Basic Mathematics for Limited Operators

Learning Objectives

At the conclusion of this chapter, you will be able to:

- Demonstrate calculations involving the use of fractions, decimals, percentages, exponents, ratios and proportions, and simple algebraic equations
- Identify and use standard measurement units, state equivalent values for measurements in both the English and metric systems, and convert measurements from one unit to another
- When given two of the milliampere-second (mAs) values (milliampere [mA], time, mAs), calculate the third
- Calculate changes in radiation intensity and required mAs for changes in source—image receptor distance (SID)
- Given a set of exposure factors, calculate the changes required to change contrast levels using the 15% rule
- Given a set of exposure factors, make appropriate adjustments for differences in patient part thickness using both kilovolt peak (kVp) and mAs
- Perform routine medication dose calculations accurately

Key Terms

algebra
base number
cubed
decimal
decimal point
denominator
difference
dividend
divisor
equation
exponent
lowest terms
mixed number
numerator
percentage
power
product
proportion
quotient
ratio
remainder
squared
square root
sum

With the rapidly advancing use of digital radiographic systems in limited practice, limited operators are required to perform fewer mathematical calculations than they were in the past. Still, determining measurements, exposure factors, radiation doses, and medication doses requires skill in using units of measurement and the ability to manipulate these numbers accurately (Fig. 3.1). Although the increasing use of computers and computerized equipment has reduced the amount of routine calculation required in radiography, an understanding of these functions helps you recognize errors when they occur and make mathematical adjustments when necessary. Certification examinations frequently test these skills by including problems that must be solved without the aid of a computer or calculator.

This chapter consists of two distinct sections: the first section addresses basic mathematical principles; the second section applies these principles to a variety of practical problems that may be encountered in radiography. How you use the first portion of this chapter will depend on your current skill level. For example, some students may be able to solve some or all of the basic math problems immediately with little or no practice, whereas others may need more intensive study to achieve the same level of competence. It is assumed that the reader can perform basic arithmetic functions: adding, subtracting, multiplying, and dividing whole numbers. At this point in the course, your instructor may want you to demonstrate a mastery of the basic mathematics used in radiography. Alternatively, you may turn to this chapter when you are presented with mathematical concepts later in the text.

In the second section of this chapter, in which mathematical skills are applied to practical problems, you will find that the content is easier to understand when you are familiar with the circumstances in which these calculations are required. Although brief explanations are provided in this chapter, the references to later chapters will be very important if you are not already familiar with these situations. For example, formulas for calculating x-ray exposure factors and for preparing medications will be easier to grasp if you are familiar with these activities or have studied the relevant chapters. How you use this portion of the chapter will also depend on your instructor's course design. It may be especially helpful to refer to this chapter in conjunction with your study of Chapter 10.

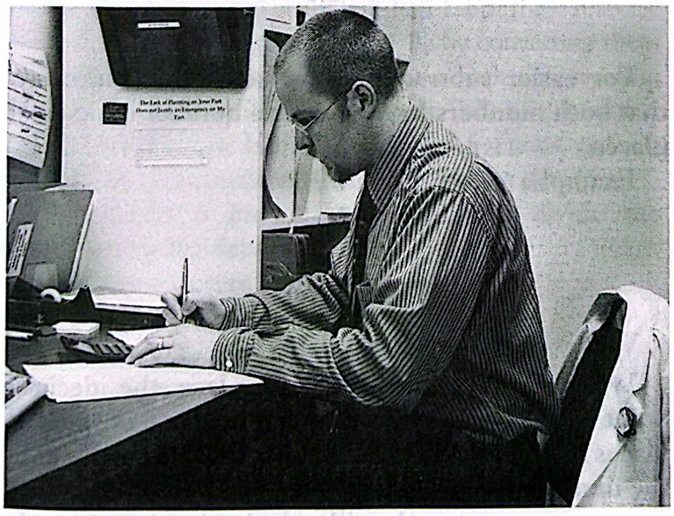

Fig. 3.1 Math skills are required for calculations in radiography.

There are many approaches to solving mathematical problems, and this chapter shows only one approach in most cases. If you are more familiar with another method, there is no reason why you should not continue to use it. The methods given can be used to perform common radiographic calculations, with or without using a calculator. Solutions to the practice problems are in Appendix C.

FUNDAMENTAL MATHEMATICAL PRINCIPLES

Terminology

Discussion of calculations is facilitated by naming the various parts of the problems. New terms are introduced throughout this chapter, but defining a few basic terms at the beginning will assist in your understanding of the sections that follow:

- **Sum**: Total; the answer to an addition problem
- **Difference**: The answer to a subtraction problem
- **Product**: The answer to a multiplication problem
- **Dividend**: The number divided into in a division problem
- **Divisor**: The number that is divided into the dividend
- **Quotient**: The answer to a division problem
- **Remainder**: The number that is "left over" when the dividend cannot be evenly divided by the divisor

Fractions and Decimals

Fractions

Fractions are parts of whole numbers. They are commonly used in our everyday lives. For example, you can easily relate to the concept of one half (½) of an orange or one quarter (¼) of a dollar. Older x-ray control panels with synchronous timers have exposure times that are expressed in fractions of seconds.

The lower number of a fraction is called the **denominator**, which indicates the number of equal parts into which the whole has been divided. The upper number is called the **numerator**, which indicates the number of parts or "pieces" of the divided whole. For example, if you cut a sheet of paper into eight equal parts, one of the parts would be $\frac{1}{8}$ of the page. Three parts would be $\frac{3}{8}$ of the page.

When both the numerator and the denominator of a fraction can be divided evenly by the same number, the resulting fraction is equal to the original fraction. This is called "reducing the fraction to lower terms." For example, if you divide both the numerator and denominator of $\frac{10}{20}$ by 5, the result is $\frac{2}{4}$, lower terms of the fraction $\frac{10}{20}$ and equal to it. The most common form of a fraction is one in which there is no number except 1 that can be divided evenly into both the numerator and the denominator. Such a fraction is said to be in its **lowest terms.** For example, the numerator and denominator of the fraction $\frac{2}{4}$ can be further divided by 2, resulting in $\frac{1}{2}$, the lowest terms of both $\frac{10}{20}$ and $\frac{2}{4}$.

To reduce a fraction to its lowest terms, divide both the numerator and the denominator by the largest number that will divide evenly into both of them.

Example: Reduce $\frac{9}{12}$ to its lowest terms.

The largest number that can be divided evenly into both the numerator and the denominator is 3.

Divide the numerator by 3: $9 \div 3 = 3$, the numerator of the new fraction.

Divide the denominator by 3: $12 \div 3 = 4$, the denominator of the new fraction.

Therefore $\frac{9}{12} = \frac{3}{4}$.

Because there is no number except 1 that can be divided evenly into both 3 and 4, $\frac{3}{4}$ is the lowest term of $\frac{9}{12}$.

The process of determining the value of a fractional portion of a whole number is calculated by multiplying the fraction times the whole number. The most common calculation of this type in radiography involves the determination of milliampere-seconds (mAs) when the exposure time is expressed as a fraction of a second.

To multiply a whole number by a fraction, multiply the whole number by the numerator of the fraction and then divide the product by the denominator of the fraction.

