengths is 7299 km. How long is each river?
- The length of a rectangle is 5 m more than its width. Its perimeter is 90 m. What are its dimensions?

- 18.** The sum of 3 consecutive numbers is 105. Find the numbers.
- 19.** The number of frost-free days per year in Quebec City is 18 less than the number in Halifax. The total number in both cities is 292. How many frost-free days are there per year in Halifax?
- 20.** The maximum life span of a brown bear is 10 times the maximum life span of a mouse. The sum of their maximum life spans is 33 years. What is the maximum life span of a mouse?
- 21.** When Canada won the Women's World Hockey Championships, the Canadian team scored 35 more goals than were scored against it. The total number of goals scored in the games that Canada played was 41.
- a)** How many goals were scored against Canada?
- b)** How many goals did Canada score?
- 22.** There are 2.25 times as many days of fog per year in Vancouver as in Winnipeg. The total number of days of fog in both cities is 65. How many days of fog does Winnipeg have per year?
- 23.** One of the equal sides of an isosceles triangle is 3 m less than twice its base. The perimeter is 44 m. Find the lengths of the sides.
- 24.** The cost of a pen is 3 times the cost of a pencil. The cost of 4 pencils and 3 pens is \$9.75. What is the cost of a pencil?
- 25.** Sally has twice as many dimes as nickels. The total value is \$3.50. How many nickels does she have?
- 26.** Aretha has \$0.85 in nickels and dimes. She has 2 more nickels than dimes. How many nickels and dimes does she have?
- 27.** A box contains 140 dimes and nickels. The total value is \$11.15. How many dimes and how many nickels are there?
- 28.** A picture is 5 cm longer than it is wide. The perimeter of the picture is 90 cm.
- a)** What is the width of the picture?
- b)** The picture is surrounded by a border, which is 6 cm wide. What are the outside dimensions of the border?
- 29.** The sum of 2 numbers is 39. Twice the first number plus 3 times the second number is 101. Find the numbers.
- 30.** One number is 5 more than another number. Three times the first plus twice the second is 30. Find the numbers.
- 31.** A jar contains \$18.50 in dimes and quarters. There are 110 coins in the jar. How many quarters are in the jar?
- 32.** Pietra sells tickets at the theatre. At the end of the evening, she had \$524.00 in \$2.00 and \$5.00 bills. The total number of bills was 145. How many of each kind of bill were there in total?
- 33.** Large pizzas cost \$12.50 and small pizzas cost \$9.00. The pizza parlor sold 38 pizzas with a total value of \$415.50. How many of each type of pizza did the parlor sell?
- 34.** Tickets to the concert cost \$9.00 for adults and \$6.50 for students. A total of 950 people paid \$7675.00 to attend. How many students attended the concert?
- 35.** A garden is 20 m by 25 m. It is surrounded by a walk-way. The outside perimeter of the walk-way is 114 m. What is the width of the walk-way?
- 36.** A picture measures 40 cm by 30 cm. The outside perimeter of the frame around the picture is 156 cm.
- a)** What is the width of the frame?
- b)** What is the area of the frame?
-  **37.** Write a problem that can be solved using an equation. Have a classmate solve your problem.

3.2 Use a Formula



Understand the Problem

Think of a Plan

Carry Out the Plan

Look Back

The word “sonar” is an acronym for **s**ound **n**avigation and **r**anging. Oceanographers use sonar to explore the floors of oceans and lakes. A sonar instrument on a ship can find the depth of the water or the distance to an object. The instrument works by measuring the time a sound signal takes for the round trip to the ocean or lake floor and back.

The speed of sound in water is 1500 m/s. The formula for the distance to an object is

$$d = \frac{1500 \times t}{2}$$

where d is the distance in metres, and t is the time in seconds for a sonar signal to reach the object and bounce back.

It takes 0.35 s for sound to reach a wreck and bounce back. How deep is the water above the wreck?

1. What information are you given?
2. What are you asked to find?
3. What formula should you use?

Substitute for t in the formula.

$$\begin{aligned} d &= \frac{1500 \times t}{2} \\ &= \frac{1500 \times 0.35}{2} \\ &= \frac{525}{2} \\ &= 262.5 \end{aligned}$$

EST	$1500 \times 0.4 = 600$ $600 \div 2 = 300$
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The water is 262.5 m deep.

To check your answer, how could you use the depth to calculate the time?

Use a Formula

1. Write the formula.
2. Replace variables with known values.
3. Calculate the remaining variable.
4. Check that your answer is reasonable.

Problems and Applications

1. The following formula gives the ideal amount of sleep needed each night by people 19 years old or younger.

$$s = \frac{35 - n}{2}$$

The amount of sleep is s hours, and n is the age in years.

a) How much sleep does a 14-year-old need?
b) Jason is 10. He gets 9 h of sleep each night. Is this enough?

2. You are a sonar operator searching for a wreck that is known to be in 900 m to 1000 m of water. Your sonar has generated the table below, which includes time measurements at 16 locations, A_1 , A_2 , and so on. Use the formula $d = \frac{1500 \times t}{2}$ to decide where the wreck might be.

	Sonar Measurements (s)			
	1	2	3	4
A	0.9	0.8	1.1	1.0
B	1.0	1.4	1.2	0.7
C	0.6	1.3	1.5	1.6
D	1.1	1.2	0.5	2.1

3. To find the distance to an object in space, a radio telescope bounces radio signals off the object and measures the time it takes for the signals to make a round trip. Radio signals travel at 298 000 km/s in space. The formula for the distance to an object is

$$d = \frac{298\,000 \times t}{2}$$

where d is the distance in kilometres, and t is the time for the round trip in seconds.

a) When the moon is closest to the Earth, the round-trip time is 2.45 s. How far is the moon from the Earth?

b) When the moon is furthest from the Earth, the round-trip time is 2.71 s. How far is the moon from the Earth?

4. In the following formula, T is the air temperature in degrees Celsius at an altitude of b metres, and t is the ground temperature in degrees Celsius.

$$T = t - \frac{b}{150}$$

If the ground temperature is 25°C, what is the temperature outside an aircraft at the following altitudes?

a) 8000 m b) 4500 m

5. The formula relates the time, t seconds, an object takes to fall to the Earth from a height of b metres.

$$b = 4.9t^2$$

The table gives the time it takes an object to fall to the ground from the top of several structures. Calculate the height of each.

Structure	Time (s)
Eiffel Tower	8.08
CN Tower	10.63
Washington Monument	5.88

6. If you stand on the Earth and jump up at a speed of 5 m/s, your approximate height, b metres, above the ground after t seconds is given by this formula.

$$b = 5t - 5t^2$$

If you stand on the moon and jump up at the same speed, the corresponding formula is

$$b = 5t - 0.8t^2$$

a) How far would you be from the Earth after 0.5 s? 1 s?

b) How far would you be from the moon after 0.5 s? 1 s? 3 s? 6 s?



c) About how much longer would you stay off the ground on the moon than on the Earth? Explain.



7. Write a problem that can be solved with one of the formulas on this page. Have a classmate solve your problem.



3.3 Working with Formulas

A squid has 8 arms and 2 tentacles. The arms and the ends of the tentacles are covered with tooth-rimmed suction pods. The squid uses its tentacles to pull prey into its arms. Giant squids prey on sharks and, in turn, are preyed upon by sperm whales.

Activity: Use the Formula

Scientists can determine the length of a giant squid by measuring the diameter of the suction-pod scars left on its prey. The formula $l = 180d$ gives the length of a squid in centimetres from the diameter of its suction pods in centimetres.

Inquire

1. How long is a squid whose suction pods have a diameter of 0.5 cm?
2. Squids found off the coast of Newfoundland have suction pods with diameters of 5 cm. How long are these squids in metres?
3. The longest squid, to date, was found in New Zealand. The suction pods were 5.4 cm in diameter. How long was this squid in metres?
4. A sperm whale was found with suction-pod scars that measured 35 cm in diameter. How many metres long was the squid that made these scars?



Example

- a) Given $P = 2(l + w)$, solve for w .
- b) Evaluate w for $P = 350$ cm and $l = 120$ cm.

