

Evaluate:

$$1) \int_0^3 (3x^2 - 4x + 1) dx$$

$$= x^3 - 2x^2 + x \Big|_0^3 = 12 - 0 = \boxed{12}$$

$$2) \int_3^6 (x^2 - 2x) dx$$

$$= \frac{x^3}{3} - x^2 \Big|_3^6 = 36 - 0 = \boxed{36}$$

$$3) \int_1^2 \left(\frac{x^2+1}{x^2}\right) dx = \int_1^2 (1 + x^{-2}) dx$$

$$= x - \frac{1}{x} \Big|_1^2 = \frac{3}{2} - 0 = \boxed{\frac{3}{2}}$$

$$4) \int_0^1 \left(\frac{z}{(z^2+1)^3}\right) dz$$

$$\int_0^1 z \cdot u^{-3} \cdot \frac{du}{2z}$$

$$\frac{1}{2} \int_0^1 u^{-3} du = -\frac{1}{4} u^{-2} \Big|_0^1$$

$$u = z^2 + 1$$

$$\frac{du}{dz} = 2z$$

$$dz = \frac{du}{2z}$$

$$-\frac{1}{4(z^2+1)^2} \Big|_0^1 = -\frac{1}{16} + \frac{1}{4} = \boxed{\frac{3}{16}}$$

$$5) \int_1^{10} \sqrt{5x-1} dx$$

$$\int_1^{10} u^{1/2} \cdot \frac{du}{5}$$

$$\frac{1}{5} \int_1^{10} u^{1/2} du = \frac{2}{15} u^{3/2} \Big|_1^{10}$$

$$u = 5x - 1$$

$$\frac{du}{dx} = 5$$

$$dx = \frac{du}{5}$$

$$\frac{2}{15} \sqrt{(5x-1)^3} \Big|_1^{10} = \frac{686}{15} - \frac{16}{15} = \boxed{\frac{134}{3}}$$

$$6) \int_{-2}^0 3w\sqrt{4-w^2} dw$$

$$3 \int_{-2}^0 u^{1/2} \cdot \frac{-du}{2w}$$

$$\frac{3}{2} \int_{-2}^0 u^{1/2} du = u^{3/2} \Big|_{-2}^0$$

$$u = 4 - w^2$$

$$\frac{du}{dw} = -2w$$

$$dw = \frac{-du}{2w}$$

$$\sqrt{(4-w^2)^3} \Big|_{-2}^0 = 8 - 0 = \boxed{8}$$

$$7) \int_0^{\pi/2} \sin 2x dx$$

$$\int_0^{\pi/2} \sin u \cdot \frac{du}{2}$$

$$\frac{1}{2} \int_0^{\pi/2} \sin u du = -\frac{1}{2} \cos u \Big|_0^{\pi/2}$$

$$u = 2x$$

$$\frac{du}{dx} = 2$$

$$dx = \frac{du}{2}$$

$$-\frac{1}{2} \cos(2x) \Big|_0^{\pi/2} = \frac{1}{2} + \frac{1}{2} = \boxed{1}$$

8) $\int_1^2 t^2 \sqrt{t^3+1} dt$

$$\int_1^2 t^2 u^{1/2} \cdot \frac{du}{3t^2}$$

$$\frac{1}{3} \int_1^2 u^{1/2} du = \frac{2}{9} u^{3/2} \Big|_1^2$$

$$u = t^3 + 1$$

$$\frac{du}{dt} = 3t^2$$

$$dt = \frac{du}{3t^2}$$

$$\frac{2}{9} \sqrt{(t^3+1)^3} \Big|_1^2 = \boxed{6 - \frac{4\sqrt{2}}{9}}$$

9) $\int_0^1 \frac{y^2+2y}{\sqrt[3]{y^3+3y^2+4}} dy$

$$\int_0^1 (y^2+2y) \cdot u^{-1/3} \cdot \frac{du}{3(y^2+2y)}$$

$$\frac{1}{3} \int_0^1 u^{-1/3} du$$

$$= \frac{1}{2} u^{2/3} \Big|_0^1$$

$$= \frac{1}{2} \sqrt[3]{(y^3+3y^2+4)^2} \Big|_0^1 = \boxed{2 - \frac{\sqrt[3]{16}}{2}}$$

$$u = y^3 + 3y^2 + 4$$

$$\frac{du}{dy} = 3y^2 + 6y$$

$$dy = \frac{du}{3(y^2+2y)}$$