

Find the volume of the solid generated by revolving the given region about the given axis. Use a piece of graph paper to graph each region.

1. The region above the curve $y = x^3$, under the line $y = 1$ and between $x = 0$ and $x = 1$ about the x-axis.

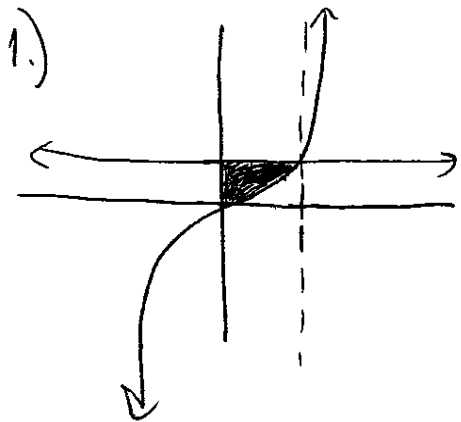
2. Revolve the above region about the y-axis.

3. The region below the line $y = 2x$, above the x-axis, and between $x = 0$ and $x = 1$ about the x-axis.

4. Revolve the above region about the y-axis.

5. The region between the parabolas $y = x^2$ and $x = y^2$ about the x-axis.

6. Revolve the above region about the y-axis.



$$V = \pi \int_0^1 (1^2 - (x^3)^2) dx = \pi \int_0^1 (1 - x^6) dx$$

$$\pi \left(x - \frac{x^7}{7} \Big|_0^1 \right) = \boxed{\frac{6\pi}{7}}$$

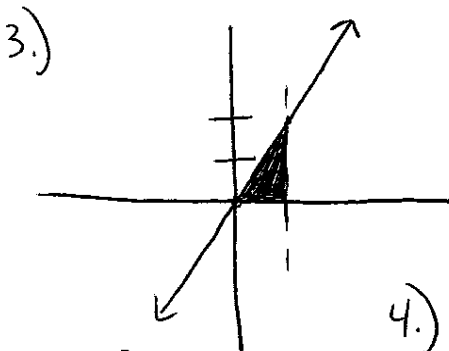
2.)

$$V = \pi \int_0^1 (y^{1/3})^2 dy = \pi \int_0^1 y^{2/3} dy$$

$$y = x^3$$

$$x = \sqrt[3]{y}$$

$$\pi \left(\frac{3}{5} y^{5/3} \Big|_0^1 \right) = \boxed{\frac{3\pi}{5}}$$



$$V = \pi \int_0^1 (2x)^2 dx = \pi \int_0^1 4x^2 dx$$

$$\pi \left(\frac{4}{3} x^3 \Big|_0^1 \right) = \boxed{\frac{4\pi}{3}}$$

4.)

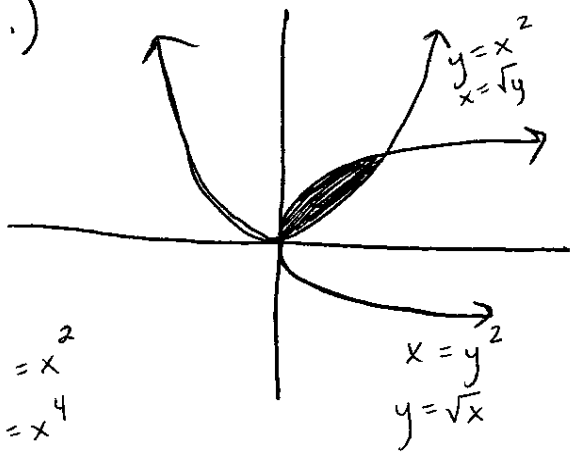
$$V = \pi \int_0^2 (1^2 - (\frac{1}{2}y)^2) dy = \pi \int_0^2 (1 - \frac{1}{4}y^2) dy$$

$$\pi \left(y - \frac{1}{12} y^3 \Big|_0^2 \right) = \boxed{\frac{4\pi}{3}}$$

$$y = 2x$$

$$x = \frac{1}{2}y$$

5.)



$$\begin{aligned} \sqrt{x} &= x^2 \\ x &= x^4 \\ x^4 - x &= 0 \\ x(x^3 - 1) &= 0 \\ x &= 0, 1 \end{aligned}$$

$$\begin{aligned} V &= \pi \int_0^1 (\sqrt{x})^2 - (x^2)^2 dx \\ &= \pi \int_0^1 x - x^4 dx \end{aligned}$$

$$\pi \left(\frac{x^2}{2} - \frac{x^5}{5} \Big|_0^1 \right) = \boxed{\frac{3\pi}{10}}$$

$$6.) \pi \int_0^1 (\sqrt{y})^2 - (y^2)^2 dy = \pi \int_0^1 y - y^4 dy$$

$$\pi \left(\frac{y^2}{2} - \frac{y^5}{5} \Big|_0^1 \right) = \boxed{\frac{3\pi}{10}}$$