Waves

TOPIC 5

How Scientists Study Waves

Are radio waves sound waves?

Radio waves are electromagnetic waves that are produced by accelerating charged particles in an antenna. Radio waves do not require a medium for transmission. They travel through space at 3.00×10^8 meters per second.

On the other hand, sound waves are longitudinal waves that are usually produced by vibrating objects. Sound waves propagate through a medium as changes in pressure. Sound waves can be transmitted by solids, liquids, and gases, but they cannot travel through a vacuum. The speed of sound is 343 meters per second in air at 20° C.

Although radio waves travel in straight lines and the Earth's surface is curved, it is possible to transmit radio waves great distances from the source of production to point of reception. Radio waves with frequencies ranging from approximately 0.3 megahertz to 30 megahertz can reflect multiple times from the ionosphere surrounding Earth and from Earth's surface.

It takes less time for a radio wave to travel from California to New York than it does for the sound wave coming from a radio to travel across a room.

TOPIC

5 Waves

Vocabulary

absolute index of refraction	incident ray	reflected ray
amplitude	interference	refracted ray
angle of incidence	law of reflection	reflection
angle of reflection	longitudinal wave	refraction
angle of refraction	medium	resonance
antinode	natural frequency	Snell's law
constructive interference	node	speed
destructive interference	normal	standing wave
diffraction	period	superposition
Doppler effect	periodic wave	transverse wave
electromagnetic spectrum	phase	vacuum
electromagnetic wave	principle of superposition	wave
frequency	pulse	wave front
hertz	ray	wavelength

Introduction to Waves

A **wave** is a vibratory disturbance that propagates through a **medium** (body of matter) or field. Every wave has, as its source, a particle vibrating or oscillating about an average position. For example, a sound wave can be produced by a vibrating tuning fork and a radio wave can be generated by accelerating electrons in a transmitter.

Waves and Energy Transfer

Waves transfer energy from one place to another by repeated small vibrations of particles of a medium or by repeated small changes in the strength of a field. The source provides the initial vibrations, but there is no actual transfer of mass from the source. Only energy is transferred from the source. The propagation of mechanical waves, such as sound and water waves, requires a material medium. Electromagnetic waves, such as visible light and radio waves, can travel through a **vacuum,** which is a region of empty space.

Pulses and Periodic Waves

A wave may be classified as either a pulse or a periodic wave. A **pulse** is a single short disturbance that moves from one position to another in a field or medium. For example, a pulse produced on a stretched rope moves horizontally along the rope, as shown in Figure 5-1.

The speed of a pulse depends upon the type and properties of the medium. Pulse speed is constant if the medium is a uniform material with the same properties throughout. If the pulse reaches an interface or boundary of a new medium, part of the pulse is transmitted through the new medium, part is absorbed, and part is reflected back to the source. **Reflection** is the rebounding of a pulse or wave as it strikes a barrier.

Ceiling tiles, draperies, and carpeting help minimize noise levels in a room. These irregularly shaped surfaces absorb some of the energy of sound waves that strike them. The reflected sound waves have less energy than the original waves.

If the right end of the rope in Figure 5-1 was attached to a fixed unyielding body, such as a wall, the pulse would be completely reflected. None of the wave energy would be absorbed or transmitted. The reflected pulse, however, would be inverted, as shown in Figure 5-2.

This inversion can be explained by Newton's third law. When the pulse in Figure 5-2 arrives at the wall, the pulse exerts an upward force on the wall. Because the wall does not move, it exerts a force of equal magnitude on the rope in the opposite direction, which is downward. This reaction force inverts the pulse just before it is reflected back through the original medium.

If the initial disturbance that causes a pulse is repeated regularly, without interruption or change, a series of regular, evenly timed disturbances in the medium is produced. This series of regularly repeated disturbances of a field or medium is called a **periodic wave.**

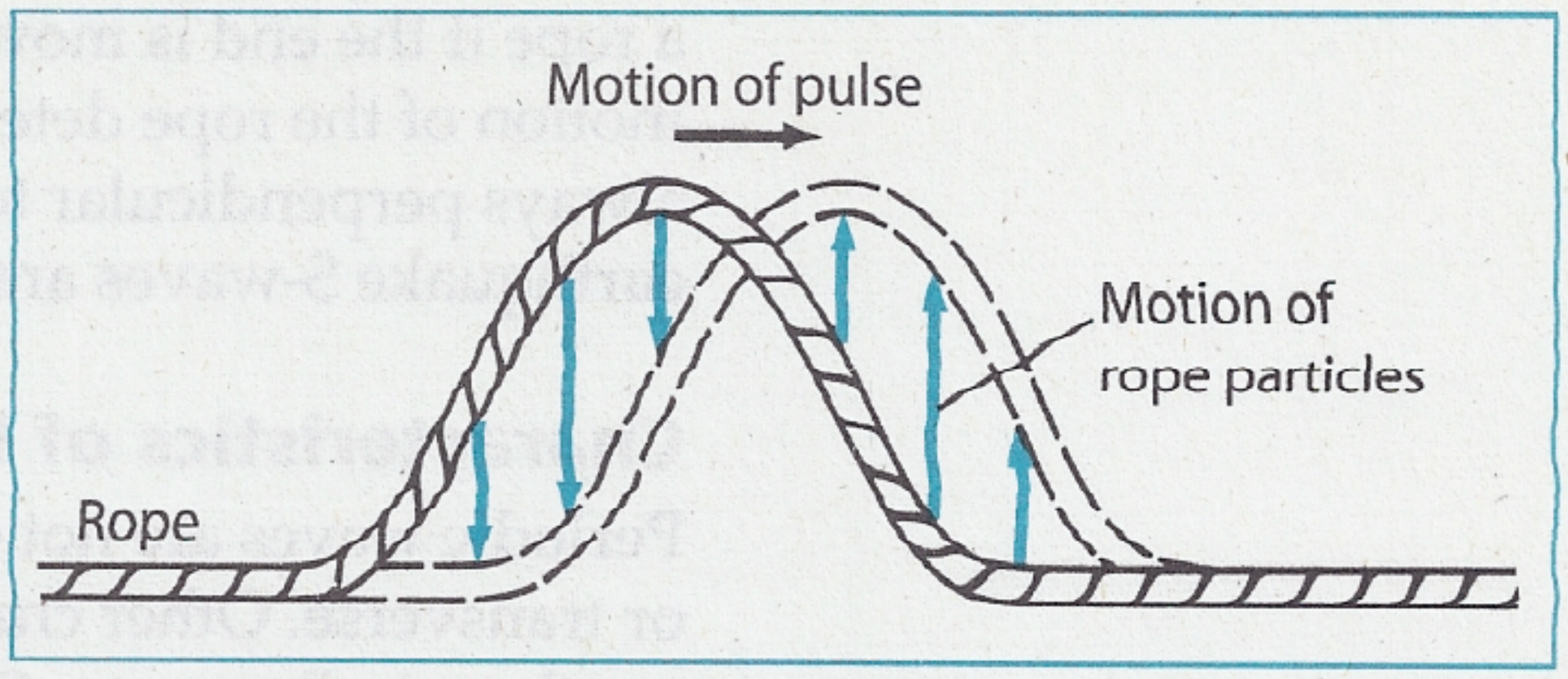


Figure 5-1. A pulse on a rope: A pulse is a single vertical disturbance transmitted horizontally at a definite speed.

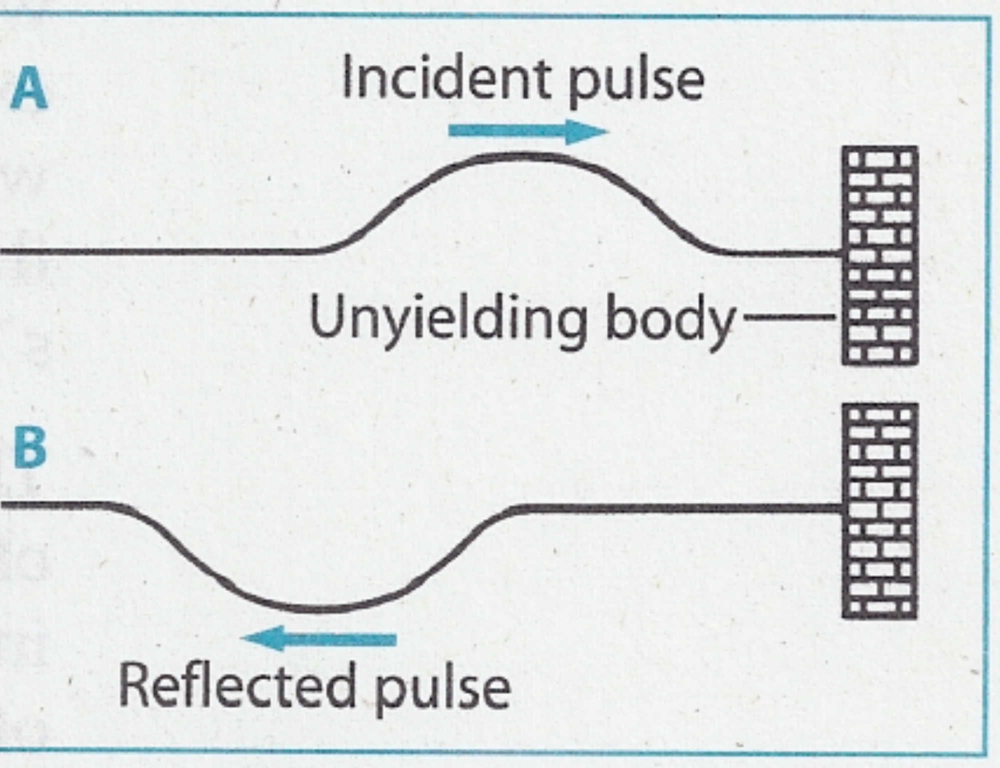


Figure 5-2. A pulse is reflected and inverted: (A) A wave pulse travels to the right along a rope attached to a brick wall. (B) When the pulse reaches the wall, it is reflected back toward the left in an inverted position.

Types of Wave Motion

A wave in which the motion of the vibratory disturbance is parallel to the direction of propagation or travel of the wave through the medium is called a **longitudinal wave.** Sound waves, compression waves in a spring, and earthquake P-waves are examples of longitudinal waves. A longitudinal wave is represented in Figure 5-3. Notice that the arrows indicating direction of motion of the wave and direction of particle motion are parallel to each other.

Another type of wave, a **transverse wave,** is one in which the motion of the vibratory disturbance is perpendicular, or at right angles to the direction of travel of the wave. An easy way to remember this is that the symbol for perpendicular lines, $\perp$, is the first letter in the word transverse, T, inverted. The transverse wave shown in Figure 5-4 is produced in

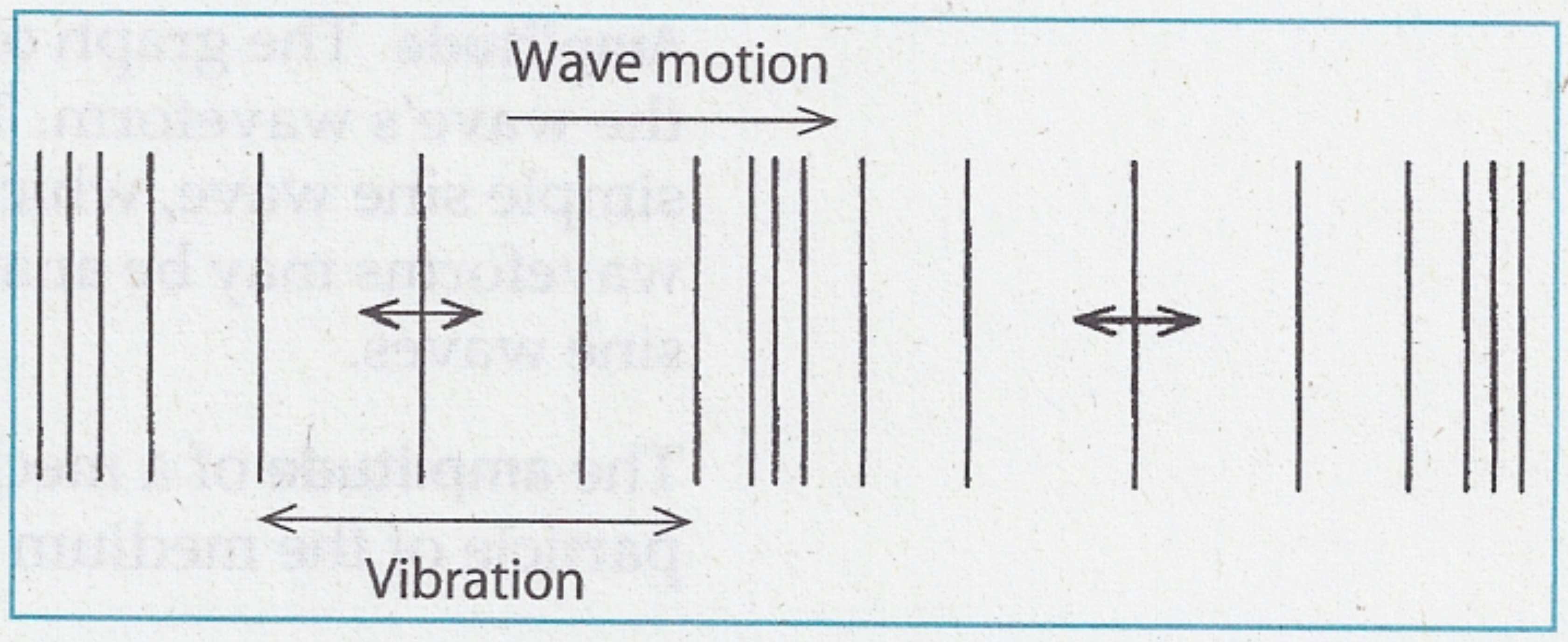


Figure 5-3. Longitudinal wave

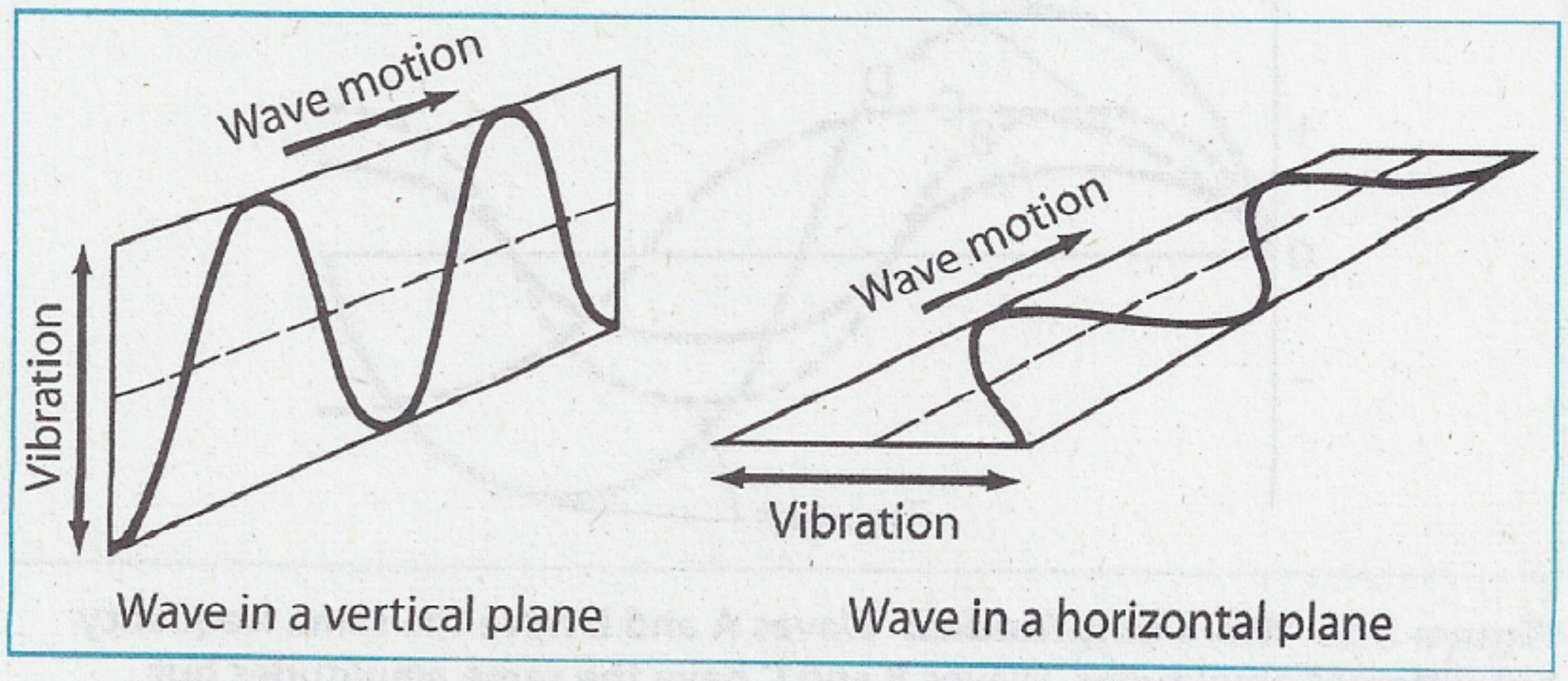


Figure 5-4. Transverse waves: These transverse waves have the same direction of travel but are in different planes.

a rope if the end is moved up and down or side to side. The direction of motion of the rope determines the plane of the wave's motion, which is always perpendicular to the rope's vibration. Electromagnetic waves and earthquake S-waves are examples of other transverse waves.

Characteristics of Periodic Waves

Periodic waves are not described solely by their type, such as longitudinal or transverse. Other characteristics distinguish an individual wave from another similar wave. Some of these characteristics are described below.

Frequency The complete series of changes at one point in a medium as a wave passes is called a cycle. The number of cycles, or complete vibrations, experienced at each point per unit time is called the **frequency,** f, of the wave. A frequency of 1 cycle per second is called 1 **hertz.** The hertz, Hz, is the derived SI unit of frequency. In fundamental units, 1 Hz equals 1/s, or s^{-1}, which can be read as *per second.*

The frequency of a sound wave determines its pitch, whereas the frequency of a light wave determines its color. The human ear can detect frequencies in the range of 20 to 20,000 hertz, and the human eye perceives frequencies of approximately 3.84×10^{14} to 7.69×10^{14} hertz.

Period The time required for one complete vibration to pass a given point in the medium is called the **period** of the wave and is denoted by T. Note that this is a capital letter. The period of a periodic wave is inversely proportional to frequency and is given by this formula.

Ⓡ

$$T = \frac{1}{f}$$

The period T is in seconds and the frequency f is in hertz or per second. The second, s, is the SI unit for period.

Amplitude The graph of the displacement of a wave versus time is called the wave's waveform. The discussion that follows treats only the relatively simple sine wave, which has the shape of a sine curve. All complex waveforms may be analyzed in terms of the interactions of many different sine waves.

The **amplitude** of a mechanical wave is the maximum displacement of a particle of the medium from its rest or equilibrium position. The amplitude of a wave in a field is the maximum change in the field strength from its normal value.

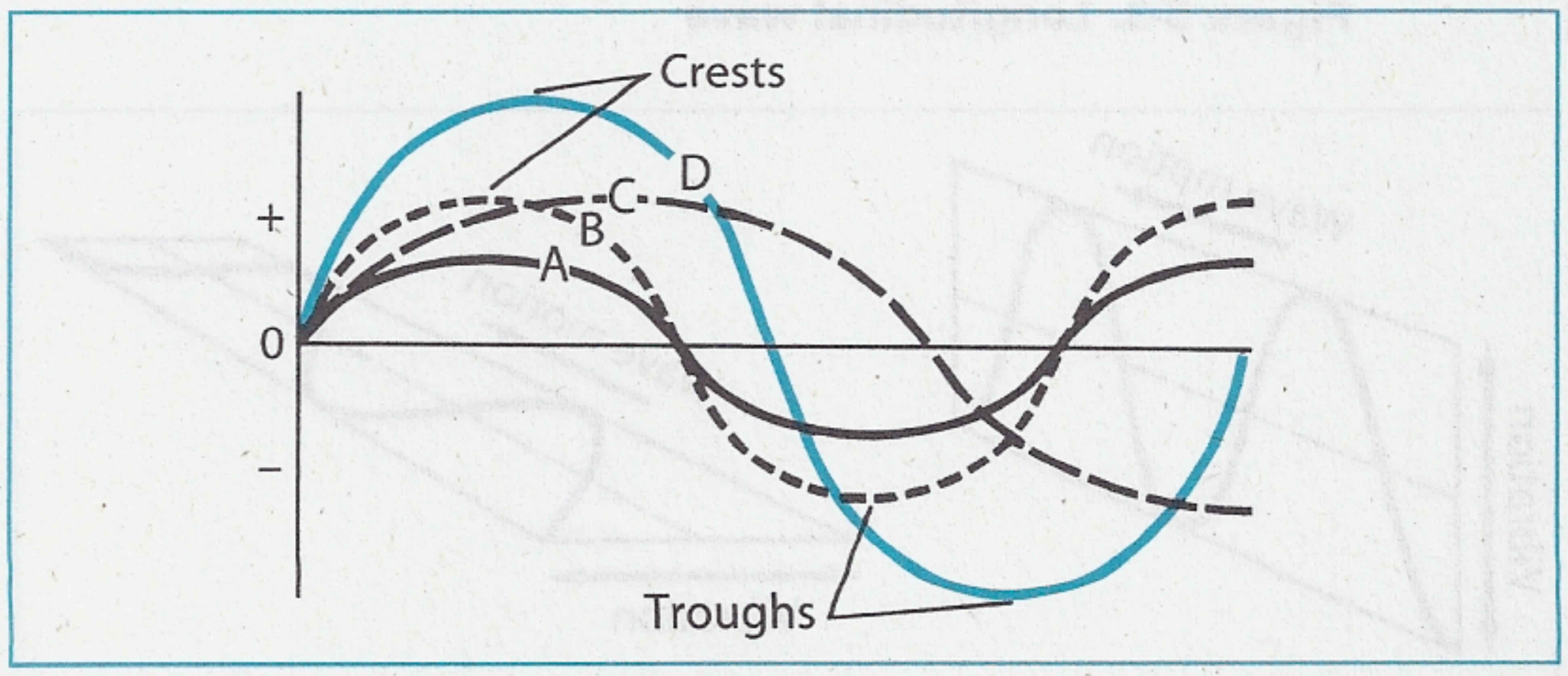


Figure 5-5. **Wave amplitudes: Waves A and B have the same frequency but different amplitudes. Waves B and C have the same amplitudes but different frequencies. Wave D has the greatest amplitude of the four waves.**

In a transverse wave, the position of maximum displacement of a particle of the medium in the positive direction (for example, upward) is called a crest. The position of maximum displacement in the negative direction (downward) is called a trough. The greater the amplitude of the wave, the higher the crests and the lower the troughs. Transverse waves of various amplitudes are shown in Figure 5-5.

In a longitudinal wave, the periodic displacements of the particles of the medium produce regions of maximum compression called condensations that alternate with regions of maximum expansion called rarefactions. The greater the amplitude of the wave, the greater the compression of the particles in the condensations and the greater the separation of the particles in the rarefactions. Figure 5-6 shows condensations and rarefactions in a longitudinal wave.

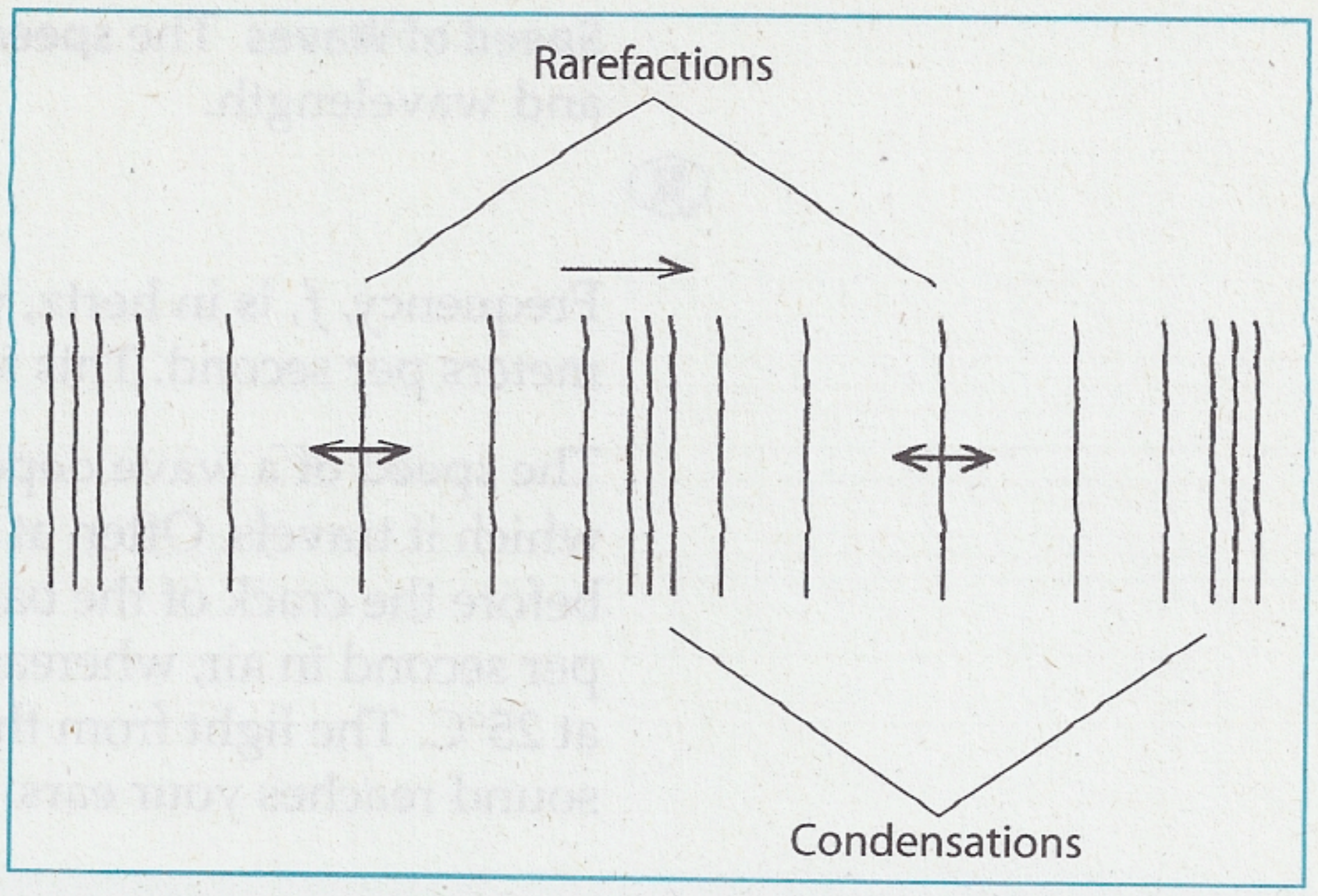


Figure 5-6. Condensations and rarefactions of a longitudinal wave

The amplitude of a wave is related to the amount of energy it transmits. The greater the amplitude of a light wave, the greater the light intensity or brightness. The greater the amplitude of a sound wave, the louder the sound. The amplitude of a sound wave is not related to its frequency or pitch.

Phase Points on successive wave cycles of a periodic wave that are displaced from their rest position by the same amount in the same direction and are moving in the same direction (away from or towards their rest positions) are said to have the same **phase,** or to be "in phase" with each other. For example, in a transverse wave, all the wave crests are in phase. In Figure 5-7 points A and E are in phase, B and F are in phase, and C and G are in phase.

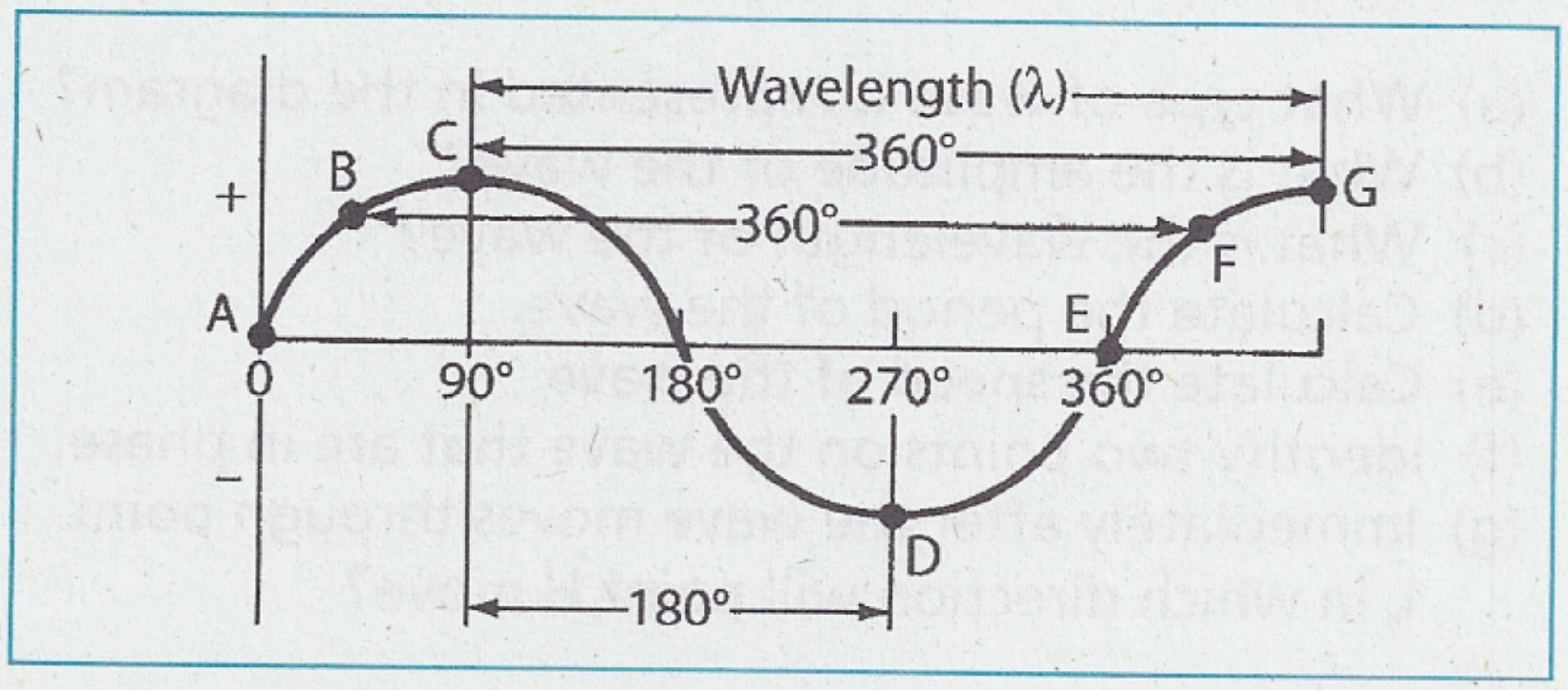


Figure 5-7. Phase relations in a wave

A simple way to determine if two points on a wave are in phase is to picture cutting out a template of the waveform between the points. If the template can be lifted, placed adjacent to one of the points, and traced without interruption to make the original sine waveform, the points are in phase.

Because there are 360° in a complete circle, one complete cycle of a periodic wave is often represented as equal to 360°. One half-cycle is then 180°. Points on a wave that are 180° apart are said to be "out of phase." In Figure 5-7, points C and D are out of phase.

Wavelength The distance between any two successive points in phase with one another in a periodic wave is called the **wavelength** of the wave. In Figure 5-7, the distance between points C and G, B and F, and A and E is one wavelength. Wavelength is represented by the symbol λ and is measured in units of length, such as meters and nanometers. If two points on a transverse wave are 180° out of phase, the distance between them is one-half wavelength or $\frac{1}{2}\lambda$.

The wavelength of a transverse wave is often measured between successive crests or troughs. The wavelength of a longitudinal wave is measured between successive condensations or rarefactions.

