



Getting Out of Line

(Function Patterns)

Objective

Students explore basic connections between graphs, tables, and symbolic representations for lines, parabolas, inverse models, and exponential models. One of the primary goals of this lesson is to help students begin to recognize patterns that are different from lines.

Overview of the Lesson

This lesson emphasizes patterns, discovery, and vocabulary by focusing on the basic connections between the graphs, tables, and symbolic representations for lines, parabolas, inverse models, and exponential models. Students investigate patterns and various models using the graphing calculator. It is designed for first-year high school students and would be an ideal lesson following a unit on lines.

Materials

- graphing calculator overhead unit
- overhead projector

For each group:

- markers
- newsprint
- *Getting Out of Line* activity sheets
- *The Shape of the Rules* activity sheets
- *Matching the Patterns* cards

Procedure

1. **Introduction:** Begin the lesson with a discussion of how high a ball might go if it were thrown into the air at a velocity of 25 meters per second. “Think About This Situation” is included in the activity sheet. Have students read and discuss the situation. How high will the ball go? How long will it be in the air? Which graph best matches the (*time, height*) data describing the ball’s flight?

Have students toss paper-wad balls up in the air to help them estimate the time that a ball is likely to remain in the air. They should then discuss the graphs and share thoughts about the different graph patterns. Through discussion help them determine a model and graph that model, relating it back to the question about the shape of the graph they expected for the (*time, height*) data describing the ball’s flight.

2. **The Shape of the Rules:** Have students work in groups to complete the activity *The Shape of the Rules*. Assign each group one of the experiments to do. Groups should work together to complete their experiments and report to the entire class. Each experiment involves graphing six equations and examining the patterns among the graphs, the tables and the symbolic forms. Ask each group to pick one of their graphs to plot on large chart paper along with the table of values so that they can display their work as they present their findings to the class.

As the groups report, the various characteristics of the four different functions—linear, quadratic, inverse, and exponential—will be discussed in very elementary terms. Students may notice such things as rates of change, symmetry, first degree terms, second degree terms, undefined values, and exponential growth. After all of the models have been discussed, label the charts with the name of the function family.

3. **Matching the Patterns:** Give each group a set of cards that have either a graph, a rule, or a table. Students should work in groups to decide where the cards belong. After the groups have made their decisions, have them tape the cards to the appropriate function model chart at the front of the classroom. Review the placement of the various cards and discuss the decisions to make sure that all cards have been properly placed.

Assessment

Students demonstrate their thinking and understanding as they work in groups on the various activities, as they participate in class discussions, and as they make presentations to the class. By observing these various activities, the teacher is able to assess their mathematical understanding.

Included in the lesson plan are other activities that can be used for assessment. The Checkpoint that follows Experiment 4 gives students an opportunity to express their thoughts either orally or in writing. The On Your Own problems following that Checkpoint offer another opportunity to check student understanding.

Extensions & Adaptations

- Students could bring in examples of the various function models to share with the class and then post these examples on the large sheets modeling the four function families. Examples might come from science books, encyclopedias, newspapers, advertisements, or any other source where they are likely to find functions represented in tables, graphs, or symbolic form.
- In the video lesson, the quadratic equation modeling the height of the ball over time is discussed by the entire class. Have students examine this model more completely and more independently by working in groups on the problems following the “Think About This Situation.”
- Use the example from *Mathematically Speaking* as an additional problem for your students to explore.

Mathematically Speaking

One of the important aspects of the study of algebra is the development of techniques for studying numerical patterns, quantitative variables, and relationships among those variables and their patterns of change. In this video lesson, students explore those patterns using their graphing calculator. Through the use of the algebraic rule, they can enter the symbolic model into the calculator and then use the capabilities of the calculator to explore patterns in both the graphs and the tables of the various models.

Being able to use the calculator effectively requires two major skills. First, the students must know the various capabilities of the machine and how to use those capabilities to get a job done. Second, the students must develop something that is called “symbol sense.” They must have an idea of what is reasonable to expect, given a certain rule, table, or graph. They must have some idea of the shape of the graph, the pattern in the table based on the symbolic model, and a reasonable window for viewing a given model.

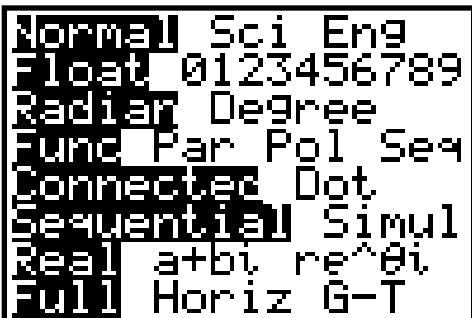
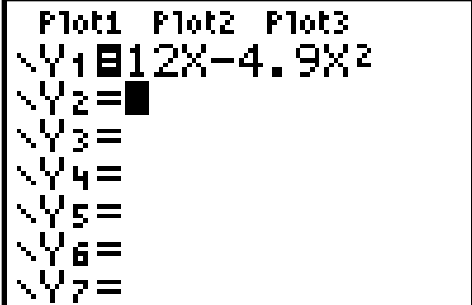
The particular activities in the video lesson are first steps in helping students develop these sorts of algebraic understandings. For students to be good mathematical problem solvers, they need to have good common sense, good number sense, and good symbol sense. All three are gained through multiple experiences over time.

What should students be able to do with the calculator? As an example, explore the following symbolic model and its tables and graphs.

A player kicks a soccer ball into the air with an initial velocity of 12 meters per second. Thus the equation representing the height of the ball, h , over time, t , is $h = 12t - 4.9t^2$. How long does it take the ball to return to the ground? What is the maximum height reached by the ball?

Exploring Tables with the Calculator

Students need to be able to do the following to explore tables.

<p>1. First enter the algebraic model as a function. Press MODE ▼ ▼ ▼ Enter to set Func graphing mode.</p>	 <pre> Normal Sci Eng Float 0123456789 Radian Degree Func Par Pol Seq Connected Dot Sequential Simul Real a+bi re^θi Full Horiz G-T </pre>
<p>2. Press Y=. Press 12 X, T, %, n = 4.9 X, T, %, n x^2 to enter the function $Y1=12X-4.9X^2$</p>	 <pre> Plot1 Plot2 Plot3 \Y1=12X-4.9X^2 \Y2= \Y3= \Y4= \Y5= \Y6= \Y7= </pre>

3. Press **2nd** [**TBLSET**] to display the **TABLE SETUP** screen. Press 0 **Enter** to set **Tbl Start**= 0. Press 1 **Enter** to set **Tbl** =1. Press **Enter** to select **Indpnt**: Auto and press **▼** **Enter** to select **Depend**: **Auto** to automatically generate the independent and dependent values.

```

TABLE SETUP
TblStart=0
ΔTbl=1
Indpnt:  AUTO  Ask
Depend:  AUTO  Ask
    
```

4. Press **2nd** [**TABLE**] to display the table screen. Press the arrow keys to see different values in the table.

X	Y1	
0	0	
1	7.1	
2	4.4	
3	-8.1	
4	-30.4	
5	-62.5	
6	-104.4	

X=6

5. Notice that the sign changes between $X = 2$ and $X = 3$. Reset the table initial value, and change the increment so that the values start at 2 and change by 0.1. Press **2nd** **[TBLSET]** to display the **TABLE SETUP** screen. Press **2** **[Enter]** to set **Tbl Start= 2**. Press **0.1** **[Enter]** to set **Tbl =0. 1**. Press **[Enter]** to select **Indpnt: Auto**, and press **▼** **[Enter]** to select **Depend: Auto** to automatically generate the independent and dependent values. Repeat this procedure, resetting the initial value and changing the increment to 0.01 to get an even closer approximation to when the ball hits the ground.

```
TABLE SETUP
TblStart=2
ΔTbl=.1
Indnt: Auto Ask
Depend: Auto Ask
```