Example: How much is $\frac{3}{5}$ of 200? This question can also be expressed as:

$$\frac{3}{5} \times 200 = ?$$

First, multiply the whole number (200) times the numerator of the fraction (3).

$$200 \times 3 = 600$$

Then, divide the product (600) by the denominator of the fraction (5).

$$600 \div 5 = 120$$

Therefore $\frac{3}{5}$ of 200 equals 120.

Decimals

A **decimal** is actually a fraction with a denominator of 10, 100, 1000, or any number that consists of a 1 followed by one or more zeros. With these specialized fractions, the denominator is not written. Its value is indicated by the position of the numerator in reference to a dot or period called the **decimal point.** Figures to the left of the decimal point are whole numbers. Figures to the right of the decimal point represent the numerator of the fraction. The unseen denominator of the decimal fraction is determined by the number of figures to the right of the decimal point. The first place to the right of the decimal point is 10ths, the second place 100ths, the third place 1000ths, and so on. For example, 0.1 indicates one tenth ($\frac{1}{10}$) and 2.05 indicates two and five hundredths ($2\frac{5}{100}$). You are familiar with the use of decimals to indicate and calculate dollars and cents. In this case, the two figures to the right of the decimal point indicate cents, or hundredths of a dollar.

Zeros between the decimal point and the figures change the places of the figures and change the value of the decimal. For example, 0.3 equals three tenths ($\frac{3}{10}$), whereas 0.003 equals three thousandths ($\frac{3}{1000}$). Zeros added to the right of all other figures in a decimal do not change the value of the decimal. For example, 0.08 is equal to 0.0800. The first is read as eight hundredths ($\frac{8}{100}$) and the second as eight hundred ten thousandths ($\frac{800}{10{,}000}$). It is customary to drop (eliminate) zeros that are to the right of all other figures in a decimal.

To add or subtract decimals, the numbers must be placed so that the decimal points form a vertical line, both in the problem and in the answer.

Examples:

1. $3.4 + 3.04 + 3.004 = ?$

$$\begin{array}{r} 3.4 \\ 3.04 \\ +\,3.004 \\ \hline 9.444 \end{array}$$

2. $7.3 - 4.7 = ?$

$$\begin{array}{r} 7.3 \\ -\,4.7 \\ \hline 2.6 \end{array}$$

For easier subtraction, add zeros as required so that both numbers have the same number of decimal places.

Example: $6.85 - 2.4315 = ?$

$$\begin{array}{r} 6.8500 \\ -\,2.4315 \\ \hline 4.4185 \end{array}$$

When multiplying decimals, place the decimal point in the product so that the number of decimal places is equal to the total number of decimal places in the numbers being multiplied. If the product ends in one or more zeros, place the decimal point correctly before dropping the zeros.

Example: 3.271 × 2.16 = ?

```
  3.271
 × 2.16
 19626
 3271
6542
7.06536
```

To divide *into* a decimal, place the decimal point of the quotient in direct alignment with the decimal point of the dividend.

Example: 15.15 ÷ 3 = ?

```
   5.05
3)15.15
```

To divide by a decimal, move the decimal point to the right of the divisor and move the decimal point of the dividend the same number of places to the right. Align the decimal point in the quotient with the new position of the decimal point in the dividend.

Example: 37.3 ÷ 0.25 = ?

```
          149.2
0.25.)37.30.0
      25
      123
      100
       230
       225
         50
         50
          0
```

Although there are methods for performing all calculations using fractions, most students find it simpler to work with decimals. This is especially true when using a calculator, because calculators are designed to use decimals for the expression of all values that are less than 1. For these reasons, this text deals with fractions principally by converting them into decimals and performing any required calculations using the methods just described for decimals. When a calculation involves fractions, or both decimals and fractions, convert the fractions to decimals and then perform the calculation.

To convert a fraction to a decimal, divide the numerator by the denominator.

Example: Convert $\frac{3}{4}$ into a decimal.

```
  0.75
4)3.00
  28
   20
   20
    0
```

Therefore, the fraction $\frac{3}{4}$ is equal to the decimal 0.75.

A **mixed number** consists of a whole number and a fraction. For example, $1\frac{1}{2}$ and $8\frac{3}{4}$ are mixed numbers.

To convert a mixed number to a decimal, calculate the decimal value of the fraction and add it to the whole number. If the fraction is a proper fraction (meaning that the denominator is greater than the numerator and therefore its value is <1), the decimal value of the fraction will be to the right of the decimal point and the whole number will be to the left of it.

Example: Convert $5\frac{3}{10}$ to a decimal.

First, convert $\frac{3}{10}$ to a decimal. Divide the numerator (3) by the denominator (10).

```
   0.3
10)3.0
```

Then, add the whole number (5) to the decimal fraction (0.3). 5 + 0.3 = 5.3

Therefore $5\frac{3}{10}$ equals 5.3.

Some calculations involving decimals will result in numbers with many decimal places. For example, when the fraction $\frac{1}{3}$ is converted into a decimal, a "repeating decimal" is created, in this case an infinite series of threes: 1 ÷ 3 = 0.33333.... If two numbers, each having three decimal places, are multiplied together, the product will have six decimal places. Most decimals used in radiography are sufficiently accurate when limited to two, three or, at the most, four decimal places. When a decimal has too many places for convenience, it can be shortened by the process of "rounding off."

To round off a decimal, simply drop the excess figures from right to left. If the last figure dropped is 5 or greater, increase the final remaining figure by 1; if the last figure dropped is 4 or less, no change is necessary.

Examples:

1. Round off 3.1416 to three decimal places.
   Drop the excess figure: 3.141~~6~~
   Increase the final figure (1) by 1 because the dropped figure (6) is greater than 5. The result is 3.142.
2. Round off 3.1416 to one decimal place.
   Drop the excess figures: 3.1~~416~~
   There is no need to increase the final figure (1), because the last figure dropped (4) is not 5 or greater. The result is 3.1.

Practice Problems Using Fractions and Decimals

1. **Reducing to lowest terms.** Reduce the following fractions to their lowest terms:
   a. $\frac{5}{15}$
   b. $\frac{3}{9}$
   c. $\frac{12}{18}$
   d. $\frac{18}{20}$
   e. $\frac{16}{24}$