Speed of Waves *The* ***speed*** *of a wave is equal to the product of its frequency and wavelength.*

$$v = f\lambda$$

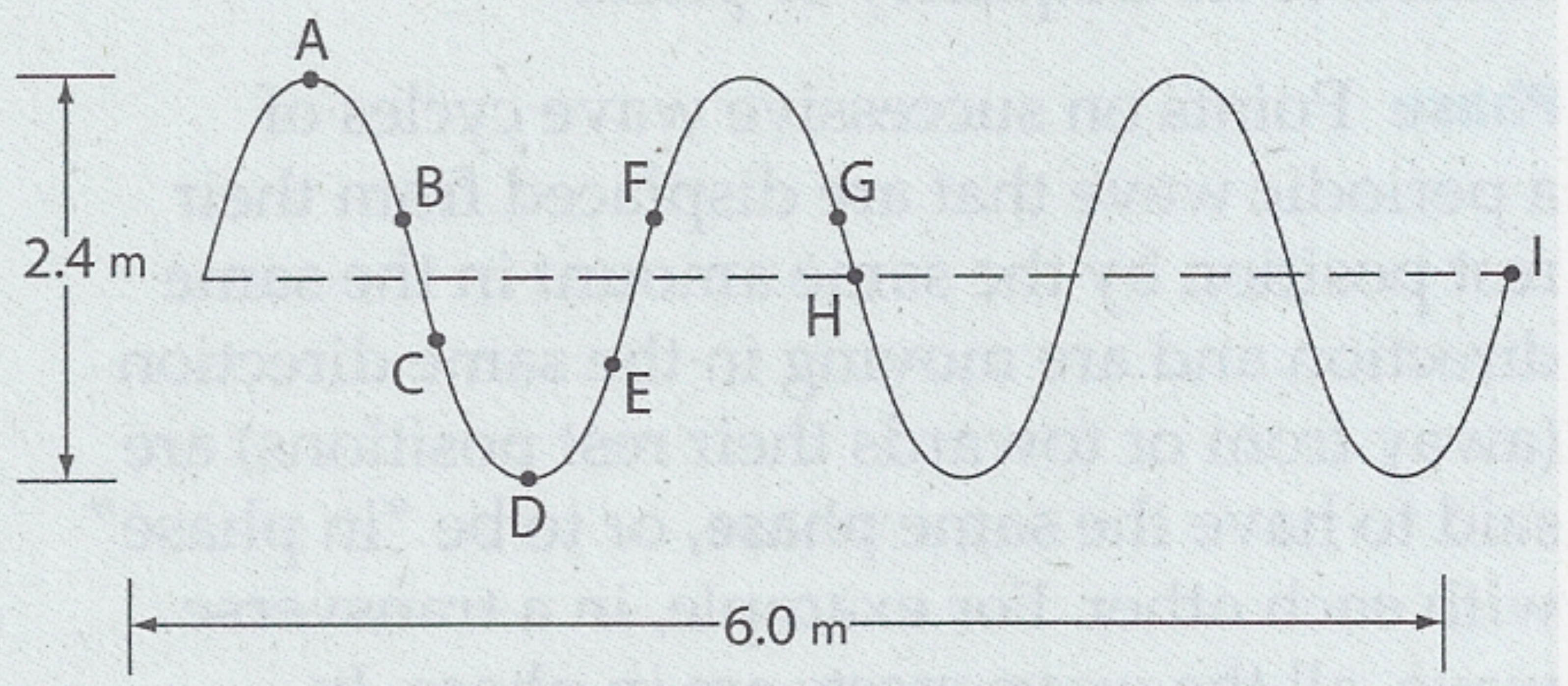
Frequency, f, is in hertz, wavelength, λ, is in meters, and speed, v, is in meters per second. This formula is valid for all waves in all media.

The speed of a wave depends upon its type and the medium through which it travels. Often at baseball games the bat is *seen* hitting the ball before the crack of the bat is *heard*. Why? Light travels at 3.00×10^8 meters per second in air, whereas sound travels only 346 meters per second in air at 25°C. The light from the bat hitting the ball reaches your eyes before the sound reaches your ears.

SAMPLE PROBLEM

The diagram shows a segment of a periodic wave in a spring traveling to the right to point I. The frequency of the wave is 2.0 hertz.

(a) What type of wave is represented in the diagram?
(b) What is the amplitude of the wave?
(c) What is the wavelength of the wave?
(d) Calculate the period of the wave.
(e) Calculate the speed of the wave.
(f) Identify two points on the wave that are in phase.
(g) Immediately after the wave moves through point I, in which direction will point H move?



SOLUTION:

(a) The particles of the medium vibrate perpendicular to the direction of wave motion. Thus, the wave is transverse.

(b) The at-rest position is represented by the horizontal dashed line. Displacement is the vertical distance from the at-rest position to the curve. Therefore, the maximum displacement is $\frac{1}{2}$ the vertical height of the diagram or 1.2 m.

(c) Three complete wavelengths are shown. Divide the given length by 3.

$$\lambda = \frac{6.0\text{ m}}{3} = 2.0\text{ m}$$

(d) Use the formula for the period $T = \frac{1}{f}$. Substitute the known values and solve.

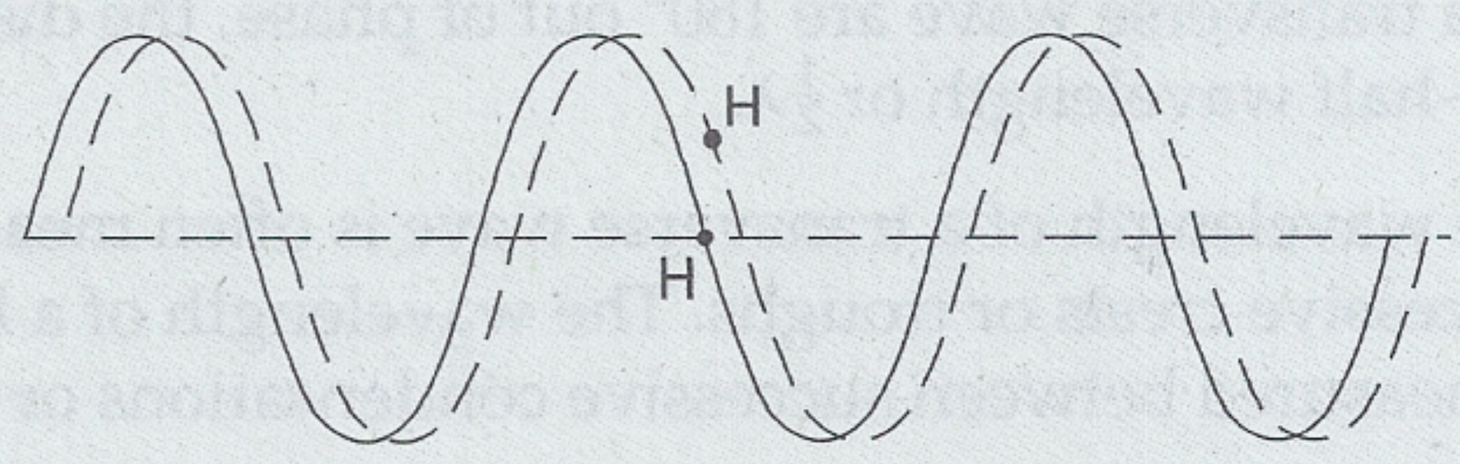
$$T = \frac{1}{f} = \frac{1}{2.0\text{ Hz}} = 0.50\text{ s}$$

(e) Write the formula for the speed of a wave $v = f\lambda$. Substitute the known values and solve.

$$v = f\lambda = (2.0\text{ Hz})(2.0\text{ m}) = 4.0\text{ m/s}$$

(f) Notice that points B and C are moving in the same direction and are the same distance from the at-rest position of the medium, but they do not have the same displacement and thus are out of phase. Points B and F have the same displacement from the at-rest position, but are moving in opposite directions, up and down, respectively, and therefore are out of phase. Points B and G are in phase because they have the same displacement and are moving in the same direction. Points B and G are separated by a distance of one wavelength.

(g) The dashed line in the diagram below shows how the entire waveform would appear in the next instant of time. Point H moves up.



Review Questions

1. A single vibratory disturbance that moves from point to point in a medium is called
(1) a node
(2) a periodic wave
(3) an antinode
(4) a pulse

2. What generally occurs when a pulse reaches a boundary between two different media?
(1) All of the pulse is reflected.
(2) All of the pulse is absorbed.
(3) All of the pulse is transmitted.
(4) Part of the pulse is reflected, part is absorbed, and part is transmitted.

3. A tuning fork vibrating in air produces sound waves. These waves are best classified as
(1) transverse, because the air molecules are vibrating parallel to the direction of wave motion
(2) transverse, because the air molecules are vibrating perpendicular to the direction of wave motion
(3) longitudinal, because the air molecules are vibrating parallel to the direction of wave motion
(4) longitudinal, because the air molecules are vibrating perpendicular to the direction of wave motion

4. When a transverse wave moves through a medium, what is the action of the particles of the medium?
(1) They travel through the medium with the wave.
(2) They vibrate in a direction parallel to the direction in which the wave is moving.
(3) They vibrate in a direction perpendicular to the direction in which the wave is moving.
(4) They remain at rest.

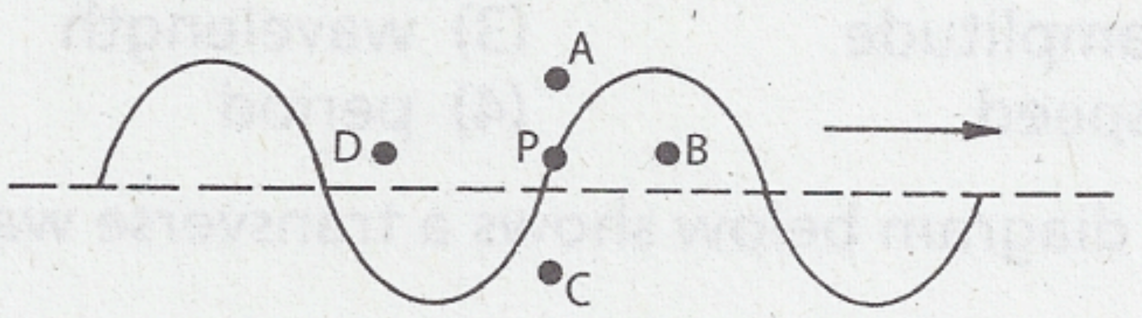
5. Compression waves in a spring are an example of
(1) longitudinal waves
(2) transverse waves
(3) elliptical waves
(4) torsional waves

6. Wave motion in a medium transfers
(1) energy only
(2) mass only
(3) both energy and mass
(4) neither energy nor mass

7. Periodic waves are produced by a wave generator at the rate of one wave every 0.50 second. What is the period of the wave?

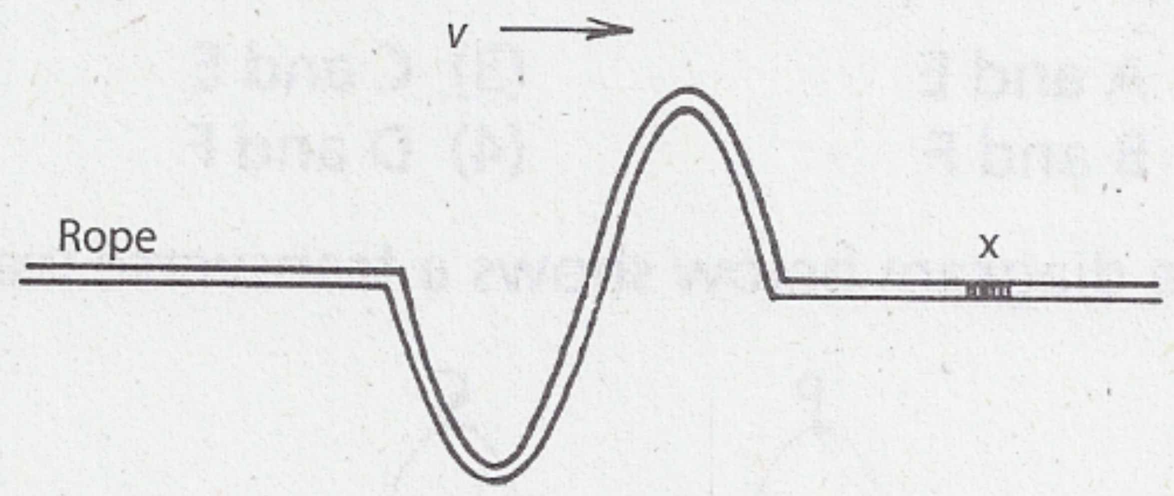
8. Which phrase best describes a periodic wave?
(1) a single pulse traveling at constant speed
(2) a single pulse traveling at varying speed in the same medium
(3) a series of pulses at irregular intervals
(4) a series of pulses at regular intervals

9. In the diagram below, the solid line represents a wave generated in a rope.



As the wave moves to the right, point P on the rope is moving towards which position?
(1) A (2) B (3) C (4) D

10. In the diagram below, a transverse wave is moving to the right on a rope.



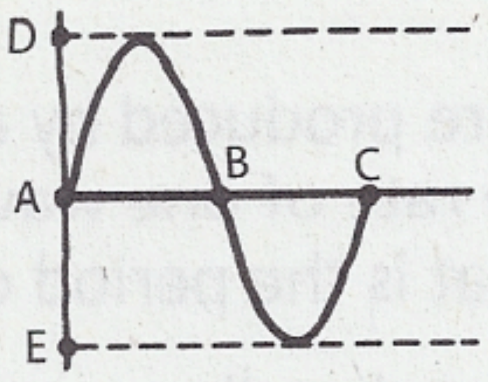
In which direction will segment x move as the wave passes through it?
(1) down, only
(2) up, only
(3) down, then up, then down
(4) up, then down, then up

11. Which wave characteristic is defined as the number of cycles of a periodic wave occurring per unit time?

12. If the frequency of a sound wave is 440. cycles per second, the period of the wave is
(1) 2.27×10^{-3} s
(2) 0.752 s
(3) 1.33 s
(4) 3.31×10^{2} s

13. If the frequency of a sound wave is doubled, the period of the sound wave is
(1) halved
(2) doubled
(3) unchanged
(4) quadrupled

14. The diagram below represents a transverse wave.



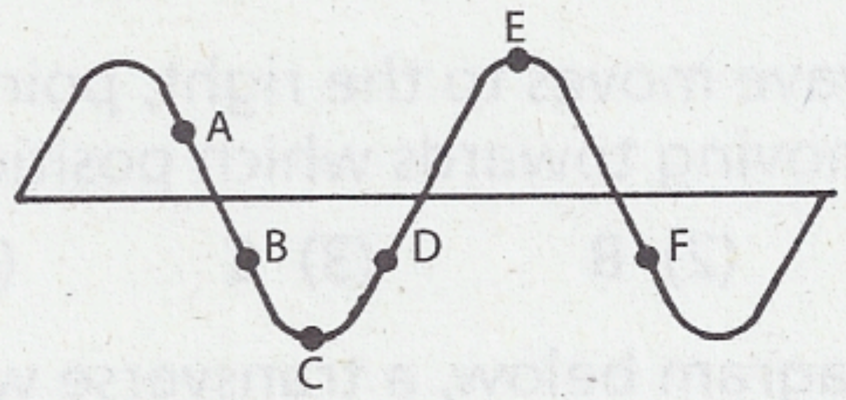
The amplitude of the wave is represented by the distance between points

(1) A and B (3) A and D
(2) A and C (4) D and E

15. If the frequency of a sound wave in air at STP remains constant, the wave's energy can be varied by changing its

(1) amplitude (3) wavelength
(2) speed (4) period

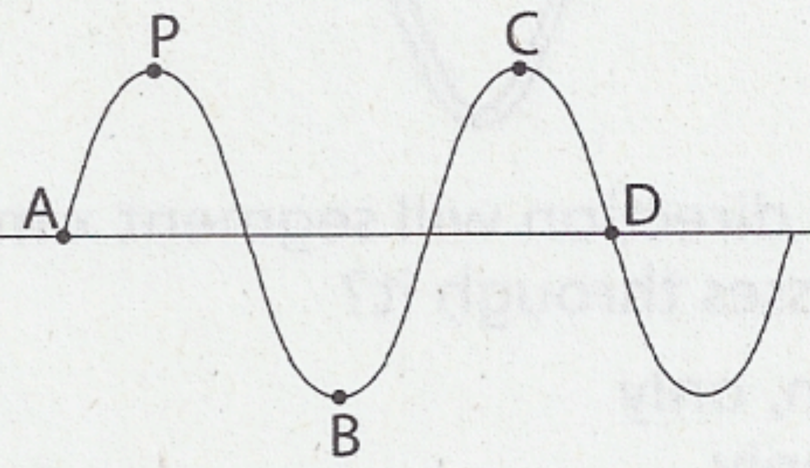
16. The diagram below shows a transverse wave.



Which two points on the wave are in phase?

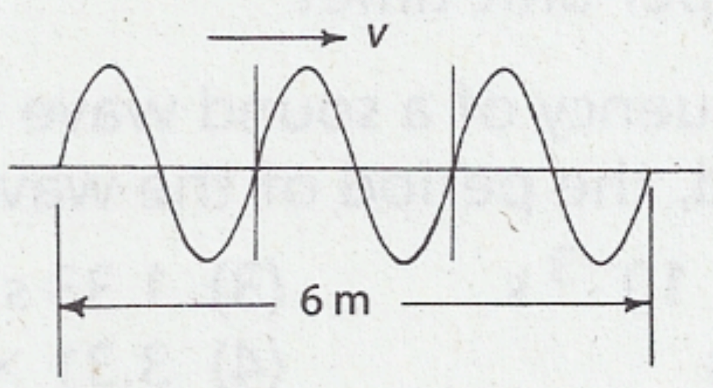
(1) A and E (3) C and E
(2) B and F (4) D and F

17. The diagram below shows a transverse wave.



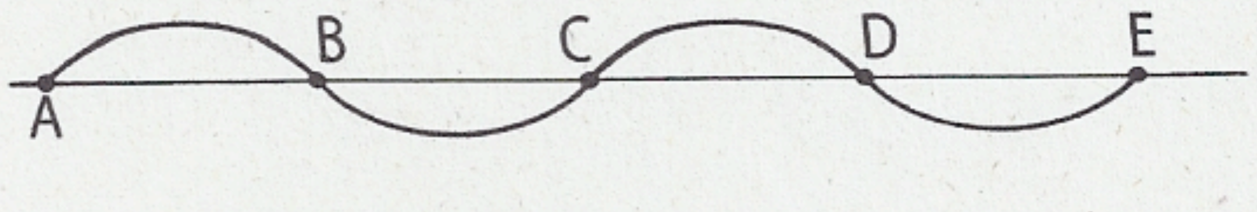
Which point on the wave is 180° out of phase with point P?

18. The diagram that follows shows a train of waves moving along a string.



What is the wavelength?

19. The wavelength of the periodic wave shown in the diagram below is 4.0 meters.

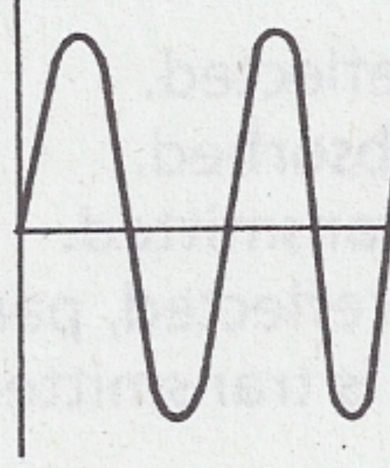


What is the distance from point B to point C?

20. An 8.0-meter long ocean wave passes the end of a dock every 5.0 seconds. Calculate the speed of the wave.

21. A sound wave travels at 340 meters per second. Determine how far from the source the wave has traveled after 0.50 second.

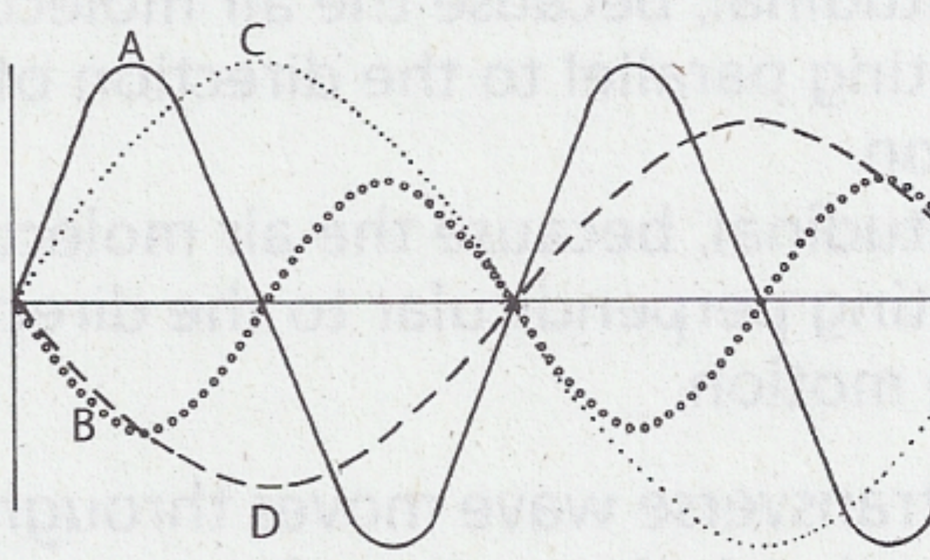
22. The diagram below represents a wave traveling in a uniform medium.



Which characteristic of the wave is constant?

(1) amplitude (3) period
(2) frequency (4) wavelength

Base your answers to questions 23 through 25 on the diagram below, which represents four transverse waves in the same medium.



23. Which two waves have the same amplitude?

24. Which two waves have the same wavelength?

25. Which two waves have the same frequency?

26. A wave has a frequency of 2.0 hertz and a speed of 3.0 meters per second. The distance covered by the wave in 5.0 seconds is

(1) 30. m (2) 15 m (3) 7.5 m (4) 6.0 m

27. A wave traveling at 5.00×10^4 meters per second has a wavelength of 2.50×10^1 meters. What is the frequency of the wave?

(1) 5.00×10^{-4} Hz (3) 5.00×10^3 Hz
(2) 2.00×10^3 Hz (4) 1.25×10^6 Hz

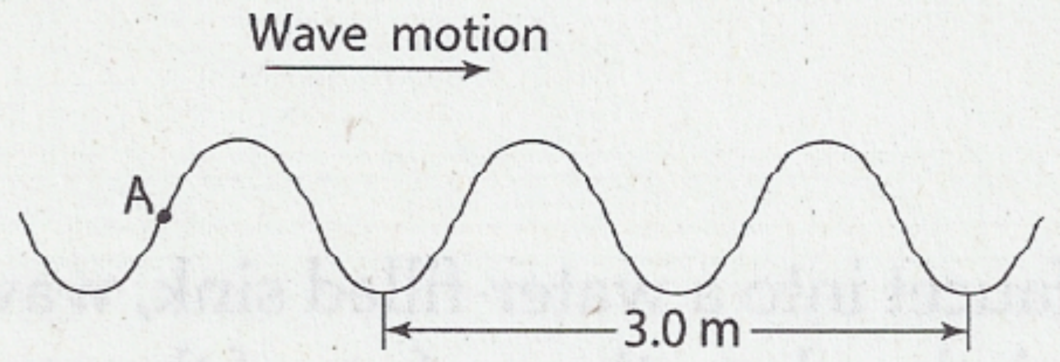
28. Sound waves with constant frequency of 250 hertz are traveling through air at STP. Calculate the wavelength of the sound waves.

29. Calculate the total distance a sound wave travels in air at STP in 3.00 seconds.

30. What type of wave is sound traveling in water?

Base your answers to questions 31 through 34 on the information and diagram below.

A periodic wave, having a frequency of 40. hertz, travels to the right in a uniform medium as shown.



31. On the diagram, draw one or more arrows to indicate the direction of motion of point A in the next instant of time.

32. On the diagram, label a point P that is in phase with point A.

33. Calculate the speed of the wave.

34. Calculate the period of the wave.

Base your answers to questions 35 and 36 on the information below.

The elapsed time between successive crests of a transverse wave passing a given point is 0.080 second.

35. Determine the period of the wave.

36. Calculate the frequency of the wave.

Base your answers to questions 37 through 39 on the information below.

The distance from one crest of a water wave to the next crest is 4.0 meters. One crest passes an observation point every 2.5 seconds.

37. Calculate the speed of the wave.

38. Calculate the time required for the wave to travel 50. meters.

39. Determine the total distance the wave travels in 4.0 seconds.

40. Write an equation that correctly relates the speed v, wavelength λ, and period T of a periodic wave.

41. A wave x meters long passes through a medium at y meters per second. The frequency of the wave could be expressed as

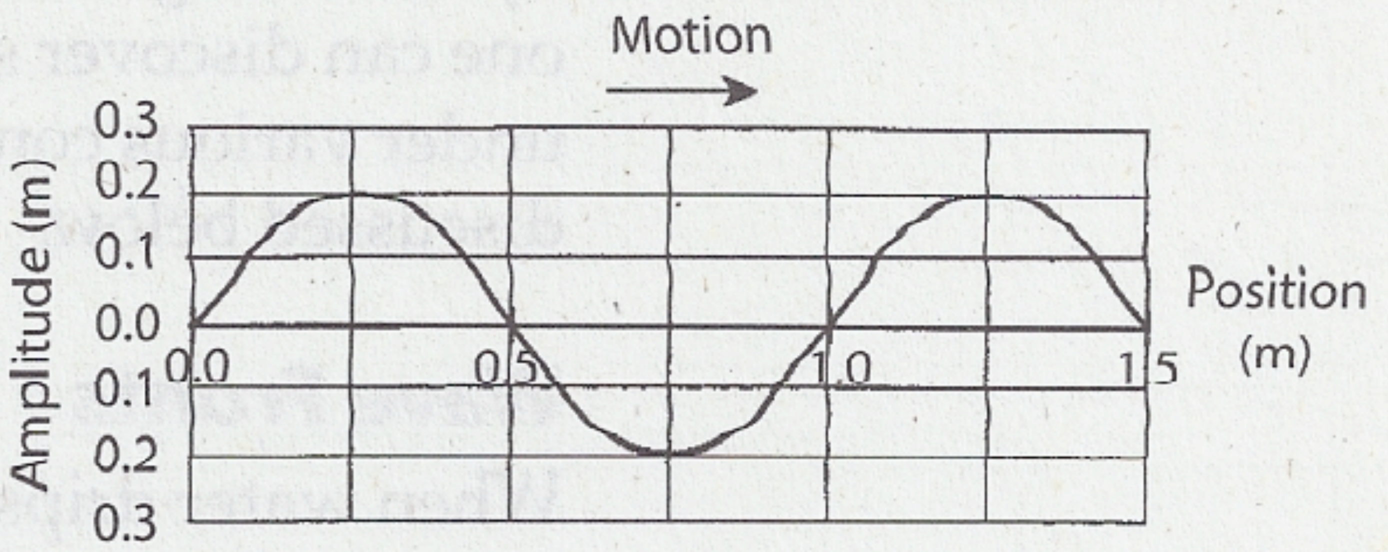
(1) $\frac{y}{x}$ Hz (3) xy Hz

(2) $\frac{x}{y}$ Hz (4) $(x + y)$ Hz

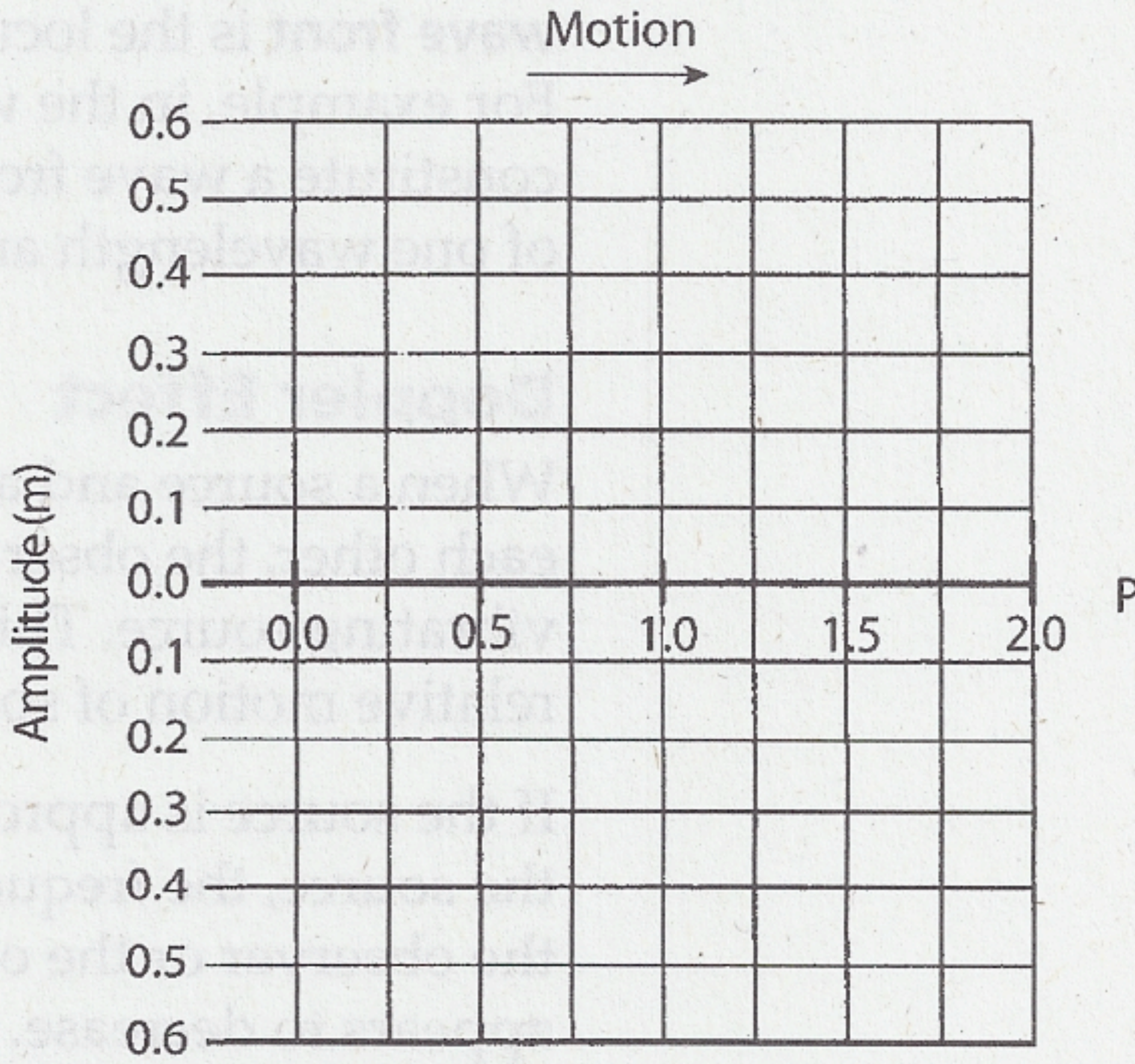
42. A unit for the amplitude of a transverse wave is

(1) m/s (2) s (3) Hz (4) m

43. The diagram below shows a periodic wave W traveling to the right in a uniform medium.



On the grid below sketch at least one cycle of a periodic wave having twice the amplitude and half the wavelength of wave W.



44. A sound wave is produced by a musical instrument for 0.40 second. If the frequency of the wave is 370 hertz, how many complete waves are produced in that time period?

45. If the frequency of a sound wave increases, the wavelength of the wave in air

(1) decreases (2) increases (3) remains the same

46. Which phrase best describes the wavelength of a sound wave in air at STP?

(1) inversely proportional to its amplitude and inversely proportional to its frequency
(2) inversely proportional to its amplitude and directly proportional to its frequency
(3) independent of its amplitude and inversely proportional to its frequency
(4) independent of its amplitude and directly proportional to its frequency

47. A water wave travels a distance of 10.0 meters in 5.0 seconds. What can be determined from this information?

(1) the speed of the wave only
(2) the period of the wave only
(3) the speed and frequency of the wave
(4) the period and frequency of the wave

Periodic Wave Phenomena

By observing two types of mechanical waves, transverse and longitudinal, one can discover some characteristics of waves and the behavior of waves under various conditions. Some of these characteristics and behaviors are discussed below.

Wave Fronts

When water drips from a leaky faucet into a water-filled sink, waves spread, or radiate, in concentric circles along the surface of the water from the point where the drips strike the surface. In a three-dimensional medium such as air, waves radiate in concentric spheres from a vibrating point. All points on a wave that are in phase comprise a wave front. A **wave front** is the locus of all adjacent points on a wave that are in phase. For example, in the waves in the sink, all of the points on one of the crests constitute a wave front. Two successive crests are separated by a distance of one wavelength and, therefore, are in phase.

Doppler Effect

When a source and an observer (receiver) of waves are moving relative to each other, the observed frequency is different from the frequency of the vibrating source. This change in observed or apparent frequency due to relative motion of source and observer is called the **Doppler effect.**

If the source is approaching the observer, or if the observer is approaching the source, the frequency appears to increase. If the source is receding from the observer or the observer is receding from the source, the frequency appears to decrease. Because the speed of the waves in the medium is not affected by the Doppler effect, it can be seen from the formula $v = f\lambda$ that the change in apparent wavelength is inversely proportional to the change in apparent frequency.

The wave front diagrams in Figure 5-8 illustrate the changes in apparent frequency and wavelength caused by the Doppler effect. In Figure 5-8A, the source is stationary, and the four successive wave fronts (1, 2, 3, and 4) are equally spaced circles in all directions. The observed wavelength and frequency are the same for all stationary observers. In Figure 5-8B, the source is moving from right to left. Each successive wave front has a different center. To a stationary observer at the left, the wavelengths appear shorter and the frequency higher; to a stationary observer at the right, the effect is the opposite.