X	Y ₁
2	4.4
2.1	3.591
2.2	2.684
2.3	1.679
2.4	.576
2.5	-.625
2.6	-1.924

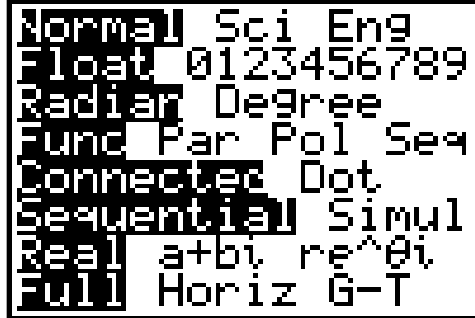
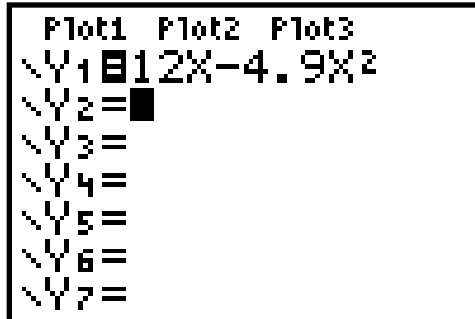
X=2

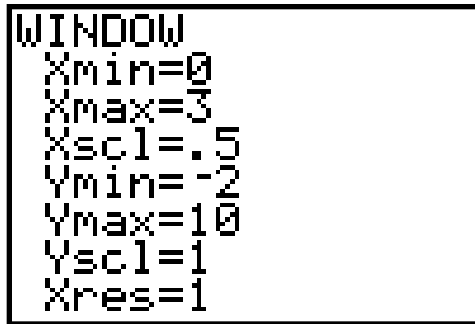
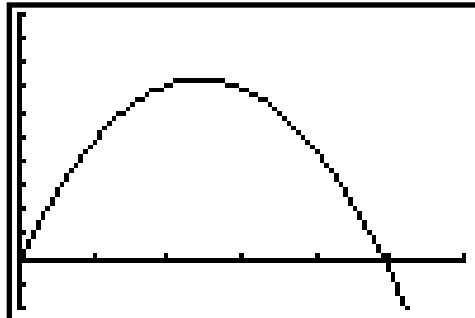
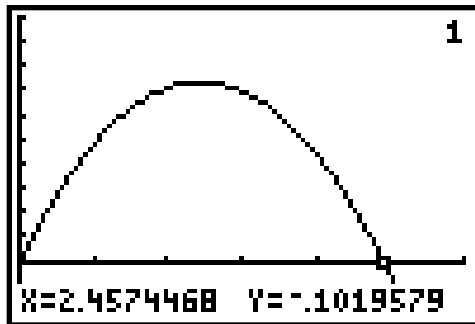
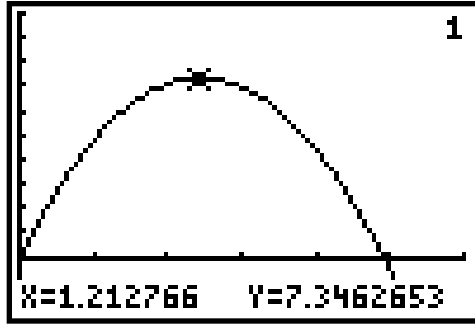
X	Y ₁
2.4	.576
2.41	.46031
2.42	.34364
2.43	.22599
2.44	.10736
2.45	-.0123
2.46	-.1328

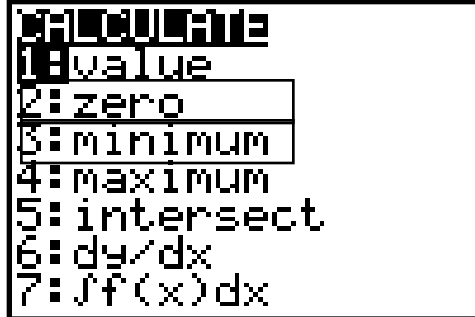
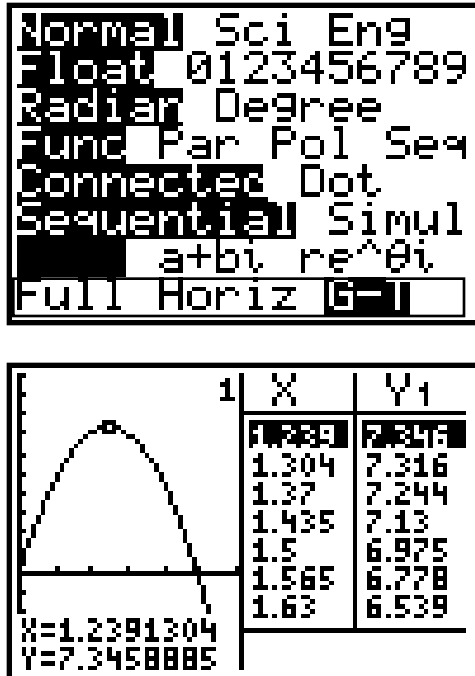
X=2.4

<p>6. Knowing the ball hits the ground at about $t = 2.45$ and knowing that the ball will reach a maximum height half way between the starting time and the time it returns to the ground should help in setting up an appropriate initial value and increment for a table that will allow you to determine the maximum height of the ball. Set the initial value at 1.2 and leave the increment at 0.01. From the information in the table, you should be able to give a good approximation for the both the maximum height of the ball and for the time at which the maximum height is reached.</p>	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="padding: 5px;">X</th> <th style="padding: 5px;">Y₁</th> </tr> </thead> <tbody> <tr><td style="padding: 5px;">1.21</td><td style="padding: 5px;">7.3444</td></tr> <tr><td style="padding: 5px;">1.22</td><td style="padding: 5px;">7.3459</td></tr> <tr><td style="padding: 5px;">1.23</td><td style="padding: 5px;">7.3468</td></tr> <tr><td style="padding: 5px;">1.24</td><td style="padding: 5px;">7.3458</td></tr> <tr><td style="padding: 5px;">1.25</td><td style="padding: 5px;">7.3438</td></tr> <tr><td style="padding: 5px;">1.26</td><td style="padding: 5px;">7.3408</td></tr> </tbody> </table> <p style="margin-top: 10px;">X=1.2</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>WINDOW</p> <p>Xmin=0</p> <p>Xmax=3</p> <p>Xscl=.5</p> <p>Ymin=-2</p> <p>Ymax=10</p> <p>Yscl=1</p> <p>Xres=1</p> </div>	X	Y ₁	1.21	7.3444	1.22	7.3459	1.23	7.3468	1.24	7.3458	1.25	7.3438	1.26	7.3408
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Exploring Graphs with the Calculator

<p>1. First enter the algebraic model as a function. Press MODE \downarrow \downarrow \downarrow Enter to set Func graphing mode.</p>	 <p>Normal Sci Eng Float 0123456789 Radian Degree Func Par Pol Seq Connected Dot Sequential Simul Real a+bi re^θi Full Horiz G-T</p>
<p>2. Press Y=. Press 12 X, T, %, n = 4.9 X, T, %, n X^2 to enter the function $Y1=12X-4.9X^2$.</p>	 <p>Plot1 Plot2 Plot3 $\sqrt{Y1=12X-4.9X^2}$ $\sqrt{Y2=}$ $\sqrt{Y3=}$ $\sqrt{Y4=}$ $\sqrt{Y5=}$ $\sqrt{Y6=}$ $\sqrt{Y7=}$</p>

<p>3. Once the function is entered, you need to set an appropriate viewing window. Using what you know about the function from having explored the table already, press WINDOW and set Xmin=0, Xmax=3, Xscl=0.5, Ymin=-2, Ymax=10, Yscl=1, Xres=1. Xres sets pixel resolution (1 through 8) for function graphs only. The default is 1. At Xres=1, functions are graphed at each pixel on the x-axis, and at Xres=8, they are graphed at every eighth pixel along the x-axis.</p>	 <pre> WINDOW Xmin=0 Xmax=3 Xscl=.5 Ymin=-2 Ymax=10 Yscl=1 Xres=1 </pre>
<p>4. Press GRAPH to display the graph with the indicated viewing window.</p>	
<p>5. Press TRACE and use the arrow keys to move the cursor along the graph until the y-value displayed at the bottom of the screen is as close to zero as possible. What is the corresponding value for x? How does this compare to the value from your table?</p>	
<p>6. Press TRACE and use the arrow keys to move the cursor along the graph until the cursor is at the top of the curve. What is the value of the function for this point? How does that correspond to your answer from the table?</p>	