f. $\frac{20}{25}$
g. $\frac{8}{10}$

2. **Multiplying whole numbers by fractions.** Multiply the following whole numbers by the indicated fractions:
   a. $\frac{1}{10} \times 300 = ?$
   b. $\frac{1}{5} \times 200 = ?$
   c. $\frac{3}{20} \times 100 = ?$
   d. $\frac{2}{5} \times 300 = ?$
   e. $\frac{1}{30} \times 100 = ?$
3. **Rounding off decimals.** Round off each of the following decimals to the number of decimal places indicated in parentheses:
   a. 0.66666... (2)
   b. 7.711 (2)
   c. 5.55 (1)
   d. 0.0101 (2)
   e. 10.2498 (3)
4. **Converting fractions and mixed numbers to decimals.** Express the following fractions and mixed numbers as decimals with up to three decimal places:
   a. $\frac{1}{20}$
   b. $\frac{2}{5}$
   c. $\frac{1}{4}$
   d. $\frac{3}{8}$
   e. $\frac{2}{15}$
   f. $2\frac{3}{4}$
   g. $10\frac{2}{3}$
   h. $1\frac{1}{8}$
5. **Addition.** To add the fractions, first convert them to decimals with two decimal places.
   a. $0.78 + 0.01 = ?$
   b. $0.24 + 0.17 + 0.06 = ?$
   c. $3.2 + 1.04 + 0.722 = ?$
   d. $31.3 + 5.007 + 0.516 = ?$
   e. $20 + 1.9 + 0.838 = ?$
   f. $\frac{1}{2} + \frac{1}{4} = ?$
   g. $\frac{3}{5} + \frac{3}{20} + \frac{3}{10} = ?$
6. **Subtraction.** To subtract the fractions, first convert them to decimals with two decimal places.
   a. $1.42 - 0.23 = ?$
   b. $2.008 - 0.71 = ?$
   c. $4.7 - 0.528 = ?$
   d. $3 - 0.54 = ?$
   e. $6.911 - 1.0007 = ?$
   f. $\frac{7}{8} - \frac{1}{4} = ?$
   g. $\frac{2}{5} - \frac{1}{20} = ?$
7. **Multiplication.** To multiply the fractions, first convert them to decimals with two decimal places.
   a. $0.33 \times 0.75 = ?$
   b. $0.2 \times 0.934 = ?$
   c. $0.03 \times 82 = ?$
   d. $0.17 \times 8524 = ?$
   e. $50 \times 0.7872 = ?$
   f. $\frac{1}{5} \times 25 = ?$
   g. $\frac{2}{3} \times 18 = ?$
   h. $\frac{4}{5} \times 0.16 = ?$
8. **Division.** To divide the fractions, first convert them to decimals with two decimal places.
   a. $0.216 \div 3 = ?$
   b. $0.12 \div 5 = ?$
   c. $36 \div 0.09 = ?$
   d. $0.49 \div 0.007 = ?$
   e. $3.44 \div 1.6 = ?$
   f. $47 \div \frac{1}{4} = ?$
   g. $\frac{1}{2} \div \frac{3}{5} = ?$

Percentages

A **percentage** is a form of fraction with a denominator of 100. The term *percent* means "per hundred" and is indicated by the % sign. For example, the statement that "52% of the population is female" means that out of every 100 people in the population, 52 of them are females. One hundred percent indicates the whole and is equal to the number 1.

Addition or subtraction involving only percentages may be performed as with whole numbers or decimals. A percent sign (%) is added to the answer.

Examples:
1. 4% + 8% = 12%
2. 35% − 15% = 20%

When multiplying or dividing percentages or performing calculations that involve a percentage and a whole number or decimal, the percentage must first be converted to a decimal. To convert a percentage to a decimal, move the decimal point two places to the left and drop the percent sign.

Examples:
1. 76% = 0.76
2. 4% = 0.04
3. 150% = 1.50 = 1.5

To convert a decimal to a percentage, move the decimal point two places to the right and add a percent sign.

Examples:
1. 0.23 = 23%
2. 0.08 = 8%
3. 1.7 = 170%
4. 0.177 = 17.7%

To determine a percentage *of* a number means to multiply the number times the percentage. The percentage is first converted to a decimal.

Example: How much is 25% of 300?
Convert the percentage to a decimal: 25% = 0.25
Multiply the number by the decimal: $300 \times 0.25 = 75$
Therefore 75 equals 25% of 300.

To determine what percentage one number is in relation to another, divide one number by the other. The dividend is the number whose percentage you are determining, the portion. The divisor is the whole. (Note that the portion may be greater than the whole, resulting in a percentage that is greater than 100%.)

Examples:

1. 15 is what percentage of 75?
   Divide the portion (15) by the whole (75): 15 ÷ 75 = 0.2
   Convert the decimal into a percentage: 0.2 = 20%
   Therefore 15 equals 20% of 75.
2. 75 is what percentage of 15?
   Divide the portion (75) by the whole (15): 75 ÷ 15 = 5
   Convert the answer to a percentage: 5 = 500%
   Therefore 75 equals 500% of 15.

To increase a number by a certain percentage, add the percentage to 100% and multiply the sum times the number to be increased.

Example: Increase 60 by 30%.
Add the percentage increase (30%) to 100%: 30% + 100% = 130%
Convert the sum of the percentages to a decimal: 130% = 1.3
Multiply the number to be increased (60) by the converted sum of the percentages (1.3): 60 × 1.3 = 78
Therefore 60 plus 30% of 60 equals 78.

To decrease a number by a certain percentage, subtract the percentage from 100% and multiply the difference times the number to be decreased.

Example: Decrease 60 by 20%.
Subtract the percentage decrease (20%) from 100%: 100% − 20% = 80%
Convert the difference between the percentages to a decimal: 80% = 0.8
Multiply the number to be decreased (60) by the converted difference between the percentages (0.8): 60 × 0.8 = 48
Therefore 60 minus 20% of 60 equals 48.

Practice Problems Using Percentages

1. Convert the following percentages to decimals:
   a. 7%
   b. 8.5%
   c. 59%
   d. 99.44%
   e. 140%
   f. 300%
   g. 21%
   h. 2.3%
   i. 100%
2. Convert the following decimals to percentages:
   a. 0.2
   b. 0.19
   c. 0.91
   d. 1.65
   e. 1.2
   f. 6.0
   g. 20.5
   h. 0.03
   i. 0.008
3. Perform the following calculations involving percentages:
   a. 37% + 33% = ?
   b. 100% + 65% = ?
   c. 25% − 13% = ?
   d. 20% × 80% = ?
   e. 150% ÷ 30% = ?
   f. ½ of 40% = ?
   g. 80% of 1.1 = ?
   h. 80% − 10% = ?
4. Calculate the value of the following percentages:
   a. 20% of 88
   b. 90% of 200
   c. 50% of 61
   d. 130% of 40
   e. 300% of 12
   f. 3% of 50
   g. 29% of 1000
   h. 15% of 90
5. Determine the following percentages. Express your answers to the nearest tenth of a percent.
   a. 13 = ? % of 63
   b. 29 = ? % of 80
   c. 43 = ? % of 200
   d. 40 = ? % of 160
   e. 50 = ? % of 35
   f. 20 = ? % of 2
   g. 100 = ? % of 25
   h. 25 = ? % of 1000
6. Calculate the solutions to the following problems that involve increasing and decreasing numbers by a percentage:
   a. Increase 100 by 15%
   b. Increase 30 by 250%
   c. Increase 150 by 25%
   d. Decrease 85 by 10%
   e. Decrease 20 by 12%
   f. Increase 15 by 40%
   g. Increase 46 by 100%
   h. Decrease 200 by 50%

Equations

Algebra is a branch of mathematics that provides a useful method of solving certain kinds of problems. Algebra problems are stated in the form of equations. An **equation** is a mathematical declaration that two mathematical statements (groups of numbers, together with their signs or mathematical functions) are equal to each other. Equations can be very complex, and it is this complexity that causes many students to feel intimidated by the thought of equations. Equations can also be quite simple. For example, 3 + 1 = 4 is an equation. The equations used in radiography are relatively simple ones.