Solution

- a) Since we want the value of w , isolate this variable.

$$P = 2(l + w)$$

$$P = 2l + 2w$$

Subtract $2l$ from both sides: $P - 2l = 2l + 2w - 2l$

$$P - 2l = 2w$$

Divide both sides by 2:

$$\frac{P - 2l}{2} = \frac{2w}{2}$$

$$\frac{P - 2l}{2} = w$$

- b) Substitute $P = 350$ and $l = 120$ into the formula.

$$\begin{aligned} w &= \frac{P - 2l}{2} \\ &= \frac{350 - 2(120)}{2} \\ &= \frac{350 - 240}{2} \\ &= \frac{110}{2} \\ &= 55 \end{aligned}$$

So, $w = 55$ cm.

Check: Substitute into $P = 2(l + w)$.

$$350 = 2(120 + 55)$$

$$350 = 2(175)$$

$$350 = 350$$

Practice

- For the formula $A = lw$,
 - find A if $l = 8$ cm and $w = 5$ cm.
 - find w if $A = 40$ m² and $l = 10$ m.
 - find l if $A = 238$ m² and $w = 14$ m.
- Assume $\pi = 3.14$. For the formula $C = 2\pi r$,
 - find C if $r = 10$ cm.
 - find r if $C = 628$ cm.
- For the formula $A = \frac{1}{2}bh$,
 - find A if $b = 6$ cm and $h = 8$ cm.
 - find b if $A = 40$ cm² and $h = 4$ cm.
 - find h if $A = 60$ m² and $b = 20$ m.
- For the formula $P = 2(l + w)$,
 - find P if $l = 9$ m and $w = 6$ m.
 - find w if $P = 60$ m and $l = 16$ m.
 - find l if $P = 84$ m and $w = 5$ m.

Solve each formula for the indicated variable.

- $A = lw$ for w
- $A = \frac{1}{2}bh$ for b
- $I = Prt$ for P
- $C = 2\pi r$ for r
- $E = mc^2$ for m
- $A = \frac{1}{2}b(a + b)$ for b

Problems and Applications

- The maximum desirable pulse rate for a person exercising can be found using the formula

$$m = 220 - a$$

where m is the pulse rate in beats per minute and a is the person's age in years.

- Copy and complete the table.

Age (years)	Maximum Desirable Pulse Rate (beats/min)
20	
27	
39	
44	
61	

- To what ages do these maximum desirable pulse rates correspond?

198 171 183

- The distance travelled by a spaceship is given by the formula

$$d = 40\,000t$$

where d is the distance in kilometres and t is the time in hours.

- How far does a spaceship travel in 12 h? in 17.5 h?
- How long does it take a spaceship to travel 130 000 km?

- The amount of food energy required per day by military personnel on active duty is given by the formula

$$E = -125T + 15\,250$$

where E is the amount of food energy in kilojoules (kJ) and T is the outside temperature in degrees Celsius. Copy and complete the table.

Temperature (°C)	Energy (kJ)
40	
23	
0	
-10	
-45	

- Shawna is at an outdoor concert in Vancouver. She is sitting 100 m from the band. The formula that gives the length of time for the band's sound to reach her is

$$t = \frac{d}{330}$$

where t is the time in seconds and d is Shawna's distance from the band in metres. Paul is listening to the same concert on a radio in Halifax. The formula that gives the length of time for the band's sound to reach him is

$$t = \frac{d}{300\,000}$$

where t is the time in seconds and d is the distance from Halifax to Vancouver in kilometres. Who hears each sound first, Shawna or Paul?

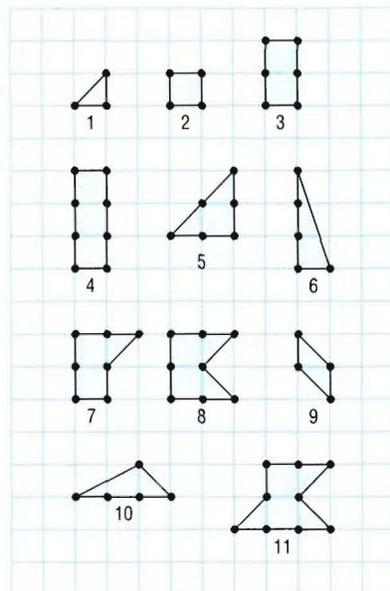
Developing Pick's Formula for Area

Activity 1 No Points in the Interior

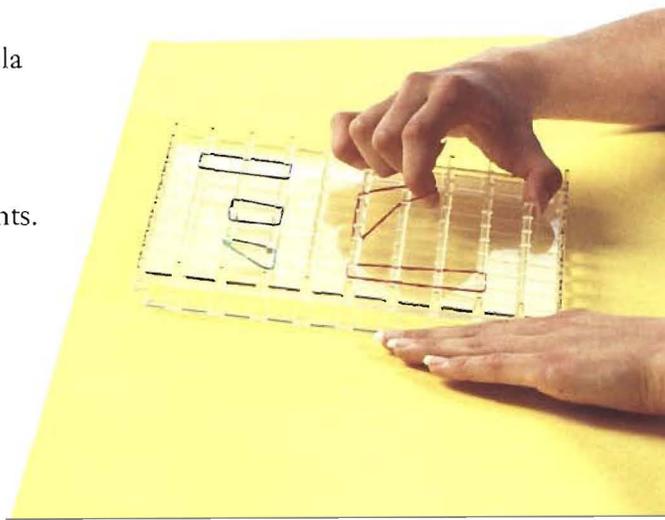
Each polygon has been constructed so that there are no points in its interior.

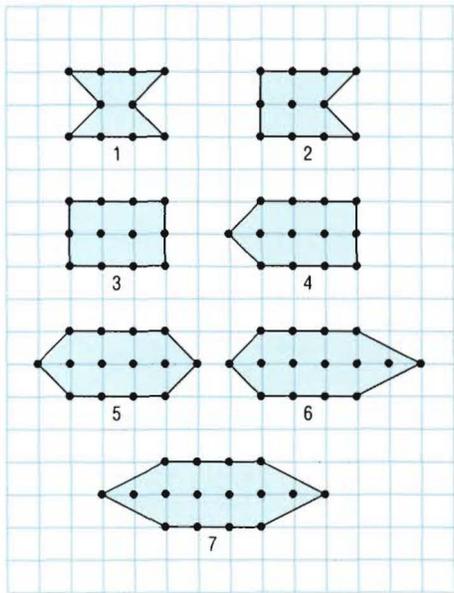
1. Construct the polygons on grid paper or on a geoboard.
2. Copy and complete this table. The first line has been completed for you.

Polygon	Points on the Perimeter (P)	Area (square units)
1	3	$\frac{1}{2}$
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		



3. If 2 figures have the same number of points on the perimeter, what can you say about their areas?
4. List the areas from smallest to largest. Do not repeat areas. The first two are:
3 points, 0.5 square units
4 points, 1 square unit
5. Predict the area of a figure with 10 points on the perimeter and no points in its interior. Check your prediction by making 2 such shapes and finding their areas.
6. Use the pattern in the table to write a formula for the area of a polygon with no points in the interior if you know the number of perimeter points.
7. Draw 3 different figures with no interior points. Verify your formula.
8. Use your formula to determine the area of a figure with 99 points on the perimeter and no interior points.





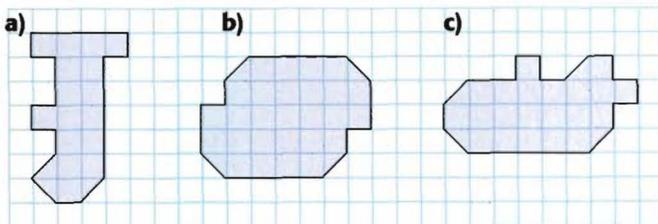
Activity 2 Points in the Interior

Each polygon has 10 points on the perimeter. The number of points in the interior, I , increases from 0 to 6.

1. Calculate the area of each polygon.
2. Copy and complete the table, calculating each area in square units.

Polygon	Points on the Perimeter (P)	Interior Points (I)	Area (square units)
1			
2			
3			
4			
5			
6			
7			

3. What happens to the area of the polygon as the number of interior points increases by 1?
4. Predict the area of figures with 10 points on the perimeter and 7 and 8 points in the interior. Check your prediction by making 2 such shapes and finding their areas.
5. Use the pattern in the table and the formula for area you found in Activity 1. Write a formula for the area of a polygon if you know the number of perimeter points and interior points. This formula is called **Pick's formula** after the mathematician who discovered it.
6. Draw 3 different figures with points in the interior. Verify your formula.
7. Use your formula to determine the areas of the following figures.



