

## Solve:

1)  $\frac{dy}{dx} = \csc^2(2x)$

subject to the initial condition  $y(\pi/8) = -1$ .

$$\int \csc^2(2x) dx \quad u=2x \quad y = -\frac{1}{2} \cot(2x) + C$$

$$\int \csc^2 u \cdot \frac{du}{2} \quad \frac{du}{dx} = 2 \quad -1 = -\frac{1}{2} \cot\left(\frac{\pi}{4}\right) + C$$

$$\frac{1}{2} \int \csc^2 u du \quad dx = \frac{du}{2} \quad -1 = -\frac{1}{2} \cdot 1 + C$$

$$y = -\frac{1}{2} \cot(2x) + C \quad C = -\frac{1}{2}$$

3)  $\frac{dy}{dx} = -3\sec^2(3x)$

$$y = -\frac{1}{2} \cot(2x) - \frac{1}{2}$$

subject to the initial condition  $y(\pi/12) = 0$ .

$$-3 \int \sec^2(3x) dx \quad u=3x$$

$$-3 \int \sec^2 u \cdot \frac{du}{3} \quad \frac{du}{dx} = 3$$

$$- \int \sec^2 u du \quad dx = \frac{du}{3}$$

$$y = -\tan(3x) + C \quad 0 = -\tan\left(\frac{\pi}{4}\right) + C$$

$$0 = -1 + C \quad C = 1$$

5)  $\frac{dy}{dx} = 2\cos(x)$

$$y = -\tan(3x) + 1$$

subject to the initial condition  $y(\pi/2) = 3$ .

$$2 \int \cos x dx \quad 3 = 2 \sin\left(\frac{\pi}{2}\right) + C$$

$$y = 2 \sin x + C \quad 3 = 2 \cdot 1 + C$$

$$C = 1$$

$$y = 2 \sin x + 1$$

7) The velocity of a particle moving along the x-axis is given by  $v(t) = 2t^2 + 3$ . Find its position as a function of  $t$  if  $s(0) = -2$ .

$$s(t) = \int 2t^2 + 3 dt = \frac{2}{3} t^3 + 3t + C$$

$$-2 = 0 + C \quad C = -2$$

$$s(t) = \frac{2}{3} t^3 + 3t - 2$$

8) A particle moving along a straight line has an acceleration  $a = 4t$ . When  $t = 0$ , the velocity  $v$  of the particle and its position  $s$  are respectively equal to 3 and 4. Find its position  $s$  as a function of time  $t$ .

$$v(t) = \int 4t dt = 2t^2 + C \quad 3 = 0 + C \quad C = 3 \quad v(t) = 2t^2 + 3$$

$$s(t) = \int 2t^2 + 3 dt = \frac{2}{3} t^3 + 3t + C \quad 4 = 0 + C \quad C = 4$$

$$s(t) = \frac{2}{3} t^3 + 3t + 4$$

2)  $\frac{dy}{dx} = -\sec^2(-3x)$

subject to the initial condition  $y(-\pi/12) = 2$ .

$$\int -\sec^2(-3x) dx \quad u = -3x$$

$$- \int \sec^2 u \cdot \frac{-du}{3} \quad \frac{du}{dx} = -3$$

$$\frac{1}{3} \int \sec^2 u du \quad dx = -\frac{du}{3}$$

$$y = \frac{1}{3} \tan(-3x) + C \quad 2 = \frac{1}{3} \tan\left(\frac{\pi}{4}\right) + C$$

$$y = \frac{1}{3} \tan(-3x) + \frac{5}{3} \quad 2 = \frac{1}{3} \cdot 1 + C$$

$$C = \frac{5}{3}$$

Solve  $\frac{dy}{dx} = -5x^2 + x - 4$

subject to the initial condition  $y(1) = -3$ .

$$\int -5x^2 + x - 4 dx$$

$$y = -\frac{5}{3} x^3 + \frac{x^2}{2} - 4x + C$$

$$-3 = -\frac{5}{3} + \frac{1}{2} - 4 + C$$

$$C = \frac{13}{6}$$

$$y = -\frac{5}{3} x^3 + \frac{x^2}{2} - 4x + \frac{13}{6}$$

6)  $\frac{dy}{dx} = -4x^2 - 4x - 2$

subject to the initial condition  $y(0) = 3$ .

$$\int -4x^2 - 4x - 2 dx$$

$$y = -\frac{4}{3} x^3 - 2x^2 - 2x + C$$

$$3 = 0 + C \quad C = 3$$

$$y = -\frac{4}{3} x^3 - 2x^2 - 2x + 3$$