The equations used in arithmetic are always stated so that the unknown quantity is to the right of the equal sign, for example, 3 + 1 = ? In algebra, the unknown quantity is usually indicated by a letter, often x, and may be at any position in the equation. The same symbols for mathematical operations used in arithmetic are also used in algebra: plus (+), minus (−), times (×), divided by (÷ or /), and equal (=). For example, $3 + x = 4$ is an algebraic equation. When a number and a letter are adjacent to each other, this indicates multiplication. For example, $5x$ means five times the value of x ($x \times 5$). A horizontal line in an equation means "divided by." For example, $\frac{2x}{3} = 6$ means 2 times x divided by 3 equals 6.

When solving algebraic problems, the equation must be kept in balance. That is, the mathematical statements on both sides of the equal sign must always be equal to each other. Operations are performed on both sides of the equation to "isolate" the unknown and determine its value. When the unknown is isolated, it is alone on one side of the equal sign and its value is on the other side.

Balance is maintained in an equation by performing the same operation on both sides of the equation.

Example: $3 + x = 4$

To isolate x on one side of the equation, subtract 3 from both sides of the equation:

$$3 - 3 + x = 4 - 3$$

To determine the value of x, perform the mathematical operations: $3 - 3 = 0$; $4 - 3 = 1$;

$$0 + x = 1$$

Therefore $x = 1$.

Other mathematical operations may be performed on both sides of an equation, depending on what is required to isolate the unknown.

Examples:

1. $3x = 27$

   To isolate x, divide both sides of the equation by 3:

   $3x \div 3 = 27 \div 3$

   Therefore $x = 9$.

2. $\frac{x}{12} = 3$

   To isolate x, multiply both sides of the equation by 12:

   $$\frac{x}{12} \times 12 = 3 \times 12$$

   Perform the calculations: $12 \div 12 = 1$; $3 \times 12 = 36$

   Therefore $1x = 36$ or $x = 36$.

3. $x - 10 = 17$

   To isolate x, add 10 to both sides of the equation:

   $$x - 10 + 10 = 17 + 10$$

   Perform the calculations:

   $$-10 + 10 = 0$$

   $$17 + 10 = 27$$

   Therefore $x = 27$.

To save steps when an equation consists of two fractions, you can eliminate the denominators from consideration by using the method of cross multiplication.

In cross multiplication, each numerator is multiplied by the denominator on the opposite side of the equation.

Example: $\frac{16}{4} \bowtie \frac{8}{x}$

Cross multiply: $16x = 4 \times 8$

Perform calculation: $16x = 32$

To isolate x, divide both sides of the equation by 16: $16x \div 16 = 32 \div 16$

Perform the mathematical operations: $16x \div 16 = x$; $32 \div 16 = 2$

Therefore $x = 2$.

Ratios and Proportions

A **ratio** expresses the operation in which one number is divided by another. A ratio may be written using a colon (3:4), a division symbol (3 ÷ 4), or a slanted line (3/4), or by placing the dividend over the divisor with a line between them: $\frac{3}{4}$.

A **proportion** is a statement that two ratios are equal to each other. An example of a proportion is 3:4 :: 9:12. This is read, "3 is to 4 as 9 is to 12." It could also be written 3/4 = 9/12.

Many mathematical relationships in radiography are proportional to each other, so you may need to solve ratio and proportion problems in which one quantity of the proportion is unknown. These problems are set up in the format $a/b = c/d$. Numerical values that are known are substituted in the formula and the rules of algebra explained in the previous section are used to solve the problem.

Example: If you walk at the rate of 3 miles/h, how far can you go in an hour and a half?

The known ratio is 3 miles/1 hour. In the second ratio, the time is known and the distance is not.

Therefore, the problem looks like this:

$$\frac{3 \text{ miles}}{1 \text{ hr}} \times \frac{x \text{ miles}}{1.5 \text{ hr}}$$

Cross multiply: $1x = 3 \times 1.5$

Perform calculation: $x = 4.5$ miles

Proportions are often designated as either *direct* or *inverse*. In a direct proportion, when one of the values increases, the other also increases. For instance, when food intake increases, body weight increases. This statement indicates that the relationship between food intake and weight gain is a direct proportion. On the other hand, if one value increases when the other decreases, this is termed an *inverse proportion*. For example, as exercise decreases, body weight increases, so this would indicate an inverse proportion between exercise and weight gain. Later in this chapter you will note both kinds of proportions in the formulas used to solve technical problems.

Practice Problems Involving Equations

Determine the value of x in each of the following equations:

1. $3x + 6 = 10 + 2$
2. $21/x = 8 - 1$
3. $x - 17 = 13$
4. $20 = 3x - 4$
5. $2x = 18/2$
6. $45 = 9x$
7. $30/x = 6/2$
8. $x/7 = 36/6$
9. $56/8 = 49/x$
10. $12/4 = x/15$

Exponents and Square Roots

Exponents

When a number is multiplied by itself, this operation may be expressed as an **exponent**. An exponent is a small superscript number (in elevated type position) that indicates how many times the number is multiplied by itself. For example, the number 64 equals $2 \times 2 \times 2 \times 2 \times 2 \times 2$. The number 2 is multiplied by itself 5 times and the number 2 is used as a multiplication factor 6 times. Using an exponent, 64 can be expressed as 2^6. In this case the repeated number, or **base number**, is 2, and the exponent or **power** of the base number is 6. This is read as "2 to the sixth power," or simply as "2 to the sixth." When the exponent is 2 or 3, special terms are commonly used. The number is said to be **squared** when it is multiplied by itself once and the exponent is 2. When the exponent is 3, the number is multiplied by itself twice and is said to be **cubed**.

Examples:

1. What is the value of 12 squared?

$$12^2 = 12 \times 12 = 144$$

2. What is the value of 2 cubed? $2^3 = 2 \times 2 \times 2 = 8$

Square Roots

The **square root** of a number is the value that, when multiplied by itself, equals the original number. The square root is represented by the radical sign, $\sqrt{\ }$.

Examples:

1. $\sqrt{4} = 2$
2. $\sqrt{100} = 10$

Square roots that are small, whole numbers, such as those in the examples given, can be easily perceived. It is a complex mathematical operation to extract square roots that are not whole numbers. Fortunately, today simple math calculators provide this function.

Practice Problems Involving Exponents and Square Roots

1. Calculate the value of the following exponential terms:
   a. 5^3
   b. 2^4
   c. 9^2
   d. 10^4
   e. 4^3
2. Determine the square roots of the following numbers:
   a. $\sqrt{49} = ?$
   b. $\sqrt{36} = ?$
   c. $\sqrt{64} = ?$
   d. $\sqrt{400} = ?$

Measurement Units and Their Conversion

In the United States many measurements are made using the English system that involves such units as pounds, feet, and gallons. Scientists worldwide prefer measurements made in the metric system, mainly because it is based on the number 10, which simplifies many calculations. You will encounter measurements in both systems and must be able to convert measurements readily from one system to the other. If you need to understand or convert measurements not found in this chapter, you can find information on measurement units used in health care in almost any medical dictionary. The Internet also provides this information; for example, www.allmath.com provides comprehensive English-to-metric conversion tables, or if you wish, you can enter your own measurements and have them converted for you at www.sciencemadesimple.com/conversions.html.