3.4 Developing Formulas

The *Bluenose* was Canada's most famous sailing ship. It was launched in Lunenburg, Nova Scotia, in 1921. It was the fastest racing schooner of its time, winning many championships until its last race in 1938.

Activity: Study the Information

In the early days of sailing ships, navigators estimated the speed of their ship by tying a rope to a log and tossing the log into the water behind the ship. After 30 s, the length of the rope was measured, showing how far the ship had travelled in this amount of time.

Inquire

1. If the length of the rope after 30 s was 90 m, what was the speed of the ship in metres per second?
2. Let the length of the rope after 30 s be l . Write a formula for the speed of the ship in metres per second.
3. If the length of the rope after 30 s was 240 m, what was the speed of the ship in metres per second?
4. Use the formula to calculate the speed of the ship if the length of the rope after 30 s is 210 m.
5. Rewrite the formula to calculate the speed of the ship in kilometres per hour.



Example

Many early navigators also estimated the speed of a ship by measuring the number of seconds it took a piece of wood to float from one end of the ship to the other.

- a) Write a formula to estimate the speed of a galleon in metres per second.
- b) Use the formula to find the speed of a 40 m long galleon if $t = 8$ s.



Solution

- a) Let the time in seconds be t . Let the speed in metres per second be s . The distance travelled in metres is the length of the galleon, l .

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$s = \frac{l}{t}$$

$$\begin{aligned} \text{b) } s &= \frac{40}{8} \\ &= 5 \end{aligned}$$

The speed of the galleon was 5 m/s.

Practice

Complete the table and state a rule for each pattern.

1.

<i>a</i>	1	2	3	4	5	6	7
<i>b</i>	3	6	9	12			

2.

<i>m</i>	2	4	6	8			
<i>n</i>	5	7	9	11	13	15	17

3.

<i>t</i>	25	24	23	22			
<i>a</i>	100	96	92	88	84	80	76

Complete the table. Then, use the variables to write a formula for each pattern.

4.

Number of Books (<i>n</i>)	1	2	3	4
Cost (<i>c</i>)	4	8	12	

5.

Hours (<i>h</i>)	5	10	15	20
Wages (<i>w</i>)	37.50	75.00	112.50	

6.

Selling Price (<i>s</i>)	300	400	500	600
Profit (<i>p</i>)	60	80	100	

Problems and Applications

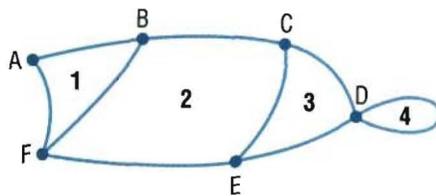
7. The cost to rent a bus is \$100 plus a certain amount per kilometre. The table gives the cost of 3 bus trips. Write a formula to calculate the cost of a bus trip in terms of distance.

Trip	Distance, <i>d</i> (km)	Cost, <i>C</i> (\$)
1	50	250
2	100	400
3	150	550

8. This table shows the cost of a taxi ride. Write a formula that relates kilometres driven, *d*, to the cost of the ride, *C*.

<i>d</i> (km)	0	5	10	15	20
<i>C</i> (\$)	3.00	10.50	18.00	25.50	33.00

9. The map shows an area where there are 6 towns, 9 roads, and 4 regions.



Roads start and end at a town. If roads cross, there is a town where they cross. A road can connect a town to itself. Regions are completely surrounded by roads. Each town is connected to at least 1 other town.

In such maps, there is a relationship between the numbers of towns, regions, and roads.

a) Make at least 5 maps, each with 4 regions. Complete a table like the one shown.

Map	Regions	Towns	Roads
1	4		
2	4		
3	4		
4	4		
5	4		

Look for a pattern in the Towns and Roads columns.

Write a formula that lets you determine the number of regions if you know the numbers of towns and roads.

Regions =

Test your formula on another map.

b) Make at least 5 maps, each with 6 towns. Complete a table like the one in part a).

Write a formula that lets you determine the number of towns if you know the numbers of roads and regions. Test your formula on another map.

c) Make at least 5 maps, each with 6 roads. Complete a table like the one in part a).

Write a formula that lets you determine the number of roads if you know the numbers of towns and regions. Test your formula on another map.

3.5 Uniform Motion Problems

Problems involving **uniform motion** are those in which the speeds of the objects do not change. The formula $D = r \times t$ is used to solve these problems, where D is the distance, r is the rate of speed, and t is the time.

Activity: Study the Information

Frank left the ranch for town on horseback. He rode at 15 km/h. One hour later, his brother Jesse left the ranch and tried to catch up with Frank. Jesse rode at 20 km/h. How long did it take Jesse to catch up with Frank?

Inquire

1. Let x be the time in hours that Frank rode before Jesse caught up with him. Write an expression for how far Frank rode in x hours.
2. If x is the time Frank rode, write an expression in terms of x for the time Jesse rode.
3. Use your expression from question 2 to write an expression for the distance Jesse rode.
4. When Jesse caught up with Frank, the distances they had ridden were the same. Write an equation stating the 2 distances were equal.
5. Solve the equation for x .
6. a) How long had Frank been riding when Jesse caught up with him?
b) How long had Jesse been riding?



Example

A plane left Montreal for Calgary, a distance of 3000 km, travelling at 550 km/h. At the same time, a plane left Calgary for Montreal travelling at 450 km/h. How long after take-off did the planes pass each other?

Solution

Set up a table and let x be the time from take-off until the planes passed. Write expressions for D using r and t .

	t (h)	r (km/h)	D (km)
Montreal to Calgary	x	550	$550x$
Calgary to Montreal	x	450	$450x$

$$\begin{aligned} \text{The sum of the distances is 3000 km. } & 550x + 450x = 3000 \\ & 1000x = 3000 \\ & x = 3 \end{aligned}$$

The planes passed each other 3 h after take-off.

Check:

In 3 h, the plane from Montreal flew 3×550 or 1650 km.
In 3 h, the plane from Calgary flew 3×450 or 1350 km.
 $1650 \text{ km} + 1350 \text{ km} = 3000 \text{ km}$

Practice

Calculate the distance travelled.

1. 3 h at 60 km/h 2. 2 h at 85 km/h
 3. $\frac{1}{2}$ h at 90 km/h 4. $\frac{3}{4}$ h at 60 km/h

How long does each trip take?

5. 40 km at 80 km/h
 6. 400 km at 50 km/h
 7. 20 km at 100 km/h
 8. 360 km at 80 km/h

Calculate each speed.

9. 300 km in 3 h 10. 400 km in 5 h
 11. 360 km in 4 h 12. 40 km in $\frac{1}{2}$ h

Complete the table.

	Distance (km)	Rate (km/h)	Time (h)
13.	450	100	
14.		65	3
15.	600		8
16.		80	x
17.		90	$x + 1$
18.		85	$x - 1$
19.	200	x	
20.	400		x
21.		r	t
22.	D	r	
23.	D		t

Problems and Applications

24. A cruise ship left Halifax for Bermuda at 20 km/h. A private boat left for Bermuda 1 h later and travelled at 25 km/h. After how long did the private boat overtake the cruise ship?
25. Two cars left a service centre at 16:30. One car travelled in one direction at 75 km/h. The other car travelled in the opposite direction at 85 km/h.
- a) After how long were they 600 km apart?
 b) At what time were they 600 km apart?

26. Two friends, one living in Winnipeg and one living in Edmonton, decided to meet on the TransCanada Highway. The distance from Edmonton to Winnipeg is about 1360 km. They both left home at 08:00 Winnipeg time. The friend from Winnipeg drove at 80 km/h, and the friend from Edmonton drove at 90 km/h.

- a) After how long did they meet?
 b) What was the time in Winnipeg when they met?
 c) What assumptions have you made?

27. A plane left Vancouver for Los Angeles at 08:30 and flew at 600 km/h. Fifteen minutes later, another plane left Vancouver for Los Angeles and flew at 700 km/h.

- a) How long did it take the second plane to overtake the first one?
 b) At what time did it happen?