<p>7. You can find several more helpful calculator functions under the CALC key. For this problem, zero and maximum allow you to determine the time at which the ball hits the ground and the maximum height of the ball.</p>																						
<p>8. One very helpful mode to use when examining tables and graphs is called graph-table (G-T) mode. This is the last option under the MODE key.</p> <p>You can use the trace function in this mode and watch the values in the table change as you trace the function.</p>	 <table border="1" data-bbox="1096 903 1315 1165"> <thead> <tr> <th>1</th> <th>X</th> <th>Y1</th> </tr> </thead> <tbody> <tr> <td></td> <td>1.304</td> <td>7.316</td> </tr> <tr> <td></td> <td>1.37</td> <td>7.244</td> </tr> <tr> <td></td> <td>1.435</td> <td>7.13</td> </tr> <tr> <td></td> <td>1.5</td> <td>6.975</td> </tr> <tr> <td></td> <td>1.565</td> <td>6.778</td> </tr> <tr> <td></td> <td>1.63</td> <td>6.539</td> </tr> </tbody> </table> <p>X=1.2391304 Y=7.3458885</p>	1	X	Y1		1.304	7.316		1.37	7.244		1.435	7.13		1.5	6.975		1.565	6.778		1.63	6.539
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Tips From Ellen

Simultaneous Sharing Techniques

When working with small groups, a common phenomenon is that each group creates a product to share with the remainder of the participants. Often each group will stand up and share sequentially. The skill of conducting classroom presentations is a very important one to foster. The following alternative formats for sharing can offer a change of pace, provide movement (particularly important in block scheduling or at low energy times of the day), involve more participants, and often speed up the process. See if one or two ideas meet your style and activities, or even suggest a different way to design a group activity.

Chalkboard/Chart Paper Sharing: As soon as each group is finished, it sends a representative to a chalkboard or chart paper to post the group's best answer or idea. This tends to offer ideas to slower groups and/or speed them to completion. Depending on the nature of the work, groups may continue working after posting an answer to see if they can further refine it.

Simultaneous Stand and Share: (Good for questions with multiple solutions). Following a short small group discussion, each group makes sure that all their members have a different idea to share. On a signal, all participants stand up. The leader starts in one area of the room and asks a participant to share (generally the leader records). As each participant shares, he or she sits down, and anyone in the room who had the same or similar idea also sits down. Sharing continues until all participants are seated.

Gallery Tour: Completed group products are displayed around the room. On a signal, groups pass from one product to another, responding as groups to the products they see by writing questions or adding ideas. It may be necessary to time each "turn."

Roam the Room: Like the Gallery Tour, except that participants move freely and individually around the room. On a signal they return to their groups and report on what they found. Often participants are given a goal to achieve while "roaming."

One-Way Jigsaw: Groups number themselves off (e.g. if each group has 4 members, each group numbers its members from 1 to 4). Then all the ones meet together, the twos meet together, etc., to share their group's ideas. If there are too many groups, make the groups larger to begin with (use 6 groups of 6 instead of 9 groups of 4) or divide the room into two smaller sharing groups. This works particularly well if carbon paper is used in groups so that group members receive a carbon of the group's ideas to take with them.

Carousel Sharing: One representative from each group stays at the group's table (or chart) while the remaining members of each group rotate from table to table (or chart to chart). The representatives shares their groups' information with other groups as they rotate through.

Many ideas in this section were adapted from Cooperative Learning & Language Arts: A Multi-Structural Approach written by Jeanne M. Stone and published by Kagan's Cooperative Learning.

Resources

Coxford, Art, James Fey, Christian Hirsch, and Harold Schoen, *Contemporary Mathematics in Context, Course 1*. Chicago, IL: Everyday Learning Corporation, 1997.

Everyday Learning Corporation, Two Prudential Plaza, Suite 1175, Chicago, IL 60601, 1-800-382-7670.

Internet location: <http://www.ti.com/calc/docs/t3.htm>

Information about "Teachers Teaching with Technology" is provided at this site. Conferences, workshops, and summer institutes are listed. In addition, classroom activities that incorporate technology are provided.

Internet location: <http://web99.ed.gov/GTEP/Program2.nsf>

The Guide to U.S. Department of Education Programs and Resources, a reference describing the department's programs and grants, is available at this site.

Internet location: <http://www.everydaylearning.com>

Everyday Learning Corporation publishes Contemporary Mathematics in Context. You can request catalogs or send email to the company from this site.

Internet location: <http://www.wmich.edu/math-stat/cpmp/>

Information about the Core-Plus Mathematics Project is available at this site. Sample lessons are available as well as an overview of the project.

Ideas for Online Discussion

(Some ideas may apply to more than one standard of the NCTM Professional Standards for Teaching Mathematics.)

Standard 1: Worthwhile Mathematical Tasks

1. In traditional algebra, students generally focus on linear and quadratic models in year one. A number of programs now emphasize having students recognize more mathematical models at an earlier point in their study of algebra. What are the advantages of this type of approach?

Standard 2: The Teacher’s Role in Discourse

2. The teacher refers to what the students know from their physical science course and brings that knowledge into the classroom. This background may come from actually working with the science teacher, or it may come from what students in previous years have told the teacher about what they have learned in physical science. How can this type of interaction or knowledge strengthen the class? What type of interaction do you have with members of the science department in your school?

Standard 3: Students’ Role in Discourse

3. At least once in this video lesson a student refers to a particular problem that was studied earlier in the course—the whale population problem. In what ways do these real-world applications help students make connections and bring other ideas into discussions?

Standard 4: Tools for Enhancing Discourse

4. Do you think the paper wads activity was effective? Have you done this or something like this with your students? What risks might be involved and how could they be avoided?
5. What impact does the use of chart paper have in this lesson?

Standard 5: Learning Environment

6. The teacher had the students working in groups of three. How did you think this worked for this lesson? How many students do you have in your groups, and how does that compare to the groups of three? Discuss the merits of different types of grouping.

Standard 6: Analysis of Teaching and Learning

7. In this video lesson, several activities could have been collected and evaluated in a different way. Each group could have done all four experiments, and they could have been collected and graded. The pattern matching cards could have been printed on an activity sheet, and all students could have evaluated all of the patterns and turned it in to be graded. Keeping assessment in mind, discuss the pros and cons of what was done in the lesson, and what might have been done.

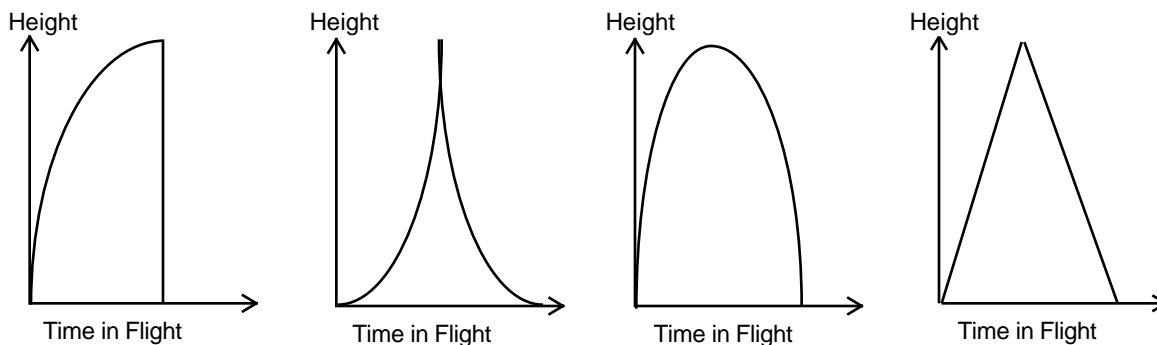
Getting Out of Line

Earthbound humans are especially fascinated by things that fly through the air. We follow the flight of balls in all kinds of sports, perhaps curious about how high they go or when and where they will come down. In some sports, people are also interested in the speed at which a ball travels. For example, baseball teams use radar guns to “clock” the speed of a pitch or the path of a long hit.

Think About This Situation

Suppose you threw a ball straight up into the air at a velocity of 25 meters per second. (Major league pitches travel at approximately 95 miles per hour, which is about 42.5 meters per second.)

- Approximately how high do you think the ball would go before it started falling back to the ground?
- Approximately how many seconds do you think it would take before the ball hit the ground? How fast do you think the ball would be traveling just before it hit the ground?
- Which of the following graphs do you think best matches the pattern of *(time, height)* data describing the ball’s flight? Explain your reasoning.



To study the flight of a thrown or hit baseball, all you need to know is the velocity with which the ball leaves the hand of the thrower or the bat of the hitter. Using principles of physics, you can predict the velocity of the ball as a function of time in flight with the following rule.

$$v = 25 - 9.8t$$

In this rule, the letter v stands for the velocity of the ball (in meters per second), and t stands for time since the ball was thrown (in seconds).

