

Find $\frac{dy}{dx}$

1) $x^2 + y^2 = 16$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

$$2y \cdot \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

2) $x^3 + y^3 = 8xy$

$$3x^2 + 3y^2 \cdot \frac{dy}{dx} = 8x \cdot \frac{dy}{dx} + 8y$$

$$3y^2 \cdot \frac{dy}{dx} - 8x \cdot \frac{dy}{dx} = 8y - 3x^2$$

$$\frac{dy}{dx} (3y^2 - 8x) = 8y - 3x^2$$

$$\boxed{\frac{dy}{dx} = \frac{8y - 3x^2}{3y^2 - 8x}}$$

3) $\frac{1}{x} + \frac{1}{y} = 1$

$$-x^{-2} - y^{-2} \cdot \frac{dy}{dx} = 0$$

$$-x^{-2} = y^{-2} \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-x^{-2}}{y^{-2}}$$

4) $\sqrt{x} + \sqrt{y} = 4$

$$\boxed{\frac{dy}{dx} = -\frac{y^2}{x^2}}$$

$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2} \cdot \frac{dy}{dx} = 0$$

$$\frac{1}{2}y^{-1/2} \frac{dy}{dx} = -\frac{1}{2}x^{-1/2}$$

$$\frac{dy}{dx} = \frac{-x^{-1/2}}{y^{-1/2}}$$

5) $x^2y^2 = x^2 + y^2$

$$\boxed{\frac{dy}{dx} = \frac{-\sqrt{y}}{\sqrt{x}}}$$

$$x^2 \cdot 2y \cdot \frac{dy}{dx} + 2xy^2 = 2x + 2y \cdot \frac{dy}{dx}$$

$$2x^2y \frac{dy}{dx} - 2y \frac{dy}{dx} = 2x - 2xy$$

$$\frac{dy}{dx} (2x^2y - 2y) = 2x - 2xy$$

6) $y = \cos(x - y)$

$$\frac{dy}{dx} = -\sin(x - y) \left(1 - \frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = -\sin(x - y) + \frac{dy}{dx} \sin(x - y)$$

$$\frac{dy}{dx} - \frac{dy}{dx} \sin(x - y) = -\sin(x - y)$$

$$\frac{dy}{dx} (1 - \sin(x - y)) = -\sin(x - y)$$

7) $\sec^2 x + \csc^2 y = 4$

$$\boxed{\frac{dy}{dx} = \frac{-\sin(x - y)}{1 - \sin(x - y)}}$$

$$= 2\sec x \cdot \sec x \cdot \tan x + 2\csc y \cdot -\csc y \cot y \cdot \frac{dy}{dx} = 0$$

$$= 2\sec^2 x \tan x = 2\csc^2 y \cot y \cdot \frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{\sec^2 x \tan x}{\csc^2 y \cot y}}$$

8) $x \sin y + y \cos x = 1$

$$x \cdot \cos y \cdot \frac{dy}{dx} + \sin y + y \cdot -\sin x + \cos x \cdot \frac{dy}{dx} = 0$$

$$x \cos y \frac{dy}{dx} + \cos x \frac{dy}{dx} = y \sin x - \sin y$$

$$\frac{dy}{dx} (x \cos y + \cos x) = y \sin x - \sin y$$

$$\boxed{\frac{dy}{dx} = \frac{y \sin x - \sin y}{x \cos y + \cos x}}$$

$$\boxed{\frac{dy}{dx} = \frac{x - xy}{x^2y - y}}$$