Metric System

The basic units of the metric system are the gram for measuring weight, the liter for measuring liquid volume, and the meter for measuring length. Various prefixes are used with these units to specify measurements that are larger or smaller than the basic units by factors that are multiples of 10. For example, *kilo* is a prefix meaning 1000. You may already be familiar with the fact that 1 kilovolt equals 1000 volts. A kilometer is equal to 1000 meters. The prefix *milli* indicates 1/1000 or 0.001 times the basic unit. For example, 1 milliliter equals 0.001 liter.

The basic units of the metric system are summarized in Table 3.1. Metric prefixes are summarized in Table 3.2.

English System

The relationships between measurements in the English system are far less orderly than those in the metric system. As stated before, the basic units are the pound (weight),

TABLE 3.1
Base Units in the Metric System

Unit	Used to Measure	Abbreviation
Meter	Length	M or m
Liter	Liquid	L or l
Gram	Weight	g or gm

the foot (length), and the gallon (liquid volume), but a number of other units are also used to measure these same parameters in the English system. For example, the ounce and the ton are also common weight measurements, the inch and the mile are also used to measure length, and liquids may be measured in ounces, pints, quarts, or gallons. Table 3.3 lists common measurement units of the English system and their relationships to one another. Table 3.4 compares the most common English measurements with their counterparts in the metric system.

TABLE 3.2
Metric Prefixes

Prefix	Meaning	Abbreviation
Kilo-	1000	K
Hecto-	100	H
Deka-	10	da
Deci-	1/10 (0.1)	d
Centi-	1/100 (0.01)	c
Milli-	1/1000 (0.001)	m
Micro-	1/1,000,000 (0.000001)	μ or mc
Nano-	1/1,000,000,000 (0.0000000001)	n

TABLE 3.3
English Units

Type of Unit	Unit	Abbreviation	Equivalents
Length	inch	in	0.0833 ft
	foot	ft	12 in
	yard	yd	3 ft
	mile	mi	5280 ft
Liquid	ounce	oz	0.0625 pt
	pint	pt	16 oz
	quart	qt	2 pt
	gallon	gal	4 qt
Weight	ounce	oz	0.0625 lb
	pound	lb	16 oz
	ton	T	2000 lb

TABLE 3.4
Common English/Metric Equivalents

Type of Unit	English Units	Metric Equivalents
Length	inch	2.54 cm
	yard	0.914 m
Liquid	ounce	30 mL
	quart	0.946 L
Weight	pound	2.2 kg

Conversions Between Measurement Units

Table 3.5 provides shortcuts for conversions between units that are commonly used. For example, to change from inches to centimeters, simply multiply the number of inches by 2.54, which is the number of centimeters in an inch. You can also use the proportion formula that follows. The method for solving proportion problems is explained earlier in this chapter.

Proportion problems for converting measurements from one unit to another are set up in this format:

$$\frac{\text{A units}}{\text{B units}} = \frac{\text{1 A unit}}{\text{B units per unit A}}$$

Examples:

1. Convert 10 km into meters.
   In this problem we designate kilometers as the A unit and meters as the B unit.
   Substitute values in the proportion formula above:

$$\frac{10 \text{ km}}{x \text{ M}} = \frac{1 \text{ km}}{1000 \text{ m per km}}$$

Cross multiply: $1 \times x = 10 \times 1000 = 10{,}000$ m

TABLE 3.5
Common Length Unit Conversion Guide

Length	inches × 2.54 = centimeters
	centimeters × 0.39 = inches
	inches ÷ 12 = feet
	feet × 12 = inches
	feet ÷ 3 = yards
	feet × 0.305 = meters
	meters × 3.28 = feet
	yards × 0.91 = meters
Weight	pounds × 0.45 = kilograms
	kilograms × 2.2 = pounds
	grams × 0.0022 = pounds
	kilograms × 35.27 = ounces
Volume	cm^3 = milliliters
	ounces × 30 = milliliters
	ounces ÷ 16 = pints
	pints × 16 = ounces
	quarts × 2 = pints
	ounces × 0.0624 = pints
	gallons × 4 = quarts
	liters × 0.26 = gallons
	liters × 2.11 = pint
Time	seconds ÷ 60 = minutes
	minutes × 60 = seconds
	minutes ÷ 60 = hours
	hours × 60 = minutes
	hours ÷ 24 = days
	days × 24 = hours

2. Convert 300 mA into amperes (A).
   In this problem we designate milliamperes as the A unit and amperes as the B unit.
   Substitute values in the proportion formula:

$$\frac{300 \text{ mA}}{x \text{ A}} = \frac{1 \text{ mA}}{0.001 \text{ A per mA}}$$

   Cross multiply: $x = 300 \times 0.001 = 0.3$ A
3. Convert 1 foot into centimeters.
   In this problem we designate inches as the A unit and centimeters as the B unit. Because we do not have a factor for converting feet directly into centimeters, we must first convert feet into inches. This is easily accomplished using Table 3.3, where we see that 1 foot = 12 inches.
   Substitute values in the proportion formula:

$$\frac{12 \text{ in}}{x \text{ cm}} = \frac{1 \text{ in}}{2.54 \text{ cm/in}}$$

   Cross multiply: $x = 12 \times 2.54$
   Calculate: $x = 30.48$ cm

Units of Time

Units that measure time are universal, and there is only one system, which is a mixture of the English and metric systems. The base unit for measuring time is the second. Milliseconds and nanoseconds are short time periods using metric prefixes to indicate thousandths and billionths of a second, respectively.

The time units greater than a second are probably quite familiar to you. Sixty seconds equal 1 minute, there are 60 minutes in an hour, and 24 hours constitute 1 day.

You can use the proportion formula from the previous section to convert from one time unit to another. Table 3.5 contains some shortcuts for converting between common units of time.

Units of Temperature

There are two common scales for measuring temperature. The English system uses the Fahrenheit scale (F), whereby using this scale, water freezes at 32°F and boils at 212°F.

The Celsius (C) temperature scale is used in the metric system. At one time this scale was called *centigrade*, and you may still encounter this term. Each degree on the Celsius scale represents a greater quantity of temperature change than a degree on the Fahrenheit scale. Water freezes at 0°C and boils at 100°C.

The following formulas are used to convert between the Fahrenheit and Celsius temperature scales:

$$F = (C \times 1.8) + 32°$$
$$C = F - 32° \div 1.8$$

Examples:

1. Convert normal body temperature (98.6°F) to the Celsius scale.
   Substitute the known Fahrenheit value in the formula for obtaining Celsius units:

$$C = (98.6°F - 32°) \div 1.8$$

   Perform calculations: $C = 66.6° \div 1.8 = 37°C$
2. Convert 25°C to the Fahrenheit scale.
   Substitute the known Celsius value in the formula for obtaining Fahrenheit units:

$$F = (25°C \times 1.8) + 32°$$

   Perform calculations: $F = 45° + 32° = 77°F$

Practice Problems Using Measurement Units

1. Conversions from one metric unit to another
   a. Convert 75 kilovolts to volts.
   b. Convert 3 meters to centimeters.
   c. Convert 10 milliliters to liters.
   d. Convert 20 grams to kilograms.
   e. Convert 15 centigrams to grams.
2. Conversions from one English unit to another
   a. Convert 16 inches to yards.
   b. Convert ½ pint to fluid ounces.
   c. Convert 84 inches to feet.
   d. Convert 18 quarts to gallons.
   e. Convert 4.8 pounds to ounces.
3. Conversions between English and metric units
   a. Convert 3 fluid ounces to milliliters.
   b. Convert 0.25 pound to grams.
   c. Convert 5 inches to meters.
   d. Convert 30 millimeters to inches.
   e. Convert 40 grams to ounces.
4. Time and temperature conversions.
   a. Convert 1/20 second to milliseconds.
   b. Convert 330 seconds to hours.
   c. Convert 3.4 days to hours.
   d. Convert 19°C to the Fahrenheit scale.
   e. Convert 50°F to the Celsius scale.