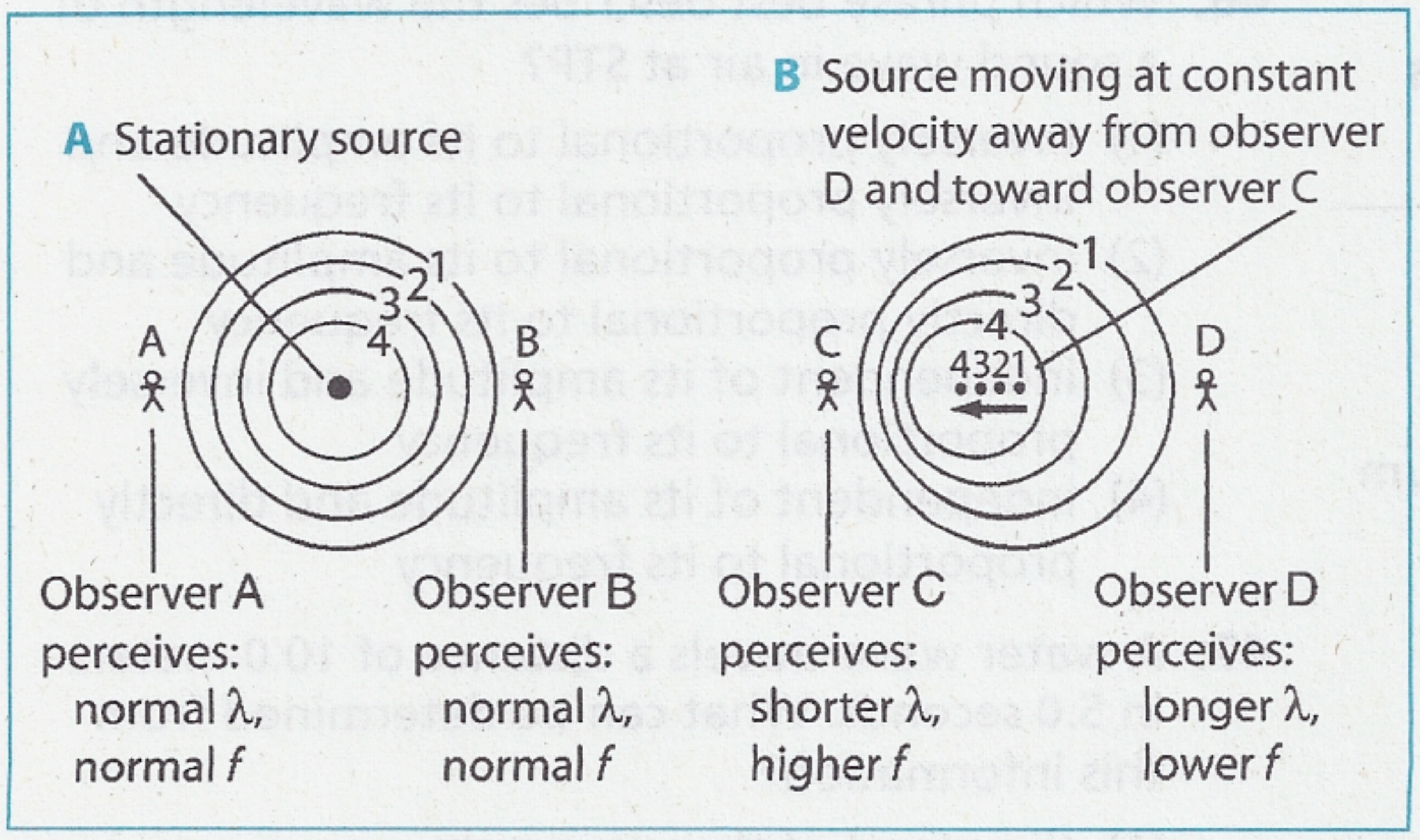


Figure 5-8. The Doppler effect: (A) When the source is stationary, the wave fronts are equally spaced in all directions. (B) When the source is moving, the wave fronts are closer together in the direction in which the source is moving.

The Doppler effect can cause changes in the apparent pitch of a sound wave because the ear perceives a sound wave of higher frequency as a sound of higher pitch. Thus the pitch of an approaching sound source is higher than its pitch when the source is stationary, and the pitch drops lower as the source passes the observer and begins to recede.

Visible light waves are subject to a similar effect. The human eye perceives light waves of different frequencies as differences in color. Light waves of the lowest frequency (longest wavelength) that the eye can detect are seen as red, while those of highest frequency (shortest wavelength) are seen as blue-violet. Other colors are distributed between these extremes in the *visible spectrum.* Because of the Doppler effect, the apparent color of an approaching light source is shifted toward the blue-violet end of the spectrum, while that of a receding source is shifted toward the red end. If the light source is a mixture of many frequencies, such as the light from a star, its light appears slightly bluer if it is approaching an observer, or slightly redder if it is receding, than it would appear if it were not moving relative to the observer.

Applications of the Doppler Effect The Doppler effect has practical applications in weather forecasting and police work. For example, the speed of a car can be determined by a computerized radar system. If a car is at rest and a beam of radio waves is directed at the car from a stationary source, the incident and reflected waves have the same frequency. If the car is moving toward the source of the radar, however, the reflected waves have a higher frequency than the waves emitted by the source. The greater the car's speed toward the radar source, the greater the Doppler shift in frequency. In a similar way, if the car is moving away from the source of radar, the frequency of the reflected waves decreases by an amount that depends upon the speed of the car. Thus, equipped with a "radar gun," a law-enforcement officer can detect speed-limit violators "coming or going."

Interference

Superposition occurs when two or more waves travel through the same medium simultaneously. The **principle of superposition** states that the resultant displacement at any point is the algebraic sum of the displacements of the individual waves. The effect of the superposition is called **interference,** which may be constructive or destructive. Although any number of waves may superpose, the discussion that follows is restricted to two waves.

Constructive interference occurs when the wave displacements of two in-phase waves in the same medium are in the same direction. The algebraic sum of the displacements is an amplitude greater than that of either of the original waves. Maximum constructive interference occurs when the waves are in phase and crest superposes on crest. Thus, maximum constructive interference occurs when the phase difference is equal to 0°, as shown in Figure 5-9A. The point of maximum displacement of a medium when two waves are interacting is called an **antinode.**

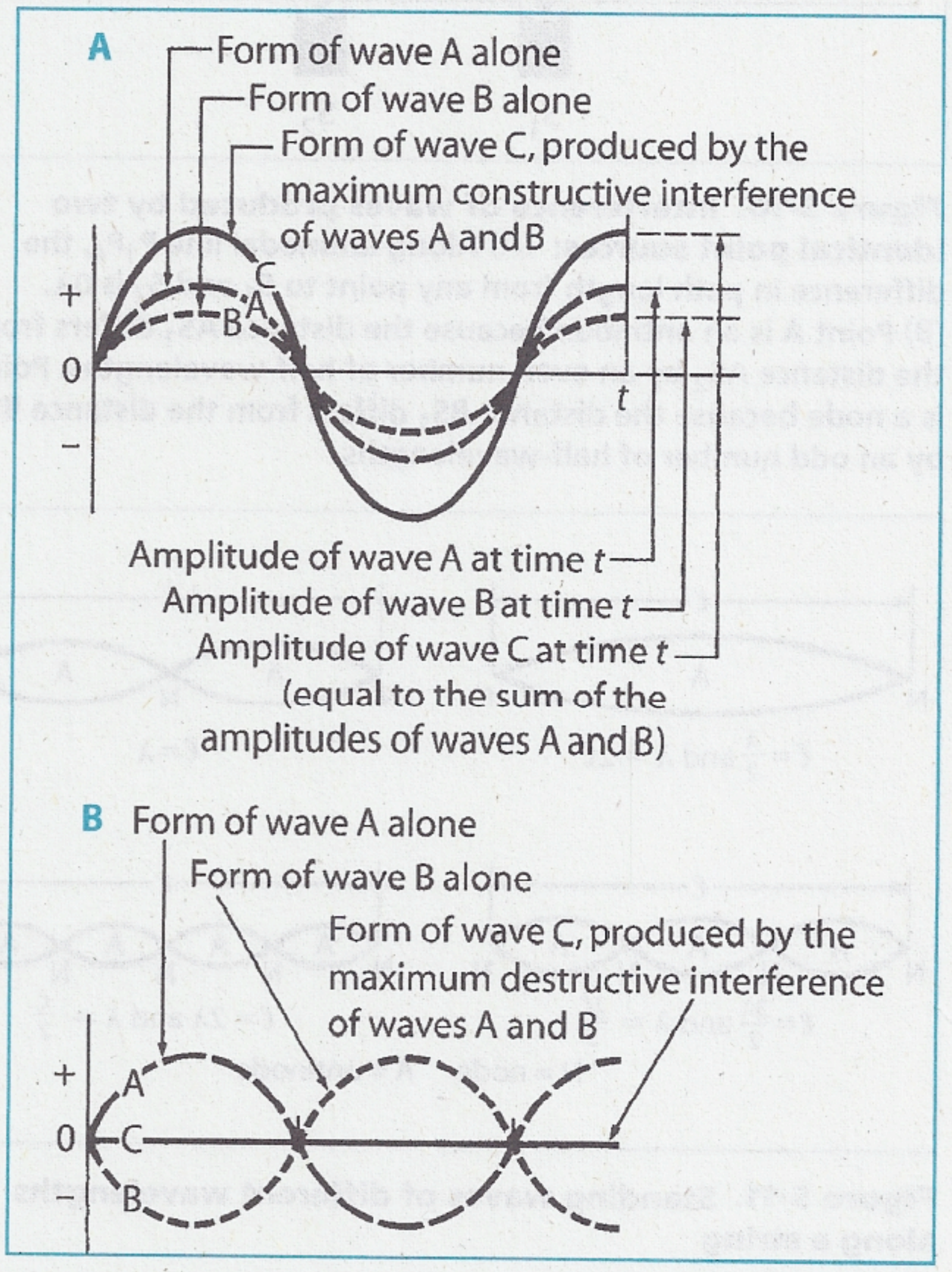


Figure 5-9. Constructive and Destructive Interference: (A) Waves A and B have the same frequency and a phase difference of 0°. As a result, they show maximum constructive interference, producing wave C. Note that the amplitudes of A and B always add up to the amplitude of C at every instant of time. This is demonstrated for the time *t* at the extreme right of the graph. (B) Waves A and B have the same frequency and the same amplitude, but a phase difference of 180°. As a result, they show maximum destructive interference. Notice that waves A and B cancel each other.

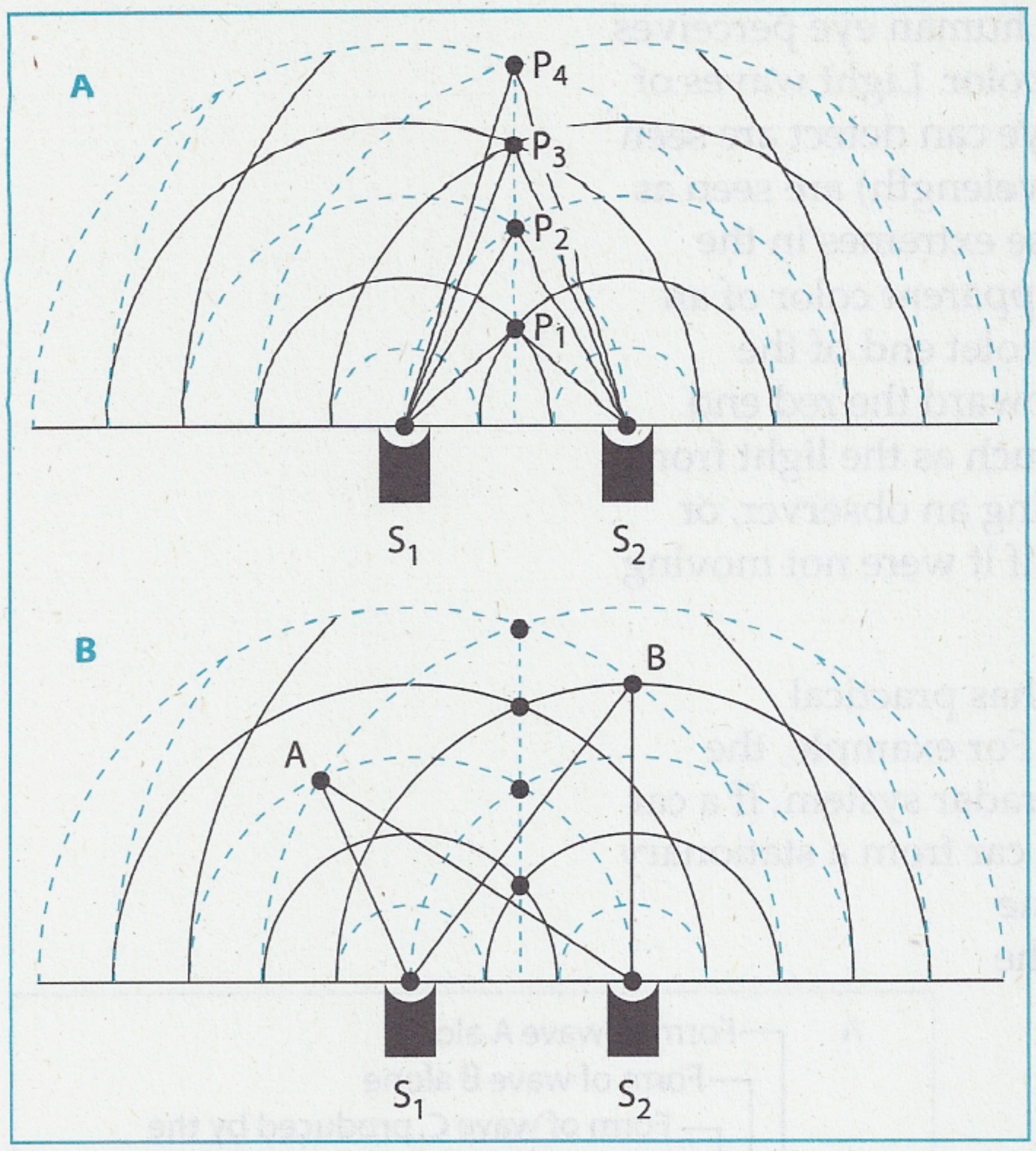


Figure 5-10. Interference of waves produced by two identical point sources: (A) Along antinodal line P_1P_4, the difference in path length from any point to S_1 and S_2 is 0λ. (B) Point A is an antinode because the distance AS_1 differs from the distance AS_2 by an even number of half-wavelengths. Point B is a node because the distance BS_1 differs from the distance BS_2 by an odd number of half-wavelengths.

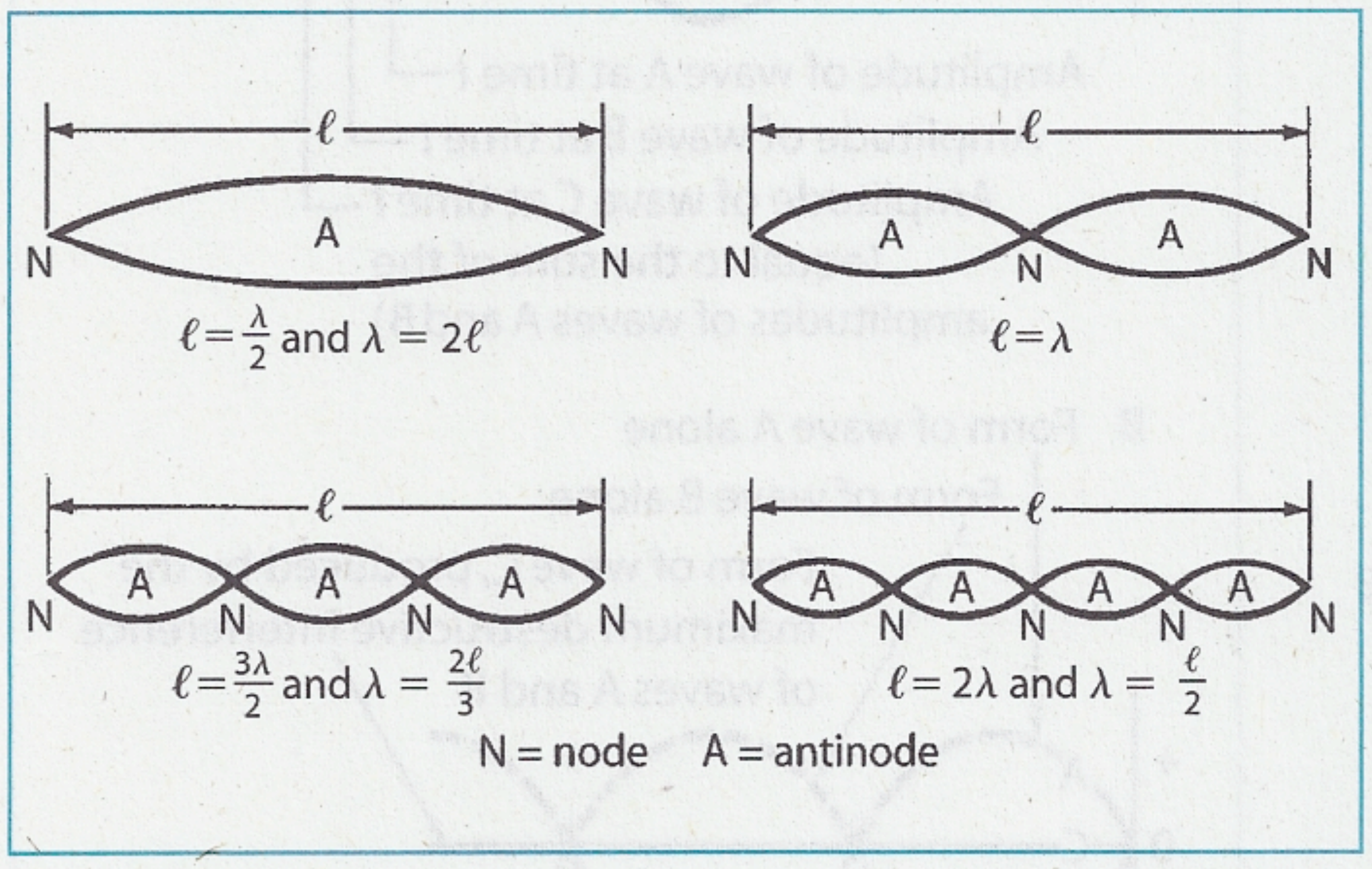


Figure 5-11. Standing waves of different wavelengths along a string

When two waves of equal frequency and amplitude whose phase difference is 180° or $\frac{1}{2}\lambda$ meet at a point (for example, crest to trough), there is maximum **destructive interference,** as shown in Figure 5-9B. Maximum destructive interference results in the formation of **nodes** (points or lines), which are regions of zero displacement of the medium. Intermediate degrees of interference occur between the regions of maximum constructive interference and maximum destructive interference.

Two Sources in Phase in the Same Medium When two in-phase point sources generate waves in the same medium, a symmetrical interference pattern results because of maximum constructive and destructive interference. Figure 5-10A shows two identical point sources, S_1 and S_2, producing wave crests (solid lines) and wave troughs (dashed lines) that interfere. The path difference from any point of constructive interference to the sources, S_1 and S_2, is an even number of half-wavelengths. For example, along antinodal line P_1P_4, the difference in path length from any point on the line to S_1 and S_2 is 0λ. In Figure 5-10B, point A is on an antinodal line because distance AS_1 differs from distance AS_2 by two half-wavelengths. On the other hand, point B is on a nodal line because distance BS_1 differs from distance BS_2 by an odd number of half-wavelengths. Nodal lines occur midway between antinodal lines.

Standing Waves

When two waves having the same amplitude and frequency travel in opposite directions through a uniform medium, a standing wave is formed. A **standing wave** is a pattern of wave crests and troughs that remains stationary in a medium. The nodes and antinodes are stationary and the wave appears to stand still. Standing waves are easily produced in a stretched string that is fixed at both ends. Wave trains traveling along the string are reflected at the ends and travel back with the same frequency and amplitude. Figure 5-11 illustrates several possible standing waves in a string. Note that a node appears at each end of the string. The distance between two successive nodes is equal to $\frac{1}{2}\lambda$.

Resonance

Every elastic body has a particular frequency called its **natural frequency** at which it will vibrate if disturbed. When a periodic force is applied to an elastic body, it absorbs energy and the amplitude of its vibration increases. The vibration of a body at its natural frequency because of the action of a vibrating source of the same frequency is called **resonance.** For example, a nonvibrating tuning fork, having a natural frequency of 512 hertz, will resonate when a vibrating tuning fork with a natural frequency of 512 hertz is brought near it. Furthermore, it is possible for an opera singer to shatter a glass by maintaining a note with a frequency equal to the natural frequency of the glass. The transfer of energy by resonance increases the amplitude of vibrations in the glass until its structural strength is exceeded. Probably the most dramatic example of resonance was the collapse of the Tacoma Narrows Bridge in the state of Washington in 1940. High winds set up standing waves in the bridge in addition to vibrations in a torsional (twisting) mode. Resonance increased the amplitude of vibrations until the bridge collapsed.

Diffraction

The spreading of waves into the region behind a barrier in the wave's path is called **diffraction.** Parallel water wave fronts incident on a small opening are diffracted to form concentric semicircular fronts. These semicircular fronts have the same wavelength as the incident wave if the medium is uniform throughout, as shown in Figure 5-12A. If the opening through which the wave is diffracted is much larger than one wavelength of the incident wave, diffraction effects are small, as shown in Figure 5-12B.

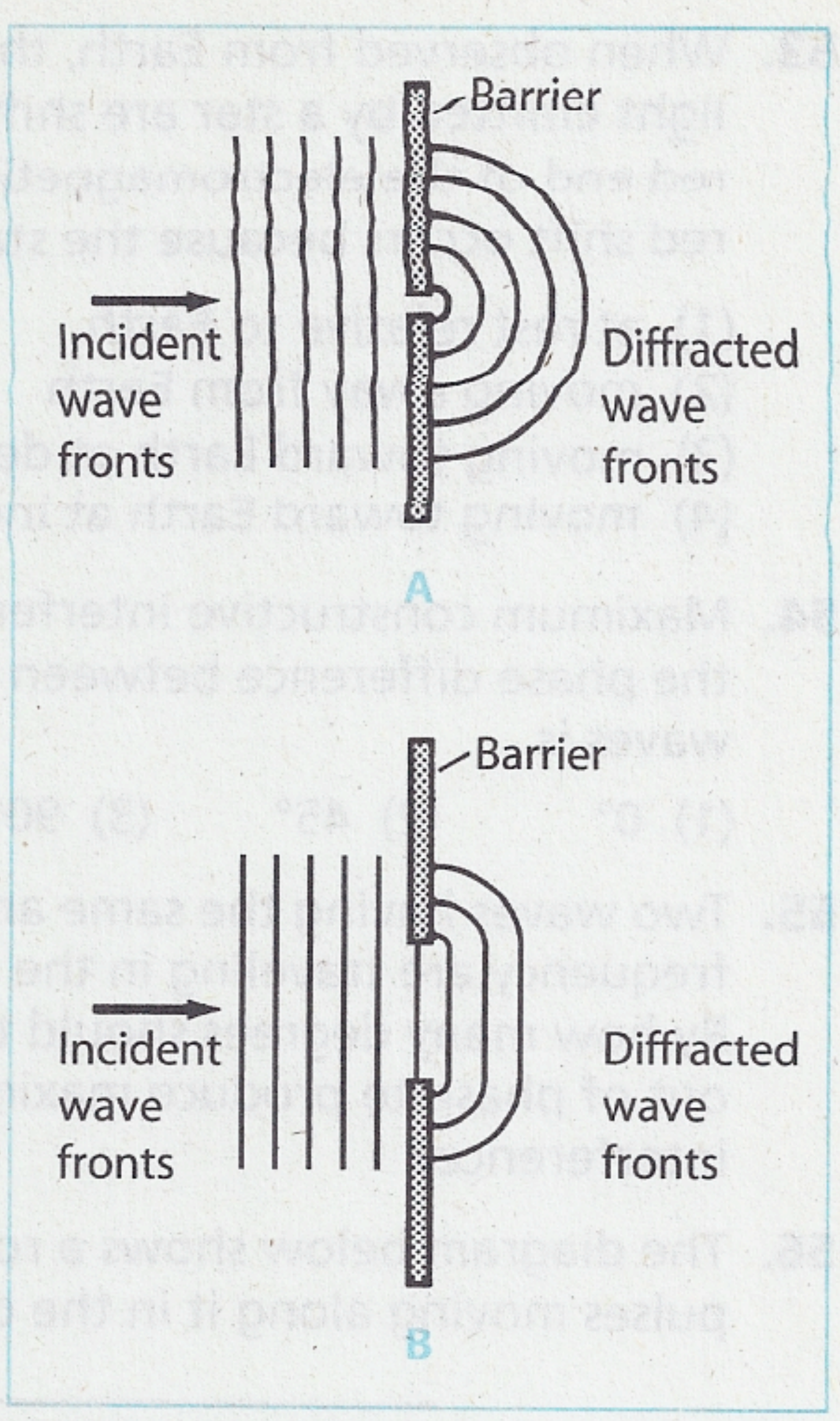


Figure 5-12. Diffraction of parallel wave fronts resulting from different sized openings in a barrier

Review Questions

48. What term describes the variations in the observed frequency of a sound wave when there is relative motion between the source and the receiver?

49. A source of waves and an observer are moving relative to each other. The observer will detect a steadily increasing frequency if

(1) he moves toward the source at a constant speed
(2) the source moves away from him at a constant speed
(3) he accelerates toward the source
(4) the source accelerates away from him

50. The driver of a car hears the siren of an ambulance that is moving away from her. If the actual frequency of the siren is 2000. hertz, the frequency heard by the driver may be

(1) 1900. Hz
(2) 2000. Hz
(3) 2100. Hz
(4) 4000. Hz

51. A police officer's stationary radar device indicates that the frequency of the radar wave reflected from an automobile is less than the frequency emitted by the radar device. This indicates that the automobile is

(1) moving toward the police officer
(2) moving away from the police officer
(3) not moving

52. A stationary person makes observations of the periodic waves produced by a moving source. When the wave source recedes from the observer, he observes an apparent increase in the wave's

(1) speed
(2) frequency
(3) wavelength
(4) amplitude

53. When observed from Earth, the wavelengths of light emitted by a star are shifted toward the red end of the electromagnetic spectrum. This red shift occurs because the star is

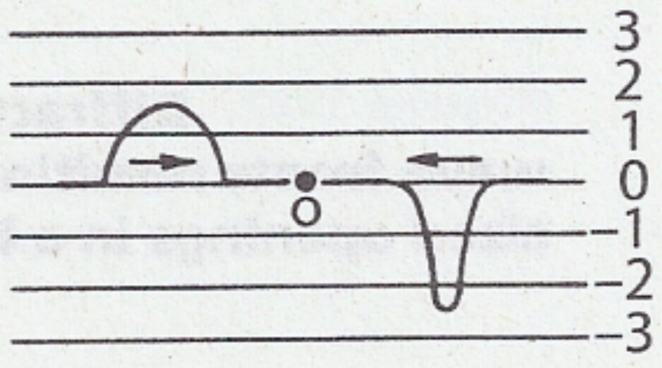
(1) at rest relative to Earth
(2) moving away from Earth
(3) moving toward Earth at decreasing speed
(4) moving toward Earth at increasing speed

54. Maximum constructive interference occurs when the phase difference between the interfering waves is

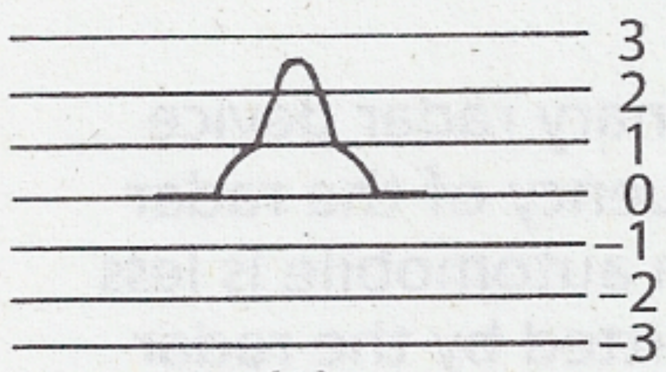
(1) 0° (2) 45° (3) 90° (4) 180°

55. Two waves having the same amplitude and frequency are traveling in the same medium. By how many degrees should the waves be out of phase to produce maximum destructive interference?

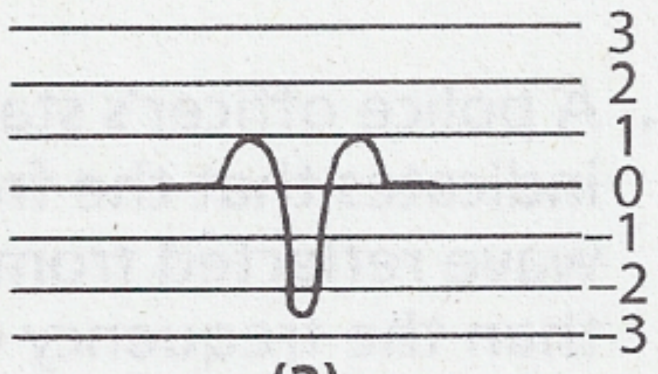
56. The diagram below shows a rope with two pulses moving along it in the directions shown.



What is the resultant wave pattern at the instant when the maximum displacement of both pulses is at point O on the rope?



(1)



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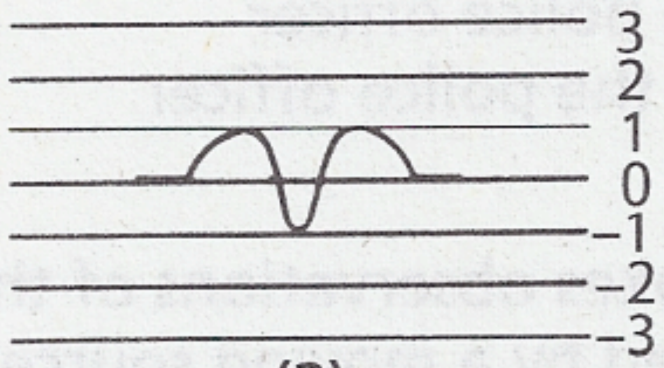

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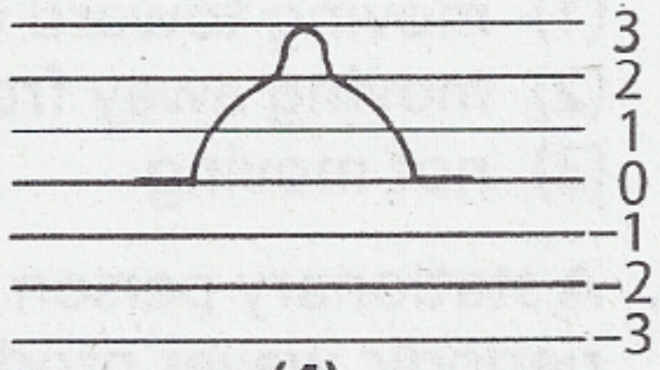
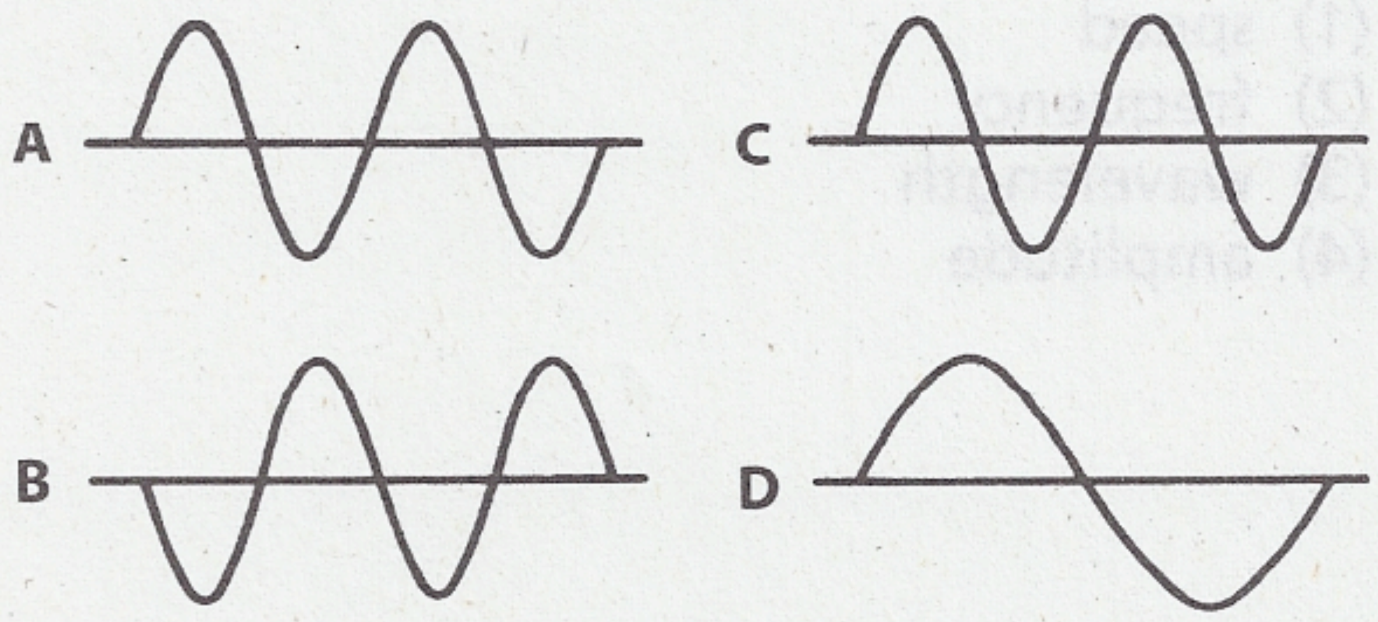


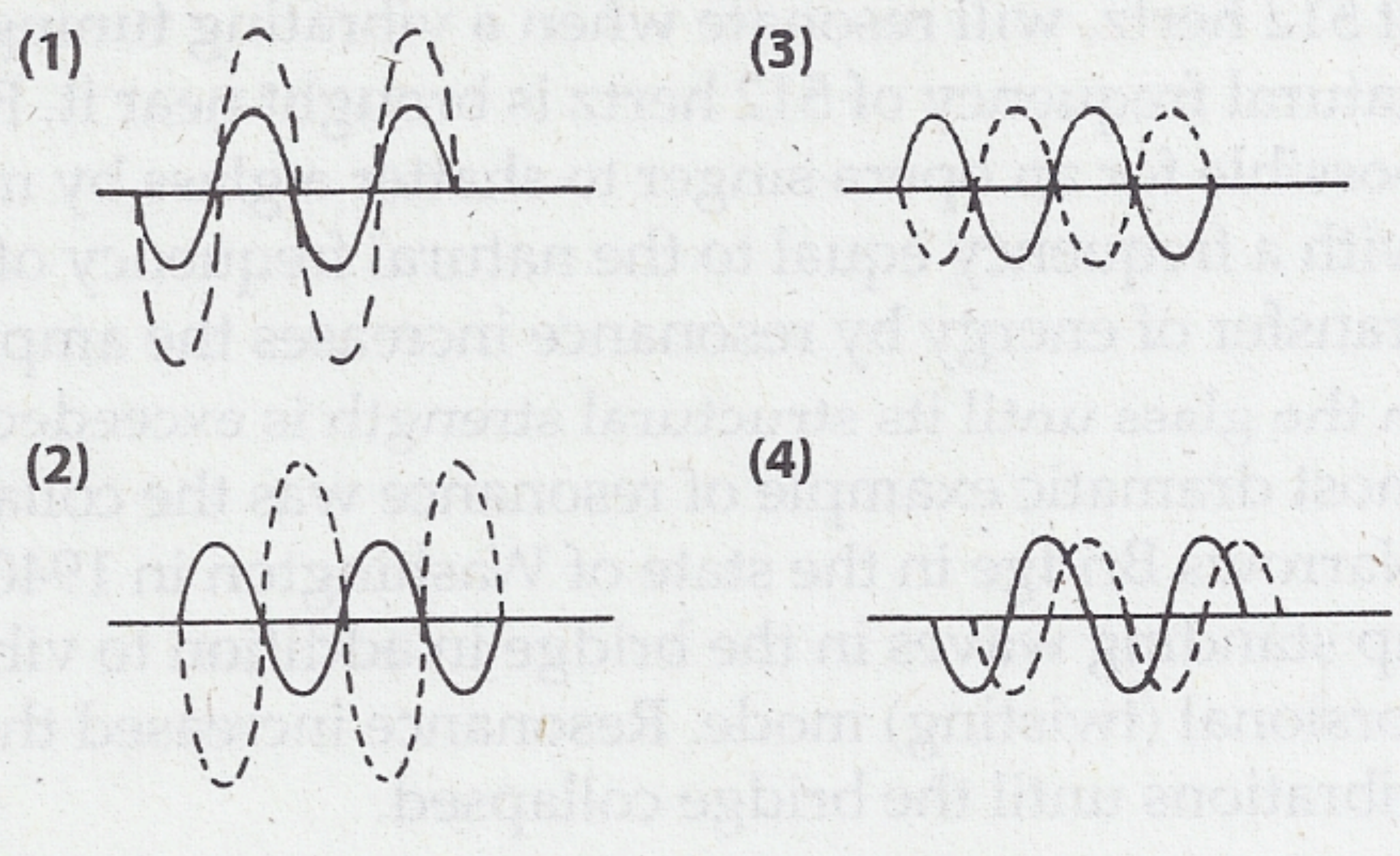
(4)

57. The diagram below shows four waves that pass simultaneously through a region.

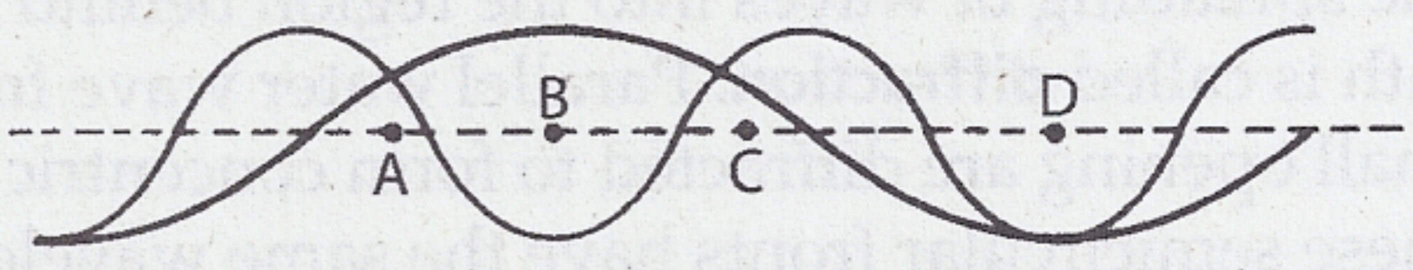


Which two waves will produce maximum constructive interference if they are combined?

