Calculus

1. Let $y = x^3 + 3x^2 - 9x + 2$. Determine the intervals where the function is monotonically increasing and decreasing.

$$y' = 3x^2 + 6x - 9$$
$$3x^2 + 6x - 9 = 0$$
$$3(x^2 + 2x - 3) = 0$$
$$(x+3)(x-1) = 0$$
$$x = -3, 1$$

	$(-\infty, -3]$	$[-3, 1]$	$[1, \infty)$
y'	$y' > 0$	$y' < 0$	$y' > 0$

$$\boxed{\begin{array}{l} \text{inc.: } (-\infty, -3] \cup [1, \infty) \\ \text{dec.: } [-3, 1] \end{array}}$$

2. Find $\dfrac{dy}{dx}$ when $xy - y^3 = 1$.

$$x \cdot \frac{dy}{dx} + y - 3y^2 \frac{dy}{dx} = 0$$
$$x\frac{dy}{dx} - 3y^2 \frac{dy}{dx} = -y$$
$$\frac{dy}{dx}(x - 3y^2) = -y$$
$$\boxed{\frac{dy}{dx} = \frac{-y}{x - 3y^2}}$$

3. Let $f(x) = \sqrt{3x^4 + 5}$. Find $f'(x)$.

$$u = 3x^4 + 5$$
$$u' = 12x^3$$
$$f(x) = u^{1/2}$$
$$f'(x) = \frac{1}{2}u^{-1/2} \cdot u' = \boxed{\frac{12x^3}{2\sqrt{3x^4+5}}}$$

4. A population of harmful insects is growing according to the formula $i(t) = 2t^2 + t + 5$, where t is measured in days and $i(t)$ is measured in thousands of insects.

a) Find the average rate of growth between $t = 1$ and $t = 3$

$$\frac{i(3) - i(1)}{3 - 1} = \frac{26 - 8}{2} = \boxed{9 \text{ thousand/day}}$$

b)Find the instantaneous rate of growth at $t = 2$

$$i'(t) = 4t + 1$$
$$i'(2) = \boxed{9 \text{ thousand/day}}$$

5. Let $f(x) = \dfrac{x^3 + 2x}{5x + 7}$ find $f'(x)$

$$f'(x) = \frac{(5x+7)(3x^2+2) - 5(x^3+2x)}{(5x+7)^2}$$

6. Find an equation of the tangent line to the curve $f(x) = 3x^2 + 4x$ at $x = 1$

$f'(x) = 6x + 4$ $(1, 7)$

$f'(1) = 10$

$7 = 10 \cdot 1 + b$ $\boxed{y = 10x - 3}$

$b = -3$

7. $\int \frac{1}{t^6}\, dt = \int t^{-6}\, dt$

$\frac{-1}{5} t^{-5} + C = \boxed{\frac{-1}{5t^5} + C}$