MATHEMATICS APPLIED TO RADIOGRAPHY

Milliampere-Seconds

The concept of mAs as an exposure factor in radiography is introduced in Chapter 5 and further expanded in Chapters 7 and 10. The unit called *milliampere-seconds*, abbreviated *mAs*, is the product of milliamperage and exposure time (seconds). Milliamperage (mA) is a unit that represents the rate at which x-rays are produced. When this number is multiplied by the exposure time in seconds, the resulting quantity is expressed in units of mAs and indicates the total quantity of radiation involved in an exposure. Several possible combinations of mA and time may be used to obtain a given mAs quantity. The manipulation of these three quantities is part of the everyday work in radiography.

The formula for determining mAs is:

$$\text{mA} \times \text{seconds} = \text{mAs}$$

Example: Determine the mAs of an exposure made using 300 mA and 0.4 second

$$300 \text{ mA} \times 0.4 \text{ second} = 120 \text{ mAs}$$

According to the principles of algebra explained earlier in this chapter, this formula can be rearranged to isolate any of its three parts.

Examples:

1. Using the mAs formula, derive a formula for determining the exposure time when the values of mA and mAs are known.

$$\frac{\cancel{\text{mA}} \times \text{second}}{\cancel{\text{mA}}} = \frac{\text{mAs}}{\text{mA}}$$

Divide the first fraction by mA, therefore reducing it to isolate seconds.

Therefore, the exposure time in seconds equals the mAs divided by the mA. The formula can be stated:

$$\text{mAs}/\text{mA} = T \text{ (second)}$$

2. If the desired mAs is 20 and you wish to use 200 mA, what should be the exposure time?

$$20 \text{ mAs} \div 200 \text{ mA} = 0.1 \text{ second or } 1/10 \text{ second}$$

Note: When calculating exposure time for controls that have exposure times in decimals, it is best to divide so as to obtain a decimal. For controls that have exposure times in fractions, formulate the problem as a fraction and reduce the fraction, if required, to obtain a fractional exposure time. See the instructions earlier in this chapter for reducing fractions to their lowest terms.

The diagram in Fig. 3.2 is a handy reminder of the relationships among mA, exposure time, and mAs. When you cover the factor you wish to obtain, the calculation required is apparent. For example, when mAs is covered, mA and time are separated by a vertical line indicating that these factors should be multiplied to obtain mAs. When mA is covered, time is beneath the horizontal line and mAs is over it, indicating that mAs is divided by time in seconds to calculate mA.

It will often be the case that you will calculate values for mA, time, or mAs that are not available on your control console. It is helpful to remember that variations in exposure of less than 20% are scarcely noticeable on the image. When the ideal exposure factor is not available, simply select the factor that is closest to the value you want. Most control consoles are designed to provide settings within a 20% range of any specific goal you may calculate.

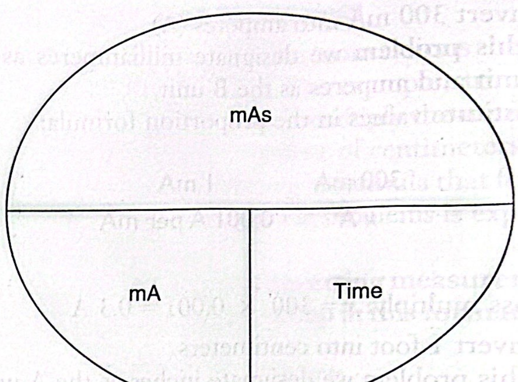

Fig. 3.2 The relationships among milliamperes *(mA)*, time, and milliampere-seconds *(mAs)*.

Practice Problems Involving Milliampere-Seconds

1. Calculate the mAs for the following exposures. Round any extended decimals to two decimal places.
   a. 200 mA, 1/40 second
   b. 300 mA, 1/20 second
   c. 100 mA, 2/15 second
   d. 500 mA, 0.02 second
   e. 50 mA, 0.3 second
   f. 150 mA, 1¼ second
   g. 400 mA, 2 milliseconds
   h. 300 mA, 1/120 second
   i. 1200 mA, 0.005 second
   j. 600 mA, 0.04 second
   k. 750 mA, 0.1 second
   l. 300 mA, 0.3 second
   m. 200 mA, 0.4 second
2. Calculate the exposure time for the following exposures. Round any extended decimals to three decimal places.
   a. 50 mA, 40 mAs
   b. 200 mA, 25 mAs
   c. 300 mA, 10 mAs
   d. 100 mA, 1 mAs
   e. 400 mA, 8 mAs
   f. 500 mA, 50 mAs
   g. 800 mA, 20 mAs
   h. 150 mA, 300 mAs
   i. 25 mA, 150 mAs
   j. 1000 mA, 5 mAs
3. Calculate the mA for the following exposures:
   a. 10 mAs, 1/30 second
   b. 25 mAs, ½ second

c. 40 mAs, 2⁄15 second
d. 5 mAs, 0.01 second
e. 8 mAs, 0.02 second
f. 100 mAs, 0.5 second
g. 20 mAs, 0.2 second
h. 3 mAs, 0.01 second
i. 60 mAs, 0.3 second

Source–Image Receptor Distance and Radiation Intensity

Source–image receptor distance (SID) is the distance in inches between the radiation source in the x-ray tube and the imaging plane. Chapter 7 explains and illustrates the relationship between radiation intensity and SID according to the inverse square law:

Radiation intensity is inversely proportional to the square of the distance.

$$\frac{I_1 \text{ (Original intensity)}}{I_2 \text{ (New intensity)}} = \frac{SID_2^2 \text{ (New distance, squared)}}{SID_1^2 \text{ (Original distance, squared)}}$$

Example: When the SID is changed from 30 to 90 inches, what is the relationship between the original radiation intensity and the intensity at the new distance?

Substitute the distances in the formula. The original intensity is assigned a relative value of 1. The new intensity is unknown:

$$\frac{I_1}{I_2} = \frac{90^2}{30^2}$$

The calculation is greatly simplified if you first reduce the fraction. $90^2/30^2$ can be reduced by dividing both the numerator and the denominator by 30:

$$\frac{I_1}{I_2} = \frac{3^2}{1^2}$$

Calculate the squares: $3^2 = 3 \times 3 = 9$; $1^2 = 1 \times 1 = 1$

Substitute these values in the formula:

$$\frac{I_1}{I_2} = \frac{9}{1}$$

Cross multiply: $9 \times I_2 = 1$

To isolate I_2, divide both sides by 9:

$$\frac{\cancel{9} \times I_2}{\cancel{9}} = \frac{1}{9}$$

Therefore $I_2 = 1/9$. That is, the new intensity is one ninth of the original intensity.