58. Which pair of waves will produce a resultant wave with the smallest amplitude?



59. The diagram below represents two waves traveling simultaneously in the same medium.

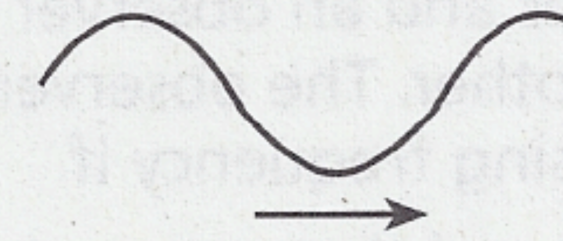


At which of the given points will maximum constructive interference occur?

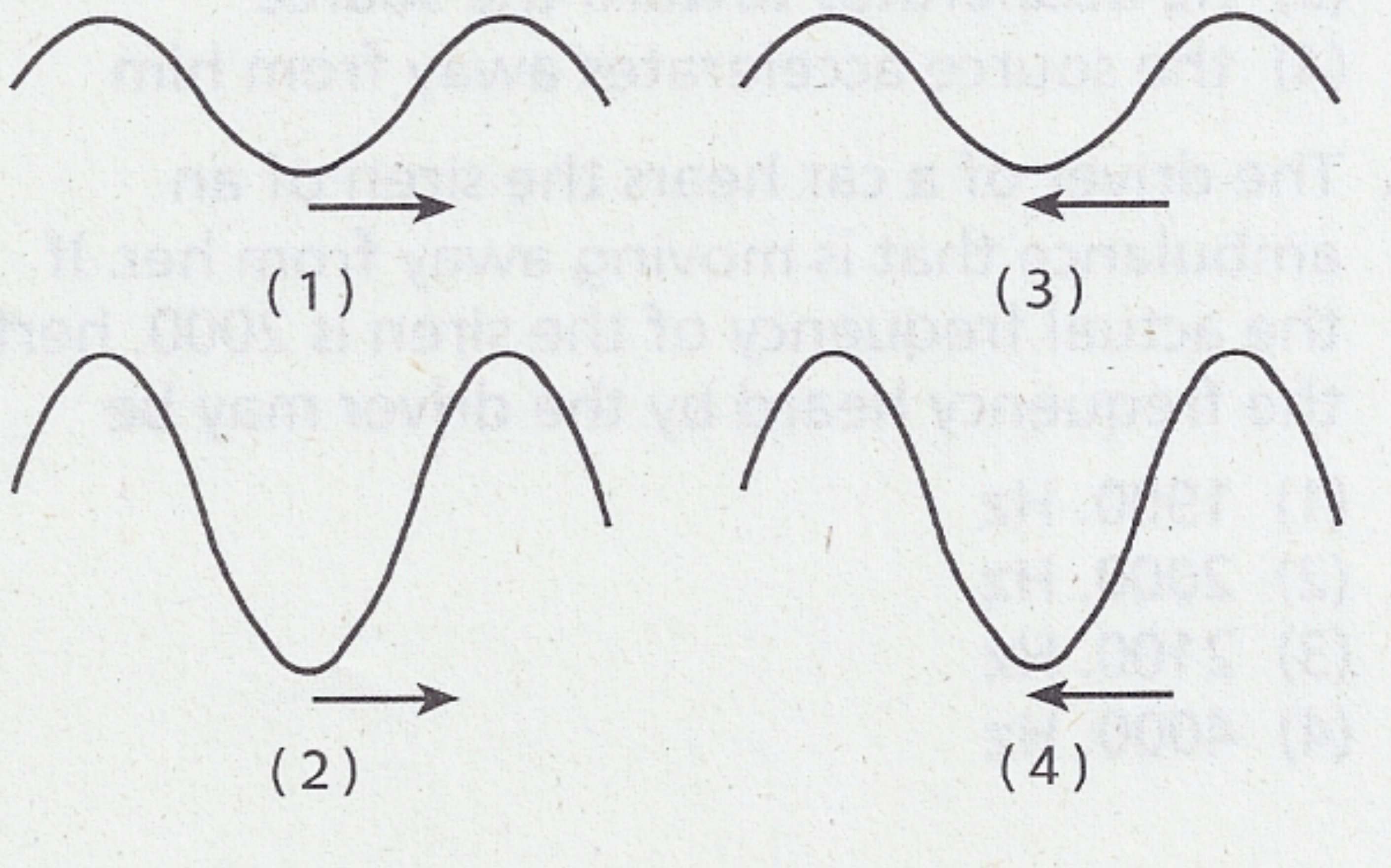
60. Standing waves are produced by two waves traveling in opposite directions in the same medium. The two waves must have

(1) the same amplitude and the same frequency
(2) the same amplitude and different frequencies
(3) different amplitudes and the same frequency
(4) different amplitudes and different frequencies

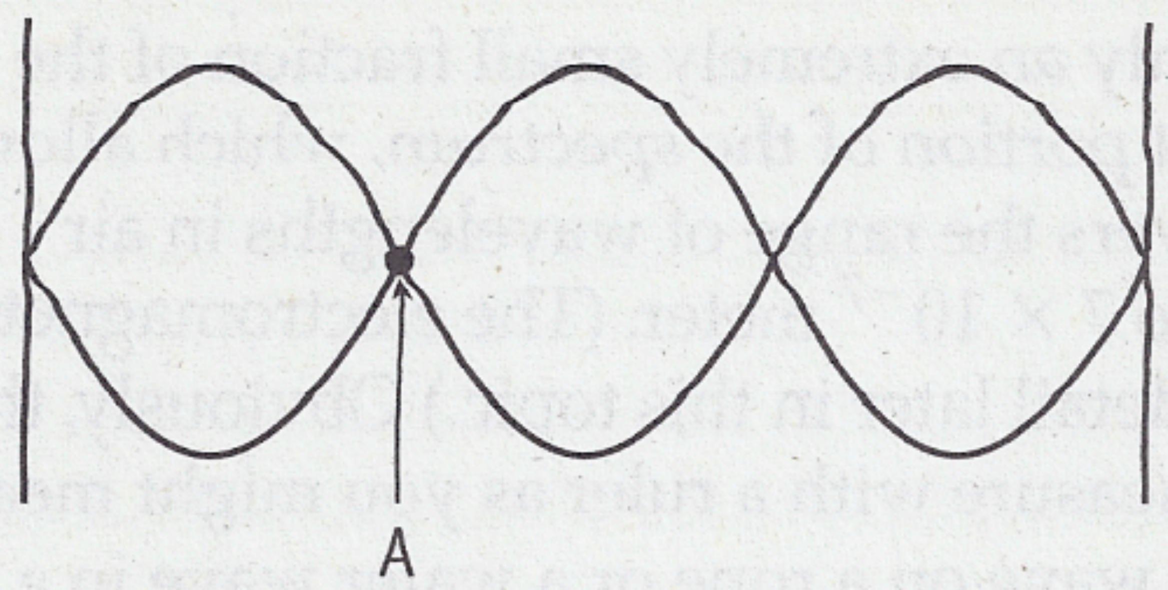
61. The diagram below represents a wave moving toward the right side of this page.



Which wave shown below could produce a standing wave with the original wave?



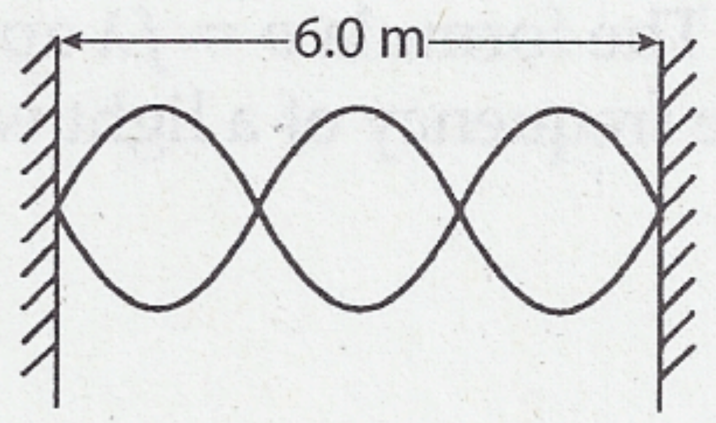
62. The diagram below shows a standing wave.



Point *A* on the standing wave is

(1) a node resulting from constructive interference
(2) a node resulting from destructive interference
(3) an antinode resulting from constructive interference
(4) an antinode resulting from destructive interference

Base your answers to questions 63 and 64 on the diagram below, which shows a standing wave in a rope.



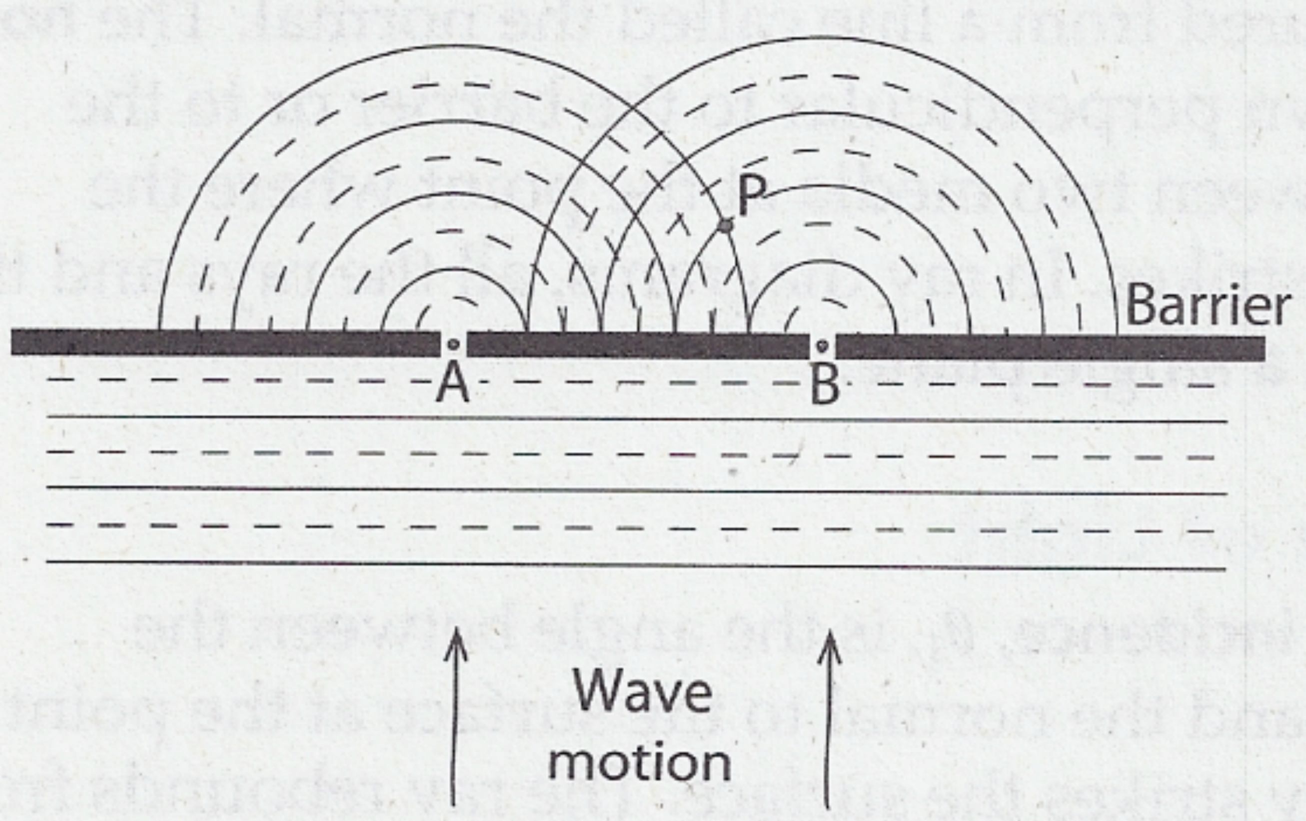
63. How many nodes are represented?

64. If the rope is 6.0 meters long, what is the wavelength of the standing wave?

65. Two waves traveling in the same medium and having the same wavelength (λ) interfere to create a standing wave. What is the distance between two consecutive nodes on this standing wave?

(1) λ (2) $\frac{3\lambda}{4}$ (3) $\frac{\lambda}{2}$ (4) $\frac{\lambda}{4}$

66. The diagram below represents shallow water waves interacting with two slits in a barrier.



Identify *two* wave phenomena illustrated in the diagram.

67. An opera singer's voice is able to break a thin crystal glass if a note sung and the glass have the same natural

(1) speed
(2) frequency
(3) amplitude
(4) wavelength

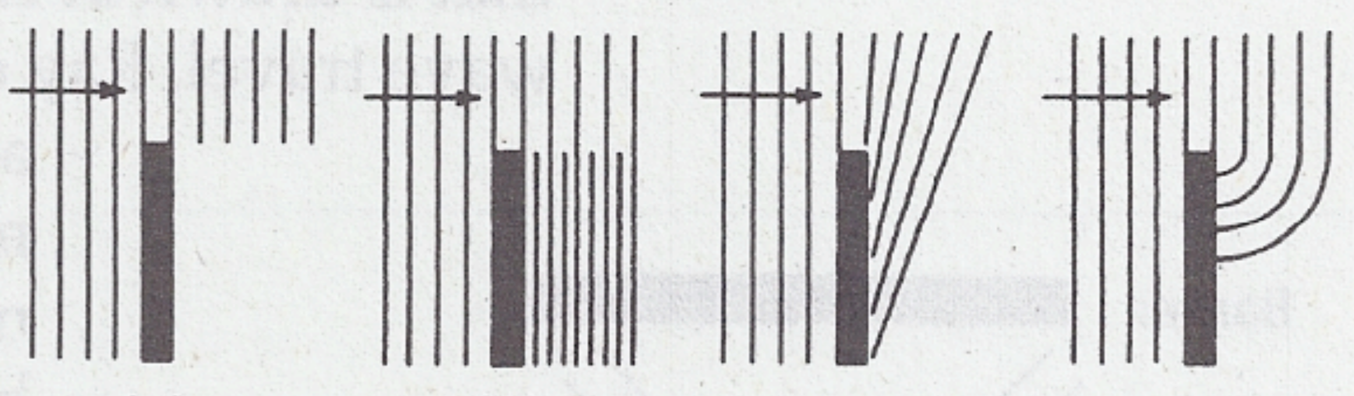
68. When an opera singer hits a high-pitch note, a glass on the opposite side of the opera hall shatters. Which statement best explains this phenomenon?

(1) The amplitude of the note increases before it reaches the glass.
(2) The singer and the glass are separated by an integral number of wavelengths.
(3) The frequency of the note and the natural frequency of the glass are equal.
(4) The sound produced by the singer slows down as it travels from the air into the glass.

69. A wave is diffracted as it passes through an opening in a barrier. The amount of diffraction that the wave undergoes depends on both the

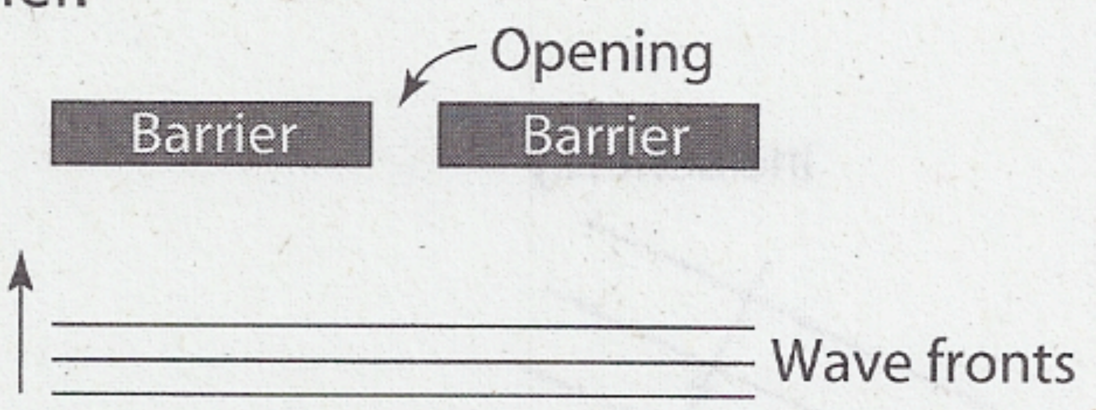
(1) amplitude and frequency of the incident wave
(2) wavelength and speed of the incident wave
(3) wavelength of the incident wave and the size of the opening
(4) amplitude of the incident wave and the size of the opening

70. Which diagram best illustrates diffraction of waves incident on a barrier?

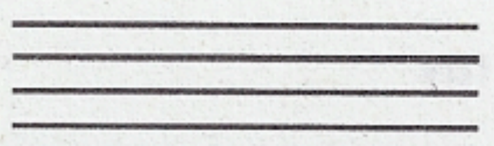


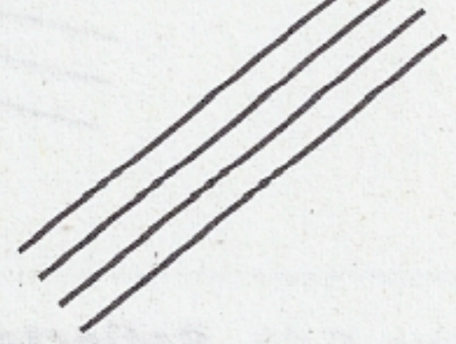
(1) (2) (3) (4)

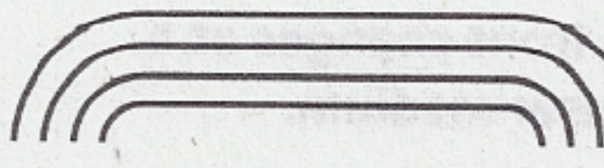
71. The diagram below represents straight wave fronts approaching a narrow opening in a barrier.



Which diagram best represents the shape of the waves after passing through the opening?

(1) 

(3) 

(2) 

(4) 

Light

The human eye can perceive only an extremely small fraction of the *electromagnetic spectrum.* That portion of the spectrum, which allows us to see, is called light and covers the range of wavelengths in air from approximately 4×10^{-7} to 7×10^{-7} meter. (The electromagnetic spectrum will be discussed in detail later in this topic.) Obviously, these wavelengths are too small to measure with a ruler as you might measure the wavelength of a transverse wave on a rope or a water wave in a shallow tank.

Speed of Light

Measurements of the speed of light to more than two or three significant figures could not be made until about 100 years ago. To three significant figures, the speed of light in a vacuum or air is 3.00×10^8 meters per second. Measurements of the speed of light are now recorded to nine significant figures. This more accurate data reveals that the speed of light in air is slightly less than it is in a vacuum. The speed of light in a vacuum is represented by the symbol c, an important physical constant.

The speed of light in a vacuum is the upper limit for the speed of any material body. No object can travel faster than c. The speed of light in a material medium is always less than c. The formula $v = f\lambda$ applies to light waves. Therefore, $c = f\lambda$, where f is the frequency of a light wave and λ is its wavelength in a vacuum.

Ray Diagrams

Because it is not possible to see individual wave fronts in a light wave, a ray is used to indicate the direction of wave travel. A **ray** is a straight line that is drawn at right angles to a wave front and points in the direction of wave travel. Ray diagrams show only the direction of wave travel, not the actual waves. An **incident ray** is a ray that originates in a medium and strikes a boundary or an interface of that medium with another medium. A **reflected ray** is a ray that has rebounded from a boundary or interface. A **refracted ray** is a ray that results from an incident ray entering a second medium of different optical density obliquely. Figure 5-13 shows these rays as well as the wave fronts whose motion they represent.

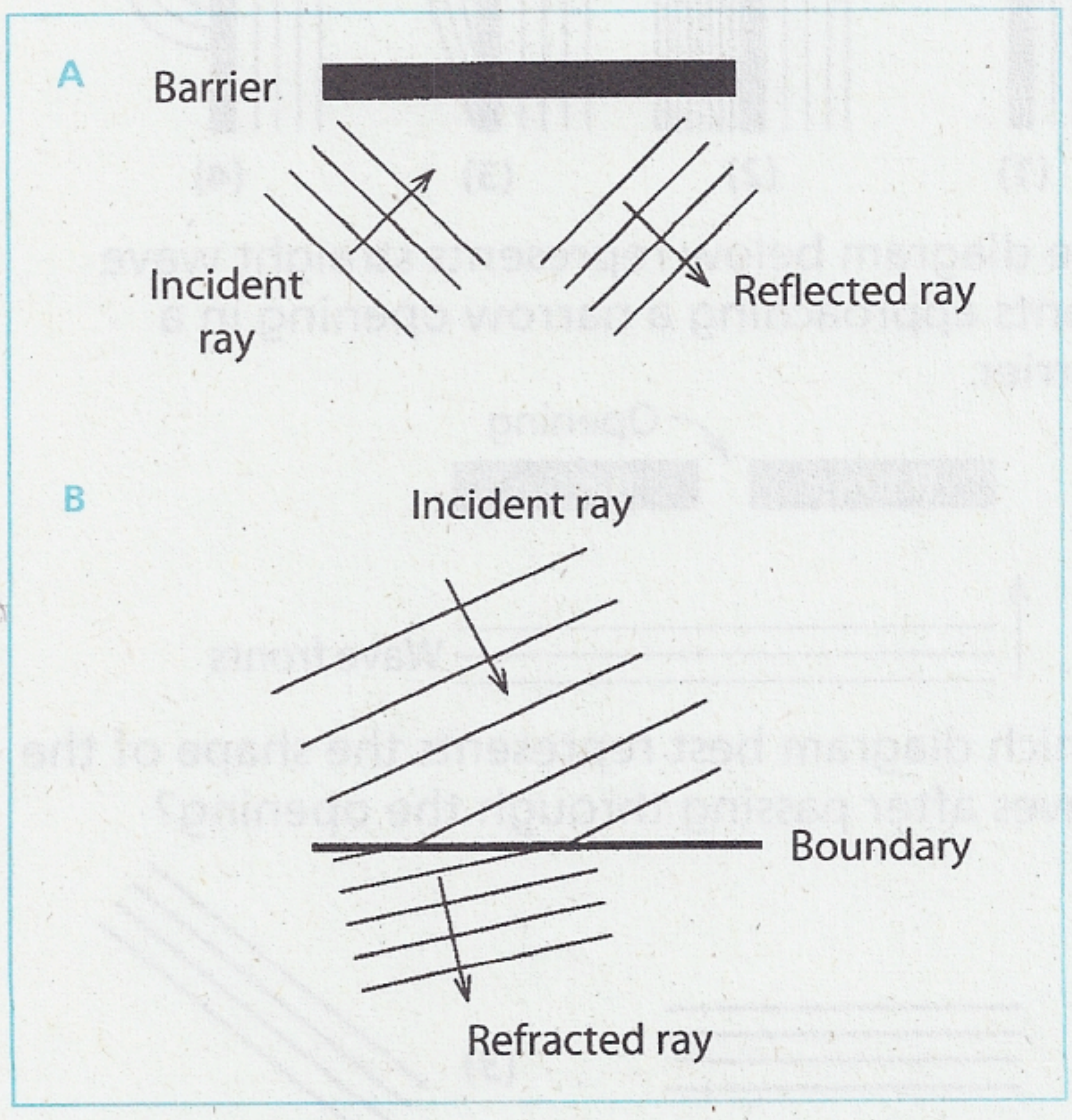


Figure 5-13. Reflected and refracted rays: (A) shows the direction of a reflected wave front at a barrier. (B) shows how a wave front changes at a boundary between air and a denser medium.

Incident, reflected, and refracted rays form corresponding angles measured from a line called the normal. The **normal** is a line drawn perpendicular to the barrier or to the interface between two media at the point where the incident ray strikes. In ray diagrams, all the rays and the normal lie in a single plane.

Reflection of Light

The **angle of incidence,** θ_i, is the angle between the incident ray and the normal to the surface at the point where the ray strikes the surface. The ray rebounds from the surface at the **angle of reflection,** θ_r, the angle between the reflected ray and the normal to the surface at the point of reflection. The **law of reflection** states that the angle of incidence is equal to the angle of reflection.

R

$$\theta_i = \theta_r$$

Figure 5-14 illustrates the law of reflection. This law is valid for all types of waves including light, water, and sound waves. The reflection of sound waves is called an echo.

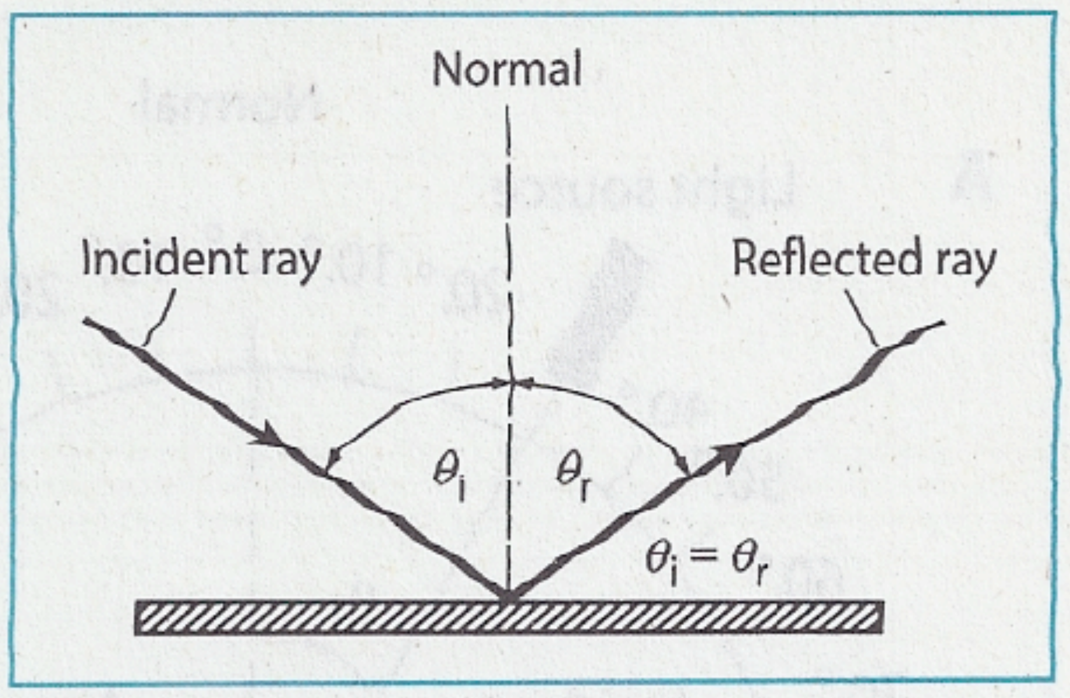


Figure 5-14. The law of reflection: The angle of incidence equals the angle of reflection.

Refraction of Light

Waves travel at different speeds in different media, so when a wave travels from one medium to another medium of different optical density, the speed of the wave changes. If the wave is incident on the interface between two media at an angle other than 90°, the direction of wave travel changes in the new medium. That means that both the speed and the direction of a wave usually change as the wave enters a new medium obliquely. The change in direction of a wave due to a change in speed at the boundary between two different media is called **refraction.** If the wave fronts of an incident wave are parallel to the interface, the angle of incidence is 0° and the wave may change speed upon entering the new medium, but the direction of the wave does not change.

The amount of refraction of a ray depends upon the properties of the two media at the interface and is measured by the angle of refraction. The **angle of refraction** is the angle between a ray emerging from the interface of two media and the normal to that interface at the point where the ray emerges.

Speed of Light and Refraction

When a light ray in air is incident on an interface with water at an angle of incidence of 0°, the ray of light slows down upon entering the more optically dense water, but does not change its direction of travel. Figure 5-15 shows an incident ray approaching the interface between air and water along the normal. The ray is not refracted as it travels from air into water. The ray travels more slowly in water than in air but its frequency remains the same. The speed of a wave is proportional to its wavelength when frequency is constant, so its wavelength in water is shorter than its wavelength in air.

