Total Area vs. Displacement

1. A particle's velocity function is given by $v(t) = 3x^2 - 2x - 1$. If $s(0) = -2$, find:

   a. The particle's displacement from t = 0 to t = 3 sec.

   $$\int_0^3 3x^2 - 2x - 1\,dx = x^3 - x^2 - x\Big|_0^3 = 15 - 0 = \boxed{15}$$

   b. The particle's position at t = 3 sec.

   $$-2 + 15 = \boxed{13}$$

   c. The total distance traveled by the particle from t = 0 to t = 3 sec.

   $$\left|\int_0^1 3x^2 - 2x - 1\,dx\right| + \left|\int_1^3 3x^2 - 2x - 1\,dx\right|$$

   $$= \left|x^3 - x^2 - x\Big|_0^1\right| \qquad \left|x^3 - x^2 - x\Big|_1^3\right| \qquad 1 + 16 = \boxed{17}$$

   $$= |-1| = 1 \quad + \quad |15 + 1| = 16$$

2. For the function $f(x) = 2x^2 - x - 6$, find:

   a. The net area under the curve from x = -3 to x = 0.

   $$\int_{-3}^0 2x^2 - x - 6\,dx = \frac{2}{3}x^3 - \frac{x^2}{2} - 6x\Big|_{-3}^0 = 0 + 4.5 = \boxed{4.5}$$

   b. The total area under the curve from x = -3 to x = 0.

   $$\left|\int_{-3}^{-1.5} 2x^2 - x - 6\,dx\right| + \left|\int_{-1.5}^0 2x^2 - x - 6\,dx\right| \qquad 10.125 + 5.625$$

   $$\left|\frac{2}{3}x^3 - \frac{x^2}{2} - 6x\Big|_{-3}^{-1.5}\right| + \left|\frac{2}{3}x^3 - \frac{x^2}{2} - 6x\Big|_{-1.5}^{0}\right| \qquad = \boxed{15.75}$$

   $$= |5.625 + 4.5| = 10.125 \qquad |0 - 5.625| = 5.625$$

   c. Find the average value of the function from x = -3 to x = 0.

   $$\frac{1}{3}\int_{-3}^0 2x^2 - x - 6\,dx = \boxed{1.5}$$

3. For f(x) = $(e^{x^2-1}) - 5$, find:

a. The net area from x = 0 to x = e

*Use Calculator (NINT)

$$\int_0^e e^{x^2-1} - 5\,dx = \boxed{105.880}$$

b. The total area from x = 0 to x = e.

$$\int_0^e \left| e^{x^2-1} - 5 \right| dx = \boxed{118.078}$$

4. A particles velocity function is given by v(t) = 6t – 7. If s(0) = 5, find:

a. The particle's displacement from t = 0 to t = 2 sec.

$$\int_0^2 6t - 7\,dt = 3t^2 - 7t \Big|_0^2 = -2 - 0 = \boxed{-2}$$

b. The particle's position at t = 2 sec.

$$5 - 2 = \boxed{3}$$

c. The total distance traveled by the particle from t = 0 to t = 2 sec.

$$\left| \int_0^{7/6} 6t - 7\,dt \right| + \left| \int_{7/6}^{2} 6t - 7\,dt \right| \qquad \frac{49}{12} + \frac{25}{12} = \boxed{\frac{37}{6}}$$

$3t^2 - 7t$ $\quad$ $3t^2 - 7t$

$49/12$ $\quad$ $-2 + 49/12 = 25/12$

d. Find the average velocity of the particle from t = 0 to t = 4 sec.

$$s(t) = \int 6t - 7\,dt = 3t^2 - 7t + C$$

$$5 = 0 + C \qquad s(t) = 3t^2 - 7t + 5$$

$$C = 5$$

$$\frac{s(4) - s(0)}{4 - 0} = \frac{28 - 5}{4} = \frac{23}{4} \text{ or } \boxed{5.75}$$

e. Find the average acceleration of the particle from t = 0 to t = 4 sec.

$$\frac{v(4) - v(0)}{4 - 0} = \frac{17 + 7}{4} = \boxed{6}$$