Use the rule relating velocity of the ball and time in flight to study the pattern of $(time, velocity)$ data that could be expected. If you enter the velocity rule into your graphing calculator or computer, the calculations will be much easier!

1.
 - a. Make a table of $(time, velocity)$ data for $t = 0$ to $t = 6$ in steps of 0.5 seconds.
 - What is the velocity of the ball 0.5 seconds after it was thrown?
 - At what time will the velocity be about 10 meters per second? About 5 meters per second?
 - What does the pair $(3, -4.4)$ mean?
 - Describe the pattern of change in velocity for every 0.5 seconds the ball is in flight.
 - b. Produce a graph of the pattern of $(time, velocity)$ data obtained in part (a). Use the table to help you choose an appropriate viewing window.
 - c. Write an explanation of what the table and graph say about the pattern of the ball's velocity. Be sure to explain any points or trends in the graph that are surprising or important.
2. Looking at the equation $v = 25 - 9.8t$, can you see how the numbers and operations in the rule relate to flight of the ball? For example, does 25 look familiar? What is the effect of the $-9.8t$ part of the rule?
3. Use your calculator table or graph tools to solve these equations and then explain what the solutions tell about the flight of the ball. Find times to the nearest 0.1 seconds.
 - a. $10 = 25 - 9.8t$
 - b. $0 = 25 - 9.8t$
 - c. $v = 25 - (9.8 \times 2)$
 - d. $-5 = 25 - 9.8t$

The equation relating time in flight and velocity of the baseball gives some information about the path of the ball, but it also leaves some interesting questions unanswered. More can be learned from a second rule that estimates height of the ball as a function of time in flight. Again, scientific principles suggest that this relation will be given by the following rule.

$$h = 25t - 4.9t^2$$

4. Make a table of $(time, height)$ data for $t = 0$ to $t = 6$ seconds in steps of 0.5 seconds. Remember to use your table to help set your viewing window. Then plot a graph of the same data. Adjust your table or graph as necessary to estimate:
 - a. The height of the ball after 3 seconds. After 5 seconds.
 - b. The time(s), to the nearest 0.1 seconds, when the ball is 20 meters above the ground.
 - c. The maximum height of the ball and the time when the ball reaches that height.
 - d. The time when the ball hits the ground.
 - e. The velocity of the ball when it hits the ground.

5. Look at the patterns in the tables and graphs for $(time, height)$ and $(time, velocity)$ data, and try to explain how those patterns relate to each other. Look at both graphs on the same axes using a viewing window of $Xmin = 0$, $Xmax = 7$, $Ymin = -40$, and $Ymax = 35$. What does the shape of the $(time, height)$ graph tell about the velocity of the ball during its flight? How is that shape illustrated by the trend of numbers in the $(time, height)$ and $(time, velocity)$ data tables?

6. Compare the shape of your $(time, height)$ graph with the choice(s) you made in answering question 1(c).

The Shape of the Rules

Finding symbolic rules for relationships between variables is a big part of mathematical model building. It is common to start the search with some data in a table or graph. But to find a rule fitting patterns in that data, it helps to know about the match between types of rules and shapes of graphs and patterns in tables. You can discover useful connections between rules and graphs by some experimentation with your graphing calculator or a computer.

Experiment 1

- For each rule listed below:
 - Produce a table of values and a graph.
 - On a separate sheet of paper, write the rule and make a sketch of the graph. How does the pattern of values in the table match the pattern in the graph?
 - Explore how the graph appears using various possible viewing windows.
 - $y = 2x - 4$
 - $y = 2x + 4$
 - $y = 0.5x + 2$
 - $y = -0.5x + 2$
 - $y = 10 - 1.5x$
 - $y = x^2 - 4$
- As a group, discuss similarities and differences in the graphs you produced.
- As a group, make some conjectures about the shapes of graphs for certain types of rules. Have individual members test various conjectures and report back to the group.

Experiment 2

- For each rule listed below:
 - Produce a table of values and a graph.
 - On a separate sheet of paper, write the rule and make a sketch of the graph. How does the pattern of values in the table match the pattern in the graph?
 - Explore how the graph appears using various possible viewing windows.
 - $y = x^2$
 - $y = x^2 - 3$
 - $y = -x^2$
 - $y = -x^2 + 5$
 - $y = (x + 3)^2$
 - $y = \frac{2}{x}$
- As a group, look for patterns in the rules that seem connected to special patterns in the graphs. Make and test some conjectures about the shapes of graphs for certain types of rules.

Experiment 3

- For each rule listed below:
 - Produce a table of values and a graph.
 - On a separate sheet of paper, write the rule and make a sketch of the graph. How does the pattern of values in the table match the pattern in the graph?
 - Explore how the graph appears using various possible viewing windows.
 - $y = \frac{1}{x}$
 - $y = \frac{3}{x}$
 - $y = \frac{5}{x}$
 - $y = \frac{-5}{x}$
 - $y = \frac{x}{3}$
 - $y = \frac{5}{x+1}$
- As a group, look for patterns in the rules that seem connected to special patterns in the graphs. Make and test some conjectures about the shapes of graphs for certain types of rules.

Experiment 4

1. For each rule listed below:
 - Produce a table of values and a graph.
 - On a separate sheet of paper, write the rule and make a sketch of the graph. How does the pattern of values in the table match the pattern in the graph?
 - Explore how the graph appears using various possible viewing windows.
 - a. $y = 2^x$
 - c. $y = 3^x$
 - e. $y = (0.5)^x$
 - a. $y = (1.5)^x$
 - d. $y = x^3$
 - f. $y = x^{0.5}$

2. As a group, look for patterns in the rules that seem connected to special patterns in the graphs. Make and test some conjectures about the shapes of graphs for certain types of rules.

√ Checkpoint

As a result of the experiments, your group probably has some hunches about matches between the form of a symbolic rule and the pattern of data in tables and the shape of graphs. Summarize your ideas in statements like this:

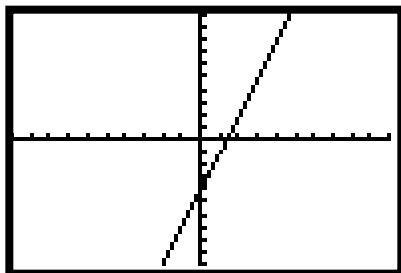
If we see a rule like ... , we expect to get a table like

If we see a rule like ... , we expect to get a graph like

Be prepared to compare your summary statements with those of other groups.

On Your Own:

As best you can, match each table or graph with one of the given rules. In each case, explain the reason for your choice.

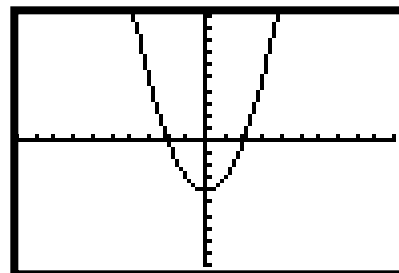


X	Y1	
0	1	
1	4	
2	16	
3	64	
4	256	
5	1024	
6	4096	

X=0

X	Y1	
1	5	
2	2.5	
3	1.6667	
4	1.25	
5	1	
6	.83333	
7	.71429	

X=1



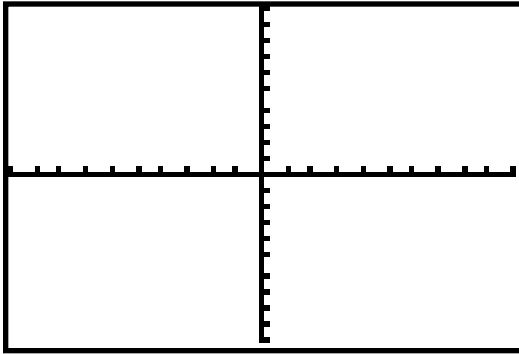
- | | | |
|----------------------|-------------------|-----------------|
| a. $y = x^2 - 4$ | c. $y = -4x - 3$ | e. $y = 4^x$ |
| b. $y = \frac{5}{x}$ | d. $y = -x^2 + 4$ | f. $y = 3x - 4$ |

In this investigation, you have explored only a few of many possible types of equations that can be used to model relationships between variables. Your study of the match between rule types and patterns in tables and graphs of sample data is an important part of your study of algebra.