The situation is different when a light ray passes obliquely from a less dense medium such as air into a more dense medium such as water. In this case, the ray is refracted towards the normal, as shown in Figure 5-16A. Upon entering the denser medium, the ray's frequency does not change, but its wavelength decreases as its speed decreases. If the path of the ray is from a more dense medium, such as water, into a less dense medium, such as air, the ray is refracted away from the normal, as shown in Figure 5-16B. Upon entering the less dense medium,

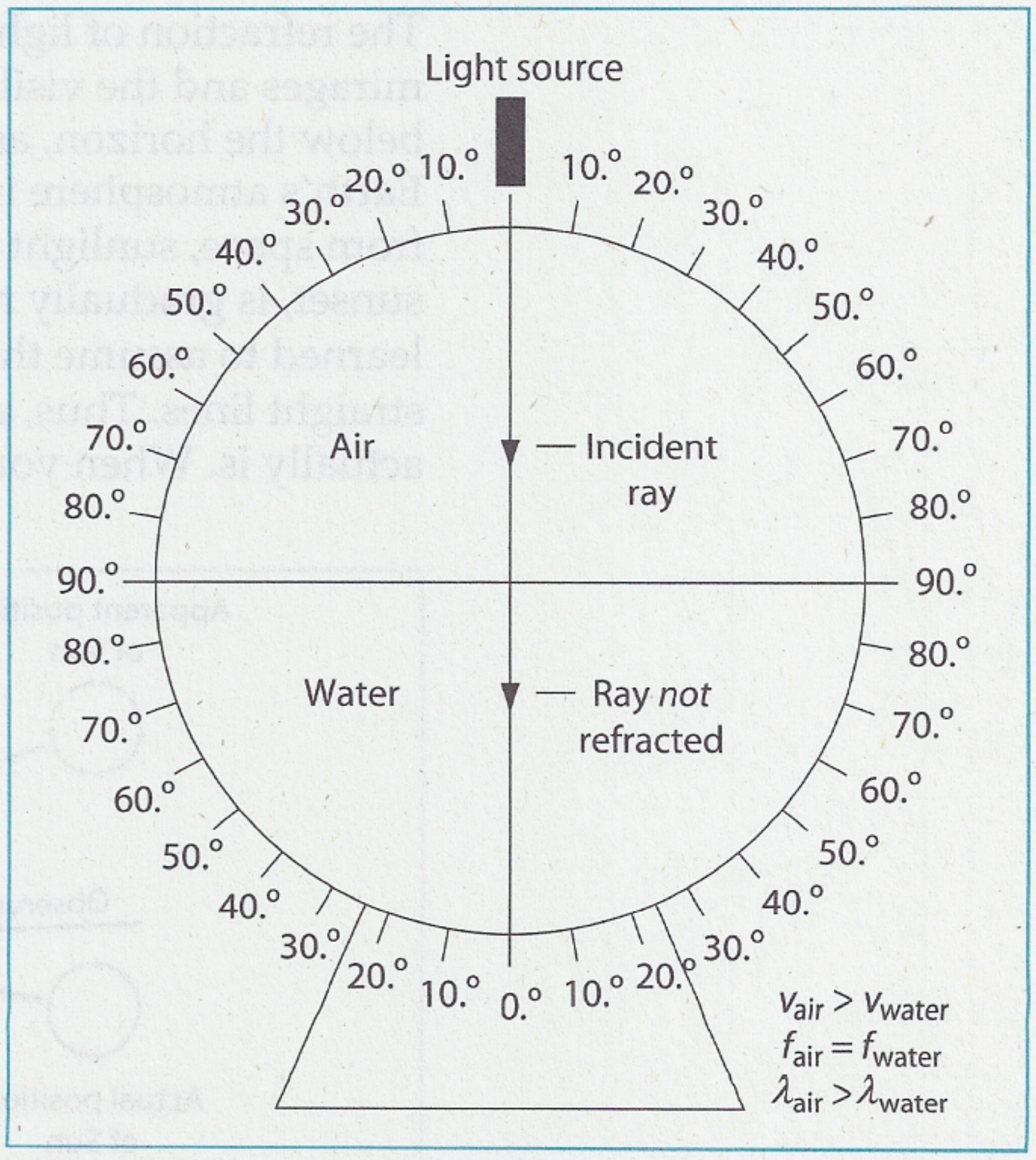


Figure 5-15. Refraction of light: A light ray passes from a less optically dense medium, air, into a more optically dense medium, water, at an angle of incidence of 0°.

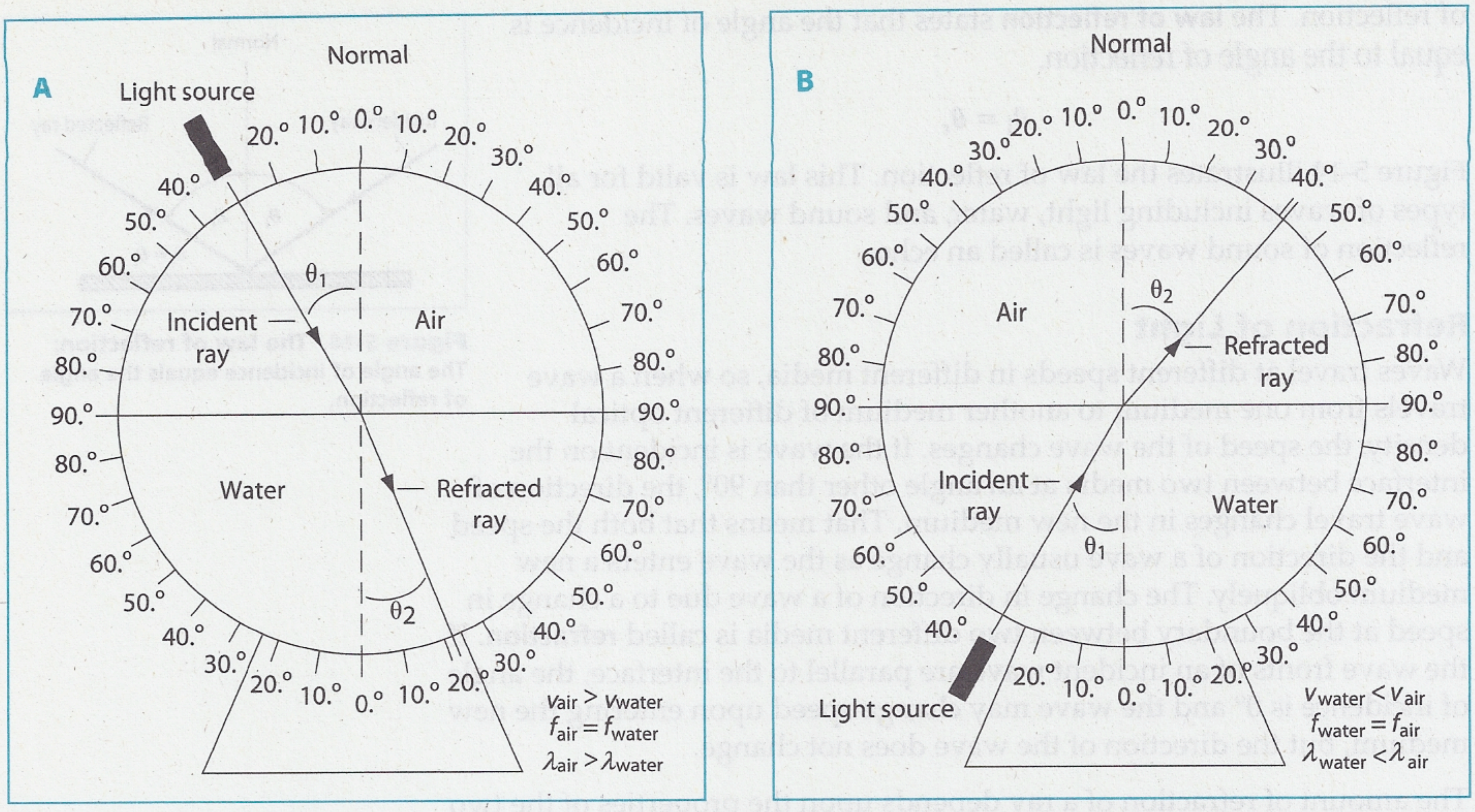


Figure 5-16. Additional examples of refraction of light: (A) A light ray passes obliquely from a less optically dense medium, air, into a more optically dense medium, water, at an angle of incidence of 30°. The ray is refracted toward the normal. (B) A light ray passes obliquely from a more optically dense medium, water, into a less optically dense medium, air, at an angle of incidence of 30°. The ray is refracted away from the normal.

the ray's frequency does not change, but its wavelength increases as its speed increases.

The refraction of light explains many everyday phenomena such as mirages and the visibility of the Sun after it has actually disappeared below the horizon, as illustrated in Figure 5-17. Because the density of Earth's atmosphere increases gradually as Earth's surface is approached from space, sunlight entering the atmosphere obliquely, as it does at sunset, is gradually refracted to produce a curved path. Your brain has learned to assume that light entering your eyes has been traveling in straight lines. Thus, at sunset you "see" the Sun higher in the sky than it actually is. When you "see" the Sun on the horizon, it has already set.

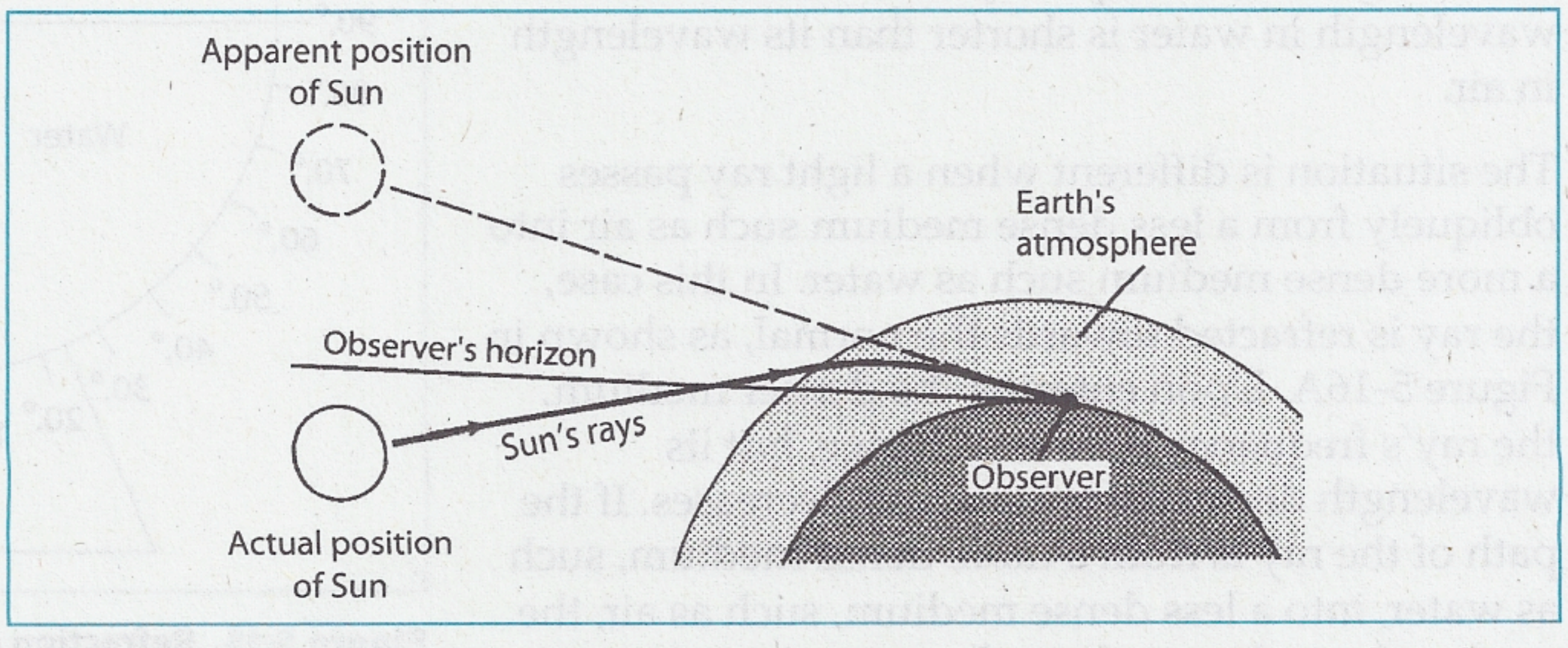


Figure 5-17. Curvature of the Sun's rays by refraction in Earth's atmosphere (not drawn to scale)

Another example of refraction is the apparent bending of a straw placed in a glass of water. The submerged portion of the straw appears to be closer to the surface than it actually is. Light from the submerged tip of the straw is bent away from the normal upon entering the less-dense air, as shown in Figure 5-18. To an observer, who interprets what is seen as light traveling in a straight line, the submerged tip of the straw seems closer to the surface than it actually is.

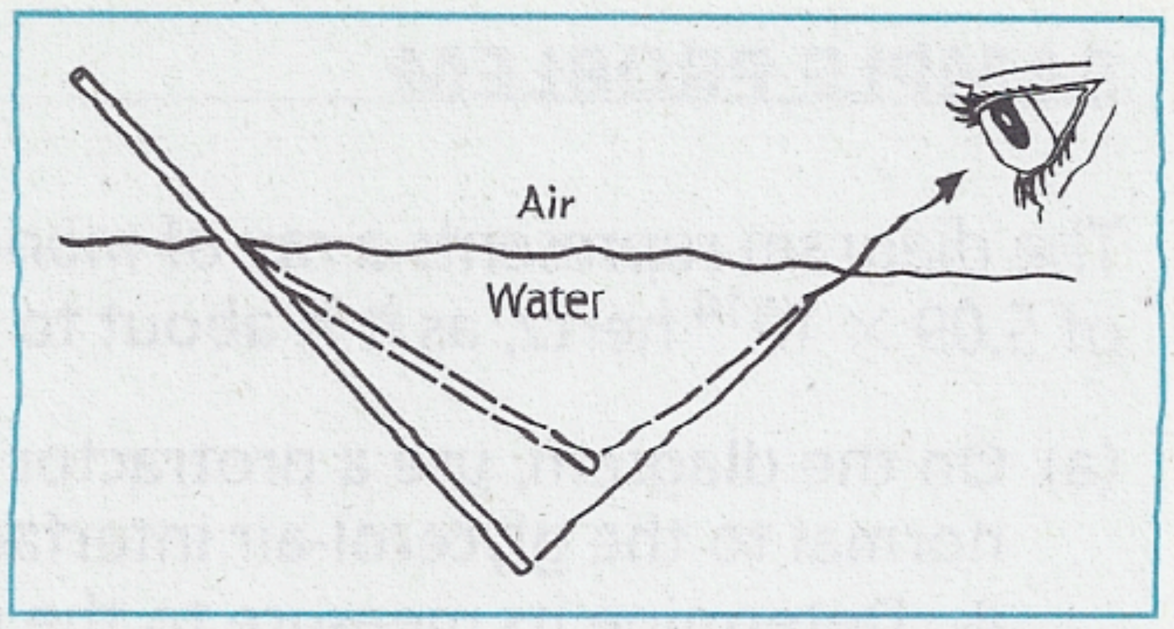


Figure 5-18. Refraction of light: Light rays from the tip of the straw are bent away from the normal as they emerge from the water. The effect is to make the straw appear to bend at the surface of the water.

Absolute Index of Refraction

The **absolute index of refraction,** n, is the ratio of the speed of light in a vacuum, c, to the speed of light in a material medium, v.

$$n = \frac{c}{v}$$

The absolute index of refraction has no units because both c and v are measured in the same units. The greater the value of n, the more optically dense the medium and the slower light travels in the medium. The absolute indices of refraction for a variety of materials are listed in the *Reference Tables for Physical Setting/Physics*.

Solving the equation for c yields $c = nv$. Thus, the following equations apply for two different media.

$$n_1 v_1 = n_2 v_2 \quad \text{or} \quad \frac{n_2}{n_1} = \frac{v_1}{v_2}$$

Also, the following equations apply for any two media.

$$v_1 = f\lambda_1 \quad \text{and} \quad v_2 = f\lambda_2$$

Note that the frequency of the wave does not change as the wave enters a new medium. Thus, the relationship between the speeds and wavelengths of the wave in the two media is this.

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

These relationships can be combined as follows.

$$\frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

Snell's Law

The mathematical relationship that governs the refraction of light as it passes obliquely from one medium to another of different optical density is called **Snell's law.**

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Angles θ_1 and θ_2 are the angles of incidence and refraction respectively, and n_1 and n_2 are the absolute indices of refraction of the incident and refractive media, respectively.

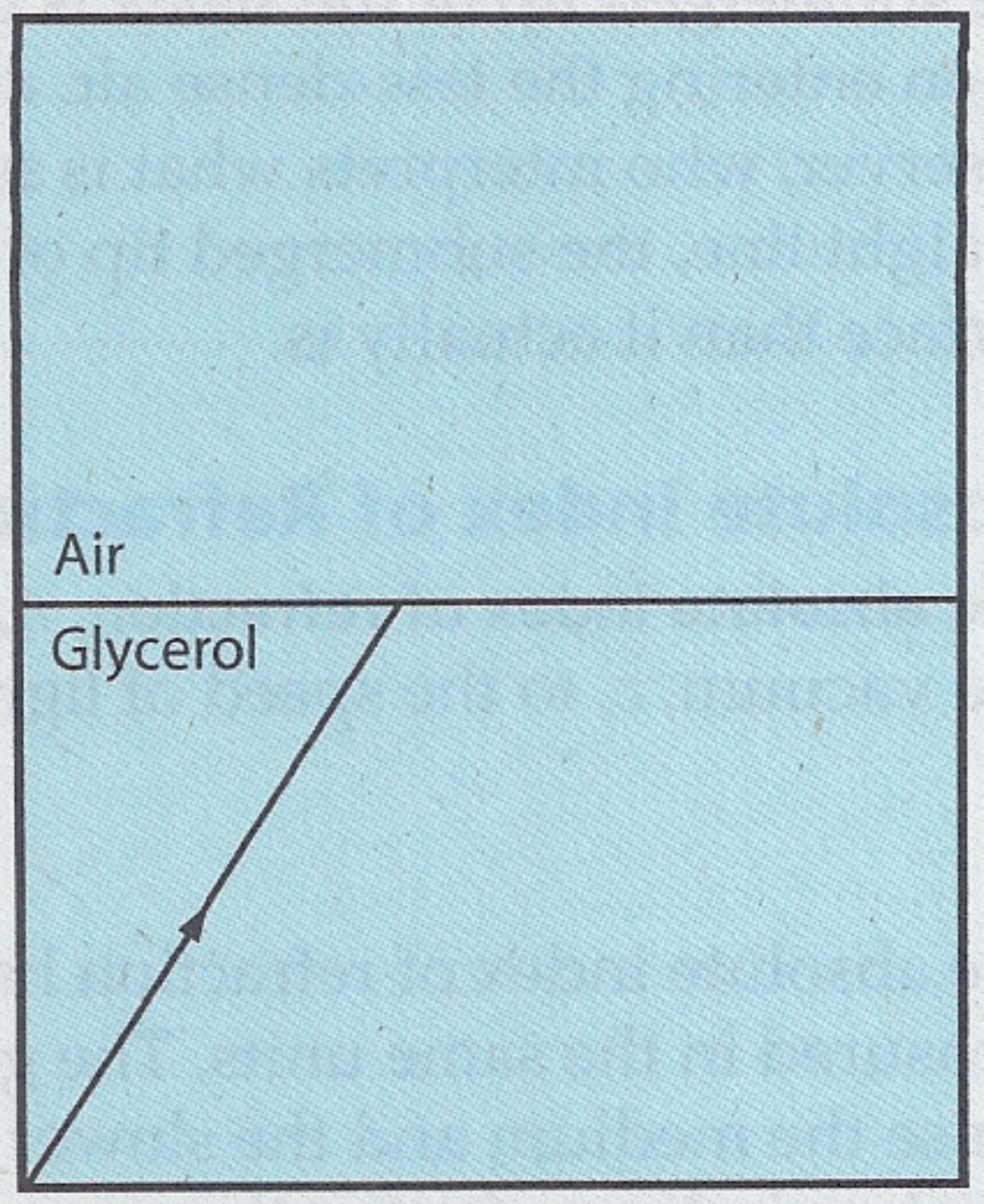
Snell's law can be rearranged in this way.

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}$$

The ratio n_2/n_1 is called the relative index of refraction for the two media.

SAMPLE PROBLEM

The diagram represents a ray of monochromatic light, having a frequency of 5.09×10^{14} hertz, as it is about to emerge from glycerol into air.

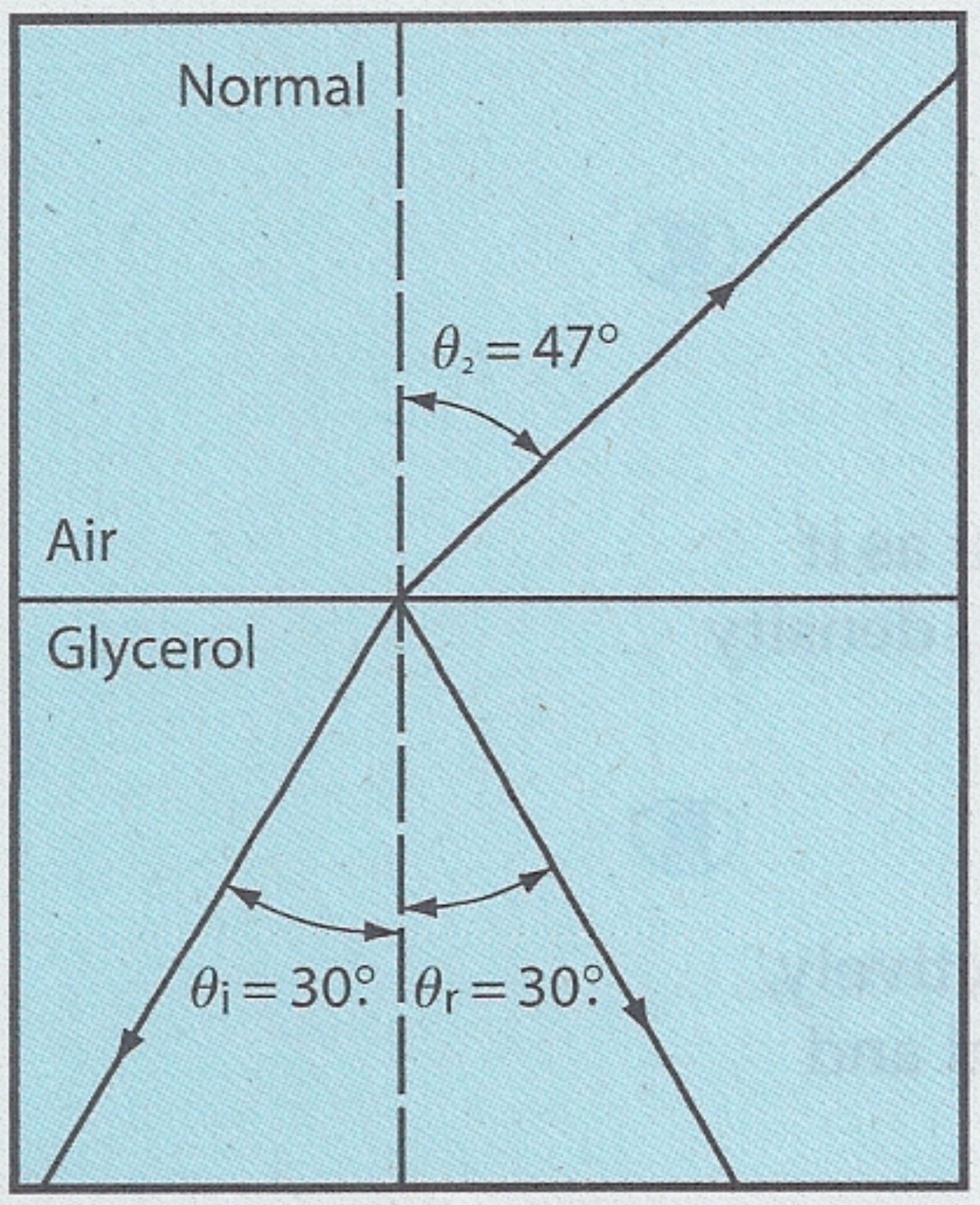


(a) *On the diagram, use a protractor and a straight edge to draw a normal to the glycerol-air interface. Label the angle of incidence θ_1. Determine its measure to the nearest degree.*
(b) Calculate the angle of refraction.
(c) On the diagram, draw the refracted light ray, label the angle of refraction θ_2, and indicate its measure to the nearest degree.
(d) At a boundary between two media, some of the incident light is always reflected. On the diagram, use a protractor and a straight edge to draw the reflected ray, label the angle of reflection θ_r, and indicate its measure to the nearest degree.
(e) Calculate the speed of the light in glycerol.
(f) Calculate the wavelength of the light in air in nanometers.
(g) Calculate the wavelength of the light in glycerol in nanometers.

SOLUTION: Identify the known and unknown values.

Known	*Unknown*
$f = 5.09 \times 10^{14}$ Hz	$\theta_1 = ?°$
$n_1 = 1.47$ (glycerol)	$\theta_2 = ?°$
$n_2 = 1.00$ (air)	$\theta_r = ?°$
$v_2 = c = 3.00 \times 10^8$ m/s	$v_1 = ?$ m/s (glycerol)
	$\lambda_2 = ?$ nm (air)
	$\lambda_1 = ?$ nm (glycerol)

(a) On the diagram, draw a normal to the surface at the point of incidence. The angle of incidence is measured from the normal. See the diagram that follows. The angle of incidence is 30°.



(b) Use the formula $n_1 \sin \theta_1 = n_2 \sin \theta_2$. Note that the subscript 1 refers to the incident medium and the subscript 2 refers to the refractive medium. Solve the equation for $\sin \theta_2$.

$$\sin \theta_2 = \frac{n_1 \sin \theta_1}{n_2}$$

Substitute the known values and solve for θ_2.

$$\sin \theta_2 = \frac{(1.47)(\sin 30.°)}{1.00}$$

$$\theta_2 = 47°$$

(c) The angle of refraction is in air and is measured from the normal using a protractor.

(d) The angle of incidence is equal to the angle of reflection. Thus, the angle of reflection is 30.°, and is measured from the normal.

(e) Solve the formula $n = c/v$ for v.

$$v = \frac{c}{n}$$

Substitute the known values and solve.

$$v = \frac{3.00 \times 10^8 \text{ m/s}}{1.47} = 2.04 \times 10^8 \text{ m/s}$$

(f) Solve the formula $v = f\lambda$ for the wavelength, λ.

$$\lambda = \frac{v}{f}$$

Substitute the known values and solve.

$$\lambda_2 = \frac{3.00 \times 10^8 \text{ m/s}}{5.09 \times 10^{14} \text{ Hz}}$$

$$\lambda_2 = 5.89 \times 10^{-7} \text{ m}$$

$$\lambda_2 = 589 \text{ nm}$$

(g) Write the formula relating absolute indices of refraction and wavelengths.

$$\frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2}$$

Solve the equation for λ_1.

$$\lambda_1 = \frac{n_2 \lambda_2}{n_1}$$

Substitute the known values and solve.

$$\lambda_1 = \frac{(1.00)(589 \text{ nm})}{1.47} = 401 \text{ nm}$$

The Electromagnetic Spectrum

Light waves are **electromagnetic waves** which consist of periodically changing electric and magnetic fields and move through a vacuum at speed $c = 3.00 \times 10^8$ meters per second. All electromagnetic waves, regardless of their frequency and wavelength, are produced by accelerating charged particles. The **electromagnetic spectrum,** which is the complete range of frequencies and wavelengths of electromagnetic waves, is shown in Figure 5-19. Notice that visible light is only a small portion of the spectrum.

There are no sharp divisions between the various kinds of electromagnetic waves. They are classified according to the methods by which they are generated or received. For example, radio waves, used for communication systems, are produced by charges accelerating in a wire. Do not confuse electromagnetic radio waves with longitudinal sound waves.

Microwaves are used in radar systems in air-traffic control, for transmitting long-distance telephone communications in outer space, and to cook food. The frequency of microwaves used in a microwave oven is the same as the natural rotational frequency of water molecules. Resonance is produced in water molecules contained in food and the resulting internal energy due to vibration heats the food.

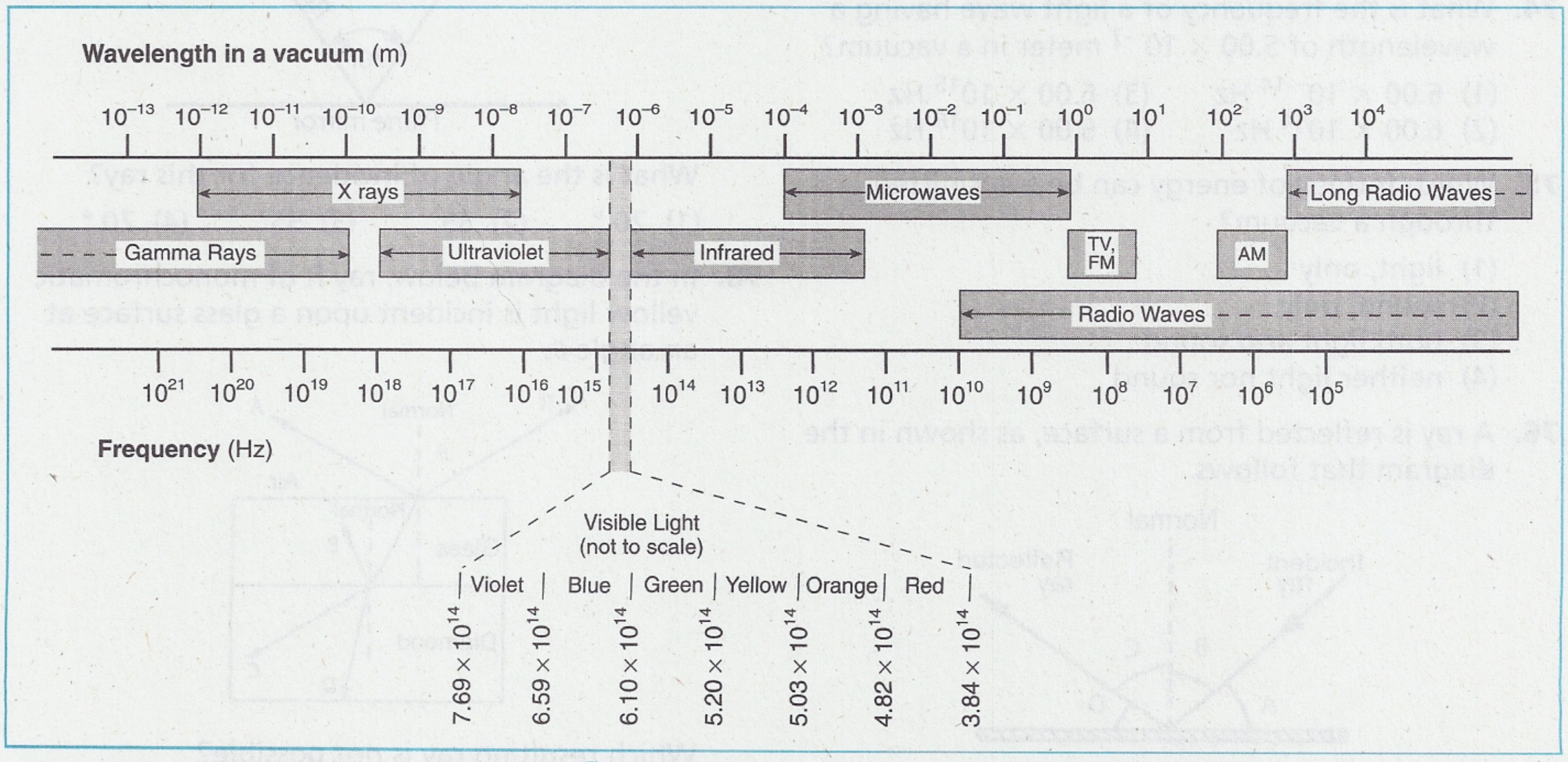


Figure 5-19. The electromagnetic spectrum R

Infrared waves appear as heat when absorbed by objects. Practical applications of the infrared portion of the electromagnetic spectrum include heat lamps used in physical therapy and infrared photography.

Visible light is approximately one percent of the electromagnetic spectrum. It is produced by the rearrangement of electrons in atoms and molecules. The wavelengths that the human eye can detect are in the range of approximately 400 to 700 nanometers.

Ultraviolet light is the part of sunlight that causes sunburns. The ozone layer of the atmosphere filters practically all of the high frequency components of ultraviolet radiation from the Sun, but the inner atmosphere readily transmits the remaining lower frequency ultraviolet radiation. Some commercial skin lotions are designed to absorb ultraviolet rays to prevent them from affecting the skin.

X rays are used as diagnostic tools by physicians. Living tissues and organisms can be destroyed by x rays, so precautions should be taken to avoid overexposure.

Gamma rays are emitted by radioactive nuclei. This electromagnetic radiation is harmful to living tissues.

Review Questions

72. How long does it take light to travel a distance of 100. meters?

(1) 3.00×10^{10} s
(2) 3.00×10^{8} s
(3) 3.33×10^{-7} s
(4) 3.33×10^{7} s

73. Calculate the wavelength in a vacuum of a light wave having a frequency of 5.3×10^{14} hertz. Express the wavelength in nanometers.

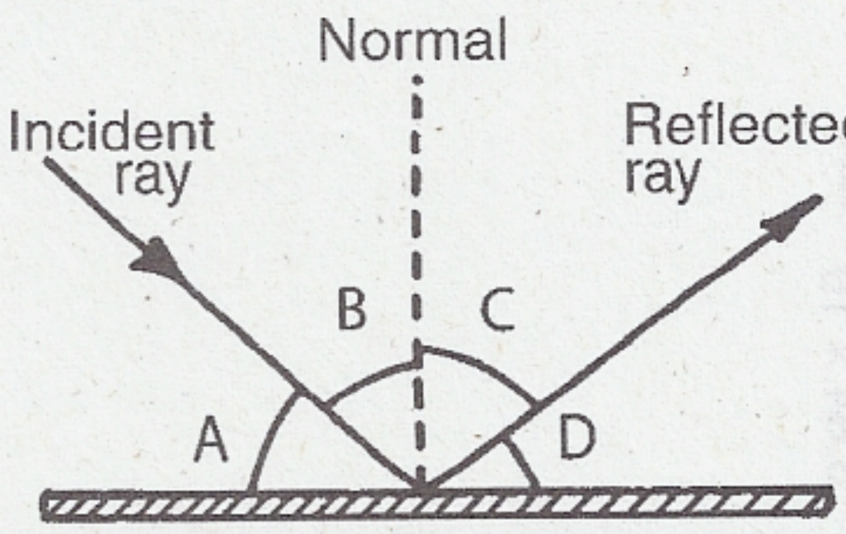
74. What is the frequency of a light wave having a wavelength of 5.00×10^{-7} meter in a vacuum?

