Solve:

1) Solve the differential equation: $y' = 3x^2y^2$.

*Substitute $\frac{dy}{dx}$ for y'.

$\frac{dy}{dx} = 3x^2y^2$

$\int \frac{1}{y^2}\,dy = \int 3x^2\,dx$

$-y^{-1} = x^3 + C$

$-\frac{1}{y} = x^3 + C$

$y = \frac{-1}{x^3 + C}$

2) Solve the differential equation: $y' = (e^x)y^2$.

$\frac{dy}{dx} = e^x \cdot y^2$

$\int \frac{1}{y^2}\,dy = \int e^x\,dx$

$-y^{-1} = e^x + C$

$-\frac{1}{y} = e^x + C$

$y = \frac{-1}{e^x + C}$

3) Solve the differential equation: $y' = (\sin x)y^2$.

$\frac{dy}{dx} = \sin x \cdot y^2$

$\int \frac{1}{y^2}\,dy = \int \sin x\,dx$

$-y^{-1} = -\cos x + C$

$-\frac{1}{y} = -\cos x + C$

$y = \frac{-1}{-\cos x + C}$

or $y = \frac{1}{\cos x + C}$

4) Solve the differential equation: $\frac{dy}{dx} = -2xy^2$.

$\int \frac{1}{y^2}\,dy = \int -2x\,dx$

$-y^{-1} = -x^2 + C$

$-\frac{1}{y} = -x^2 + C$

$y = \frac{-1}{-x^2 + C}$ or $y = \frac{1}{x^2 + C}$

5) Solve the differential equation: $y' = 3xy$.

$\frac{dy}{dx} = 3xy$

$\int \frac{1}{y}\,dy = \int 3x\,dx$

$\ln y = \frac{3}{2}x^2 + C$

$y = Ce^{3/2x^2}$

6) A bacteria grows at a constant proportional rate of 0.3 members per member per year. How long will it take for the number of members to double? ($k = 0.3$)

$2y_0 = y_0 e^{.03t}$

$2 = e^{.03t}$

$\ln 2 = .03t$

$t = \frac{\ln 2}{.03} =$ **4.2 years**

7) A bacteria culture initally contains 400 members and grows at a rate proportional to its size. One year later, it contains 1400 members. Find the number of bacteria after 3 years.

$1400 = 400e^{k \cdot 1}$

$3.5 = e^k$

$\ln 3.5 = k$

$k = 1.253$

$A = 400e^{1.253 \cdot 3}$

17,162.200

8) The half life of Rutherfordium is 13.1 hours. We began with a 1000 mg sample, and now we have a 300 mg sample. Find the amount of time that has past.

$13.1 = \frac{\ln 2}{k}$

$k = \frac{\ln 2}{13.1} = .053$

$300 = 1000e^{-.053 \cdot t}$

$.30 = e^{-.053t}$

$\ln .30 = -.053t$

$t = \frac{\ln .30}{-.053} =$ **22.7 hours**

9) The half life of Dubnium is 5.56 hours. We began with a 900 mg sample, and now we have a 400 mg sample. Find the amount of time that has past.

$5.56 = \frac{\ln 2}{k}$

$k = \frac{\ln 2}{5.56} = .125$

$400 = 900e^{-.125t}$

$.444 = e^{-.125t}$

$\ln .444 = -.125t$

$t = \frac{\ln .444}{-.125} =$ **6.5 hours**

10) The half life of Ununquadium is 1.33 years. If we start with 200 mg, find the amount that will be left after 3 years.

$1.33 = \frac{\ln 2}{k}$

$k = \frac{\ln 2}{1.33} = .521$

$A = 200e^{-.521 \cdot 3}$

$=$ **41.901 mg**