The Shape of the Rules

EXPERIMENT 1

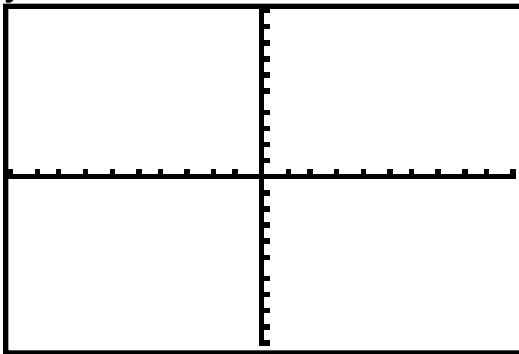
a. $y = 2x - 4$



Window _____

x	y

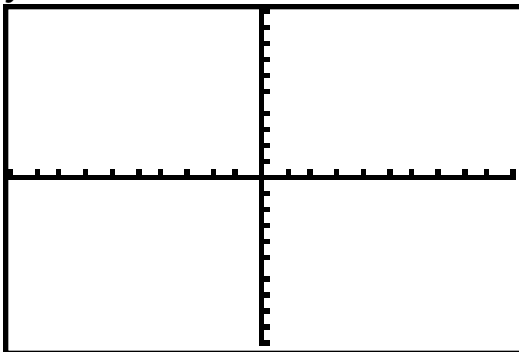
b. $y = 2x + 4$



Window _____

x	y

c. $y = 0.5x + 2$

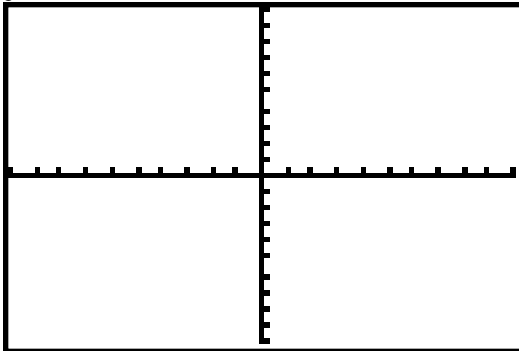


Window _____

x	y

EXPERIMENT 1 (Continued)

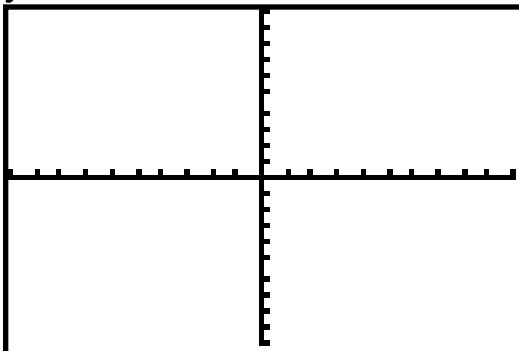
d. $y = -0.5x + 2$



Window _____

x	y

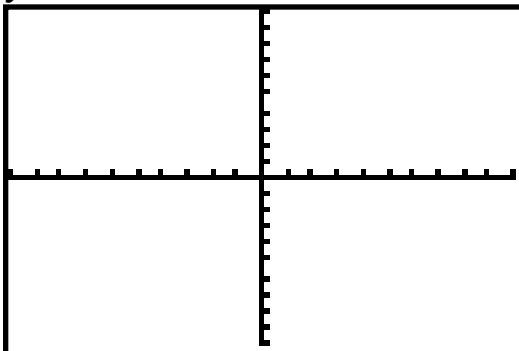
e. $y = 10 - 1.5x$



Window _____

x	y

f. $y = x^2 - 4$



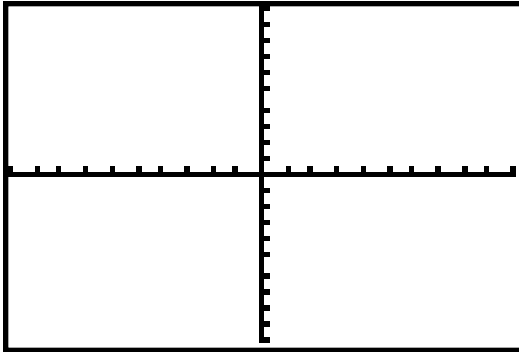
Window _____

x	y

The Shape of the Rules

EXPERIMENT 2

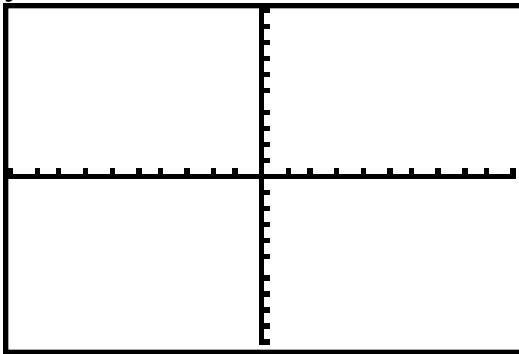
a. $y = x^2$



Window _____

x	y

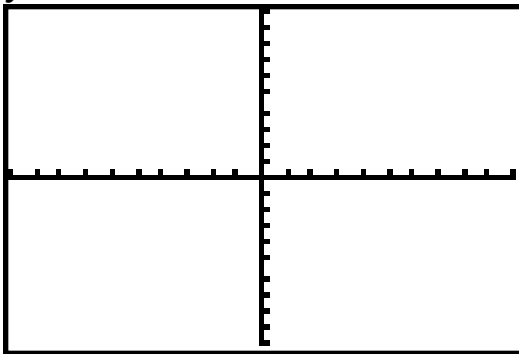
b. $y = x^2 - 3$



Window _____

x	y

c. $y = -x^2$

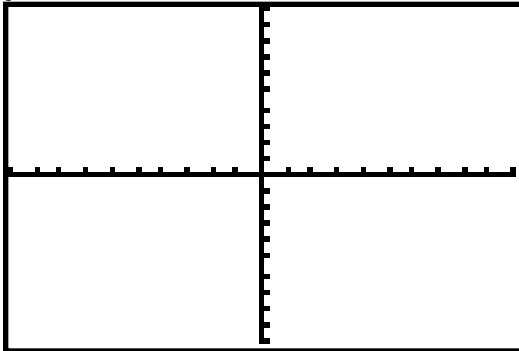


Window _____

x	y

EXPERIMENT 2 (Continued)