(1) 6.00×10^{-14} Hz
(2) 6.00×10^{14} Hz
(3) 6.00×10^{15} Hz
(4) 6.00×10^{16} Hz

75. Which form(s) of energy can be transmitted through a vacuum?

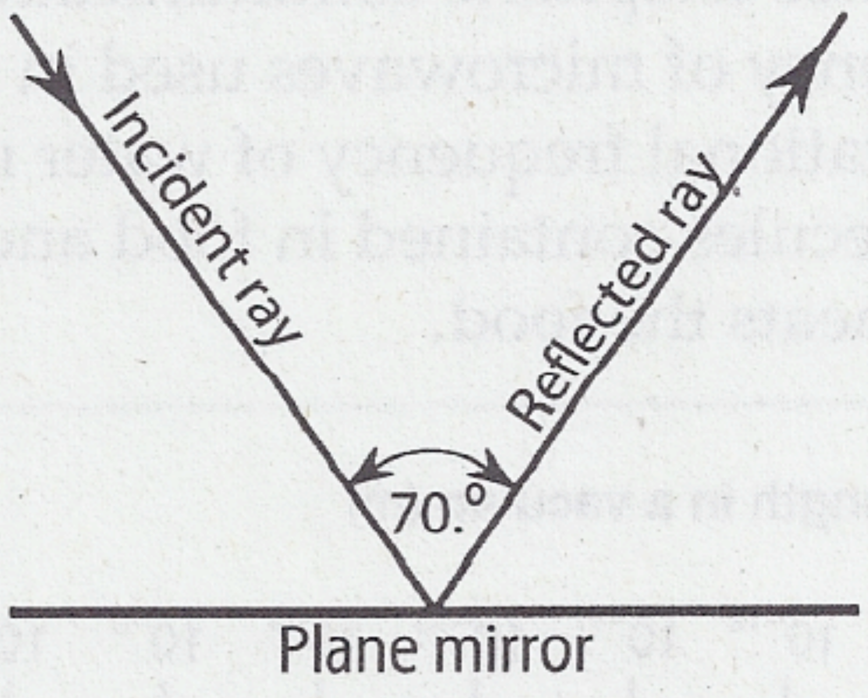
(1) light, only
(2) sound, only
(3) both light and sound
(4) neither light nor sound

76. A ray is reflected from a surface, as shown in the diagram that follows.



Which letter represents the angle of incidence?

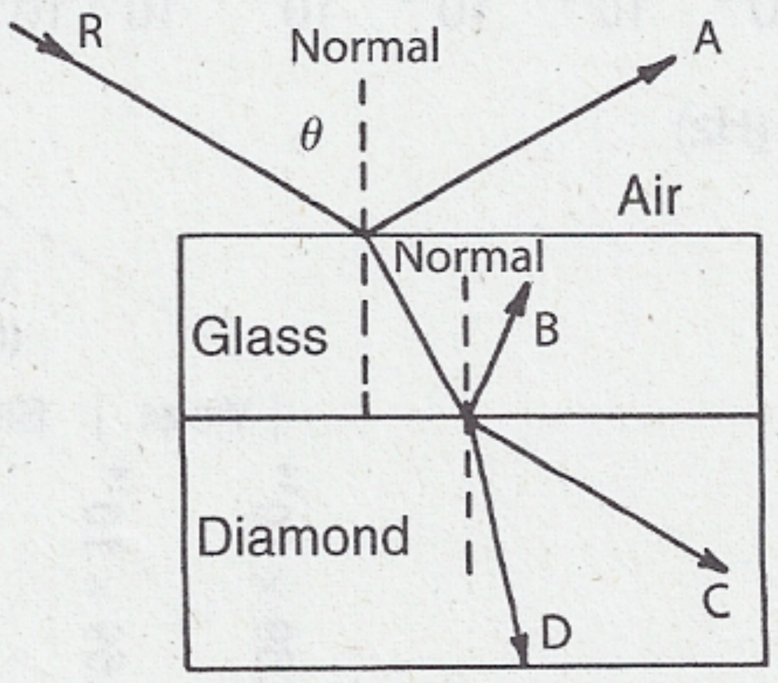
77. The diagram below represents a light ray being reflected from a plane mirror. The angle between the incident and reflected ray is 70.°



What is the angle of incidence for this ray?

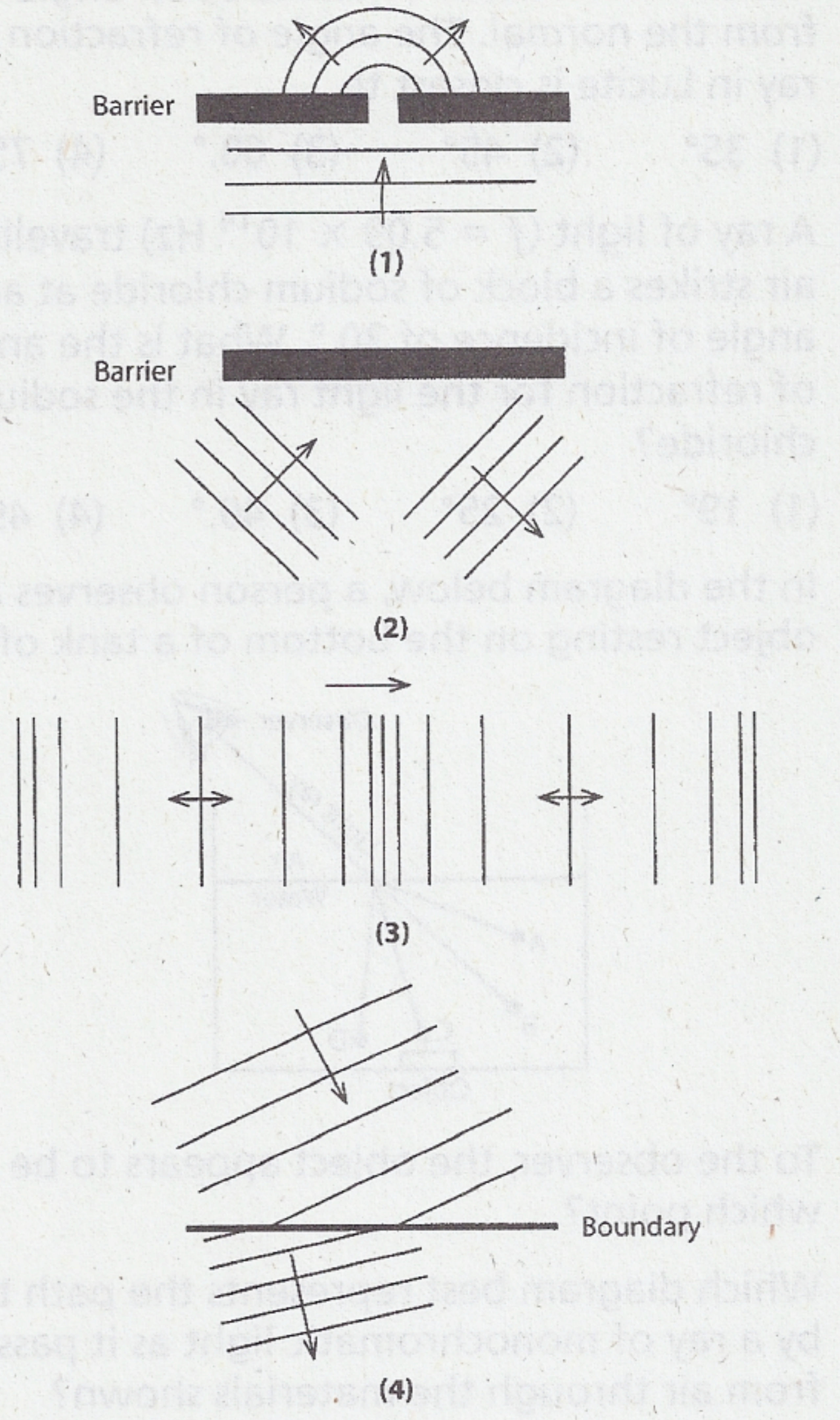
(1) 20.° (2) 35° (3) 55° (4) 70.°

78. In the diagram below, ray R of monochromatic yellow light is incident upon a glass surface at an angle θ.



Which resulting ray is *not* possible?

79. Which diagram best represents wave reflection?



80. When a ray of light strikes a mirror perpendicular to its surface, what is the angle of reflection?

81. A ray of light passes from air into glass at an angle of incidence of 0°. Which statement best describes the speed and direction of the light ray as it passes into the glass?

(1) Only speed changes.
(2) Only direction changes.
(3) Both speed and direction change.
(4) Neither speed nor direction changes.

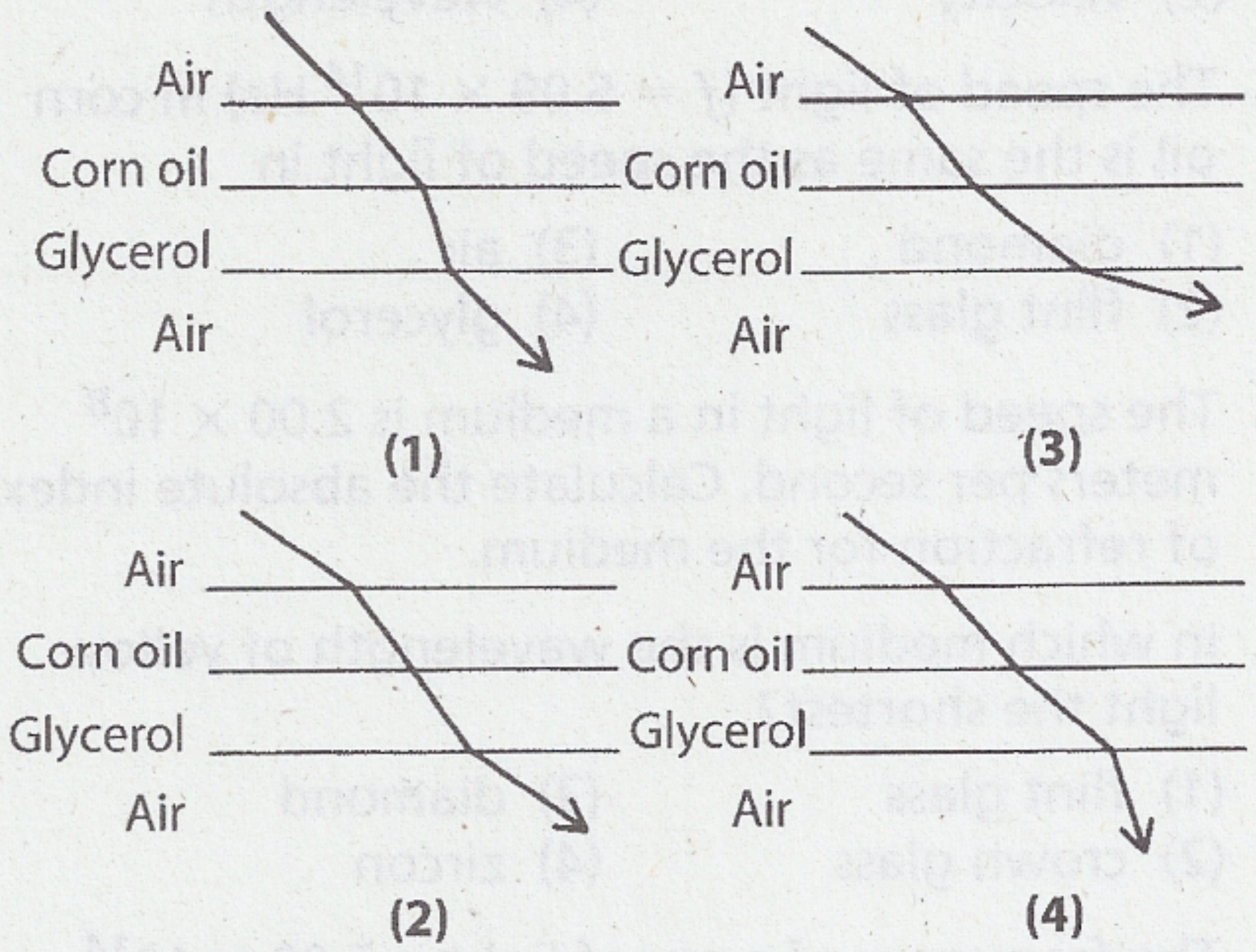
82. As a wave enters a new medium, there may be a change in the wave's

(1) frequency (3) period
(2) speed (4) phase

83. The speed of a ray of light traveling through a substance having an absolute index of refraction of 1.1 is

(1) 1.1×10^8 s (3) 3.0×10^8 s
(2) 2.7×10^8 s (4) 3.3×10^8 s

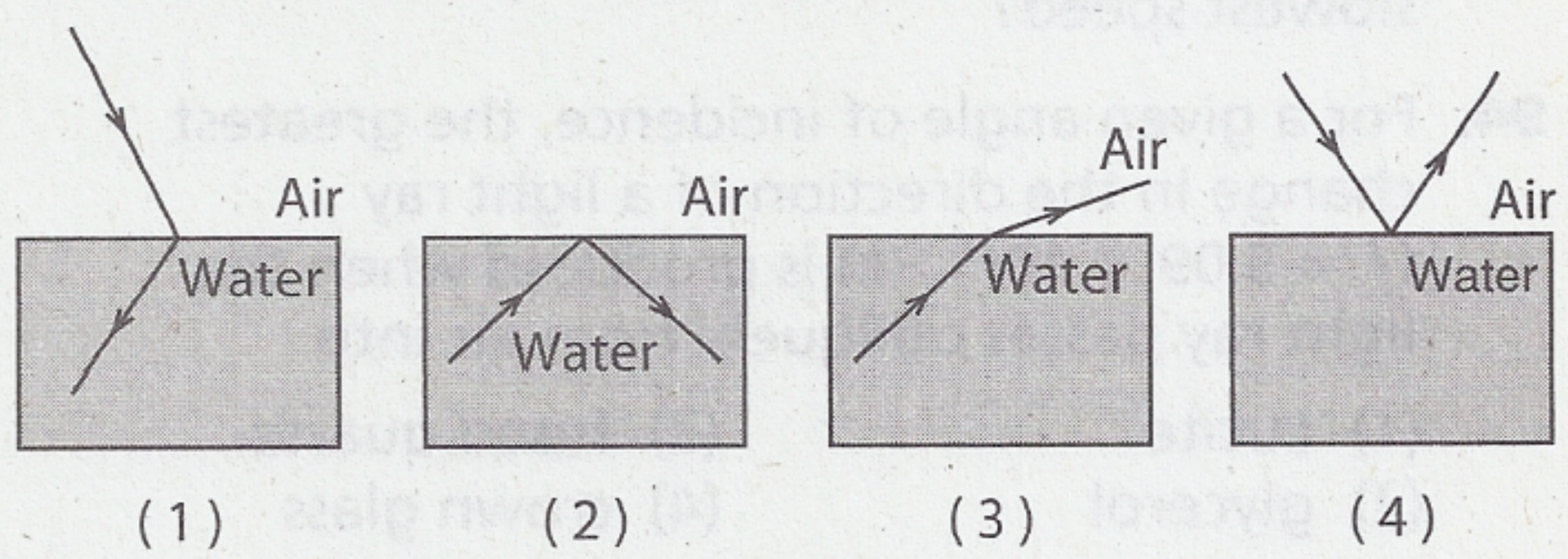
84. Which arrow best represents the path that a monochromatic ray of light ($f = 5.09 \times 10^{14}$ Hz) travels as it passes through air, corn oil, glycerol, and back into air?



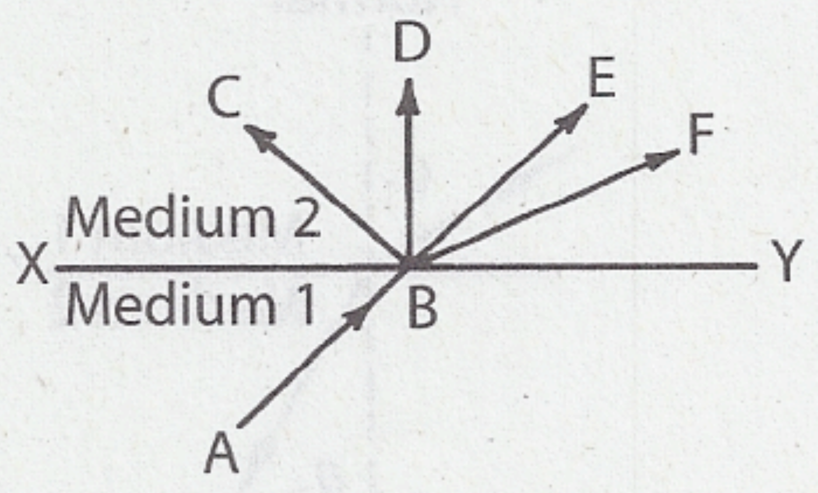
85. What happens to the speed, wavelength, and frequency of light when it passes from water into flint glass?

(1) Its speed decreases, its wavelength becomes shorter, and its frequency remains the same.
(2) Its speed decreases, its wavelength becomes shorter, and its frequency increases.
(3) Its speed increases, its wavelength becomes longer, and its frequency remains the same.
(4) Its speed increases, its wavelength becomes longer, and its frequency decreases.

86. Which ray diagram best represents the phenomenon of refraction?



87. In the diagram below, ray AB is incident on surface XY at point B.



If medium 2 has a lower index of refraction than medium 1, through which point will the ray most likely pass?

88. A beam of monochromatic red light passes obliquely from air into water. Which characteristic of the light does *not* change?

(1) direction (3) frequency
(2) velocity (4) wavelength

89. The speed of light ($f = 5.09 \times 10^{14}$ Hz) in corn oil is the same as the speed of light in

(1) diamond (3) air
(2) flint glass (4) glycerol

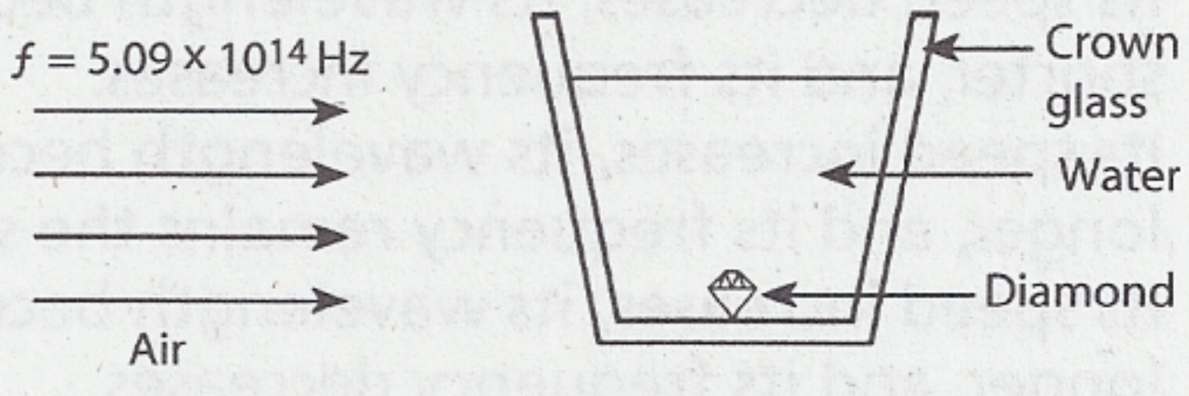
90. The speed of light in a medium is 2.00×10^8 meters per second. Calculate the absolute index of refraction for the medium.

91. In which medium is the wavelength of yellow light the shortest?

(1) flint glass (3) diamond
(2) crown glass (4) zircon

92. The frequency of a ray of light is 5.09×10^{14} hertz. What is the ratio of the speed of this ray in diamond to its speed in zircon?

93. In the diagram below, monochromatic light ($f = 5.09 \times 10^{14}$ Hz) in air is about to travel through crown glass, water, and diamond.



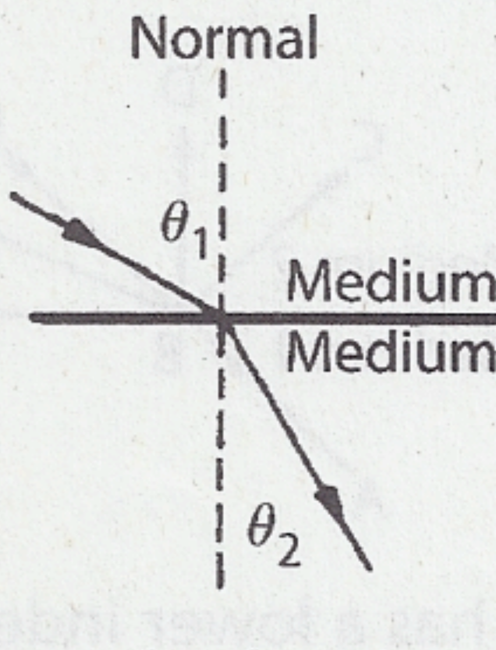
(not drawn to scale)

In which substance does the light travel at the slowest speed?

94. For a given angle of incidence, the greatest change in the direction of a light ray ($f = 5.09 \times 10^{14}$ Hz) is produced when the light ray passes obliquely from air into

(1) Lucite (3) fused quartz
(2) glycerol (4) crown glass

95. The diagram below represents a wave traveling from medium 1 to medium 2.



The relative index of refraction may be determined by calculating the ratio of

(1) $\frac{\theta_1}{\theta_2}$ (2) $\frac{\sin \theta_2}{\sin \theta_1}$ (3) $\frac{\sin \theta_1}{\sin \theta_2}$ (4) $\frac{n_1}{n_2}$

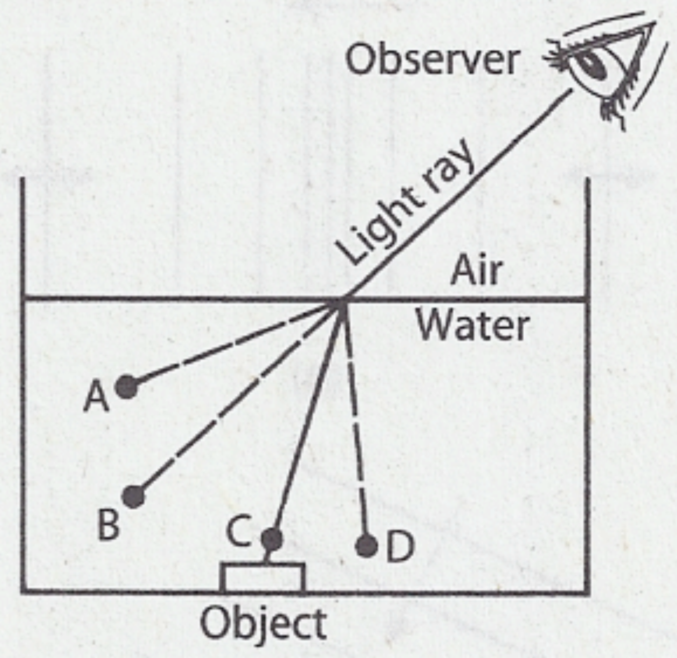
96. A ray of light ($f = 5.09 \times 10^{14}$ Hz) in air is incident on a block of Lucite at an angle of 60.° from the normal. The angle of refraction of this ray in Lucite is closest to

(1) 35° (2) 45° (3) 60.° (4) 75°

97. A ray of light ($f = 5.09 \times 10^{14}$ Hz) traveling in air strikes a block of sodium chloride at an angle of incidence of 30.°. What is the angle of refraction for the light ray in the sodium chloride?

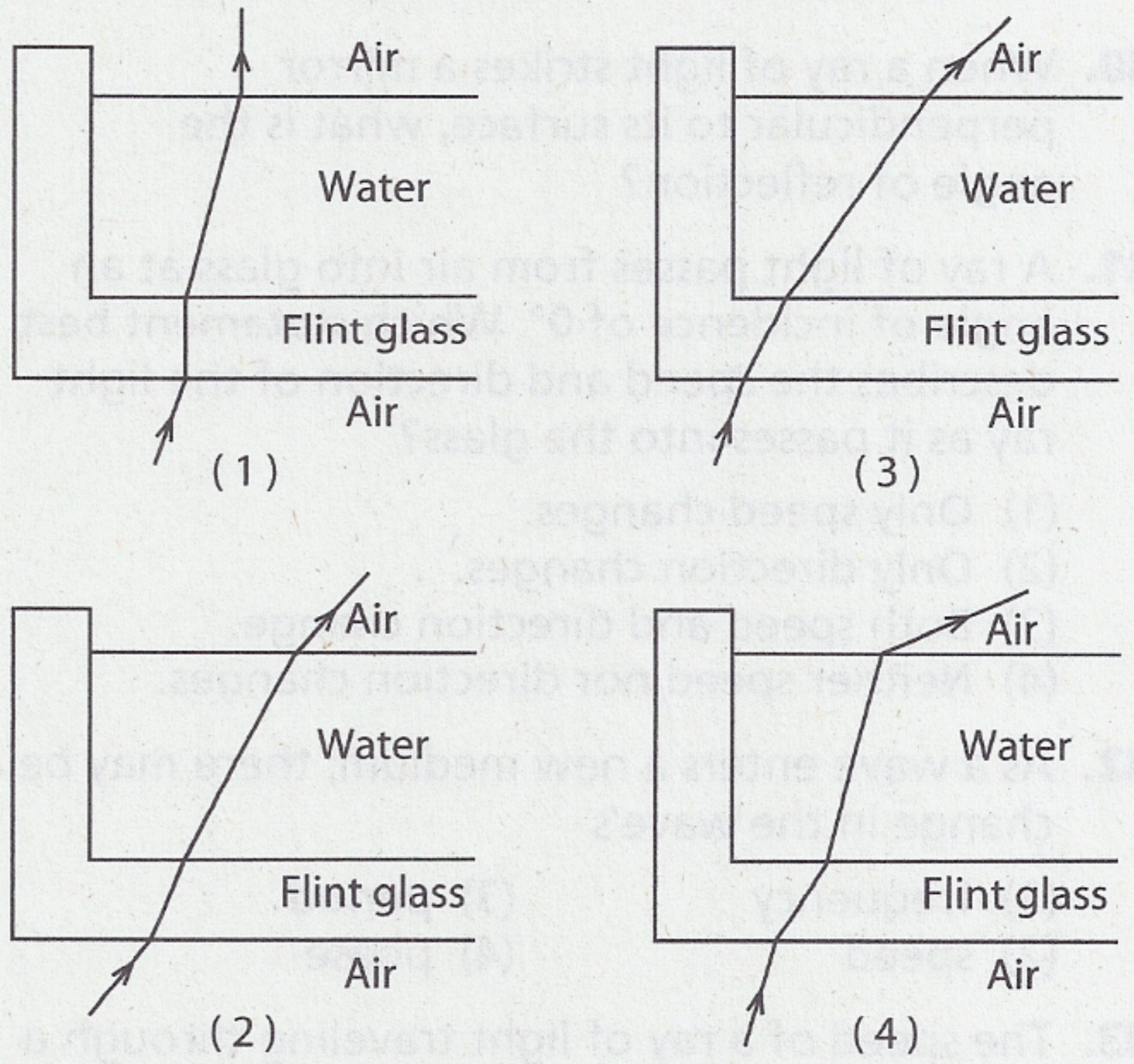
(1) 19° (2) 25° (3) 40.° (4) 49°

98. In the diagram below, a person observes an object resting on the bottom of a tank of water.



To the observer, the object appears to be at which point?

99. Which diagram best represents the path taken by a ray of monochromatic light as it passes from air through the materials shown?



100. Which electromagnetic radiation has the shortest wavelength?

(1) infrared (3) gamma
(2) radio (4) ultraviolet

101. The electromagnetic spectrum does *not* include

(1) light waves (3) sound waves
(2) radio waves (4) x rays

102. Electrons oscillating with a frequency of 2.0×10^{10} hertz produce electromagnetic waves. These waves would be classified as

(1) infrared (3) microwave
(2) visible (4) x ray

103. A microwave and an x ray are traveling in a vacuum. Compared to the wavelength and period of the microwave, the x ray has a wavelength that is

(1) longer and a period that is shorter
(2) longer and a period that is longer
(3) shorter and a period that is longer
(4) shorter and a period that is shorter

104. Which wavelength is in the infrared range of the electromagnetic spectrum?

(1) 100 nm (2) 100 mm (3) 100 m (4) 100 μm

105. Radio waves are propagated through the interaction of

(1) nuclear and electric fields
(2) electric and magnetic fields
(3) gravitational and magnetic fields
(4) gravitational and electric fields

106. In a vacuum, all electromagnetic waves have the same

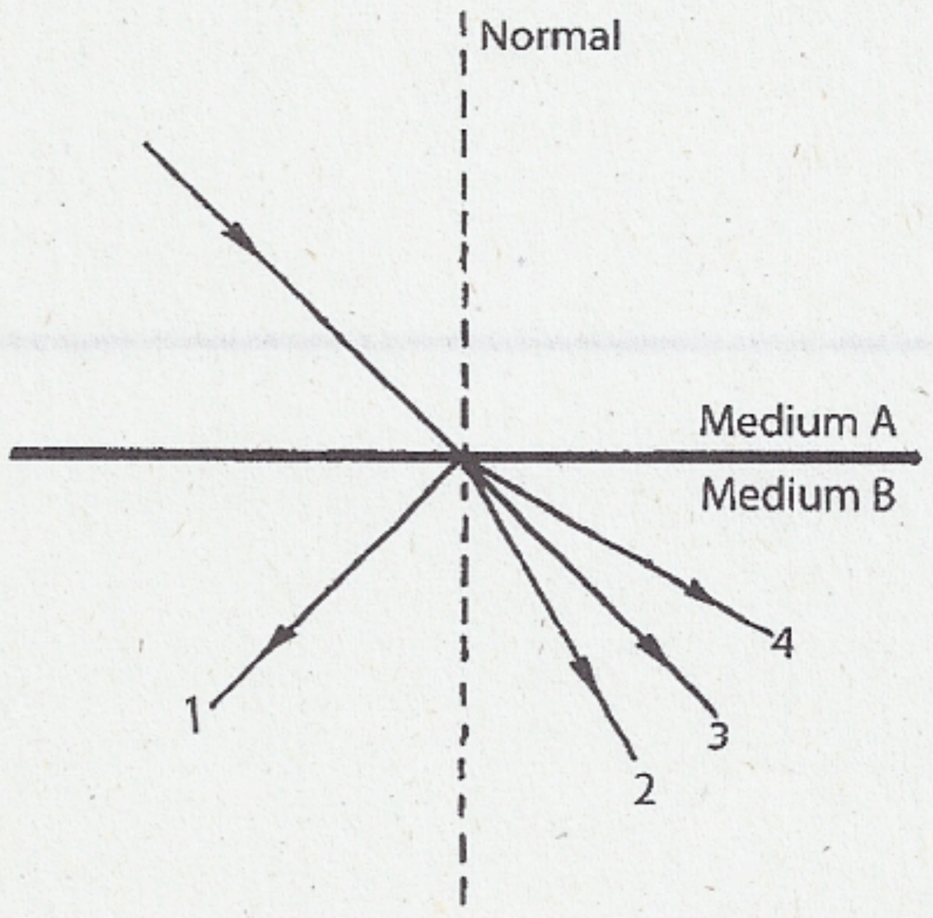
(1) frequency (3) speed
(2) wavelength (4) energy

107. A monochromatic beam of light with a frequency of 5.45×10^{14} hertz travels in a vacuum. What is the color of the light?

108. The wavelength of a typical AM radio wave is 3×10^3 meters. Determine the order of magnitude of its frequency.

Base your answers to questions 109 through 112 on the information and diagram below.

When a ray of monochromatic light passes from medium A to medium B, its speed decreases.



109. Which arrow best represents the path of the ray in medium B?

110. Compared to the frequency of the light in medium A, the frequency of the light in medium B is

(1) lower (2) higher (3) the same

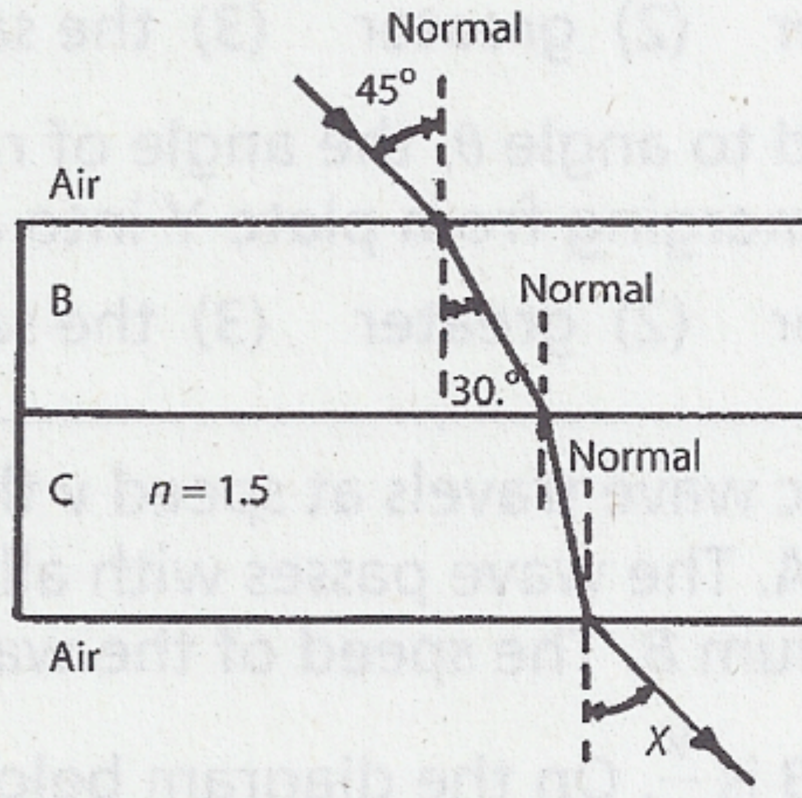
111. Compared to the wavelength of the light in medium A, the wavelength of the light in medium B is

(1) shorter (2) longer (3) the same

112. According to information listed in the *Reference Tables for Physical Setting/Physics*, what could be the identity of substance B if medium A is corn oil?