28. A car left a garage on the highway at 100 km/h. Fifteen minutes later, a police cruiser left the same garage at 120 km/h in pursuit of the car. How long did it take the cruiser to catch up with the car?

29. Write a problem involving uniform motion and ask a classmate to solve it.

NUMBER POWER

Place the digits from 1 to 9 in the boxes to make the statements true. Use the order of operations.

$$\square + \square - \square = 10$$

$$\square \div \square + \square = 10$$

$$\square - \square + \square = 10$$

3.6 Rate of Work Problems

Activity: Study the Process

Tania and Justine own a cruise boat that takes people out to the Atlantic Ocean to show them how lobsters are caught. At the end of the day, Tania and Justine take turns cleaning the boat for the next day's cruise. Tania is older and cleans the boat in 2 h. It takes Justine 3 h. We want to know how long it would take them to clean the boat together. Problems of this type are known as **rate of work problems**.



Inquire

1. What fraction of the boat does Tania clean in 1 h?
2. What fraction of the boat does Justine clean in 1 h?
3. What is the sum of these fractions?
4. If they clean $\frac{5}{6}$ of the boat in 1 h, how long will it take to clean the last $\frac{1}{6}$?
5. What is the total time to clean the boat?

Example

After the restaurant closes, Roberto takes 2 h to clean it. Bill is new to the job and takes 4 h. How long would it take them if they worked together?

Solution

One way to solve this type of problem is to use an equation. Let x hours represent the time the job takes if they work together. Then, $\frac{x}{2}$ represents the part cleaned by Roberto and $\frac{x}{4}$ the part cleaned by Bill. Together, the 2 parts must add to the whole job.

$$\begin{aligned}\frac{x}{2} + \frac{x}{4} &= 1 \\ \text{Multiply by 4: } 4 \times \frac{x}{2} + 4 \times \frac{x}{4} &= 4 \times 1 \\ 2x + x &= 4 \\ 3x &= 4 \\ x &= \frac{4}{3} \text{ or } 1\frac{1}{3}\end{aligned}$$

Roberto and Bill can clean the restaurant in $1\frac{1}{3}$ h by working together.

Check:

In $\frac{4}{3}$ h, the fraction of the restaurant cleaned by Roberto is $\frac{\frac{4}{3}}{2}$ or $\frac{4}{3} \times \frac{1}{2} = \frac{2}{3}$.

In $\frac{4}{3}$ h, the fraction of the restaurant cleaned by Bill is $\frac{\frac{4}{3}}{4}$ or $\frac{4}{3} \times \frac{1}{4} = \frac{1}{3}$.
 $\frac{2}{3} + \frac{1}{3} = 1$ or the entire restaurant

Practice

Write 2 fractions for each question.

1. Ahmed takes 2 h to mow his lawn. His brother, Sami, takes 4 h. What fraction of the lawn does each mow in 1 h?
2. Athena takes 6 h to paint an apartment. Helena takes 8 h to do the same job. What fraction of the apartment do they each paint in 1 h?

Problems and Applications

Solve. Round answers to the nearest tenth, where necessary.

3. Julio can fill a water tank in 4 min using a large hose. He takes 6 min using a smaller hose. How long will he take if he uses both hoses?
4. Andrea can deliver 500 handbills in 2 h. Althea can deliver the same number in 3 h. How long will they take to deliver 500 handbills if they work together?
5. Murray can tile a floor in one hour. His partner can do the same job in half the time. How long will it take them to tile the floor if they work together?
6. Mario can take inventory at the store in 30 min. His partner, Carmen, can take inventory in 20 min. If they work together, how long will the inventory take?
7. Ken and Milan are office cleaners. Ken earns \$10/h and takes 8 h to clean an office. Milan earns \$8/h and takes 10 h to clean it.
 - a) How long will it take Ken and Milan to clean the office together?
 - b) What is the cost of cleaning the office using only Ken? only Milan? Ken and Milan together?

8. Uri, Max, and Boris work for a company that installs carpet tiles in offices. Uri can install 2000 tiles in 10 h. It takes Max 12 h to do the same job. Boris installs 2000 tiles in 15 h. How long would it take them to install 2000 tiles if they work together?

9. Mary takes 3 h to complete a task. Mary and Jim together take 2 h to complete the same task. How long will it take Jim to complete the task working alone?

 10. Dan and Brad are brothers who attend the same school. Brad can walk to school in 15 min. Dan takes 20 min.

a) How long will it take them to walk to school together?

 b) What makes this problem different from the others in this section?

 11. Write a rate of work problem. Have a classmate solve your problem.

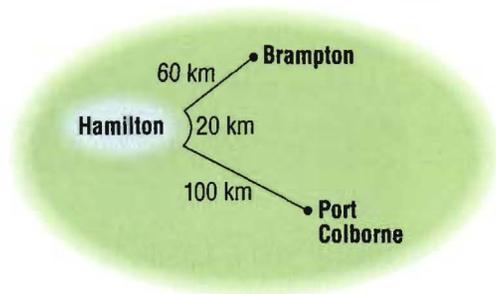
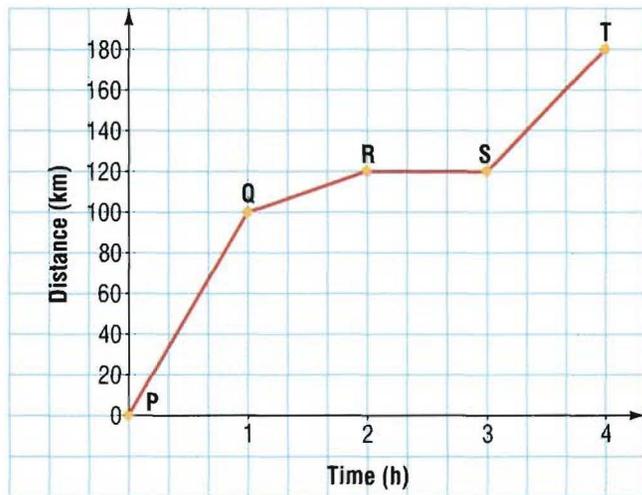
LOGIC POWER

Draw the grid and place 3 pennies, a nickel, and a dime on it as shown. By sliding one coin at a time into a neighbouring empty square, make the nickel and the dime change places. You can move horizontally or diagonally. Make the switch in as few moves as possible.



3.7 Interpret Graphs

Much of the information you receive from newspapers and magazines is displayed on graphs. Therefore, the ability to interpret graphs correctly is an important life skill. The graph shows the distance from Port Colborne to Brampton and the time taken for a car to travel this distance. What is happening between P and Q; Q and R; R and S; S and T?



1. What information are you given?
 2. What are you asked to find?
 3. Do you need an exact or approximate answer?
- Read the information from the graph. Use it to solve the problem.
- From P to Q* The car covered 100 km in 1 h, so it travelled at 100 km/h from Port Colborne to Hamilton.
- From Q to R* The car took 1 h to drive 20 km around Hamilton. It averaged 20 km/h.
- From R to S* The car stopped for 1 h on the outskirts of Hamilton on the Brampton side.
- From S to T* The car travelled the 60 km from Hamilton to Brampton in 1 h. The car travelled at 60 km/h.
- Does the answer seem reasonable?

Understand the Problem

Think of a Plan

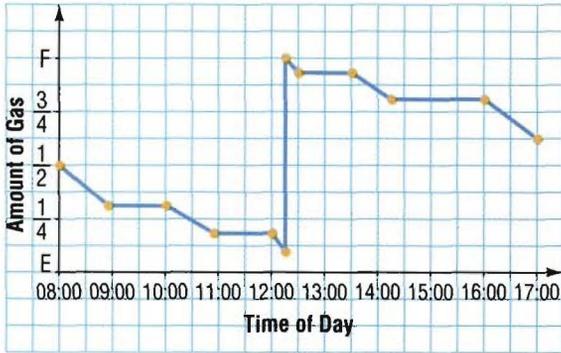
Carry Out the Plan

Look Back

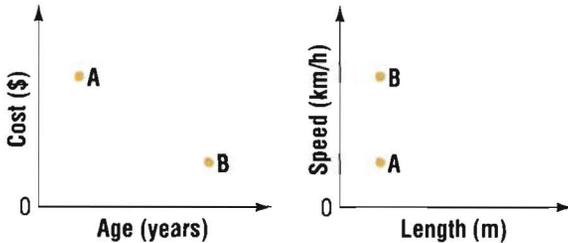
- Interpret Graphs**
1. Interpret the graph.
 2. Use the information to solve the problem.
 3. Check that the answer is reasonable.