Although knowing this relationship enhances understanding, it is not of great practical value. Assuming that the original radiation intensity was satisfactory, the more important question is how to maintain a constant radiation intensity when the distance changes. The mAs is used to compensate for changes in distance. The correct change in mAs enables you to maintain the same radiation intensity when the distance is changed. Because the intensity decreases when the distance increases (an inverse proportion), the mAs must be *increased* when the distance increases (a direct proportion).

To maintain a constant radiographic intensity, the mAs must be directly proportional to the square of the distance. The formula for changing mAs to maintain a constant radiation intensity when the distance changes is as follows:

$$\frac{mAs_1 \text{ (Original)}}{mAs_2 \text{ (New)}} = \frac{SID_1^2 \text{ (Original distance, squared)}}{SID_2^2 \text{ (New distance, squared)}}$$

Example: A satisfactory radiograph is made using 20 mAs at 40 inches SID. How much mAs is required to produce a similar radiograph at 60 inches SID?

Substitute in the formula:

$$\frac{20 \text{ mAs}}{mAs_2} = \frac{40^2}{60^2}$$

To simplify the calculation, reduce the fraction (divide both numerator and denominator on the right side of the equation by 20):

$$\frac{20 \text{ mAs}}{mAs_2} = \frac{2^2}{3^2}$$

Calculate the squares: $2^2 = 2 \times 2 = 4$; $3^2 = 3 \times 3 = 9$

Substitute these values in the formula:

$$\frac{20 \text{ mAs}}{mAs_2} = \frac{4}{9}$$

Cross multiply: $4 \times mAs_2 = 20 \text{ mAs} \times 9$

Perform the calculation: $4 \times mAs_2 = 180 \text{ mAs}$

To isolate mAs_2, divide both sides by 4:

$$\frac{\cancel{4} \times mAs_2}{\cancel{4}} = \frac{\cancel{180}\ 45 \text{ mAs}}{\cancel{4}}$$

Therefore the new mAs required at 60 inches SID is 45 mAs.

Practice Problems Involving Distance Changes

1. What is the relative change in radiation intensity when the distance changes from 40 inches SID to 30 inches SID?
2. What is the relative change in radiation intensity when the distance is changed from 72 inches SID to 40 inches SID?
3. A satisfactory radiograph is made using 15 mAs at 40 inches SID. How much mAs is required to produce a similar radiograph at 48 inches SID?
4. A satisfactory radiograph is made using 40 mAs at 72 inches SID. How much mAs is required to produce a similar radiograph at 84 inches SID?

5. A satisfactory radiograph is made using 10 mAs at 40 inches SID. How much mAs is required to produce a similar radiograph at 72 inches SID?
6. What is the relative change in radiation intensity when the distance is changed from 72 inches SID to 40 inches SID?
7. A satisfactory radiograph is made using 25 mAs at 40 inches SID. How much mAs is required to produce a similar radiograph at 60 inches SID?
8. A satisfactory radiograph is made using 100 mAs at 72 inches SID. How much mAs is required to produce a similar radiograph at 60 inches SID?
9. A satisfactory radiograph is made using 20 mAs at 60 inches SID. How much mAs is required to produce a similar radiograph at 48 inches SID?
10. A satisfactory radiograph is made using 8 mAs at 72 inches SID. How much mAs is required to produce a similar radiograph at 40 inches SID?

Exposure Adjustments for Patient Size

Alteration of Kilovoltage for Patient Part Size Changes

A single set of exposure factors is not adequate for all sizes of patients. For the purpose of determining correct exposure factors, body parts are measured with calipers in centimeters. Adjustments in kilovoltage for variations in part size are discussed in Chapter 10. As stated there, kilovolts peak (kVp) is a useful adjustment only for relatively small variations from normal because large changes in kVp cause significant alteration in the appearance of the image.

Below 85 kVp, an adjustment of 2 kVp/cm will compensate for small changes in part size. Above 85 kVp, a change of 3 kVp/cm is necessary.

Example: A wrist measures 4 cm in posteroanterior diameter, and an exposure of 5 mAs at 56 kVp produces a satisfactory radiograph. How much kVp adjustment is required for the lateral projection, which measures 6 cm?

The kVp range is below 85, so the adjustment is 2 kVp/cm.

The size difference is 2 cm (6 cm− 4 cm = 2 cm).

2 cm × 2 kVp/cm = 4 kVp increase.

The original kVp (56) plus the increase (4) equals the new kVp (60).

Therefore, the exposure for the lateral wrist projection is 5 mAs at 60 kVp.

Alteration of Milliampere-Seconds for Patient Part Size Changes

As explained in Chapter 10, mAs is the usual and best choice of factors to adjust when compensating for differences in patient part size. The mAs is increased by 30% for a 2 cm increase in part size and decreased by 20% for a 2 cm decrease in part size. These percentages may be added or subtracted, respectively, from 100% and the result multiplied by the original mAs. *These changes compound, much like compound interest, and must be applied 2 cm at a time.*

For a 2 cm increase in patient part size, increase the mAs by 30% (multiply the mAs by 1.3). For a 2 cm decrease in patient part size, decrease the mAs by 20% (multiply the mAs by 0.8).

Examples:

1. If 20 mAs is a satisfactory exposure for a patient part measuring 20 cm, how much mAs is required to produce a similar image of a patient part measuring 24 cm?

   Multiply the mAs by 1.3 for the first 2 cm increase:

   20 mAs × 1.3 = 26 mAs for a 22-cm patient part

   Multiply the mAs for 22 cm (26) by 1.3 to obtain the mAs for 24 cm:

   26 mAs × 1.3 = 33.8 mAs for a 24-cm patient part

2. If 60 mAs is a satisfactory exposure for a patient part measuring 28 cm, how much mAs is required for a patient part measuring 26 cm?

   Multiply the mAs by 0.8 for a 2 cm decrease:

   60 mAs × 0.8 = 48 mAs for a 26-cm patient part

Practice Problems Involving Patient Part Size Changes

1. A satisfactory radiograph is made using 96 kVp on a body part measuring 30 cm. Adjust the kVp to compensate for a body part size decrease to 27 cm.
2. A satisfactory radiograph is made using 74 kVp on a body part measuring 16 cm. Adjust the kVp to compensate for a body part size increase to 18 cm.
3. A satisfactory radiograph is made using 50 mAs on a body part measuring 24 cm. Adjust the mAs to compensate for a body part size decrease to 18 cm.
4. A satisfactory radiograph is made using 30 mAs on a body part measuring 19 cm. Adjust the mAs to compensate for a body part size increase to 25 cm.
5. A satisfactory radiograph is made using 10 mAs on a body part measuring 12 cm. Adjust the mAs to compensate for a body part size increase to 16 cm.
6. A satisfactory radiograph is made using 55 kVp on a body part measuring 4 cm. Adjust the kVp to compensate for a body part size increase to 7 cm.
7. A satisfactory radiograph is made using 100 mAs on a body part measuring 36 cm. Adjust the mAs to compensate for a body part size decrease to 30 cm.
8. A satisfactory radiograph is made using 4 mAs on a body part measuring 19 cm. Adjust the mAs to compensate for a body part size increase to 21 cm.
9. A satisfactory radiograph is made using 40 mAs on a body part measuring 20 cm. Adjust the mAs to compensate for a body part size increase to 24 cm.