Base your answers to questions 113 through 116 on the information and diagram below.

A ray of light ($f = 5.09 \times 10^{14}$ Hz) moves from air through substance B, through substance C, and back into air. The surfaces of substances B and C are parallel.



113. Calculate the index of refraction of substance B.

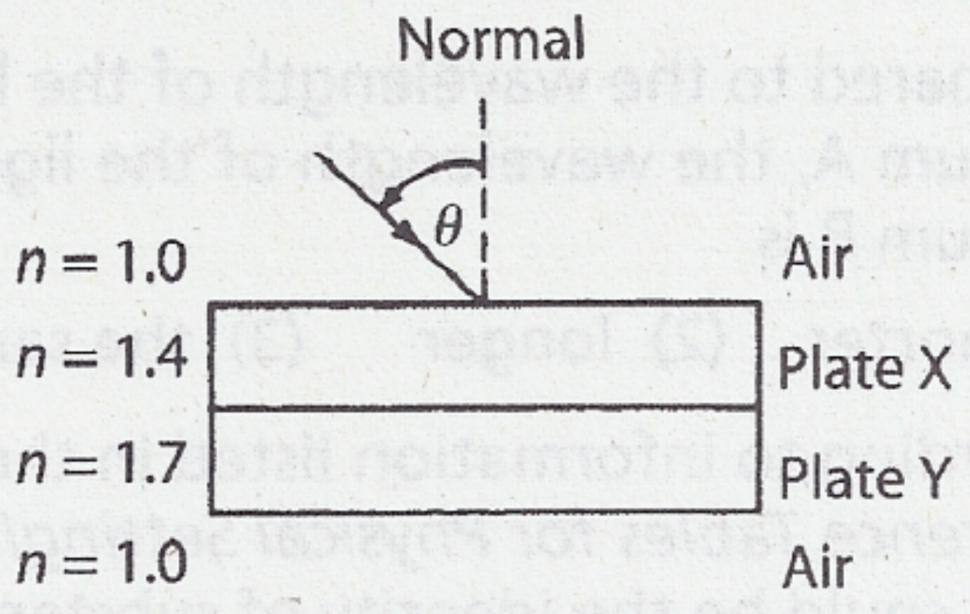
114. Calculate the speed of light in substance C.

115. If the angle of incidence of the light ray in air is increased, what happens to the angle of refraction in substance B?

116. What is the measure of angle X?

Base your answers to questions 117 through 120 on the information and diagram below.

A ray of monochromatic light ($f = 5.09 \times 10^{14}$ Hz) traveling in air is incident upon the surface of plate X. The values of n in the diagram represent absolute indices of refraction.



117. What is the relative index of refraction of the light going from plate X to plate Y?

(1) $\frac{1.0}{1.7}$ (2) $\frac{1.0}{1.4}$ (3) $\frac{1.7}{1.4}$ (4) $\frac{1.4}{1.7}$

118. Calculate the speed of the light ray in plate X.

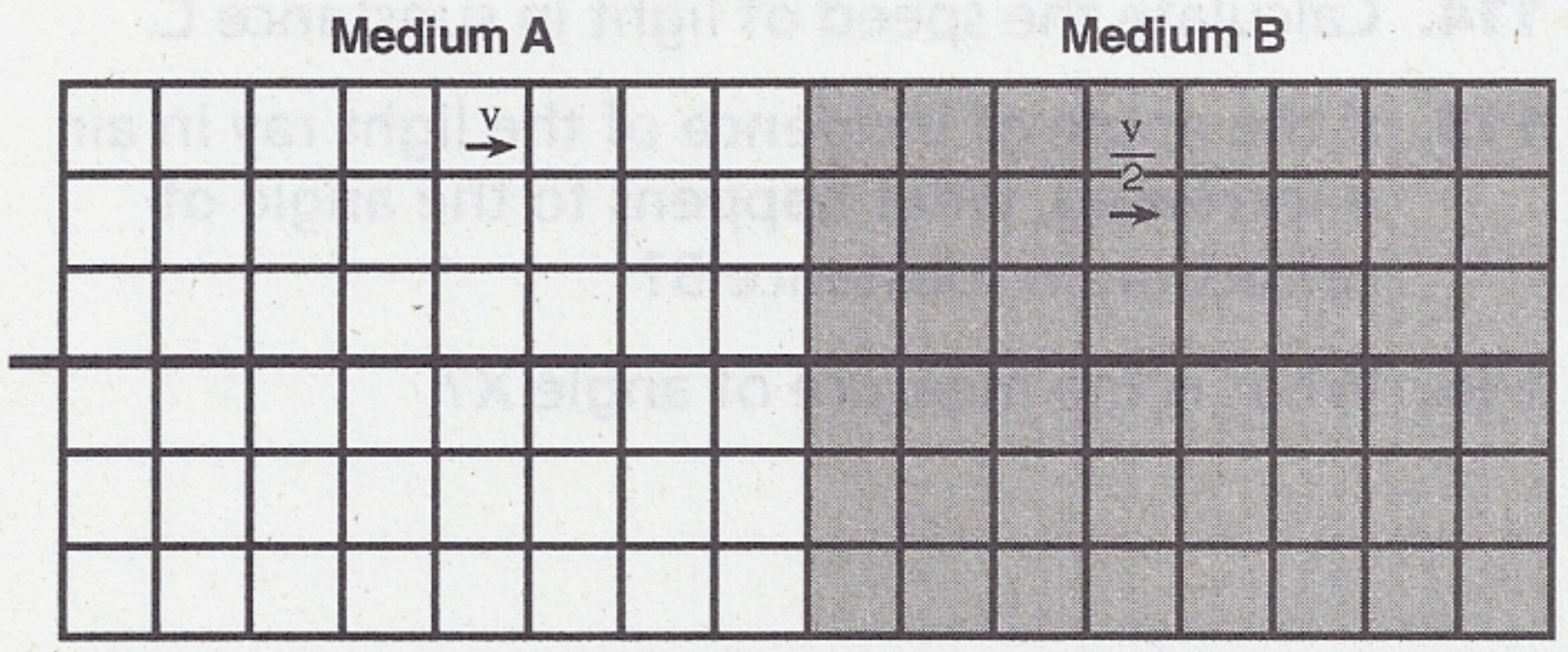
119. Compared to angle θ, the angle of refraction of the light ray in plate X is

(1) smaller (2) greater (3) the same

120. Compared to angle θ, the angle of refraction of the ray emerging from plate Y into air is

(1) smaller (2) greater (3) the same

121. A periodic wave travels at speed v through medium A. The wave passes with all its energy into medium B. The speed of the wave through medium B is $\frac{v}{2}$. On the diagram below draw the wave as it travels through medium B. [Show at least one full wave.]



Base your answers to questions 122 through 124 on the information below.

A stationary research ship uses sonar to send a 1.18×10^3-hertz sound wave down through the ocean water. The reflected sound wave from the flat ocean bottom 324 meters below the ship is detected 0.425 second after it was sent from the ship.

122. Calculate the speed of the sound wave in the ocean water.

123. Calculate the wavelength of the sound wave in the ocean water.

124. Determine the period of the sound wave in the ocean water.

Base your answers to questions 125 through 127 on the information below.

A beam of monochromatic light having a wavelength of 5.89×10^{-7} meters in air is incident on the surface of a diamond at an angle of 0°.

125. Calculate the wavelength of this light in the diamond.

126. Determine the angle of refraction of this light as it enters the diamond.

127. Compare the frequency and speed of this light in the diamond to the frequency and speed of this light in air.

Practice Questions for the New York Regents Exam

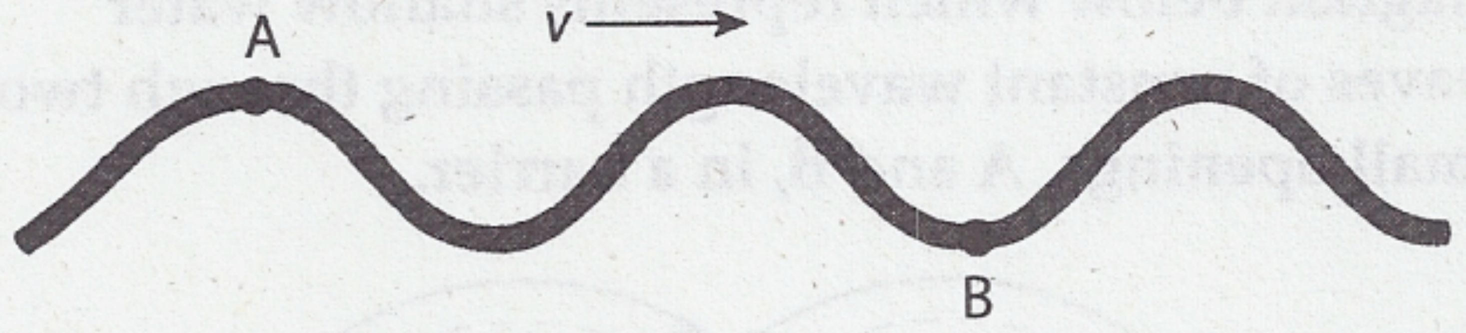
TOPIC 5

Directions

Review the Test-Taking Strategies section of this book. Then answer the following questions. Read each question carefully and answer with a correct choice or response.

Part A

1 A periodic wave travels through a rope, as shown in the diagram below.



As the wave travels, what is transferred between points A and B?

(1) mass only
(2) energy only
(3) both mass and energy
(4) neither mass nor energy

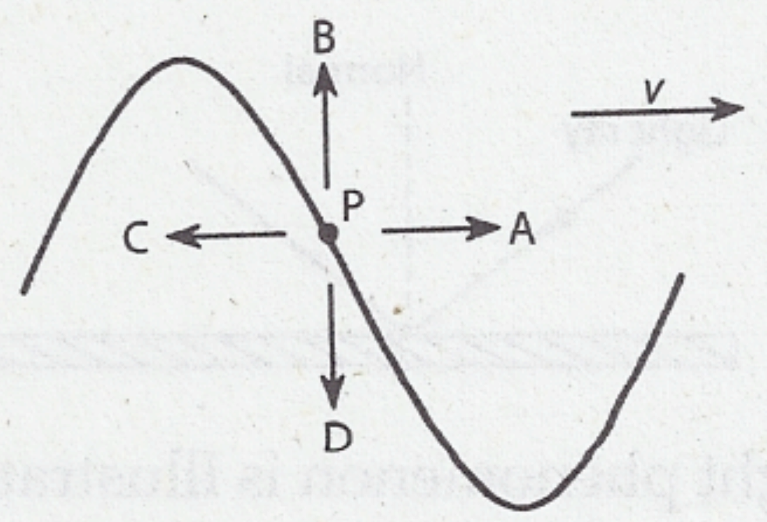
2 In which wave type is the disturbance parallel to the direction of wave travel?

(1) torsional
(2) longitudinal
(3) transverse
(4) circular

3 A tuning fork oscillates with a frequency of 256 hertz after being struck by a rubber hammer. Which phrase best describes the sound waves produced by this oscillating tuning fork?

(1) electromagetic waves that require no medium for transmission
(2) electromagnetic waves that require a medium for transmission
(3) mechanical waves that require no medium for transmission
(4) mechanical waves that require a medium for transmission

4 The diagram below shows a transverse water wave moving in the direction shown by velocity vector *v*.



At the instant shown, a cork at point P on the water's surface is moving toward

(1) A (2) B (3) C (4) D

5 The energy of a sound wave is most closely related to its

(1) period
(2) amplitude
(3) frequency
(4) wavelength

6 The product of a wave's frequency and its period is

(1) one
(2) its velocity
(3) its wavelength
(4) Planck's constant

7 What is the period of a wave with a frequency of 250 hertz?

(1) 1.2×10^{-3} s
(2) 2.5×10^{-3} s
(3) 9.0×10^{-3} s
(4) 4.0×10^{-3} s

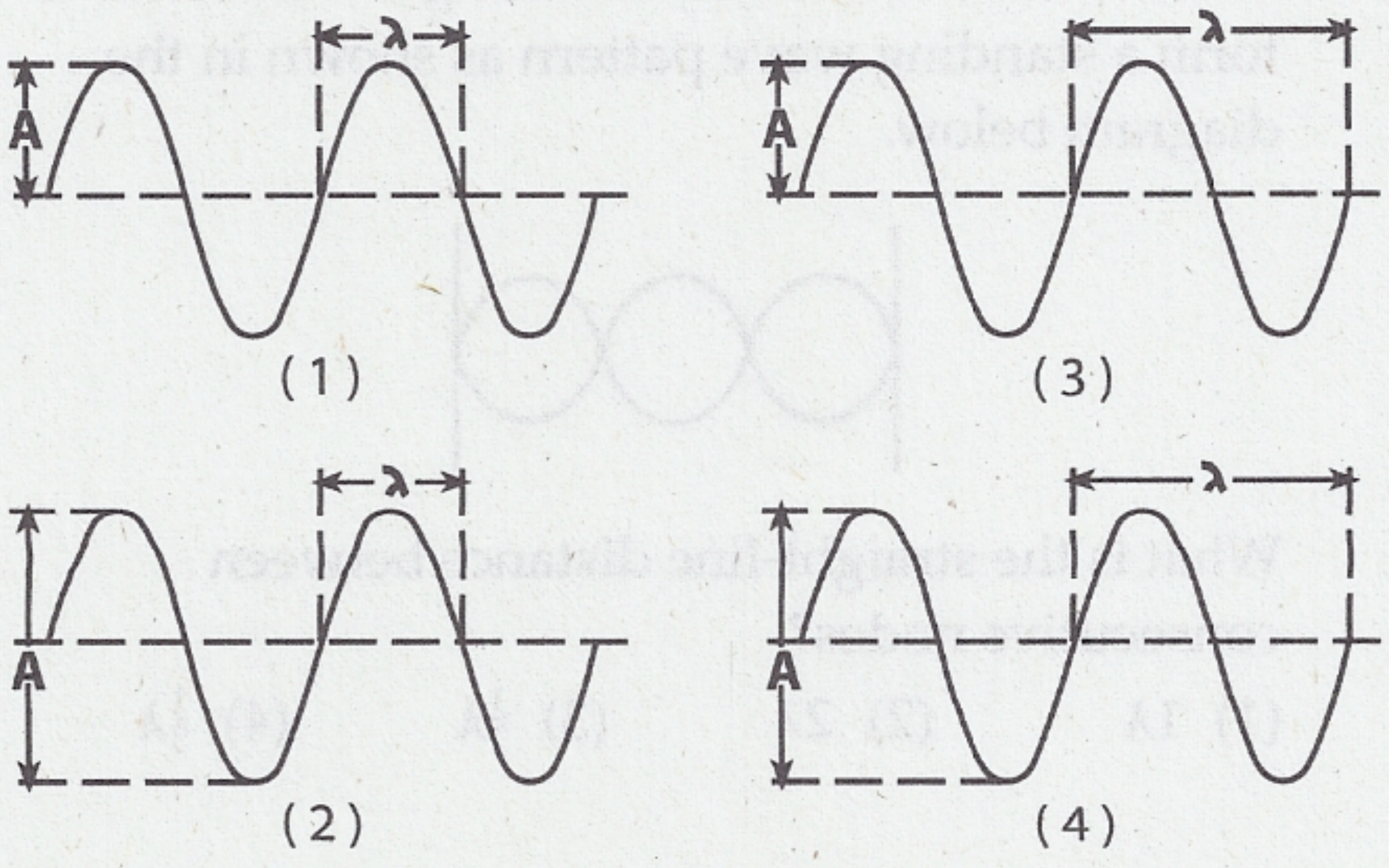
8 The reciprocal of the frequency of a periodic wave is the wave's

(1) period
(2) amplitude
(3) intensity
(4) speed

9 Two points on a transverse wave that have the same magnitude of displacement from equilibrium are in phase if the points also have

(1) the same direction of displacement and the same direction of motion
(2) the same direction of displacement and the opposite direction of motion
(3) the opposite direction of displacement and the same direction of motion
(4) the opposite direction of displacement and the opposite direction of motion

10 Which wave diagram has *both* wavelength (λ) and amplitude (*A*) labeled correctly?



11 A source of sound waves approaches a stationary observer through a uniform medium. Compared to the frequency and wavelength of the emitted sound, the observer would detect waves with a
(1) higher frequency and shorter wavelength
(2) higher frequency and longer wavelength
(3) lower frequency and shorter wavelength
(4) lower frequency and longer wavelength

12 Which phenomenon is produced by two or more waves passing simultaneously through the same region?
(1) refraction (3) interference
(2) diffraction (4) reflection

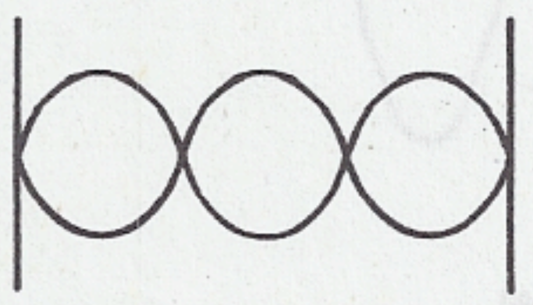
13 Maximum constructive interference between two waves of the same frequency could occur when their phase difference is
(1) 1λ (2) $\frac{\lambda}{2}$ (3) $\frac{3\lambda}{2}$ (4) $\frac{\lambda}{4}$

14 Which wave phenomenon could *not* be demonstrated with a single wave pulse?
(1) a standing wave (3) reflection
(2) diffraction (4) refraction

15 If two identical sound waves arriving at the same point are in phase, the resulting wave has
(1) an increase in speed
(2) an increase in frequency
(3) a larger amplitude
(4) a longer period

16 Standing waves are produced by the interference of two waves of the same
(1) frequency and amplitude, but opposite directions of travel
(2) frequency and direction of travel, but different amplitudes
(3) amplitude and direction of travel, but different frequencies
(4) frequency, amplitude, and direction of travel

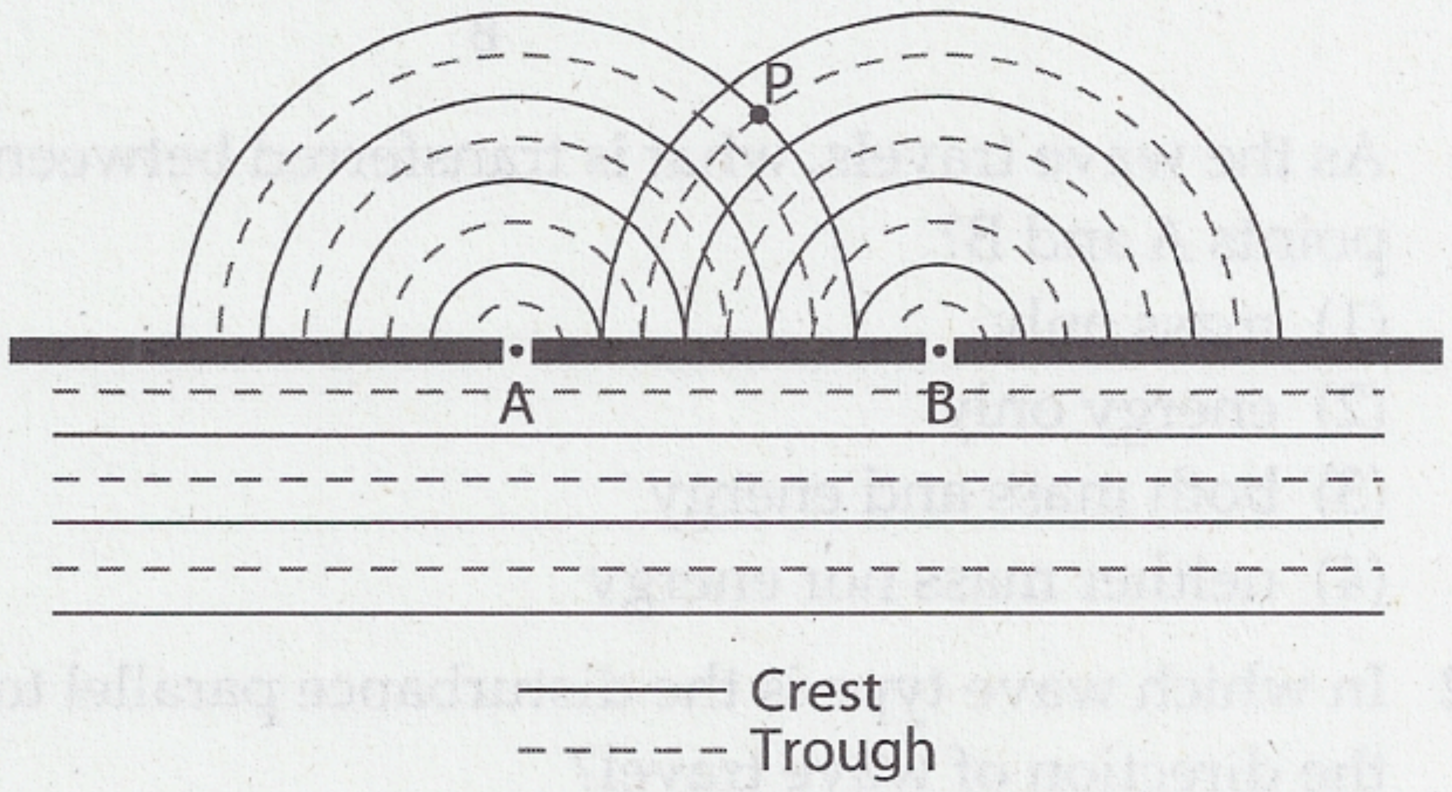
17 Two waves of the same wavelength λ interfere to form a standing wave pattern as shown in the diagram below.



What is the straight-line distance between consecutive nodes?
(1) 1λ (2) 2λ (3) $\frac{1}{2}\lambda$ (4) $\frac{1}{3}\lambda$

18 Two identical guitar strings are tuned to the same pitch. If one string is plucked, the other nearby string vibrates with the same frequency. This phenomenon is called
(1) resonance
(2) reflection
(3) refraction
(4) destructive interference

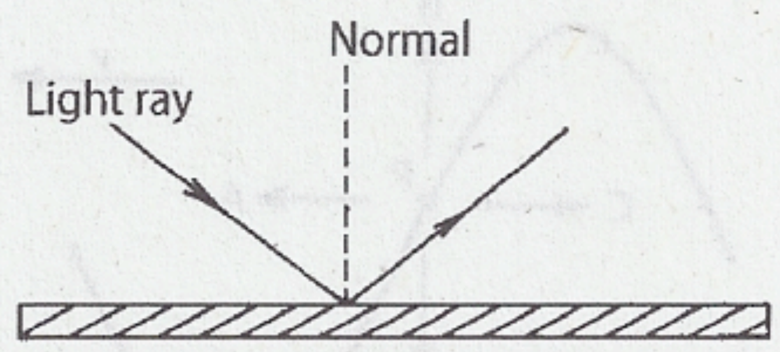
Base your answers to questions 19 and 20 on the diagram below which represents shallow water waves of constant wavelength passing through two small openings, A and B, in a barrier.



19 Compared to the length of path BP, the length of path AP is
(1) 1λ longer (3) $\frac{1}{2}\lambda$ longer
(2) 2λ longer (4) the same

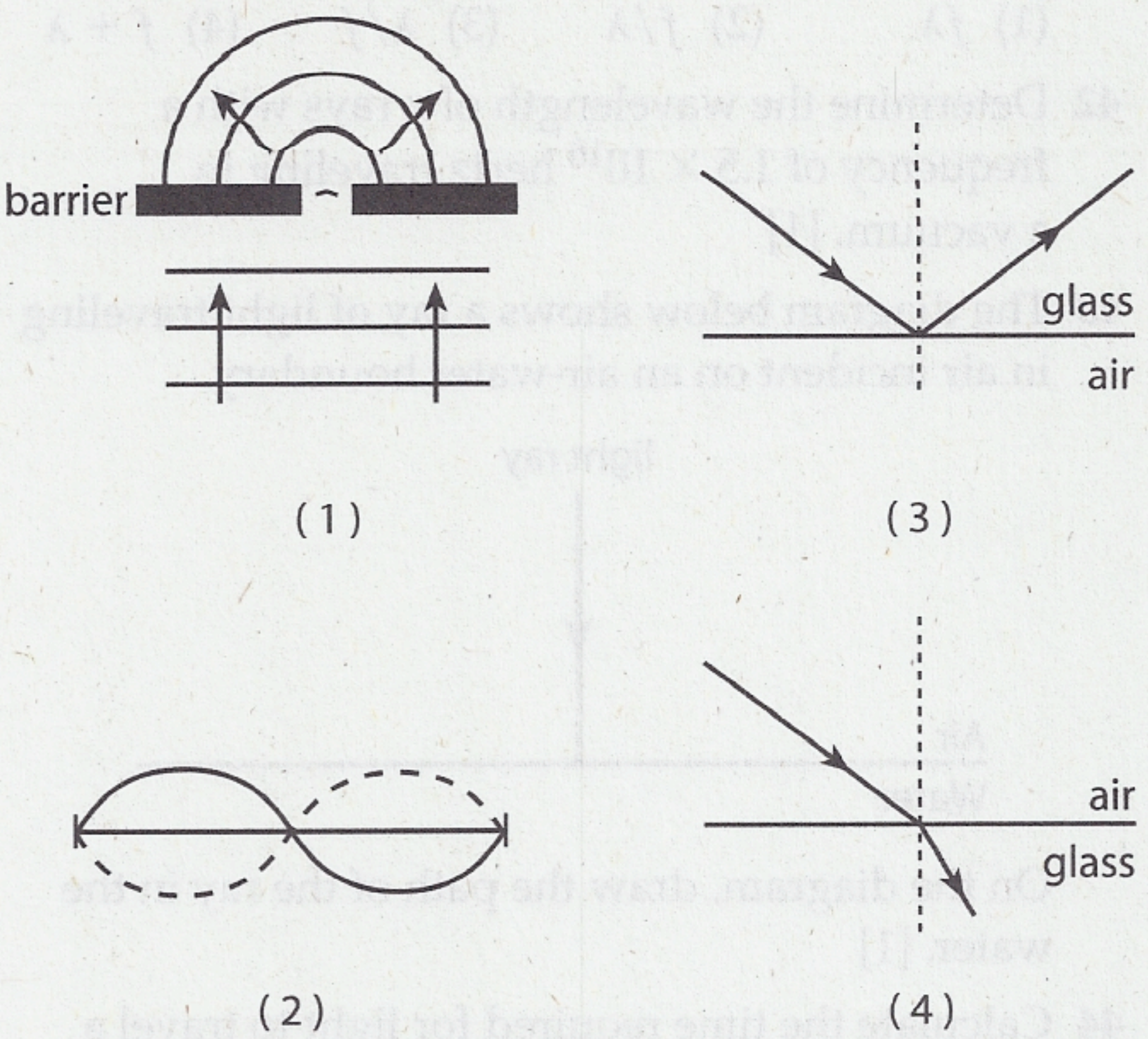
20 Which statement best describes the interference at point *P*?
(1) It is constructive, and causes a longer wavelength.
(2) It is constructive, and causes an increase in amplitude.
(3) It is destructive, and causes a shorter wavelength.
(4) It is destructive, and causes a decrease in amplitude.

21 The diagram below shows a light ray interacting with a barrier.



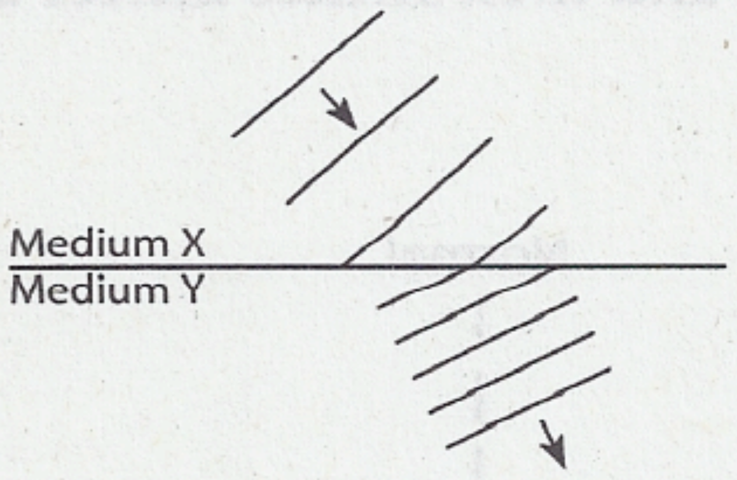
Which light phenomenon is illustrated?
(1) diffraction (3) refraction
(2) interference (4) reflection

22 Which diagram best represents the phenomenon of diffraction?



23 A ray of monochromatic light is incident on a plane mirror at an angle of 30.° The angle of reflection for the light ray is
(1) 15° (2) 30.° (3) 60.° (4) 90.°

24 The diagram below represents wave fronts traveling from medium X into medium Y.



All points on any one wave front shown must be
(1) traveling with the same speed
(2) traveling in the same medium
(3) in phase
(4) superposed

25 What is the approximate speed of light in alcohol?
(1) 1.4×10^8 m/s (3) 3.0×10^8 m/s
(2) 2.2×10^8 m/s (4) 4.4×10^8 m/s

26 What is the color of light with a frequency of 5.65×10^{14} hertz?
(1) green (2) red (3) violet (4) yellow

27 Which color of light has the lowest frequency?
(1) violet (2) green (3) yellow (4) red

28 Which waves are *not* electromagnetic?
(1) radio (3) light
(2) ultraviolet (4) sound

29 What is the speed of a radio wave in a vacuum?
(1) 0 m/s (3) 1.13×10^3 m/s
(2) 3.31×10^2 m/s (4) 3.00×10^8 m/s

30 Electromagnetic radiation is produced by
(1) an accelerating electron
(2) an accelerating neutron
(3) an electron at constant velocity
(4) a neutron at constant velocity

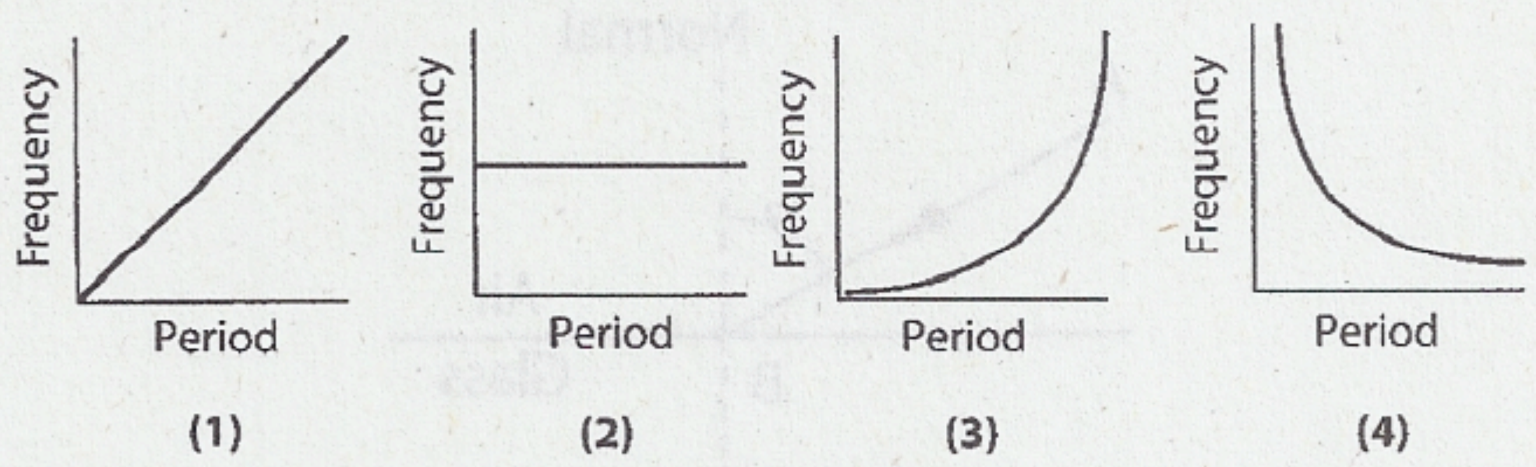
31 Which form of electromagnetic radiation has the shortest wavelength in air?
(1) ultraviolet (3) infrared
(2) visible (4) radio

32 How much time does it take light from a flash camera to reach a subject 6.0 meters across a room?
(1) 5.0×10^{-9} s (3) 5.0×10^{-8} s
(2) 2.0×10^{-8} s (4) 2.0×10^{-7} s

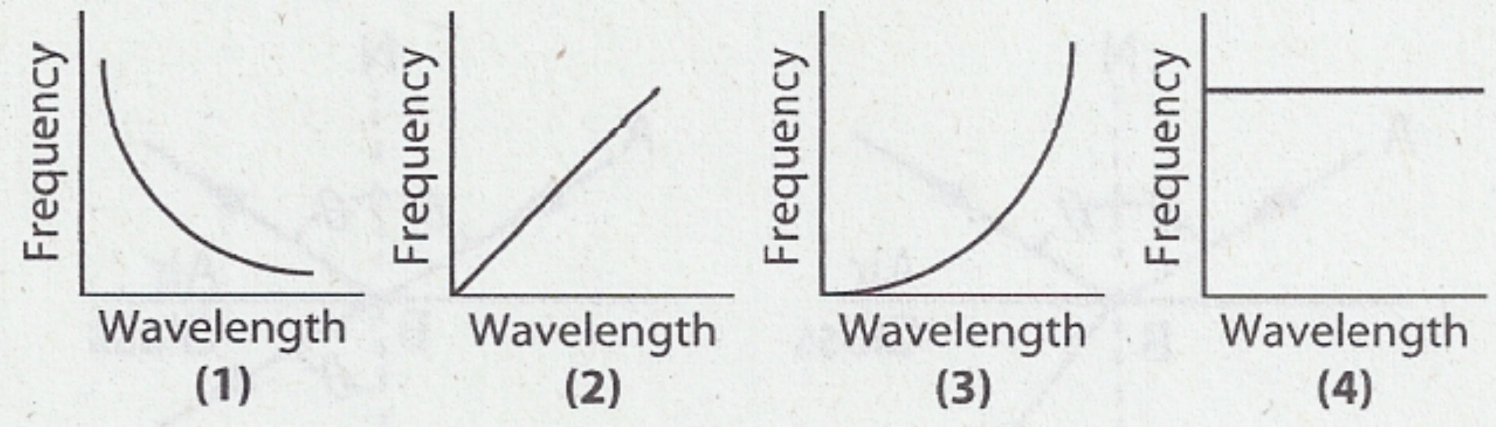
33 Electromagnetic radiation having a wavelength of 1.3×10^{-7} meter would be classified as
(1) infrared (3) blue
(2) orange (4) ultraviolet

Part B

34 Which graph best represents the relationship between the frequency and period of a wave?



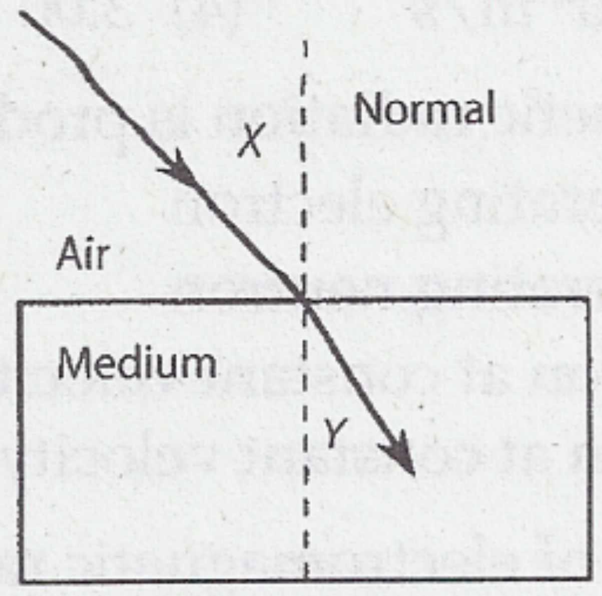
35 Which graph best represents the relationship between frequency and wavelength for microwaves in a vacuum?