Problems and Applications

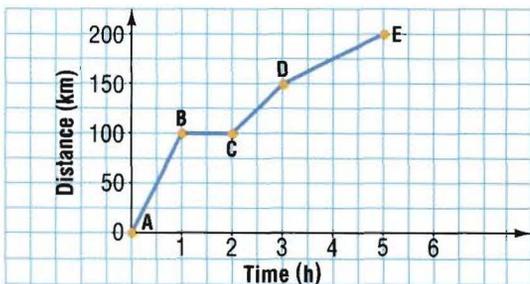
-  1. The graph shows the amount of gasoline in the tank of a salesperson's car at different times of a work day. Describe how the salesperson might have spent the day.



2. The 2 graphs describe 2 pleasure boats. Compare the boats by age, cost, speed, and length.

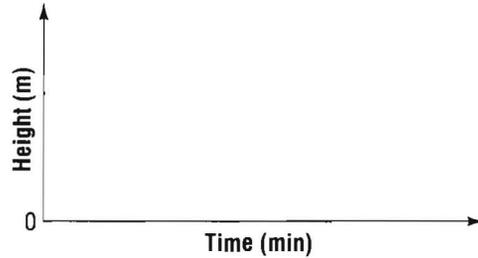


-  3. a) The graph shows the distance versus time for a car travelling from Fredericton to Moncton. Describe what is happening between A and B; B and C; C and D; D and E.

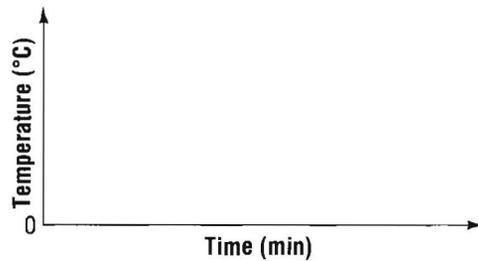


- b) Sketch a graph of the car's speed versus time.

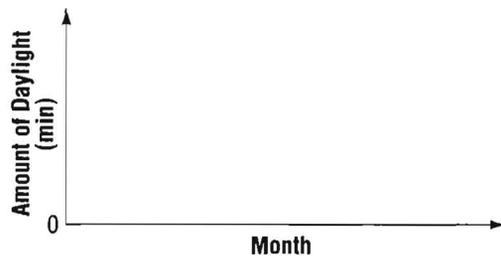
4. You are the first person to get on and off a Ferris wheel with 8 cars. Once the cars are loaded, the Ferris wheel makes 3 revolutions before letting people off. Sketch a graph of your height versus time.



5. Suppose someone turned on the hot-water tap on a kitchen sink and left it running. Sketch a graph of the water temperature versus time as the tap is running.



6. Sketch a graph of the number of minutes of daylight versus the month of the year.



7. Write a graph-sketching problem that involves each of the following. Have a classmate solve each problem.

- the temperature of water in a kettle versus time
- the depth of water in a bathtub versus time

3.8 Solving Inequalities

Dr. Roberta Bondar was the second Canadian in space. She wore a space suit that protected her against pressure and temperature changes. The mass of a space suit is 23 kg. The combined mass of an astronaut plus space suit must be less than 110 kg.

Activity: Complete the Table

Copy and complete the table.

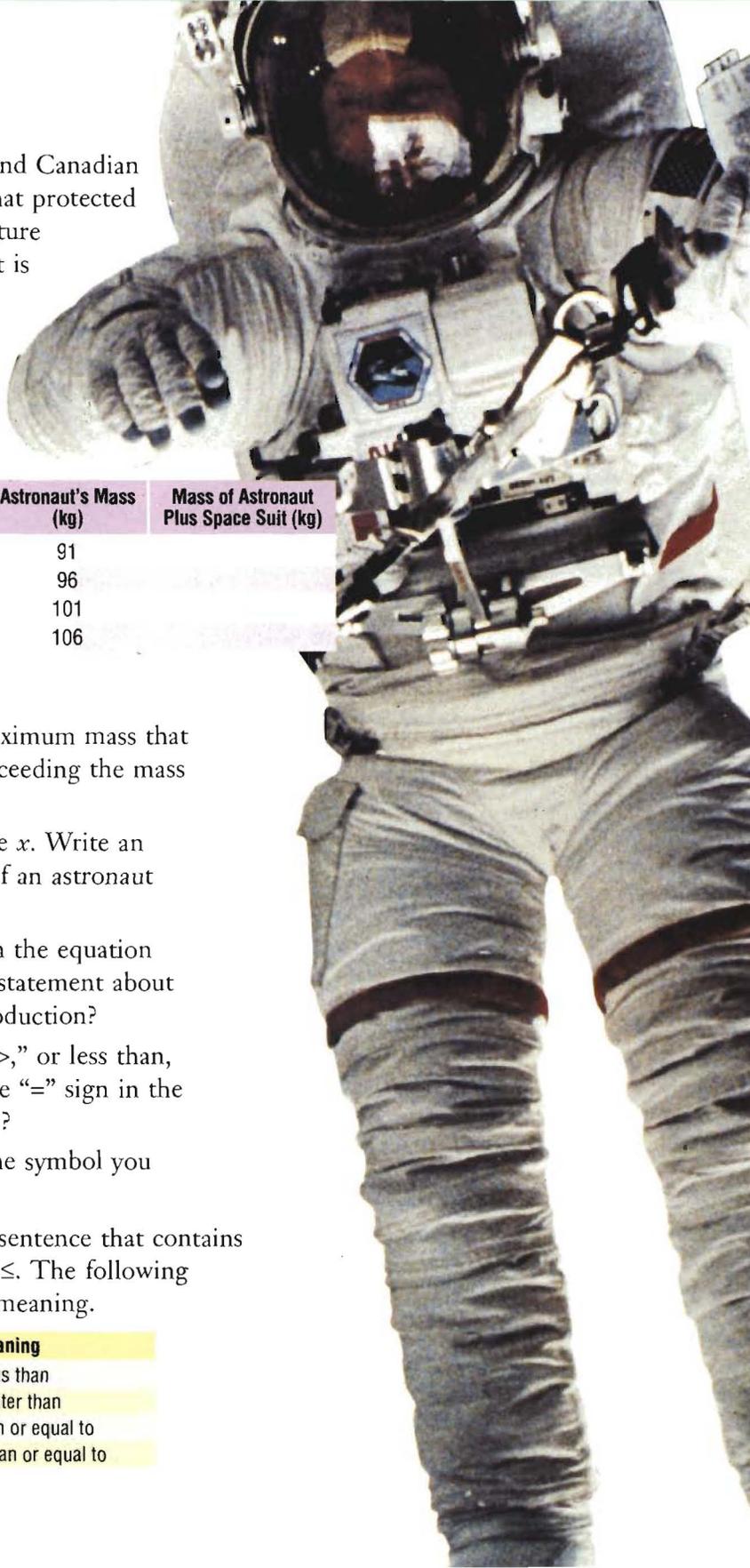
Astronaut's Mass (kg)	Mass of Astronaut Plus Space Suit (kg)	Astronaut's Mass (kg)	Mass of Astronaut Plus Space Suit (kg)
71		91	
76		96	
81		101	
86		106	

Inquire

1. From the table, what is the maximum mass that an astronaut can have without exceeding the mass restriction?
2. Let the mass of an astronaut be x . Write an equation to show that the mass of an astronaut plus space suit can equal 110 kg.
3. What is the difference between the equation you wrote in question 2 and the statement about combined mass given in the introduction?
4. Which symbol, greater than, “ $>$,” or less than, “ $<$,” should you use to replace the “ $=$ ” sign in the equation you wrote in question 2?
5. Rewrite your equation using the symbol you chose in question 4.

An **inequality** is a mathematical sentence that contains one of these symbols: $>$, $<$, \geq , or \leq . The following table shows each symbol and its meaning.

Symbol	Meaning
$<$	is less than
$>$	is greater than
\leq	is less than or equal to
\geq	is greater than or equal to



Activity: Discover the Relationships

Copy the table.
Complete it by writing the correct inequality after each operation.

