A Sampling of Multiple-Choice Questions—Differential Equations

1. 1985 BC 33

If $\frac{dy}{dt} = -2y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{2}$?

(A) $-\frac{\ln 2}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{\ln 2}{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) $\ln 2$

2. 1985 BC 44

At each point (x, y) on a certain curve, the slope of the curve is $3x^2y$. If the curve contains the point $(0,8)$, then its equation is

(A) $y = 8e^{x^3}$ (B) $y = x^3 + 8$ (C) $y = e^{x^3} + 7$
(D) $y = \ln(x+1) + 8$ (E) $y^2 = x^3 + 8$

3. 1988 BC 39

If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x} + 4$ (B) $e^{\tan x} + 5$ (C) $5e^{\tan x}$
(D) $\tan x + 5$ (E) $\tan x + 5e^x$

4. 1988 BC 43

Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

(A) $\frac{3\ln 3}{\ln 2}$ (B) $\frac{2\ln 3}{\ln 2}$ (C) $\frac{\ln 3}{\ln 2}$ (D) $\ln\left(\frac{27}{2}\right)$ (E) $\ln\left(\frac{9}{2}\right)$

5. 1993 AB 33

If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

6. 1993 AB 42

A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

(A) 4.2 pounds (B) 4.6 pounds (C) 4.8 pounds

(D) 5.6 pounds (E) 6.5 pounds

7. 1993 BC 13

If $\frac{dy}{dx} = x^2 y$, then y could be

(A) $3\ln\left(\frac{x}{3}\right)$ (B) $e^{\frac{x^3}{3}} + 7$ (C) $2e^{\frac{x^3}{3}}$ (D) $3e^{2x}$ (E) $\frac{x^3}{3} + 1$

8. 1993 BC 38

During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

(A) 343 (B) 1,343 (C) 1,367 (D) 1,400 (E) 2,057

10. 1997 BC 83 (calculator allowed)

If $\frac{dy}{dx} = (1 + \ln x)y$ and if $y = 1$ when $x = 1$, then $y =$

(A) $e^{\frac{x^2-1}{x^2}}$ (B) $1 + \ln x$ $\ln x$ (C) $e^{2x + x\ln x - 2}$ (D) $e^{x\ln x}$ (E)

11. 1998 AB 21

If $\frac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt} + 3$ (D) $kty + 5$ (E) $\frac{1}{2}ky^2 + \frac{1}{2}$

12. 1998 AB 84 (calculator allowed)

Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

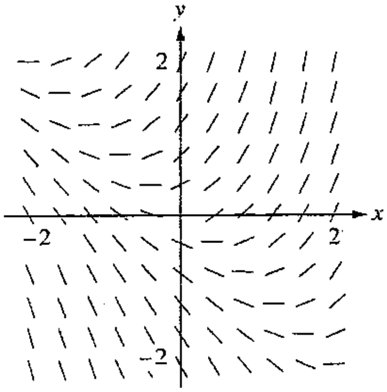
(A) 0.069 (B) 0.200 (C) 0.301 (D) 3.322 (E) 5.000

13. **1988 BC 8**

If $\frac{dy}{dx} = \sin x \cos^2 x$ and if $y = 0$ when , $x = \frac{\pi}{2}$ what is the value of y when $x = 0$?

(A) -1 (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) 1

14. 1998 BC 24



Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$ (D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

15. 1998 BC 26

The population $P(t)$ of a species satisfies the logistic differential equation, $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$ where the initial population $P(0) = 3{,}000$ and t is the time in years.
What is $\lim_{t \to \infty} P(t)$?

(A) 2,500 (B) 3,000 (C) 4,200 (D) 5,000 (E) 10,000

A Sampling of Free-Response Questions—Differential Equations

Please note that the free-response questions and scoring guidelines from recent years can be downloaded from AP Central. Sets of free-response questions and official solutions for 1969-1997 are available for sale in the College Board Store.

apcentral.collegeboard.com/calculusab
apcentral.collegeboard.com/calculusbc
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2001 AB 6

The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6-2x)$

(a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

(b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6-2x)$ with the initial condition $f(3) = \frac{1}{4}$.

2000 AB 6 (no calculator)

Consider the differential equation $\frac{dy}{dx}=\frac{3x^2}{e^{2y}}$.

(a) Find a solution $y=f(x)$ to the differential equation satisfying $f(0)=\frac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).

1993 AB 6

Let $P(t)$ represent the number of wolves in a population at time t years, when $t \geq 0$. The population $P(t)$ is increasing at a rate directly proportional to $800-P(t)$, where the constant of proportionality is k.

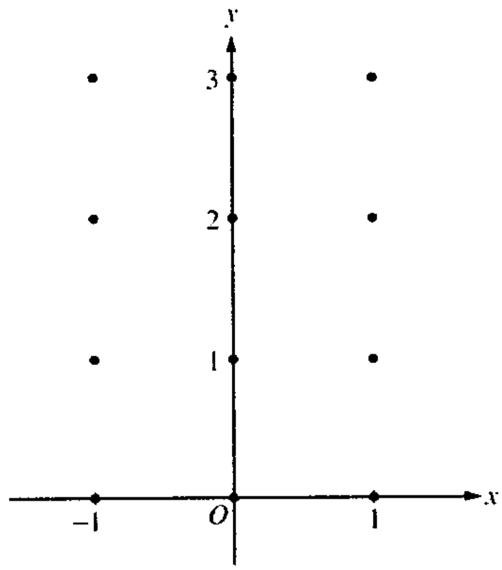
(a) If $P(0)=500$, find $P(t)$ in terms of t and k.

(b) If $P(2)$ = 700, find k.

(c) Find $\lim_{t\to\infty} P(t)$.

6. Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the pink test booklet.)**



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

END OF EXAMINATION