Calculus Review

1. Find $\lim_{x \to \infty} \frac{5x^3 + 3x^2}{7x^5 + 8x}$ $= 0$

2. Find $\lim_{x \to (\pi/4)} \frac{1 - \cos x}{x}$ $= \frac{1 - \cos\left(\frac{\pi}{4}\right)}{\frac{\pi}{4}} = \frac{1 - \frac{\sqrt{2}}{2}}{\frac{\pi}{4}} = \frac{2 - \sqrt{2}}{2} \cdot \frac{4}{\pi} = \boxed{\frac{4 - 2\sqrt{2}}{\pi}}$

3. A cylindrical tank with radius 5m is being filled with water at a rate of $3m^3/min$. How fast is the height of the water increasing? Note: $V = \pi r^2 h$.

$V = \pi \cdot 25 \cdot h$

$\frac{dV}{dt} = 25\pi \cdot \frac{dh}{dt}$

$3 = 25\pi \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \boxed{.038 \text{ m/min.}}$

4. Find $f'(x)$ if $f(x) = 4x^3 - 2x^2 + e^x$.

$f'(x) = 12x^2 - 4x + e^x$

5. Find $\lim_{x \to \infty} \frac{3x^4 - 8x^2 + 2}{7x^4}$ $= \frac{3}{7}$

6. Let $f(x) = \log_7(x^2 + 5)$. Find $f'(x)$.

$f(x) = \log_7 u$ $\quad u = x^2 + 5$

$u' = 2x$

$f'(x) = \frac{1}{u \cdot \ln 7} \cdot u' = \boxed{\frac{2x}{\ln 7 (x^2 + 5)}}$

7. Find the derivative of $f(x) = \cos x$ at $x = 0$.

$f'(x) = -\sin x$

$f'(0) = -\sin 0 = \boxed{0}$

8. Find the derivative of $f(x) = \tan^3(3x^3 - 3)$.

$f(x) = u^3$ $\quad u = \tan(3x^3 - 3)$

$f'(x) = 3u^2 \cdot u'$ $\quad u' = 9x^2 \cdot \sec^2(3x^3 - 3)$

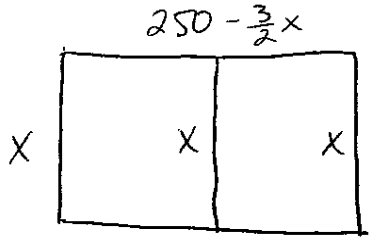
$= 3\tan^2(3x^3 - 3) \cdot 9x^2 \sec^2(3x^3 - 3)$

$= \boxed{27x^2 \tan^2(3x^3 - 3)\sec^2(3x^3 - 3)}$

9. Evaluate $\int 5\sqrt{x^2} = \int x^{2/5}\,dx$

$\boxed{\frac{5}{7}x^{7/5} + C}$

10. A farmer is given 500 ft. of fencing. He will be constructing a rectangular corral to keep pigs and sheep. However, he wants to keep them separate so there will be a wall of fence in the middle of the corral parallel to two of the sides. What are the dimensions of the corral that will maximize the area, and what is the area?



$A = x\left(250 - \frac{3}{2}x\right) = 250x - \frac{3}{2}x^2$

$A' = 250 - 3x$

$250 - 3x = 0$

$3x = 250$

$x = 83\frac{1}{3}$

	$(-\infty, 83\frac{1}{3})$	$(83\frac{1}{3}, \infty)$
A'	$A' > 0$	$A' < 0$

$\boxed{83\frac{1}{3}\text{ ft.} \times 125\text{ ft.}}$

$\boxed{A = 208\frac{1}{3}\text{ ft.}^2}$

11. Let $f(x) = (2x^4 + 8x)(e^{x^3} - 2)$. Find $f'(x)$.

$f'(x) = (2x^4 + 8x) \cdot 3x^2 e^{x^3} + (e^{x^3} - 2) \cdot (8x^3 + 8)$