Inequality	Operation on Both Sides	New Inequality	Inequality	Operation on Both Sides	New Inequality
$6 > -4$	Add 2		$6 > -4$	Add (-2)	
$6 > -4$	Subtract 2		$6 > -4$	Subtract (-2)	
$6 > -4$	Multiply by 2		$6 > -4$	Multiply by (-2)	
$6 > -4$	Divide by 2		$6 > -4$	Divide by (-2)	

Inquire

- Repeat the table for the inequality $-8 < -2$.
- Does the direction of the inequality sign stay the same when you do each of the following?
 - add the same positive or negative number to both sides
 - subtract the same positive or negative number from both sides
 - multiply both sides by the same positive number
 - divide both sides by the same positive number
 - multiply both sides by the same negative number
 - divide both sides by the same negative number

Example 1

Solve the inequality
 $5x - 2 \geq 2x + 4$
and graph the solution.

Solution

Add 2 to both sides:

$$5x - 2 + 2 \geq 2x + 4 + 2$$

$$5x \geq 2x + 6$$

Subtract $2x$ from both sides:

$$5x - 2x \geq 2x + 6 - 2x$$

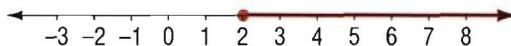
$$3x \geq 6$$

Divide both sides by 3:

$$\frac{3x}{3} \geq \frac{6}{3}$$

$$x \geq 2$$

The solution is $x \geq 2$.



The closed dot at $x = 2$ means that 2 is included in the solution.

Example 2

- Solve the inequality
 $2(x - 3) > 4x + 2$
and graph the solution.
- Check the solution.

Solution

a)

$$2(x - 3) > 4x + 2$$

$$2x - 6 > 4x + 2$$

$$2x - 4x > 2 + 6$$

$$-2x > 8$$

Divide both sides by -2 :

$$\frac{-2x}{-2} < \frac{8}{-2}$$

$$x < -4$$

Remember to reverse the symbol when multiplying or dividing by a negative number.



The open dot at $x = -4$ means that -4 is not included in the solution. CONTINUED

b) Substitute -4 for x in the original inequality, $2(x - 3) > 4x + 2$.

$$\begin{array}{ll} \text{L.S.} & 2(-4 - 3) = 2(-7) \\ & = -14 \end{array} \qquad \begin{array}{ll} \text{R.S.} & 4(-4) + 2 = -16 + 2 \\ & = -14 \end{array}$$

‡ means "is not greater than"

As $-14 \nless -14$, -4 is not a solution.

To check solutions less than -4 , substitute -5 for x in $2(x - 3) > 4x + 2$.

$$\begin{array}{ll} \text{L.S.} & 2(-5 - 3) = 2(-8) \\ & = -16 \end{array} \qquad \begin{array}{ll} \text{R.S.} & 4(-5) + 2 = -20 + 2 \\ & = -18 \end{array}$$

As $-16 > -18$, -5 is a solution.

Practice

1. Which inequalities are true?

- a) $5 > 4$ b) $10 > 6$ c) $15 \geq 17$
d) $6 < 8$ e) $7 \leq -7$ f) $11 < 13$
g) $-3 > -2$ h) $-1 > -5$ i) $-1 \geq -2$
j) $-4 \leq -4$ k) $-5 < -8$ l) $-6 < -5$

2. Which of the x values shown make each inequality true?

- a) $x + 2 < 8$ (3, 6) b) $-5 > x - 2$ (-5, 0)
c) $x + 5 < 7$ (4, 1) d) $x - 3 > 3$ (0, 8)
e) $x + 3 \leq 8$ (7, -1) f) $x - 5 \geq -9$ (-5, 1)
g) $6 \leq -1 + x$ (-4, 7) h) $x + 2 \leq -2$ (-2, -4)

Solve each inequality and graph its solution.

3. $x - 2 > 0$ 4. $10 > x + 5$
5. $x - 3 < 1$ 6. $x - 5 > 2$
7. $y - 4 > -3$ 8. $y + 3 < 4$
9. $1 < -2 + z$ 10. $z - 5 < -2$

Solve, then check your solution.

11. $4x < 8$ 12. $2y > -8$ 13. $4m < 20$
14. $5n < -20$ 15. $3s > 0$ 16. $21 \geq 7y$
17. $6p \leq -12$ 18. $-15 \geq 5t$ 19. $8b \leq 24$

Solve.

20. $-3m < 9$ 21. $-4n > 12$
22. $-15 > -5x$ 23. $-6 > -2y$
24. $-3t \geq -18$ 25. $-10y \leq -50$
26. $21 \geq -7x$ 27. $4x \geq -12$

Solve. Graph the solution.

28. $2x < 10$ 29. $2y > -8$ 30. $-8 > 4m$
31. $2n > 6$ 32. $5t \leq -15$ 33. $4 \leq 4s$
34. $3y \leq -6$ 35. $8x \leq 40$ 36. $2x \geq -2$

Solve and check.

37. $4x + 2 < 3x + 5$ 38. $2x - 4 > x + 2$
39. $7y - 4 \geq 6y + 3$ 40. $5y + 7 \leq 4y - 2$
41. $2t + 7 < t - 1$ 42. $10a + 4 > 9a + 2$
43. $3n + 4 < 2n + 2$ 44. $5n + 2 > 4n - 1$

Solve.

45. $4(x - 3) > 3x + 1$
46. $3y + 9 < 2y + 12$
47. $4(2m - 1) \geq 7m - 3$
48. $3x - 10 > 2x - 9$
49. $3y + 2 \leq 2y + 1$
50. $3m + 14 < 2(m + 6)$
51. $17 - 6x > 12 - 7x$
52. $20m - 7 < 19m - 2$
53. $2(5 - 3b) \geq -7b + 2$
54. $5w - 7 \leq 4w - 3$
55. $7p + 11 \geq 6p + 17$
56. $10x + 10 \leq 9x + 15$
57. $19 - 13x \geq -14x + 16$

Solve. Graph the solution.

58. $7x + 4 < 5x + 8$ 59. $5y - 2 \leq 2y + 7$
60. $2m - 3 < 9 - 2m$ 61. $3x + 10 < -2 - 3x$
62. $4y + 5 \geq 6y - 1$ 63. $6t - 5 \leq 8t + 3$
64. $7 + y < 4y + 13$ 65. $6x - 13 \geq 8x - 15$

Solve.

66. $6(y - 2) + 7 > 8y - 25$
67. $4(t - 1) < 8t + 20$
68. $8(x - 2) > 8 - 4x$
69. $24x + 18 > 12 + 7(3x - 3)$
70. $3(5 - 5x) + 3 \geq 34 - 7x$
71. $2(7x - 11) \leq 9x - 32$
72. $15 + 4x \geq 5(2x - 1) - 10$
73. $2(8x - 13) + 4 < 18x + 26$
74. $3t + 45 \geq 6(t - 4)$
75. $6(m - 4) - 2(m + 2) < 7(m - 4) - 6$
76. $5(y + 1) - 2(y + 3) \leq 5(y - 1)$
77. $3(2x - 5) \geq 2(1 + 2x) + 5$

Problems and Applications

78. In her last math test, Giselle got 10 marks more than on any other test this year. Her mark was less than 100. Solve the inequality $x + 10 < 100$ to find her highest possible mark on any other test.

79. On Saturdays, Campus Clothes sells at least 25 more jackets than on any other day of the week. The store has never sold more than 84 jackets in one day. Solve the inequality $84 \geq n + 25$ to find the number of jackets that could have been sold on any other day of the week.

80. Matt wants to keep his annual travel expenses under \$4680. Solve the inequality $52x < 4680$ to find how much Matt can spend each week.

81. Bianca must keep her phone bill below \$55/month. The basic charge is \$15, and it costs her \$2/min to phone her friend in Taiwan. Solve the inequality $2t + 15 < 55$ to determine how long Bianca can spend talking to her friend each month.

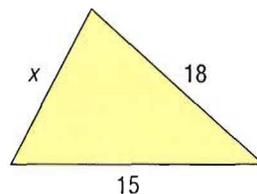
82. A peregrine falcon can dive at up to 350 km/h. This speed is 50 km/h faster than 3 times the top speed of the fastest land animal, the cheetah.

a) Solve the inequality $350 \geq 3s + 50$ to determine the speed of a cheetah.



