Calculus Worksheet: Differentiation of Functions (2)

Find the first derivative of the functions

1. $y=(4x-4)^4$

$y=u^4$

$y'=4u^3\cdot u'$

$y'=4(4x-4)^3\cdot 4 = \boxed{16(4x-4)^3}$

$u=4x-4$

$u'=4$

2. $y=\cos(\sin(x)+3x)$

$y=\cos u$

$y'=-\sin u\cdot u'$

$\boxed{y'=-(\cos x+3)\cdot\sin(\sin x+3x)}$

$u=\sin x+3x$

$u'=\cos x+3$

3. $y=e^{x^2+4x}$

$y=e^u$

$y'=e^u\cdot u'$

$\boxed{y'=(2x+4)\cdot e^{x^2+4x}}$

$u=x^2+4x$

$u'=2x+4$

4. $y=\sin^{-1}(\sqrt{x}-1),\ 0\le x\le 4$

$y=\sin^{-1}u$

$y'=\dfrac{1}{\sqrt{1-u^2}}\cdot u'$

$\boxed{y'=\dfrac{1}{2\sqrt{x}\cdot\sqrt{1-(\sqrt{x}-1)^2}}}$

$u=\sqrt{x}-1$

$u'=\frac{1}{2}x^{-1/2}=\dfrac{1}{2\sqrt{x}}$

5. $y = \dfrac{2}{\sqrt{(1-x^3)^5}}$

$y = 2u^{-1/2}$ $\qquad u = (1-x^3)^5$

$y' = -u^{-3/2} \cdot u'$ $\qquad u' = 5(1-x^3)^4 \cdot -3x^2$

$u' = -15x^2(1-x^3)^4$

$y' = -\left[(1-x^3)^5\right]^{-3/2} \cdot (-15x^2(1-x^3)^4)$

$= 15x^2(1-x^3)^{-15/2} \cdot (1-x^3)^4 = 15x^2(1-x^3)^{-7/2} = \boxed{\dfrac{15x^2}{\sqrt{(1-x^3)^7}}}$

6. $y = \ln(2x^2+4)^2$

$y = \ln u$ $\qquad u = (2x^2+4)^2$

$y' = \dfrac{1}{u} \cdot u'$ $\qquad u' = 2(2x^2+4) \cdot 4x$

$u' = 8x(2x^2+4)$

$y' = \dfrac{8x(2x^2+4)}{(2x^2+4)^2}$

$y' = \dfrac{8x}{2x^2+4} = \boxed{\dfrac{4x}{x^2+2}}$

7. $y = \ln^2(2x^2+4)$

$y = u^2$ $\qquad u = \ln(2x^2+4)$

$y' = 2u \cdot u'$ $\qquad u' = \dfrac{1}{2x^2+4} \cdot 4x = \dfrac{2x}{x^2+2}$

$y' = \dfrac{2\ln(2x^2+4) \cdot 2x}{x^2+2}$

$= \boxed{\dfrac{4x \cdot \ln(2x^2+4)}{x^2+2}}$

8. $y = 3u^{\frac{1}{2}} - 2u^{\frac{1}{3}}$ with $u = 2x^3 + x$

$y' = \left(\frac{3}{2}u^{-1/2} - \frac{2}{3}u^{-2/3}\right) \cdot u'$ $\qquad u' = 6x^2 + 1$

$y' = \left(\frac{3}{2\sqrt{2x^3+x}} - \frac{2}{3\sqrt[3]{(2x^3+x)^2}}\right) \cdot (6x^2+1)$

9. $y = \sqrt[3]{(2x^2+x-1)^2 - 2}$

$y = u^{1/3}$ $\qquad u = (2x^2+x-1)^2 - 2$

$y' = \frac{1}{3}u^{-2/3} \cdot u'$ $\qquad u' = 2(2x^2+x-1) \cdot (4x+1)$

$y' = \dfrac{2(4x+1)(2x^2+x-1)}{3\sqrt[3]{\left[(2x^2+x-1)^2-2\right]^2}}$