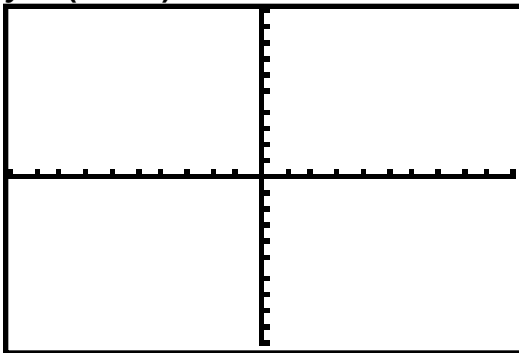
d. $y = -x^2 + 5$



Window _____

x	y

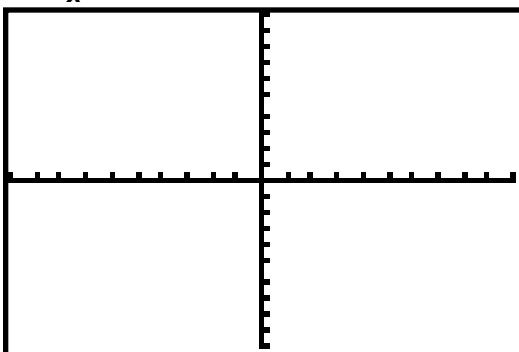
e. $y = (x + 3)^2$



Window _____

x	y

f. $y = \frac{2}{x}$



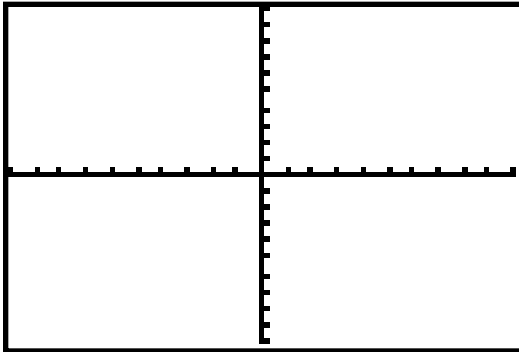
Window _____

x	y

The Shape of the Rules

EXPERIMENT 3

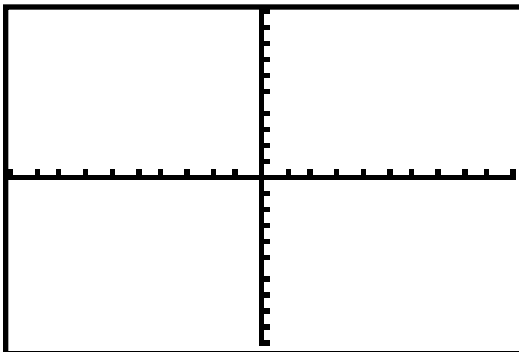
a. $y = \frac{1}{x}$



Window _____

x	y

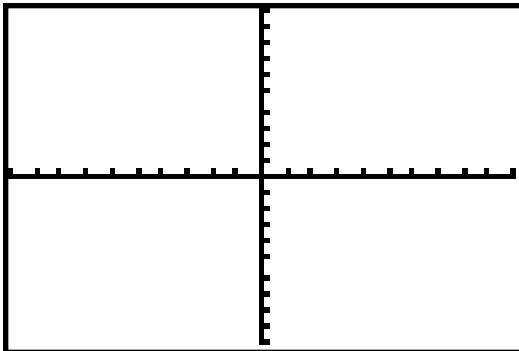
b. $y = \frac{3}{x}$



Window _____

x	y

c. $y = \frac{5}{x}$

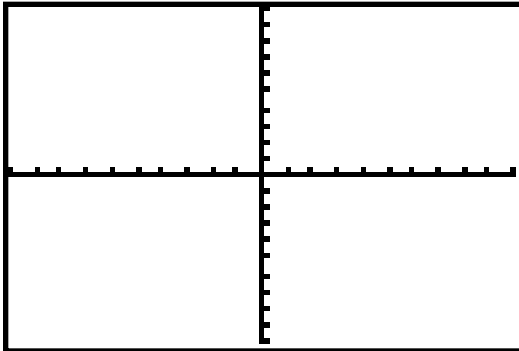


Window _____

x	y

EXPERIMENT 3 (Continued)

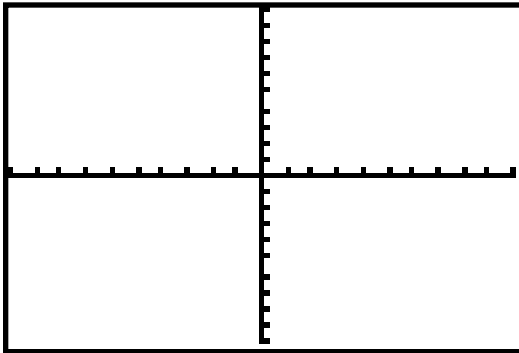
d. $y = \frac{-5}{x}$



Window _____

x	y

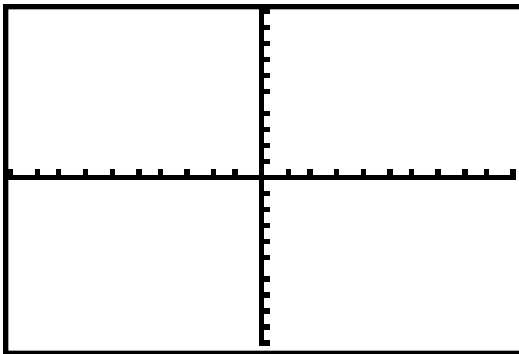
e. $y = \frac{5}{x + 1}$



Window _____

x	y

f. $y = \frac{x}{3}$



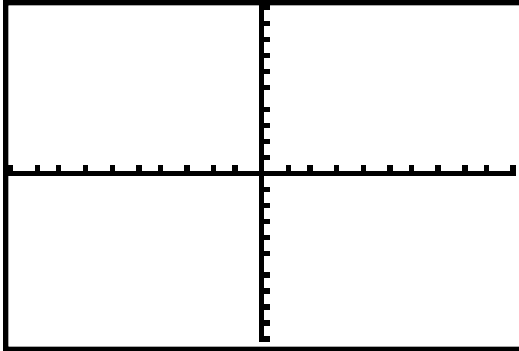
Window _____

x	y

The Shape of the Rules

EXPERIMENT 4

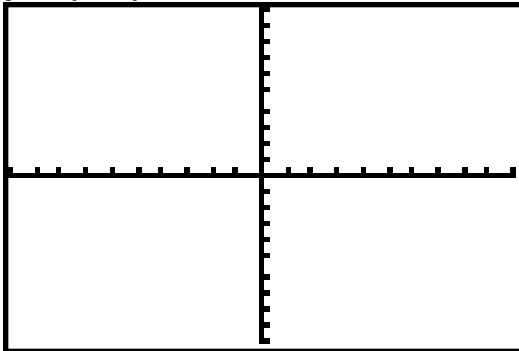
a. $y = 2^x$



Window _____

x	y

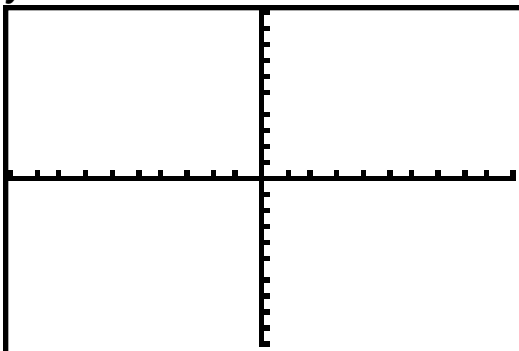
b. $y = (1.5)^x$



Window _____

x	y

c. $y = 3^x$

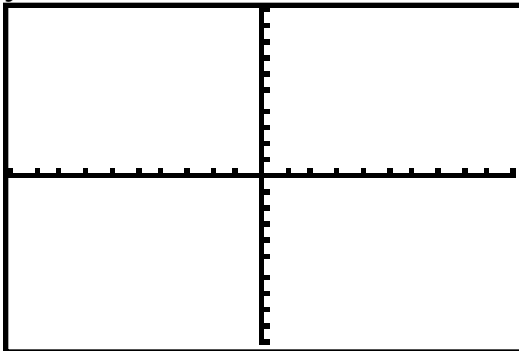


Window _____

x	y

EXPERIMENT 4 (Continued)

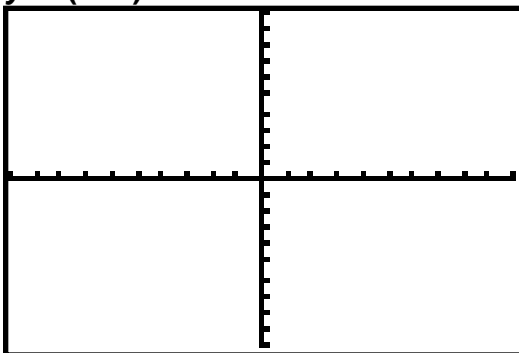
d. $y = x^3$



Window _____

x	y

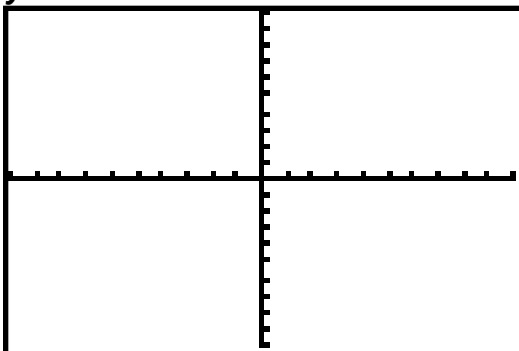
e. $y = (0.5)^x$



Window _____

x	y

f. $y = x^{0.5}$



Window _____

x	y

Matching the Patterns

The following items involving graphs, tables, symbols and stories should be cut apart. They may be mounted on heavier cards and distributed to the groups. Group members should work together to determine which function family each pattern matches.

1.