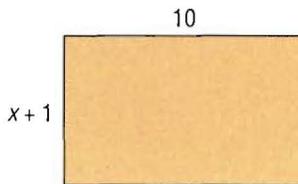
b) Does a cheetah have a minimum speed? Explain.

83. a) Solve the inequality $x + 15 + 18 \leq 50$ to find the values of x that give this triangle a perimeter of no more than 50.



b) Does x have a minimum value? Explain.

84. a) Solve the inequality $10(x + 1) < 50$ to find values of x that give this rectangle an area of less than 50.



b) Does x have a minimum value? Explain.



85. Write an inequality that has variables and numbers on both sides and has a solution of $x \leq -1$. Have a classmate solve your inequality.

Solving Problems Using Spreadsheets

A spreadsheet consists of rows and columns. It allows you to quickly analyze and change data. A spreadsheet can be an important problem solving tool.

The following problem solving activities show both the power and simplicity of a spreadsheet.

Activity 1

A collection of 48 coins has a total value of \$7.32. The collection consists of pennies, nickels, dimes, quarters, and half dollars. How many coins of each type are there?

1. Set up the following spreadsheet. Each formula in Column C is typed into that particular cell as indicated.

	A	B	C
1	Coin	Number	Value
2	Penny		$0.01 * B2$
3	Nickel		$0.05 * B3$
4	Dime		$0.1 * B4$
5	Quarter		$0.25 * B5$
6	Half Dollar		$0.5 * B6$
7	Totals	@SUM(B2..B6)	@SUM(C2..C6)

2. Enter the number of each type of coin into Column B of the spreadsheet until the total number of coins is 48, and the total value of all the coins is \$7.32.

3. What strategy did you use to solve the problem?
4. Is there more than one solution?

Activity 2

You can use a spreadsheet to help select a store for shopping.

1. Obtain advertising flyers from several stores.
2. Find several items that are common to all stores.
3. Set up a spreadsheet similar to the one shown to determine which store gives you the best value for your money.

	A	B	C	D
1	ITEM	STORE 1	STORE 2	STORE 3
2		Cost	Cost	Cost
3	Toothpaste			
4	Shampoo			
.				
.				
	Totals			

ICES!

SAVE 60%

▶ 1.77

toothpaste
cleans teeth
and brightens



Save 40%

entire Selection
Costume
Jewellery

reg. 1.99 to \$16.

9 to 9.60

SUPER SAVING

TOOTH PASTE	SHAMPOO	FILM
2.49	2.77	3.99
TOOTH PASTE with a free toothbrush	NO TEARS SHAMPOO 450 ml	COLOUR FILM 35 mm



Activity 3

1. Make up a problem of your own that can be solved using a spreadsheet.
2. Give your problem to a classmate to solve.

SAVE!

Fingerprints

Because your fingerprints never change and are different from everyone else's, they can be used to identify you. There are 3 different patterns of fingerprints, known as arches, whorls, and loops.



Plain Arch

The ridge lines enter on one side and curve up and exit on the other side.



Tented Arch

The ridge lines are the same as a plain arch, except that they make a sharp point or tent in the middle.



Whorls

The ridge lines are circles or ovals.



Loops

The ridge lines enter on one side of the print, curve, and exit on the same side. An **ulnar loop** slants toward the ulna bone in the wrist. A **radial loop** slants toward the radius bone in the wrist.



Right-hand ulnar loop or left-hand radial loop.



Right-hand radial loop or left-hand ulnar loop.

The RCMP receives many sets of prints with requests to identify the people they belong to. It is impossible to check the prints against all the prints on file. The RCMP first classifies the prints.

Activity 1

1. Identify each of your fingerprints as an arch, a loop, or a whorl.
2. According to fingerprint experts, there are 5% arches, 65% loops, and 30% whorls on fingerprints. Are these percents true for your class?



Activity 2

The primary fingerprint classification is based on the number of whorls. Prints are assigned the following values.

Whorl = 1 Arch = 0 Loop = 0

These values are substituted into the following formulas.

$$(\text{Rt Index}) \times 16 + (\text{Rt Ring}) \times 8 + (\text{Lt Thumb}) \times 4 + (\text{Lt Middle}) \times 2 + (\text{Lt Little}) \times 1 + 1$$

$$(\text{Rt Thumb}) \times 16 + (\text{Rt Middle}) \times 8 + (\text{Rt Little}) \times 4 + (\text{Lt Index}) \times 2 + (\text{Lt Ring}) \times 1 + 1$$

For example, Yoshiko has whorls on her left thumb, left little finger, and right thumb. All her other prints are arches and loops. Her primary classification is as follows.

$$(0) \times 16 + (0) \times 8 + (1) \times 4 + (0) \times 2 + (1) \times 1 + 1$$

$$(1) \times 16 + (0) \times 8 + (0) \times 4 + (0) \times 2 + (0) \times 1 + 1$$

$$0 + 0 + 4 + 0 + 1 + 1 = 6$$

$$16 + 0 + 0 + 0 + 0 + 1 = 17$$

Her primary classification is $\frac{6}{17}$

1. What is your primary classification?
2. How many different primary classifications are there in your class?
3. How many different primary classifications are possible?

Activity 3

The secondary classification is a fraction made up of letters based on the prints on the index fingers. The fraction is

$$\frac{\text{right index finger}}{\text{left index finger}}$$

The letters are A for plain arch, T for tented arch, W for whorl, U for ulnar loop, and R for radial loop. If the right index finger is a whorl and the left index finger is a radial loop, the secondary classification is $\frac{W}{R}$.

1. What is your secondary classification?
2. How many different secondary classifications are there in your class?
3. How many different secondary classifications are possible?

Activity 4

The combined classification of a set of prints is written in the form $\frac{6 W}{17 R}$.

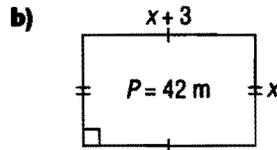
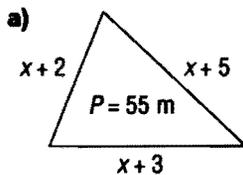
1. What is your combined classification?
2. How many different combined classifications are there in your class?
3. How many different combined classifications are possible?



Review

Solve.

- One number is five times another and their sum is 36. What are the numbers?
- The length of a rectangle is 3 m greater than the width. The perimeter is 26 m. What are the dimensions of the rectangle?
- The sum of three consecutive numbers is 183. Find the numbers.
- The length of a rectangular field is three times the width. The perimeter is 1688 m. What are the dimensions of the field?
- The lengths of the sides of a triangle are 3 consecutive numbers. The perimeter of the triangle is 126 m. How long is each side?
- One number is 4 times another. If you subtract 5 from each number, the sum is 50. What are the numbers?
- The mass of synthetic material in a hockey puck is 9 times the mass of natural rubber. A regulation hockey puck has a mass of 170 g. What mass of natural rubber is there in a hockey puck?
- Find the length of each side.



- The sum of two numbers is 51. Twice the first plus 4 times the second is 128. What are the numbers?
- One number is 5 more than another number. Four times the larger number plus 3 times the smaller is 97. Find the numbers.
- A jar contains \$36.25 in dimes and quarters. There are 250 coins in the jar. How many quarters are in the jar?

- In a pile of coins, there are 15 more quarters than loonies. The total value of the coins is \$21.25. How many quarters are there?

Solve. Graph each solution.

- $2x < 10$
- $3y > -15$
- $4 + x > 8$
- $8 > 4x$
- $3x \geq 12$
- $-3 \leq 3x$
- $-2x < 6$
- $-5x > -5$
- $-8x \leq 16$

Solve and check.

- $3(x - 2) \leq 3$
- $18 \geq 10 + 4x$
- $4t - 3 \geq -15$
- $4 - 2x \geq 8$
- $7x - 8 < 4x + 1$
- $7t + 7 \leq 10t - 14$
- $6t - 5 > 2t - 1$
- $3(s + 6) \leq -9$

Solve.

