

Solve:

- 1) Find the intervals on which $w(x) = -2x^2 - 2x - 4$ is increasing or decreasing.

$$w'(x) = -4x - 2$$

$$-4x - 2 = 0$$

$$x = -\frac{1}{2}$$

$(-\infty, -\frac{1}{2}]$	$[-\frac{1}{2}, \infty)$
$f' > 0$	$f' < 0$

inc.: $(-\infty, -\frac{1}{2}]$
 dec.: $[-\frac{1}{2}, \infty)$

- Find the intervals on which $a(x) = x^2 - 4x + 2$ is increasing or decreasing.

$$a'(x) = 2x - 4$$

$$2x - 4 = 0$$

$$x = 2$$

$(-\infty, 2]$	$[2, \infty)$
$f' < 0$	$f' > 0$

inc.: $[2, \infty)$
 dec.: $(-\infty, 2]$

- 3) Find the intervals on which $a(x) = x^3 + 3x^2 - 9x + 3$ is increasing or decreasing.

$$a'(x) = 3x^2 + 6x - 9$$

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

$$x = -3, 1$$

$(-\infty, -3]$	$[-3, 1]$	$[1, \infty)$
$f' > 0$	$f' < 0$	$f' > 0$

inc.: $(-\infty, -3] \cup [1, \infty)$
 dec.: $[-3, 1]$

- 4) Apply the first derivative test on $c(x) = x^2 - 4x$ to find all local maximum and minimums.

$$c'(x) = 2x - 4$$

$$2x - 4 = 0$$

$$x = 2$$

$(-\infty, 2]$	$[2, \infty)$
$f' < 0$	$f' > 0$

min.: $(2, -4)$

- 5) Apply the first derivative test on $c(x) = 4x^2 - 2x - 2$ to find all local maximum and minimums.

$$c'(x) = 8x - 2$$

$$8x - 2 = 0$$

$$x = \frac{1}{4}$$

$(-\infty, \frac{1}{4}]$	$[\frac{1}{4}, \infty)$
$f' < 0$	$f' > 0$

min.: $(\frac{1}{4}, -\frac{9}{4})$

- 6) Apply the first derivative test on $h(x) = -x^3 + 9x^2 - 27x + 3$ to find all local maximum and minimums.

$$h'(x) = -3x^2 + 18x - 27$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)(x-3) = 0$$

$$x = 3$$

$(-\infty, 3]$	$[3, \infty)$
$f' < 0$	$f' < 0$

none

- 7) Starting at a height of 0 feet, a ball is thrown upwards with an initial velocity of 128 feet/sec. The ball's height after t seconds is $s(t) = -16x^2 + 128x$. Find the time when the ball is at its maximum height.

(when $v(t) = 0$)

$$v(t) = -32x + 128$$

$$-32x + 128 = 0$$

$$x = 4 \text{ sec.}$$

- 8) Starting at a height of 2 feet, a ball is thrown upwards with an initial velocity of 96 feet/sec. The ball's height after t seconds is $s(t) = -16x^2 + 96x + 2$. Find the maximum height of the ball.

$$v(t) = -32x + 96$$

$$-32x + 96 = 0$$

$$x = 3 \text{ sec.}$$

$$s(3) = 146 \text{ ft.}$$

- 9) Find the intervals on which $h(x) = x^3 - 6x^2 + 9x - 2$ is concave up or down.

- 10) Find the intervals on which $k(x) = -5x^3 - x^2 + 4x - 4$ is concave up or down.

*Work on next page.

$$h'(x) = 3x^2 - 12x + 9$$

$$h''(x) = 6x - 12$$

$$6x - 12 = 0$$

$$x = 2$$

$$\text{C.U.: } [2, \infty)$$

$$\text{C.D.: } (-\infty, 2]$$

$(-\infty, 2]$	$[2, \infty)$
$f'' < 0$	$f'' > 0$

11) Find the intervals on which

$y(x) = x^3 - 12x^2 + 45x - 1$ is concave up or down.

$$y'(x) = 3x^2 - 24x + 45$$

$$y''(x) = 6x - 24$$

$$6x - 24 = 0$$

$$x = 4$$

$(-\infty, 4]$	$[4, \infty)$
$f'' < 0$	$f'' > 0$

$$\text{C.U.: } [4, \infty)$$

$$\text{C.D.: } (-\infty, 4]$$

13) Find all inflection points of

$$m(x) = x^4 - 2x^3 - 5x - 3.$$

$$m'(x) = 4x^3 - 6x^2 - 5$$

$$m''(x) = 12x^2 - 12x$$

$$12x^2 - 12x = 0$$

$$12x(x - 1) = 0$$

$$x = 0, 1$$

$(-\infty, 0]$	$[0, 1]$	$[1, \infty)$
$f'' > 0$	$f'' < 0$	$f'' > 0$

$$(0, -3) + (1, -9)$$

15) Apply the second derivative test on

$w(x) = x^3 - 3x^2 - 24x + 3$ to find all local maximum and minimums.

$$w'(x) = 3x^2 - 6x - 24$$

$$w''(x) = 6x - 6$$

$$x^2 - 2x - 8 = 0$$

$$(x - 4)(x + 2) = 0$$

$$x = 4, -2$$

$$w''(4) = 18$$

$$w''(-2) = -18$$

$$\text{min.: } (4, -77)$$

$$\text{max.: } (-2, 31)$$

$$k'(x) = -15x^2 - 2x + 4$$

$$k''(x) = -30x - 2$$

$$-30x - 2 = 0$$

$$x = -\frac{1}{15}$$

$(-\infty, -\frac{1}{15}]$	$[-\frac{1}{15}, \infty)$
$f'' > 0$	$f'' < 0$

$$\text{C.U.: } (-\infty, -\frac{1}{15}]$$

$$\text{C.D.: } [-\frac{1}{15}, \infty)$$

12) Find all inflection points of

$$y(x) = 3x^4 - 5x^3 + 4x - 2$$

$$y'(x) = 12x^3 - 15x^2 + 4$$

$$y''(x) = 36x^2 - 30x$$

$$36x^2 - 30x = 0$$

$$6x(6x - 5) = 0$$

$$x = 0, \frac{5}{6}$$

$(-\infty, 0]$	$[0, \frac{5}{6}]$	$[\frac{5}{6}, \infty)$
$f'' > 0$	$f'' < 0$	$f'' > 0$

$$(0, -2) + (\frac{5}{6}, -\frac{49}{72})$$

14) Find all inflection points of

$$r(x) = 3x^5 - 5x^4 + 3x - 3.$$

$$r'(x) = 15x^4 - 20x^3 + 3$$

$(-\infty, 0]$	$[0, 1]$	$[1, \infty)$
$f'' < 0$	$f'' < 0$	$f'' > 0$

$$(1, -2)$$

$$r''(x) = 60x^3 - 60x^2$$

$$60x^3 - 60x^2 = 0$$

$$60x^2(x - 1) = 0$$

$$x = 0, 1$$

16) Apply the second derivative test on

$u(x) = -x^2 + 4x + 4$ to find all local maximum and minimums.

$$u'(x) = -2x + 4$$

$$u''(x) = -2$$

$$-2x + 4 = 0$$

$$x = 2$$

$$u''(2) = -2$$

$$\text{max.: } (2, 8)$$

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