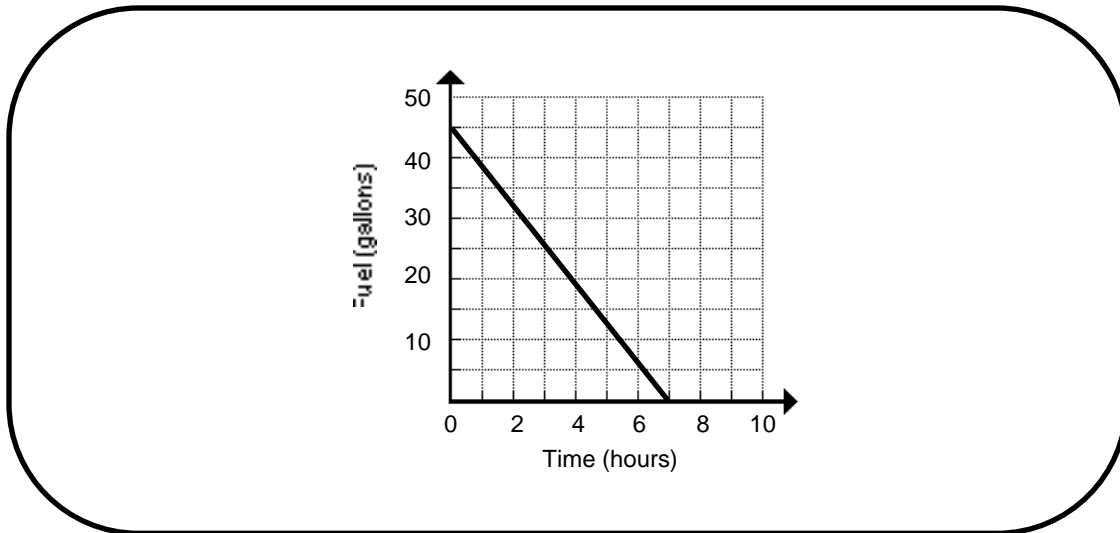
Price Charged	\$20	\$30	\$40	\$50	\$60
Daily Customers	100	70	40	20	10

2.

When businesses do market research for new products, they are interested in the relation between the prices they charge and the income they will receive from sales. For example, suppose that the owners of Video City are trying to set the best rental price for video game cartridges. Their market research staff might produce a recommendation that says: *Profit (in dollars per week) depends on charge per rental (in dollars) according to the rule*

$$P = -750 + 900C - 150C^2$$

3.



4.

In public transportation systems of most U. S. cities, the fare is generally related to distance traveled and time of day (higher in morning and evening rush hours). The fare pattern for one city's subway system is illustrated in the following table.

Distance (in miles)	1	2	3	4	5	6	7	8
Fare (in \$)	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80

5.

In the NASA explorations of space, six missions took astronauts to landings on the Moon. One other mission landed a spacecraft, but not astronauts, on Mars. The time required to reach a destination depends on the distance and the speed of the spacecraft. For example, the Moon is about 400,000 kilometers from the Earth. You can calculate time T for that trip from average speed S by the rule $T = \frac{400,000}{S}$.

6.

Number of Tickets Sold	Dollars of Income
0	0
50	125
100	250
150	375
200	500
250	625
300	750

7.

A soccer kick had height a function of time in flight that could be modeled well by the rule

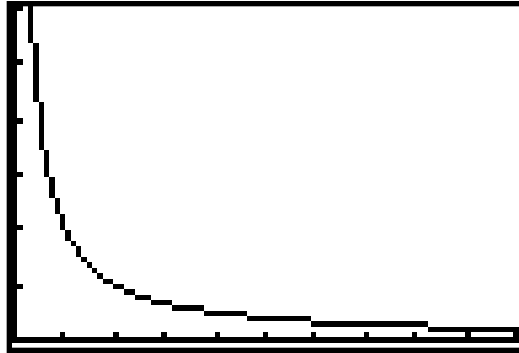
$$h(t) = -16t^2 + 50t.$$

8.

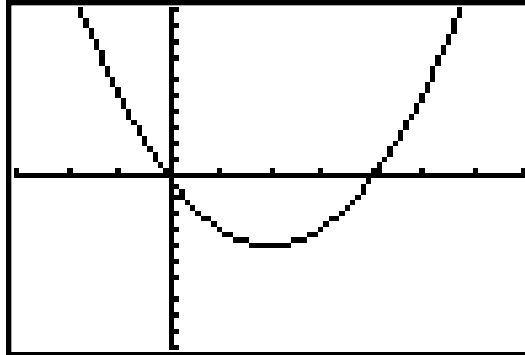
As the number of employees increases, the cost per employee decreases.

```

WINDOW FORMAT
Xmin=0
Xmax=500
Xscl=50
Ymin=0
Ymax=300
Yscl=50
  
```



9.



10.

Suppose that for a spring used in weight training equipment its length L (in feet) is related to the force pulling on it F (in pounds) by the rule $L = 2 + 0.01F$.

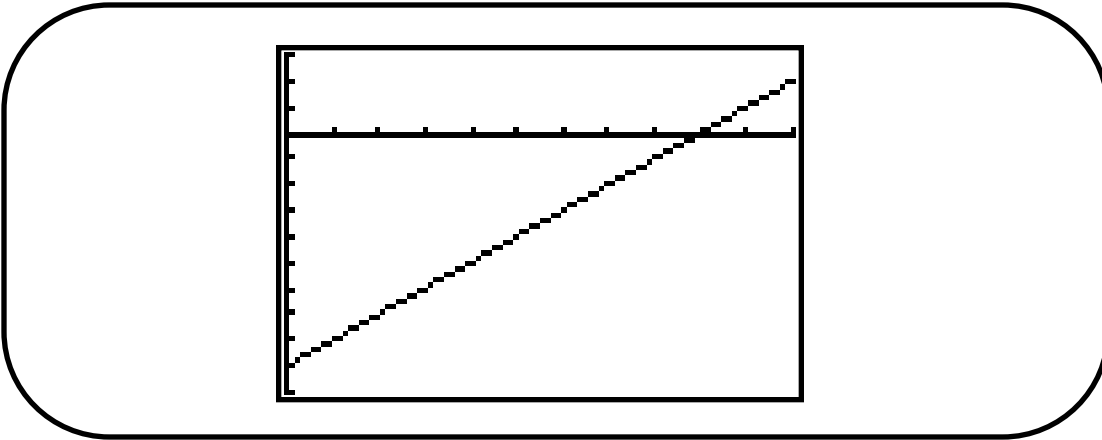
11.

The stopping distance D of a car depends on reaction distance R and braking distance B . Each of these variables is a function of the speed s of the car. You can calculate reaction distance and braking distance in *feet* by using the rules:

$$R = 1.1s \quad \text{and} \quad B = 0.05s^2$$

where s is the speed of the car in miles per hour.

12.



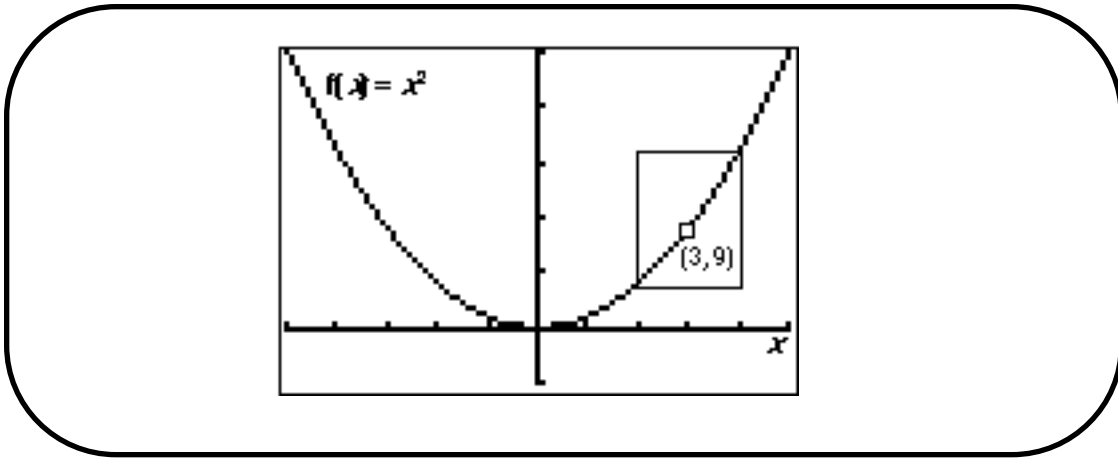
13.

The bowhead whale population is represented by the equation $y = 5700(1.03)^x$, where x represents the number of years after 1992.

14.

x	1	2	3	4	5	6	7	8
y	1	2	4	8	16	32	64	128

15.



Getting Out of Line

Selected Answers

Think About This Situation

This situation allows the students to do some thinking about projectile motion, in particular to see if they can separate the message of a graph from a picture of the phenomenon being graphed. This first question is simply an open invitation to the students to do some guessing—in part to see what the character of their thinking might be, but also to get them to experience the conjecture and test pattern that is so fundamental to science.