- $4y - 2 \leq 2y + 8$
- $5(x + 6) - 2(x - 1) > 2$
- $2(x + 4) \leq 5 - 11$
- $4(x - 5) - (x + 1) < 3$
- $7 - 3(2x - 5) - x > 1$
- The world's largest rodent is the capybara of South America. The mass of a capybara can be up to 113 kg. This mass is 8 kg more than 5 times the greatest known mass of a domestic cat. Solve the inequality $5m + 8 \leq 113$ to find the masses of domestic cats.
- The formula $Q = 12t + 35$ gives the amount of water in a tank after t minutes. The tank originally contained 35 L of water, and water runs into it at a rate of 12 L/min.
 - How much water will be in the tank after 5 min? 8 min?
 - How long will it take to fill the tank if it holds 203 L?

37. A hot water tank holds 200 L of water. When the tap is opened, the water drains at a rate of 15 L/min. The amount of water left in the tank after t minutes is given by $A = 200 - 15t$.

- a) How much water will be in the tank after 5 min? 10 min?
- b) How long, to the nearest minute, will it take to drain the tank?

38. Marie started jogging at 08:30 at 9 km/h. Heather started jogging 15 min later in the same direction. Heather jogged at 12 km/h.

- a) How long did it take Heather to catch up to Marie?
- b) At what time did she catch up to her?

39. Two cars left the same highway restaurant at the same time but in opposite directions. One travelled at 65 km/h, the other at 55 km/h. After how long were they 600 km apart?

40. Heidi and Kelly drove in an antique car rally. Kelly left a checkpoint at 09:00 travelling at 45 km/h. One hour later, Heidi left the same checkpoint travelling at 50 km/h.

- a) After how long did Heidi overtake Kelly?
- b) At what time did Heidi overtake Kelly?

41. A plane left Darwin for Beijing, a distance of 6000 km, travelling at 450 km/h. At the same time, a plane left Beijing for Darwin travelling at 550 km/h. How long after take-off did the planes pass each other?

42. It takes Renate 3 h to cut the lawn. It takes her older sister Adrianna 2 h to cut the same lawn. How long will it take them to cut the lawn if they cut it together?

43. A small gas barbecue will cook for 4 h on a tank of propane. A larger barbecue will only cook for 3 h on a tank of propane. For how long can you cook if both barbecues are attached to the same tank?

Group Decision Making Researching Medical Careers

1. Brainstorm as a class possible careers to investigate. They could include careers like dentist, doctor (general practitioner), doctor (specialist), nurse, X-ray technician, psychologist, dietitian, dental hygienist, or physiotherapist. Decide as a class on 6 careers.

2. Go to home groups. Decide as a group what career each member will research.

1	2	3	4	5	6	1	2	3	4	5	6
---	---	---	---	---	---	---	---	---	---	---	---

Home Groups

1	2	3	4	5	6	1	2	3	4	5	6
---	---	---	---	---	---	---	---	---	---	---	---

3. Form an expert group with students who have the same career as you to research. Decide on the questions you want answered, including how math is used in the career. Research the career in your expert group.

1	1	1	1	2	2	2	2	3	3	3	3
---	---	---	---	---	---	---	---	---	---	---	---

Expert Groups

4	4	4	4	5	5	5	5	6	6	6	6
---	---	---	---	---	---	---	---	---	---	---	---

4. Return to your home group and tell the others what you found out in your expert group. Ask for questions and comments from your home group.

5. Return to your expert group and report what you have learned from your home group. In your expert group, prepare a report on the career. The form of the report is to be decided by the group.

6. In your expert group, evaluate the process and your report.

Chapter Check

- When 47 is subtracted from a certain number, the result is 34. Find the number.
- The length of a rectangle is 15 m longer than the width. The perimeter of the rectangle is 74 m. Find the length and width.
- The lengths of the sides of a triangle are 3 consecutive whole numbers. The perimeter of the triangle is 102 m. Find the lengths.

Solve and graph.

4. $-3x > 21$ 5. $-2x + 5 < 9$

6. $4r + 4 > r + 1$

Solve.

7. $2(3x + 1) - 2(x - 1) \geq -16$

8. $4(x - 1) - 6(x + 1) < 10$

9. A Boeing 747 is about 60 m long. This length is about 4 m longer than 8 times the length of the longest known earthworm. Solve the inequality $8x + 4 \leq 60$ to find the lengths of earthworms.

10. The formula for the perimeter of a rectangle is $P = 2(l + w)$. Find w if $P = 200$ m and $l = 56$ m.

11. The formula for the area of a triangle is $A = \frac{1}{2}bh$. Solve for b .

12. A cargo ship left Montreal for Halifax at 15 km/h. One hour later, a patrol boat left Montreal at 20 km/h, trying to overtake the cargo ship. After how long did the patrol boat overtake the cargo ship?

13. Mark lived 350 km from his agent. Mark had to sign a contract his agent had. Mark and his agent left their apartments at the same time in their cars and drove along the same road toward each other. Mark drove at 65 km/h and his agent drove at 75 km/h. After how long did they meet?

14. You see lightning flash before you hear thunder because light travels much faster than sound. The light reaches you almost instantaneously. The sound travels at a speed of 330 m/s. Write a formula to find your distance, d metres, from a thunderstorm in terms of the number of seconds between the lightning flash and when you hear the thunder.

15. Sunil and Kim both do volunteer work at Ronald McDonald House. It takes Sunil 6 h to cut the lawns and trim the hedges, and it takes Kim 4 h to do the same. How long would it take them to cut the lawns and trim the hedges if they worked together?

THE FAR SIDE

By GARY LARSON



"Yes, yes, I know that, Sidney ... everybody knows that! ... But look: Four wrongs squared, minus two wrongs to the fourth power, divided by this formula, do make a right!"

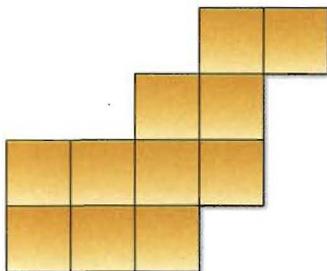
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Using the Strategies

1. If the pattern continues, find the following product.

$$\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \dots \left(1 - \frac{1}{24}\right)$$

2. The figure is made up of 11 identical squares. The area of the figure is 539 cm^2 .



What is the perimeter of the figure?

3. Your plane is scheduled to leave for Casablanca at 18:15. You must arrive at the airport 25 min before the flight to pick up your tickets. It takes a taxi 30 min to drive to the airport from your hotel. Before you leave for the airport, the naval attaché is delivering some documents to you in the lobby of your hotel. It will take the attaché 10 min to explain the documents to you. Before meeting the attaché, you need 1 h 15 min to shower, pack, change clothes, and eat. It is now noon. You want to spend some time sightseeing. What is the latest time you can return to the hotel?

4. A train travelling at 90 km/h passed a car sitting at a railway crossing. It took 45 s for the train to pass. How long was the train in metres?

5. An engine plant built 485 automobile engines. There were 2 models of engines, 4-cylinder and 6-cylinder. The total number of cylinders in the engines was 2224. How many of each model of engine did the plant build?

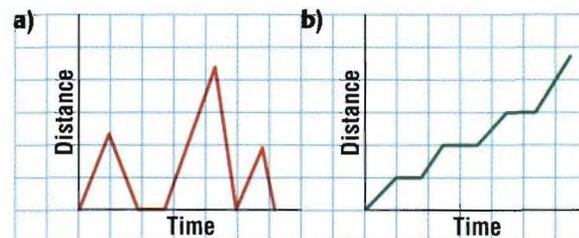
6. A 2-digit number is changed by decreasing the tens digit by 1 and increasing the units digit by 1. When the new number is added to the original number, the result is 59. What was the original number?

7. If you disregard the year on a calendar, only a limited number of calendars is necessary. For example, you need only one calendar for any year in which January 1 is on a Monday, providing the year is not a leap year.

- How many different calendars are necessary?
- Start with the year 2001 and determine how many years must pass before each of the calendars you found in part a) is used at least once.



8. The graphs show the motion of Xenia's car. The vertical axis shows the distance from her house and the horizontal axis shows time. Write a story to explain each graph.



DATA BANK

1. Before he left to sail down the Nile River, Livingstone, who was in Cairo, called his friend Stanley, who was in Edmonton. Livingstone called at 10:00 Cairo time. What time was it in Edmonton?



2. Use the Data Bank on pages 364 to 369 to write a uniform motion problem. Have a classmate solve it.

