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Calculus – Integration by Substitution

Name: ____________ **Date:** ________ **Period:** ______ **Score:** ______

Evaluate the indefinite integral. Check your result by differentiation.

1) $\int \sqrt{1-4y}\, dy =$

$u = 1-4y$, $\frac{du}{dy} = -4$, $dy = \frac{-du}{4}$

$\int u^{1/2} \cdot \frac{-du}{4}$

$-\frac{1}{4}\int u^{1/2} du$

$= -\frac{1}{6} u^{3/2} + C = \boxed{-\frac{1}{6}\sqrt{(1-4y)^3} + C}$

2) $\int x^3\sqrt[3]{x^2-9}\, dx =$

$u = x^2-9$, $\frac{du}{dx} = 2x$, $dx = \frac{du}{2x}$

$\int x (u^{1/3}) \cdot \frac{du}{2x}$

$\frac{1}{2}\int u^{1/3} du$

$= \frac{3}{8} u^{4/3} + C = \boxed{\frac{3}{8}\sqrt[3]{(x^2-9)^4} + C}$

3) $\int x^2(x^3-1)^{10}\, dx =$

$u = x^3-1$, $\frac{du}{dx} = 3x^2$, $dx = \frac{du}{3x^2}$

$\int x^2 u^{10} \cdot \frac{du}{3x^2}$

$\frac{1}{3}\int u^{10} du$

$= \frac{1}{33} u^{11} + C = \boxed{\frac{1}{33}(x^3-1)^{11} + C}$

4) $\int \frac{y^3}{(1-2y^4)^5}\, dy =$

$u = 1-2y^4$, $\frac{du}{dy} = -8y^3$, $dy = \frac{-du}{8y^3}$

$= \int y^3 u^{-5} \cdot \frac{-du}{8y^3}$

$= -\frac{1}{8}\int u^{-5} du$

$= \frac{1}{32} u^{-4} + C = \boxed{\frac{1}{32(1-2y^4)^4} + C}$

5) $\int (x-2)(x^2-4x+4)^{4/3}\, dx =$

$u = x^2-4x+4$, $\frac{du}{dx} = 2x-4$, $dx = \frac{du}{2(x-2)}$

$\int (x-2) u^{4/3} \cdot \frac{du}{2(x-2)}$

$= \frac{1}{2}\int u^{4/3} du$

$= \frac{3}{14} u^{7/3} + C$

$= \boxed{\frac{3}{14}(x^2-4x+4)^{7/3} + C}$

6) $\int x\sqrt{3-2x^2}\, dx =$

$u = 3-2x^2$, $\frac{du}{dx} = -4x$, $dx = \frac{-du}{4x}$

$\int x \cdot u^{1/2} \cdot \frac{-du}{4x}$

$-\frac{1}{4}\int u^{1/2} du$

$= -\frac{1}{6} u^{3/2} + C = \boxed{-\frac{1}{6}\sqrt{(3-2x^2)^3} + C}$

7) $\int \cos 4\theta\, d\theta =$

$u = 4\theta$, $\frac{du}{d\theta} = 4$, $d\theta = \frac{du}{4}$

$\int \cos u \cdot \frac{du}{4}$

$\frac{1}{4}\int \cos u\, du$

$= \frac{1}{4}\sin u + C$

$= \boxed{\frac{1}{4}\sin 4\theta + C}$

8) $\int 6x^2 \sin x^3\, dx =$

$u = x^3$, $\frac{du}{dx} = 3x^2$, $dx = \frac{du}{3x^2}$

$6\int x^2 \sin u \cdot \frac{du}{3x^2}$

$2\int \sin u\, du$

$= -2\cos u + C$

$= \boxed{-2\cos x^3 + C}$

9) $\int \sec^2 5x\, dx =$

$u = 5x$, $\frac{du}{dx} = 5$, $dx = \frac{du}{5}$

$\int \sec^2 u \cdot \frac{du}{5}$

$\frac{1}{5}\int \sec^2 u\, du$

$= \frac{1}{5}\tan u + C$

$= \boxed{\frac{1}{5}\tan 5x + C}$

10) $\int \frac{\sin x}{\cos^2 x} dx$

$u = \cos x$, $\frac{du}{dx} = -\sin x$, $dx = \frac{-du}{\sin x}$

$\int \sin x \cdot u^{-2} \cdot \frac{-du}{\sin x}$

$-\int u^{-2} du$

$= u^{-1} + C$

$= \boxed{\frac{1}{\cos x} + C}$

11) $\int y \csc 3y^2 \cot 3y^2 dy =$

$u = 3y^2$

$\frac{du}{dy} = 6y$

$dy = \frac{du}{6y}$

$\int y \csc u \cot u \cdot \frac{du}{6y}$

$\frac{1}{6}\int \csc u \cot u \, du$

$= -\frac{1}{6}\csc u + C$

$= \boxed{-\frac{1}{6}\csc 3y^2 + C}$

12) $\int \cos x(2+\sin x)^5 dx =$

$u = 2 + \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$\int \cos x \, u^5 \cdot \frac{du}{\cos x}$

$\int u^5 du$

$= \frac{1}{6}u^6 + C = \boxed{\frac{1}{6}(2+\sin x)^6 + C}$

13) $\int \sqrt{1+\frac{1}{3x}}\frac{dx}{x^2} =$

$u = 1 + (3x)^{-1}$

$\frac{du}{dx} = -(3x)^{-2}\cdot 3$

$\frac{du}{dx} = -\frac{3}{3x^2}$

$-3dx = 3x^2 du$

$dx = -x^2 du$

$\int \frac{u^{1/2}}{x^2} \cdot -x^2 du$

$-\int u^{1/2} du$

$= -\frac{2}{3}u^{3/2} + C$

$\boxed{-\frac{2}{3}\sqrt{\left(1+\frac{1}{3x}\right)^3} + C}$

14) $\int 2\sin x \sqrt[3]{1+\cos x} \, dx =$

$u = 1 + \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{-du}{\sin x}$

$2\int \sin x \, u^{1/3} \cdot \frac{-du}{\sin x}$

$-2\int u^{1/3} du$

$= -\frac{3}{2}u^{4/3} + C = \boxed{-\frac{3}{2}\sqrt[3]{(1+\cos x)^4} + C}$

15) $\int \cos^2 t \sin t \, dt =$

$u = \cos t$

$\frac{du}{dt} = -\sin t$

$dt = \frac{-du}{\sin t}$

$\int u^2 \cdot \sin t \cdot \frac{-du}{\sin t}$

$-\int u^2 du$

$= -\frac{1}{3}u^3 + C$

$= \boxed{-\frac{1}{3}\cos^3 t + C}$