- a. Answers will certainly vary for this questions. The ball actually will travel a little more than 40 meters up before it starts to return.
- b. Students will probably not know that the ball will hit the ground shortly after five seconds have passed, but they should be guessing relatively small amounts of time.
- c. The actual relation is most likely to follow the third sketch. However, at this point do not “give the answer away.” Instead, let students make some intelligent guesses and defend their choices. Students will come back to these graphs in this Investigation item 7, after they have more experience with velocity.

For your information . . .

The first sketch suggests rise to a maximum height with gradually slowing speed, but then an instantaneous fall to earth.

The second sketch suggests an accelerating rise (slope gets steeper as time passes) but then a very rapid descent at first, gradually slowing as earth approaches (not the way gravity acts).

The third graph shows rapid rise at first, with gradual slowing as the ball nears its peak; then the descent begins slowly and gradually picks up speed.

The fourth graph suggests constant speed of ascent, an instantaneous stop at the peak, and a constant rate of descent.

Projectile motion problems are among the classic examples of variables related by quadratic functions. While application of that model to many situations theoretically depends on assumption of motion in a vacuum (ignoring air resistance), that assumption is not crucial for motion at modest speeds.

Table building is an extremely important skill which will be used throughout this unit. Especially important is the TI-83 capability to change TblMin and Tbl to “zoom-in” on desired values. Encourage your students to use this technique in situations such as Items 3 and 4 where times are given in the nearest tenths of seconds.

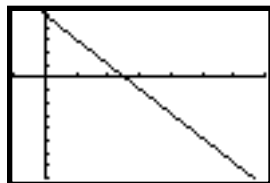
Tables can also help students choose appropriate viewing windows for graphs.

1. a.

T	V
0.0	25
0.5	20.1
1.0	15.2
1.5	10.3
2.0	5.4
2.5	0.5
3.0	-4.4
3.5	-9.3
4.0	-14.2
4.5	-19.1
5.0	-24.0
5.5	-28.9
6.0	-33.8

- $v = 20.1$ meters per second.
- $t = 1.5$ seconds; $t = 2.0$ seconds.
- After 3 seconds, the velocity is -4.4 meters per second which means that the ball is dropping toward earth with a speed of 4.4 meters per second.
- Every 0.5 seconds, the velocity decreases by 4.9 meters per second.

b.



- c. From 0 to about 2.5 seconds the ball is rising and becoming slower. Somewhere between 2.5 and 3.0 seconds the ball stops, then begins dropping and picking up speed.

2. The 25 is the initial velocity at which you threw the ball. It is also the y -intercept on the graph. The $-9.8t$ is the rate of change in the velocity. It causes the graph of the line to slope downward from left to right.

3. Encourage students to “zoom-in” on their table or graph to find times to the nearest tenth of a second.

- (a) $t = 1.5$ seconds; The ball is traveling upward with a velocity of 10 meters per second when time is 1.5 seconds.
- (b) $t = 2.6$ seconds; The ball has stopped moving (it is at the highest point) when time is about 2.6 seconds.
- (c) $v = 5.4$ meters per second; After 2 seconds the velocity is -4.4 m/s which means the ball is falling toward earth.
- (d) $t = 3.1$ seconds; The ball is falling toward earth with a velocity of -5 m/s.

Note: Once the ball hits the ground, $h = 0$, which means the ball cannot go any lower. The data points $(5.5, -10.73)$ and $(6.0, -26.4)$ in the table do not make sense in this situation. This is an excellent opportunity to discuss the need for domain restrictions with your students.

4.

t	h
0.0	0.0
0.5	11.275
1.0	20.1
1.5	26.475
2.0	30.4
2.5	31.875
3.0	30.9
3.5	27.475
4.0	21.6
4.5	13.275
5.0	2.5
5.5	-10.73
6.0	-26.4

- (a) $h = 30.9$ meters after 3 seconds; $h = 2.5$ meters after 5 seconds.
- (b) When $t = 4.1$ seconds, $h = 20.1$ meters.
- (c) Responses may vary. When $t = 2.5$ and $t = 2.6$, $h = 31.875$. This might lead some students to see that, by symmetry, the maximum height = 31.888 when $t = 2.55$ seconds.
- (d) At $t = 5.1$ seconds, the ball hits the ground because height is approximately zero (using the table or substituting in the equation gives $h = .051$). Some students may try to “zoom-in” for more decimal places. For example, when $t = 5.102$, $h = .00102$.

- (e) If the velocity equation and the height equation are entered in the $Y =$ menu, you can read from the table that the velocity is approximately -25 meters per second when $t = 3.1$ and $h = .051$.
5. The graph of (time, height) rises steeply at first, levels off at around 2.55 seconds, then begins to descend more steeply as time increases. This corresponds to a positive velocity between 0 to 2.55 seconds, and a negative velocity after 2.55 seconds.
6. At this point, students should be able to understand more fully why 1(c) is the proper choice.

The Shape of the Rules

Selected Answers

The process of finding models for patterns in data depends on the student having a repertoire of function rules that can be tried as matches to the data patterns. Thus it is helpful to explore the shape of tables and graphs for different symbolic rule patterns. This investigation is intended to begin that important agenda of discovery and, ultimately, formal algebraic study. However, the exploration is written in a way that is intended to have students use their calculators, work in groups, and pool results for conjecture about generalities. No formal naming of patterns or proof of results is intended.

When addressing this section with your class, encourage groups to divide the work among team members who would record their table and graph findings and then pool their individual results and come up with some group pattern conjectures.

Experiment 1:

In this set of 6 rules, 5 will give linear graphs and constant rates of change in the tables of values. One quadratic is thrown into the collection for contrast. While some students might notice patterns about slope or intercepts and their relation to the rule patterns for linear functions, we do not intend any formal confirmation of those conjectures in this unit. We would suggest that you suggest they make up and test some other examples with the same patterns of rules to see if the graphs and tables follow as predicted.

Experiment 2:

This set has five quadratics and one inverse variation. None will have constant rate of change in the tables or straight line graphs. The common value that holds the four parabola cases together is an exponent of two.

Experiment 3:

Five of these six have the variable in the denominator of a fraction. Those five will have hyperbolic graphs and tables that have undefined values for zeroes of the denominator and asymptotes on the x -axis $y = \frac{x}{3}$ is thrown in to show that having the variable in the denominator is significant and that $y = \frac{x}{3}$ is really the same as $y = \frac{1}{3}x$, an observation that is often very hard for algebra students to notice.

Experiment 4:

Four of the six examples in this set have the variable used as an exponent. The tables will have the classical pattern of geometric series and can be described very naturally in an iterative (NOW Æ NEXT) pattern. Though one of the exponentials is decreasing toward an x -axis asymptote, all 4 are concave up graphs. The square root rule ($y = x^{0.5}$) will be concave down and the cubic will be point symmetric around the origin and go to infinity in opposite directions at either end of the x -axis.

✓ Checkpoint

This item is designed to let students share what they have observed with the class. If anyone doubts a conjecture, they should be able to suggest an example to test it out. While some student groups might have developed conjectures that you know to be false, rather than immediately pointing out the example that defeats the conjecture, try to emphasize in all cases the fact that a few examples do not prove anything. Leave every conjecture open to future confirmation or denial. You might reproduce a list of the class's conjectures with a challenge to everyone to find counter-examples.

It is quite reasonable that you might be afraid that by not immediately pointing out the error of a conjecture you will have allowed a false conclusion to take root. Such concerns might be well-founded. However, we believe that if you keep the list of conjectures open to future challenge you will create an atmosphere that is much more genuinely like the way mathematical ideas are proposed and tested. You might return to any questionable conjectures in subsequent class meetings with the phrase, "Someone in another class suggested this example as a counter-example to the conjecture that said.... Are they right?"

On Your Own:

- Graph 1 matches rule f.
- Table 2 matches rule e.
- Table 3 matches rule b.
- Graph 4 matches rule a.

Matching Patterns

- | | |
|--------------|-----------------|
| 1. Inverse | 9. Quadratic |
| 2. Quadratic | 10. Linear |
| 3. Linear | 11. Quadratic |
| 4. Linear | 12. Linear |
| 5. Inverse | 13. Exponential |
| 6. Linear | 14. Exponential |
| 7. Quadratic | 15. Quadratic |
| 8. Inverse | |