Alteration of Contrast with Kilovoltage: The 15% Rule

Kilovoltage may be altered to modify the appearance of the radiographic image by changing the scale of contrast. Radiographic contrast is discussed in Chapter 7 and its relationship to kilovoltage is explained in Chapter 10. The kVp is *increased* to lengthen the scale of contrast; this change decreases contrast, increases latitude, and creates a grayer image. The kVp is *decreased* to shorten the scale of contrast; this change increases contrast, producing a more black-and-white appearance. When kVp is changed, however, the radiographic density, or darkness of the image, is also affected. If the original radiographic density was satisfactory and you wish only to change the level of contrast, mAs must be used to compensate for the density change that occurs when kVp is altered. An increase in kVp with a corresponding decrease in mAs results in a lower patient dose. As explained in Chapter 10, the 15% rule provides guidelines for altering the kVp while maintaining a constant radiographic density.

To use the 15% rule to increase contrast, decrease the kVp by 15% and multiply the mAs by 2.

Example: A radiograph made using 40 mAs and 90 kVp has satisfactory radiographic density but is lacking in contrast. Suggest a new technique that will provide more contrast with similar radiographic density.

To decrease kVp by 15%, multiply by 0.85 (100% − 15% = 85% or 0.85):

90 kVp × 0.85 = 77 kVp

Note: When calculating kVp changes, round off to the nearest whole kilovolt.

Multiply the mAs by 2: 40 mAs × 2 = 80 mAs

Therefore, the new technique is 80 mAs at 77 kVp.

To use the 15% rule to decrease contrast, increase latitude, lower patient dose, increase the kVp by 15%, and divide the mAs by 2.

Example: A satisfactory radiograph is made using 60 mAs and 76 kVp. Suggest a new technique that will provide more latitude and lower the patient dose.

To increase kVp by 15%, multiply by 1.15 (100% + 15%): 76 kVp × 1.15 = 87 kVp

Divide the mAs by 2: 60 mAs ÷ 2 = 30 mAs

Therefore, the new technique is 30 mAs at 87 kVp.

Note: The 15% rule may be reapplied to the new technique if the first calculation does not produce sufficient change.

Practice Problems Using the 15% Rule

(When changing kVp, round off your answers to the nearest whole number.)

1. An exposure made using 25 mAs and 86 kVp has satisfactory radiographic density. Suggest a new technique that will provide more contrast.
2. An exposure made using 100 mAs and 80 kVp has satisfactory radiographic density. Suggest a new technique that will decrease the patient dose.
3. An exposure made using 10 mAs and 66 kVp has satisfactory radiographic density. Suggest a new technique that will provide more latitude.
4. An exposure made using 40 mAs and 94 kVp has satisfactory radiographic density. Suggest a new technique that will provide more contrast.
5. An exposure made using 60 mAs and 72 kVp has satisfactory radiographic density. Suggest a new technique that will provide less contrast.
6. An exposure made using 80 mAs and 96 kVp has satisfactory radiographic density. Suggest a new technique that will provide more contrast.
7. An exposure made using 20 mAs and 70 kVp has satisfactory radiographic density. Suggest a new technique that will decrease the patient dose.
8. An exposure made using 5 mAs and 54 kVp has satisfactory radiographic density. Suggest a new technique that will provide more latitude.
9. An exposure made using 30 mAs and 120 kVp has satisfactory radiographic density. Suggest a new technique that will provide more contrast.
10. An exposure made using 50 mAs and 68 kVp has satisfactory radiographic density. Suggest a new technique that will provide less contrast.

Medication Dosage Calculations

Calculation of medication dosage is discussed in Chapter 23. This section highlights that information and provides practice problems.

Medications are provided in many forms, but the most common forms are tablets and liquids. Regardless of form, each medication is provided in specific strengths. The strength of the medication indicates the amount of active drug contained in a certain volume of the medication. For example, a liquid medication may have 5 mcg of solid dissolved in each milliliter of liquid and the strength would be stated as 5 mcg/mL. A tablet may contain 2 mg of the active drug and would be labeled simply as 2 mg/tablet.

The basic formula for determining the correct quantity of any drug is as follows:

$$\frac{\text{Dose}}{\text{Strength}} = \text{Volume}$$

In this formula, *dose* refers to the prescribed amount of the active ingredient in the medication. *Strength* refers to the amount of active ingredient per unit of volume as described in the preceding paragraph. *Volume* refers to the total quantity of medication that is administered.

Examples:

1. The physician has prescribed a dose of 50 mg of meperidine (Demerol) intramuscularly. The available

stock has a strength of 20 mg/mL. How much of this solution should you draw up into the syringe?
Substitute the known quantities in the formula and perform a calculation to determine the volume.

$$\frac{50 \text{ mg}}{20 \text{ mg/mL}} = 2.5 \text{ mL (Volume to be injected)}$$

2. A toddler got into the medicine cabinet and ate 4 acetaminophen tablets. The strength of the tablets is 500 mg. What dose did the toddler receive?
Substitute the known quantities in the formula:

$$\frac{x \text{ mg}}{500 \text{ mg/tablet}} = 4 \text{ tablets (Volume taken)}$$

To isolate the dose, multiply both sides of the equation by 500:

$$\frac{x \text{ mg} \times \cancel{500 \text{ mg/tablet}}}{\cancel{500 \text{ mg/tablet}}} = 500 \text{ mg} \times 4 \text{ tablets}$$

$$\text{Dose} = 500 \text{ mg} \times 4 = 2000 \text{ mg (2 g)}$$

Practice Problems Calculating Medication Dosage

1. The prescribed dose is 100 mg. The available stock is in the form of 25 mg tablets. How many should be given?
2. The prescribed dose is 250 mg. The available stock has a strength of 50 mg/mL. How much should be given?
3. The prescribed dose is 40 mcg. The available stock has a strength of 80 mcg/tablet. How much should be given?
4. The prescribed dose is 5 mg. The available stock has a strength of 1 mg/mL. How much should be given?
5. A patient reports taking 8 tablets of ibuprofen (Advil) a day. The tablets have a strength of 200 mg. What is the patient's daily dose?

SUMMARY

The information in this chapter supplements that in later chapters regarding mathematical relationships and technique formulation.

Fractions are portions of whole numbers and are sometimes used to measure exposure times. They are most commonly reduced to lowest terms and can easily be calculated when converted to decimals. Decimals are fractions with denominators of 10, 100, 1000, and so on, and are written differently from other fractions. The most familiar decimals are those used to indicate dollars and cents. Percentages are specific decimals with denominators of 100.

Exponents are used to shorten the notation of very large and very small numbers. A positive exponent indicates the number of times the base number is multiplied by itself. Negative exponents designate quantities smaller than 1.

Equations are numerical expressions of two quantities that are equal to each other. They can be manipulated to determine the value of an unknown quantity, but the balance of the equation must be maintained. Special equations of the type $a/b = c/d$ are called *proportions*. They represent relationships between quantities that may be either direct or inverse and are a common way of expressing relationships between technical factors in radiography.

These problem-solving skills are important, whether or not you need to use them every day. They are used to calculate mAs values and to adjust exposure factors for changes in patient size, SID, and kVp, as well as to accurately determine dose for medication administration.