

Differentiate and simplify:

1) $f(x) = \ln(4 + 5x)$
 $f(x) = \ln u$
 $f'(x) = \frac{1}{u} \cdot u' = \boxed{\frac{5}{4+5x}}$

$u = 4 + 5x$
 $u' = 5$

2) $h(x) = \ln \sqrt{4 + 5x}$
 $h(x) = \ln u$
 $h'(x) = \frac{1}{u} \cdot u'$
 $= \boxed{\frac{5}{2(4+5x)}}$

$u = \sqrt{4+5x}$
 $u' = \frac{1}{2}(4+5x)^{-1/2} \cdot 5$
 $u' = \frac{5}{2\sqrt{4+5x}}$

3) $f(t) = \ln(3t + 1)^2$
 $f(t) = \ln u$
 $f'(t) = \frac{1}{u} \cdot u'$
 $= \boxed{\frac{6}{3t+1}}$

$u = (3t+1)^2$
 $u' = 2(3t+1) \cdot 3$
 $u' = 6(3t+1)$

4) $g(t) = \ln^2(3t + 1)$
 $g(t) = u^2$
 $g'(t) = 2u \cdot u'$
 $g'(t) = \boxed{\frac{6 \ln(3t+1)}{3t+1}}$

$u = \ln(3t+1)$
 $u' = \frac{3}{3t+1}$

5) $f(x) = \ln(\sqrt[3]{4-x^2})$
 $f(x) = \ln u$
 $f'(x) = \frac{1}{u} \cdot u'$
 $= \frac{1}{\sqrt[3]{4-x^2}} \cdot \frac{-2x}{3\sqrt[3]{(4-x^2)^2}}$

$u = (4-x^2)^{1/3}$
 $u' = \frac{1}{3}(4-x^2)^{-2/3} \cdot -2x$
 $u' = \frac{-2x}{3\sqrt[3]{(4-x^2)^2}}$

6) $f(y) = \ln(\sin 5y)$
 $f(y) = \ln u$
 $f'(y) = \frac{1}{u} \cdot u'$
 $= \frac{5 \cos 5y}{\sin 5y}$ or $\boxed{5 \cot 5y}$

$u = \sin 5y$
 $u' = 5 \cos 5y$

$\frac{-2x}{3\sqrt[3]{(4-x^2)^3}}$ or $\frac{-2x}{3(4-x^2)}$

7) $f(x) = \cos(\ln x)$
 $f(x) = \cos u$
 $f'(x) = -\sin u \cdot u'$
 $= \boxed{\frac{-\sin(\ln x)}{x}}$

$u = \ln x$
 $u' = \frac{1}{x}$

8) $G(x) = \ln(\sec 2x + \tan 2x)$
 $G(x) = \ln u$
 $G'(x) = \frac{1}{u} \cdot u'$
 $= \frac{2 \sec 2x (\tan 2x + \sec 2x)}{\sec 2x + \tan 2x} = \boxed{2 \sec 2x}$

$u = \sec 2x + \tan 2x$
 $u' = 2 \sec 2x \tan 2x + 2 \sec^2 2x$

9) $f(x) = \ln \sqrt{\tan x}$
 $f(x) = \ln u$
 $f'(x) = \frac{1}{u} \cdot u'$
 $= \frac{\sec^2 x}{2 \tan x}$ or $\frac{1}{2 \sin x \cos x}$
 $u' = \frac{\sec^2 x}{2 \sqrt{\tan x}}$

$u = \sqrt{\tan x}$
 $u' = \frac{1}{2}(\tan x)^{-1/2} \cdot \sec^2 x$

10) $f(w) = \ln(3w + 1/2w - 5)$
 $f(w) = \ln u$
 $f'(w) = \frac{1}{u} \cdot u'$
 $= \frac{3 - 5}{3w+1} \cdot \frac{-17}{(2w-5)^2} = \frac{-17}{(3w+1)(2w-5)^2}$

$u = \frac{3w+1}{2w-5}$
 $u' = \frac{3(2w-5) - 2(3w+1)}{(2w-5)^2}$

11) $f(x) = x/\ln x$ $\frac{x}{\ln x} \rightarrow$ Quotient Rule
 $f'(x) = \frac{\ln x - (x \cdot \frac{1}{x})}{\ln^2 x}$
 $= \boxed{\frac{\ln x - 1}{\ln^2 x}}$