36 If the period of a wave is doubled, its wavelength is
(1) halved (3) unchanged
(2) doubled (4) quartered

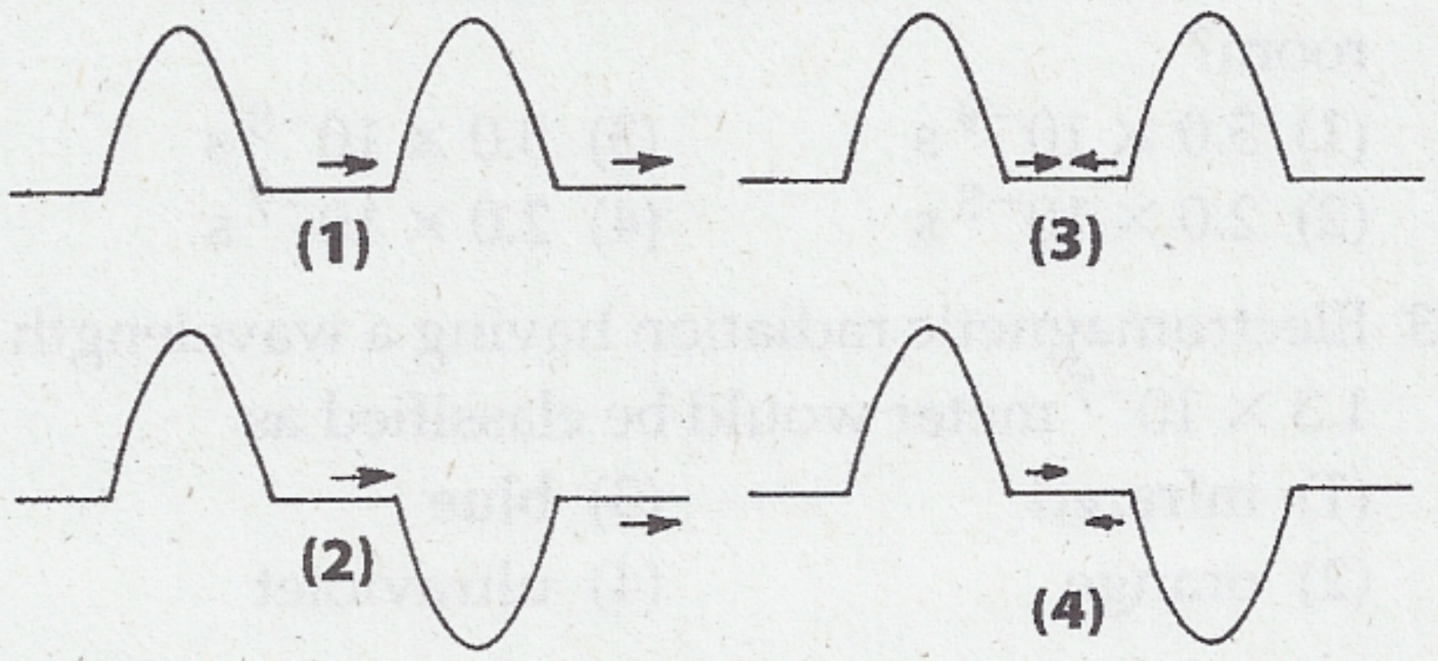
37 Periodic waves with a wavelength of 0.50 meter move with a speed of 0.30 meter per second in medium A. When the waves enter medium B, they travel at 0.15 meter per second. Calculate the wavelength of the waves in medium B. [2]

38 In the diagram below, a ray of light enters a *transparent medium from air.*

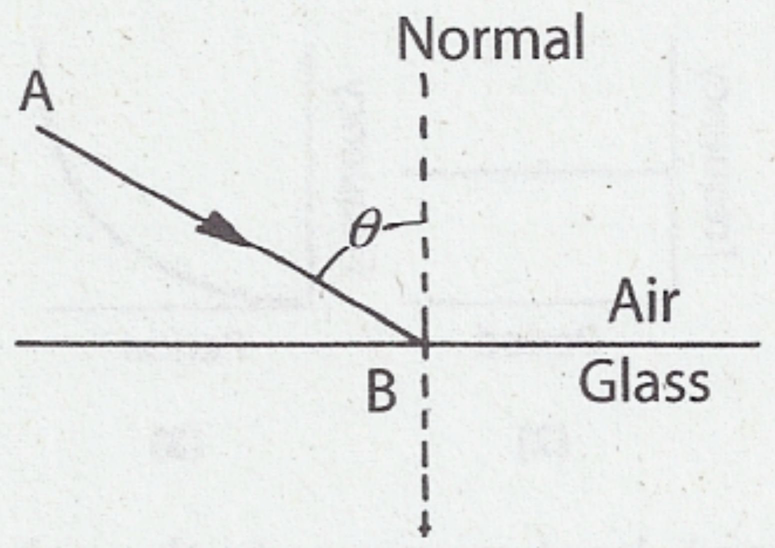


If angle X is 45° and angle Y is 30.°, what is the absolute index of refraction of the medium?

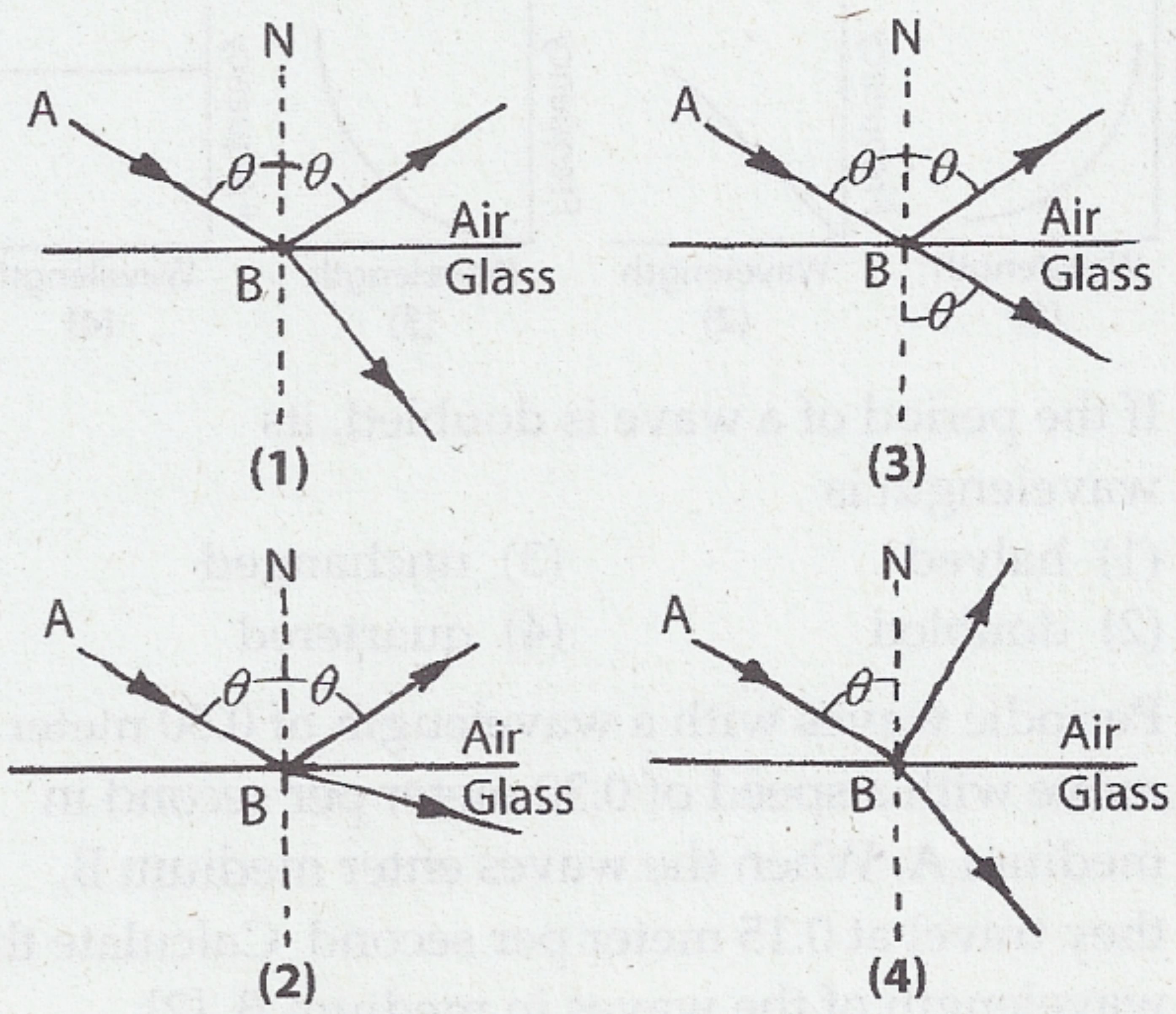
39 Which pair of moving pulses in a rope will produce destructive interference?



40 A ray of monochromatic light AB in air strikes a piece of glass at an incident angle θ, as shown in the diagram below.



Which diagram best illustrates the ray's interaction with the glass?

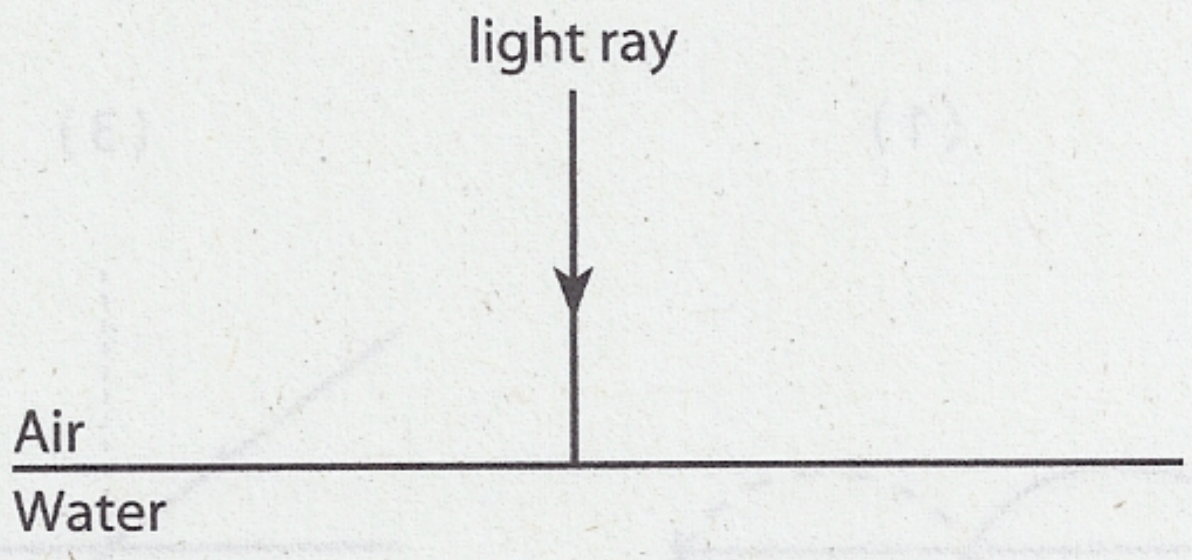


41 Which expression represents a constant for light waves of different frequencies in a vacuum?

(1) $f\lambda$ (2) f/λ (3) λ/f (4) $f + \lambda$

42 Determine the wavelength of x rays with a frequency of 1.5×10^{18} hertz traveling in a vacuum. [1]

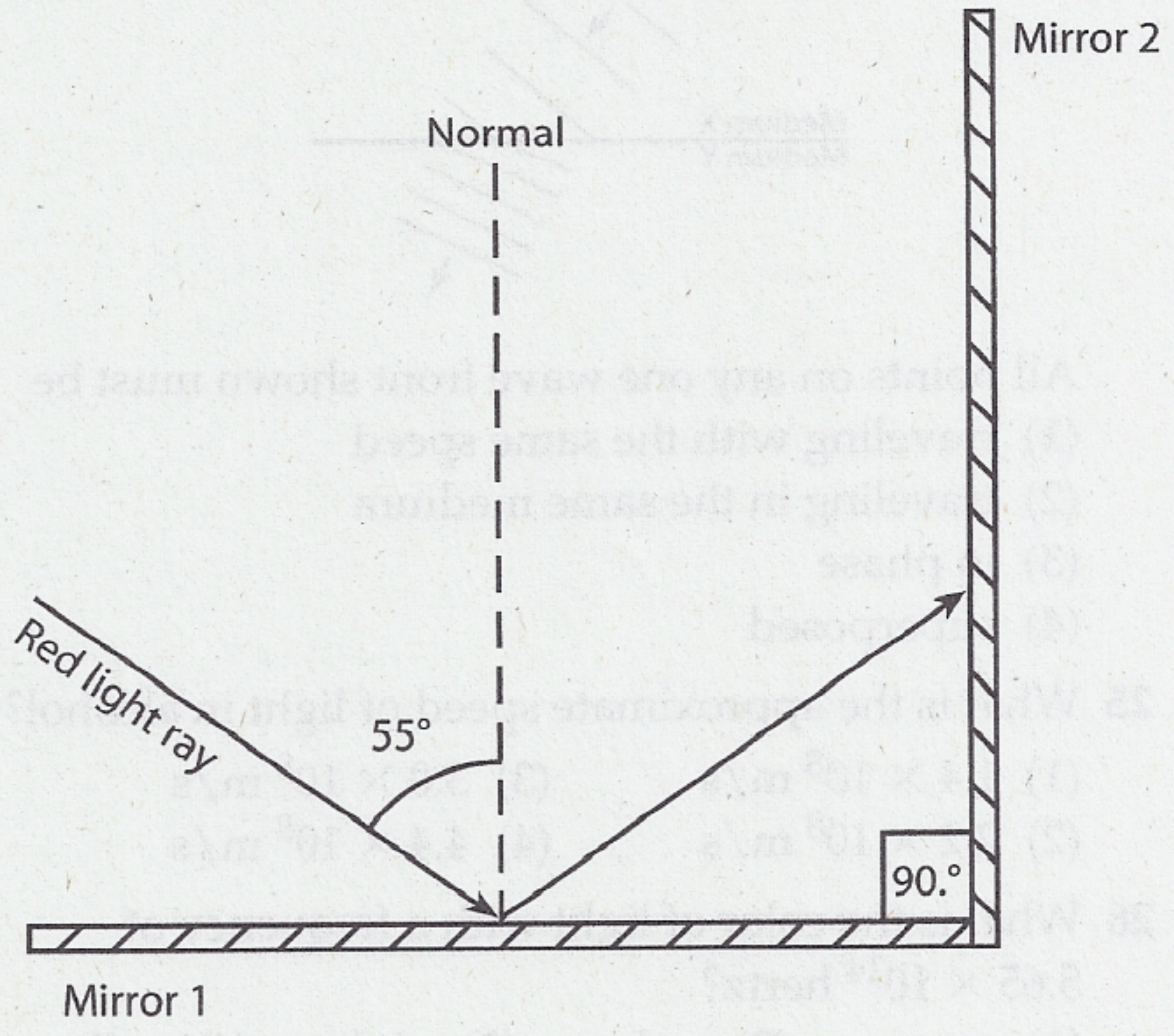
43 The diagram below shows a ray of light traveling in air incident on an air-water boundary.



On the diagram, draw the path of the ray in the water. [1]

44 Calculate the time required for light to travel a distance of 1.50×10^{11} meters. [2]

Base your answers to questions 45 and 46 on the information and diagram below. Two plane mirrors are positioned perpendicular to each other as shown. A ray of monochromatic red light is incident on mirror 1 at an angle of 55°. This ray is reflected from mirror 1 and then strikes mirror 2.

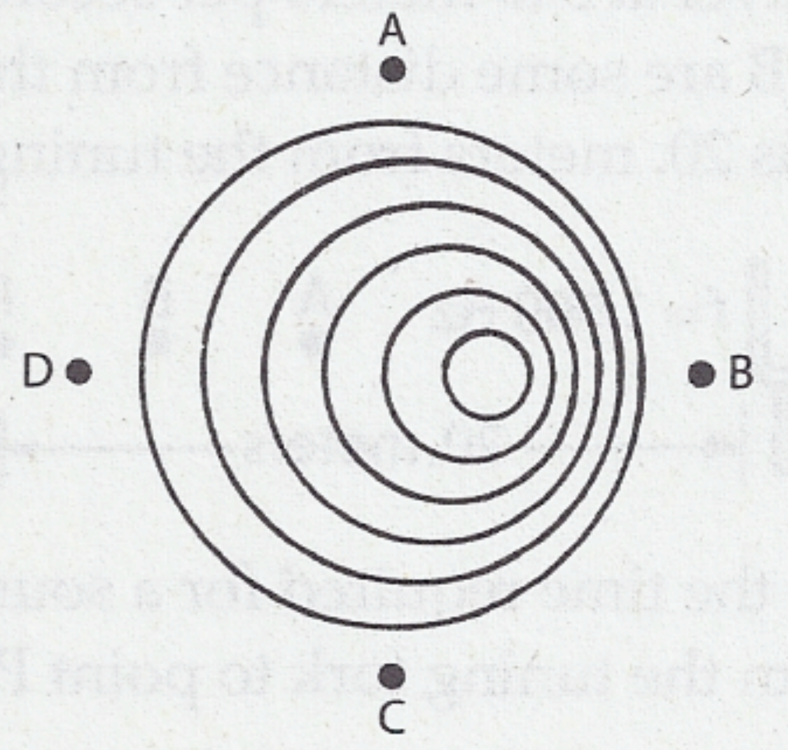


45 Determine the angle at which the ray is incident on mirror 2. [1]

46 On the diagram, use a protractor and a straightedge to draw the ray of light as it is reflected from mirror 2. [1]

47 Determine the speed of a ray of light ($f = 5.09 \times 10^{14}$ Hz) traveling through a block of sodium chloride. [1]

Base your answers to questions 48 through 51 on the diagram below, which represents the wave pattern produced by a vibrating source of constant frequency moving linearly in a shallow tank of water. The pattern is viewed from above and the lines represent crests.



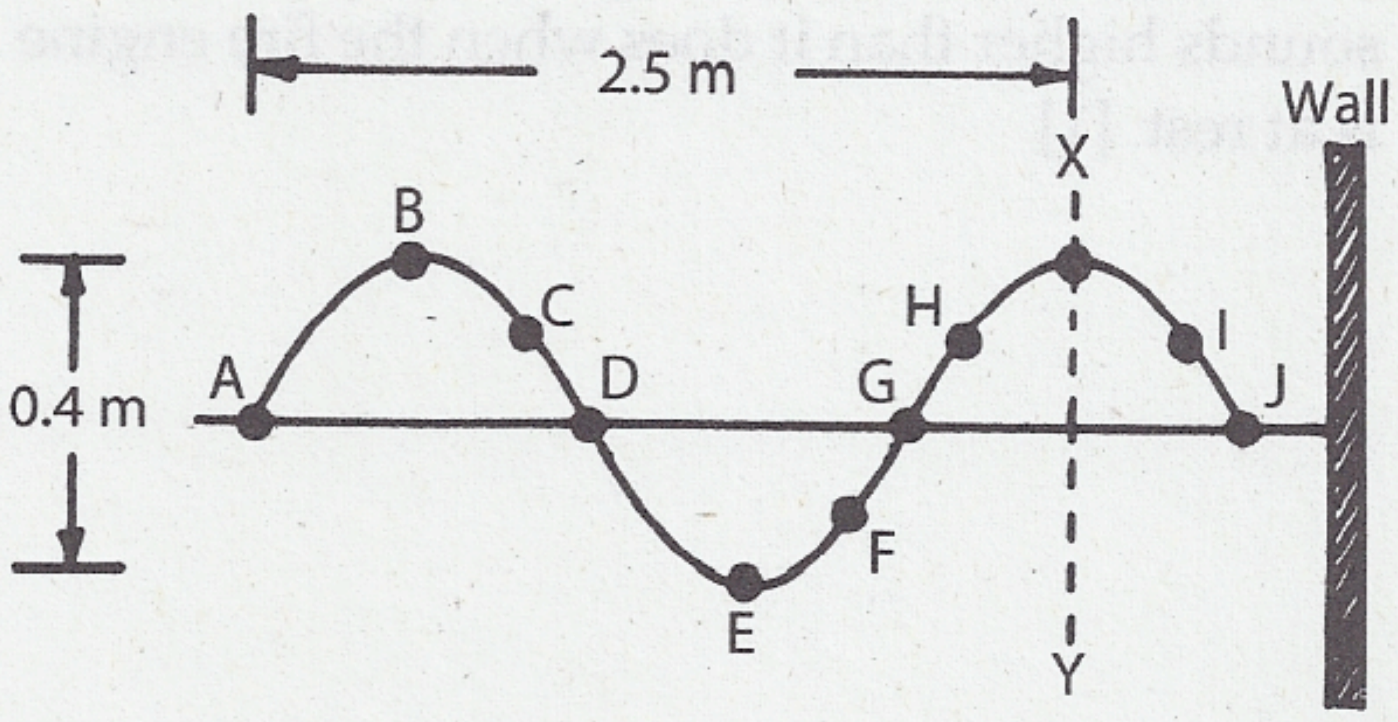
48 Towards which point is the source moving? [1]

49 What wave phenomenon is illustrated by the wave pattern? [1]

50 Compare the frequency of the waves observed at point B to the frequency of the waves observed at point D. [1]

51 Describe the wavelength of the waves observed at point D if the magnitude of the velocity of the source is increased. [1]

Base your answers to questions 52 through 58 on the diagram below, which represents a segment of a periodic wave traveling to the right in a steel spring. A crest passes line XY every 0.40 second.



52 What is the amplitude of the wave? [1]

53 What is the wavelength of the wave? [1]

54 How many cycles of the wave are shown? [1]

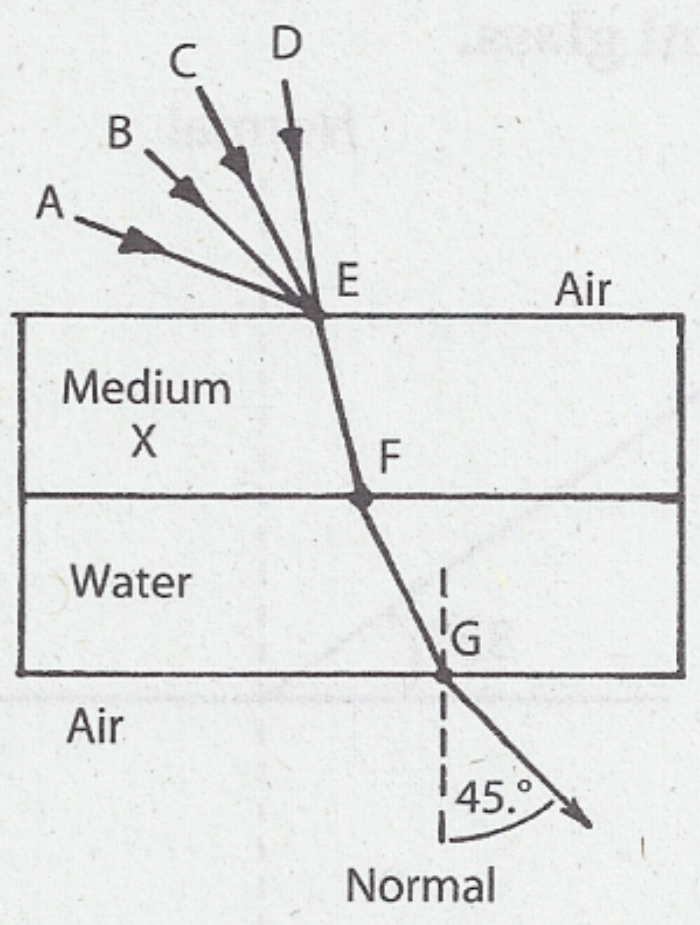
55 What is the frequency of the wave? [1]

56 Calculate the speed of the wave. [2]

57 Identify two points on the wave that are in phase. [1]

58 *In which direction will point H move in the next instant of time?* [1]

Base your answers to questions 59 through 64 on the diagram below, which represents two media with parallel surfaces in air and a ray of light ($f = 5.09 \times 10^{14}$ Hz) passing through them.



59 Calculate the approximate speed of the light in water. [2]

60 Calculate the angle of incidence in water, if the angle of refraction in air is 45°. [2]

61 Which line best represents the incident ray in air? [1]

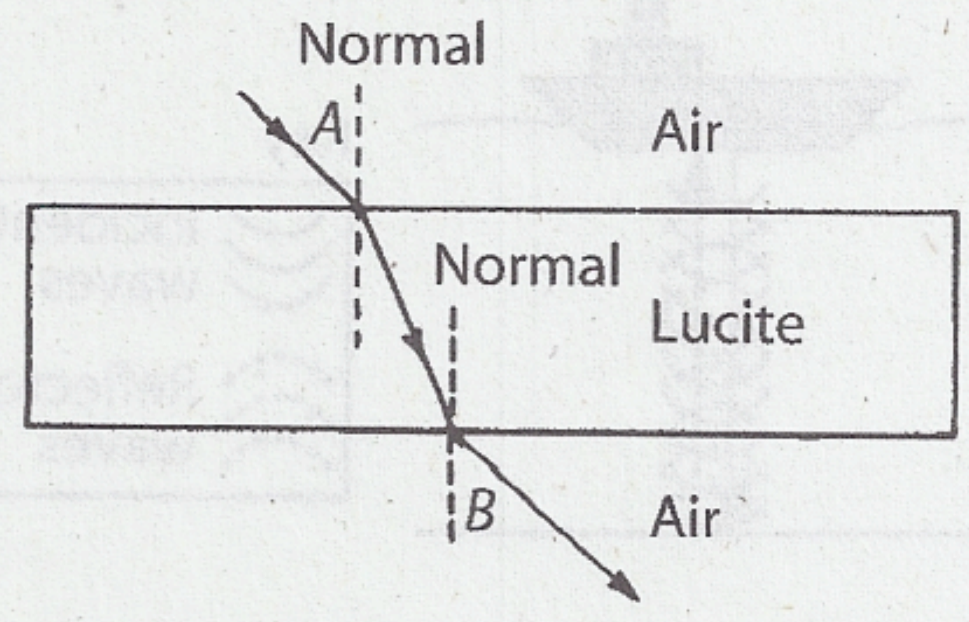
62 Compare the speed of light in water to the speed of light in medium X. [1]

63 Identify an absolute index of refraction for medium *X* that would make ray *EFG* a straight line. [1]

64 Calculate the wavelength of the light in water. [2]

Base your answers to questions 65 through 69 on the information and diagram below.

A ray of monochromatic light having a wavelength of 4.00×10^{-7} meter in air passes from air through Lucite and then into air again.



65 Calculate the frequency of the light in air. [2]

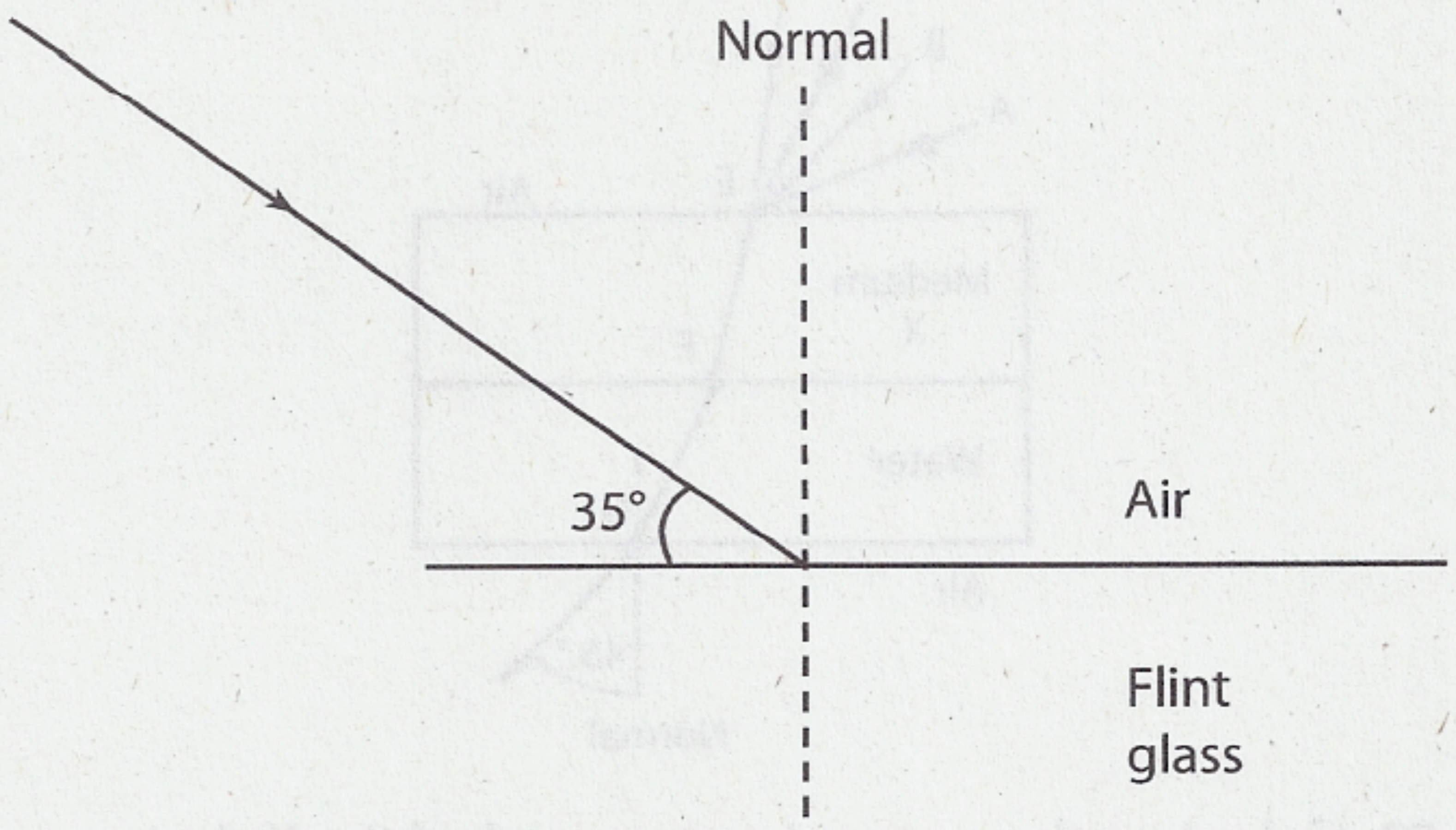
66 Identify the color of the light. [1]

67 Calculate the wavelength of the light in Lucite. [2]

68 Compare the measure of angle *A* to the measure of angle *B*. [1]

69 If angle *A* was increased, what would happen to the angle of refraction in the Lucite? [1]

Base your answers to questions 70 through 73 on the diagram below, which represents a ray of monochromatic light ($f = 5.09 \times 10^{14}$ Hz) in air incident on flint glass.



70 Determine the angle of incidence of the light ray in air. [1]

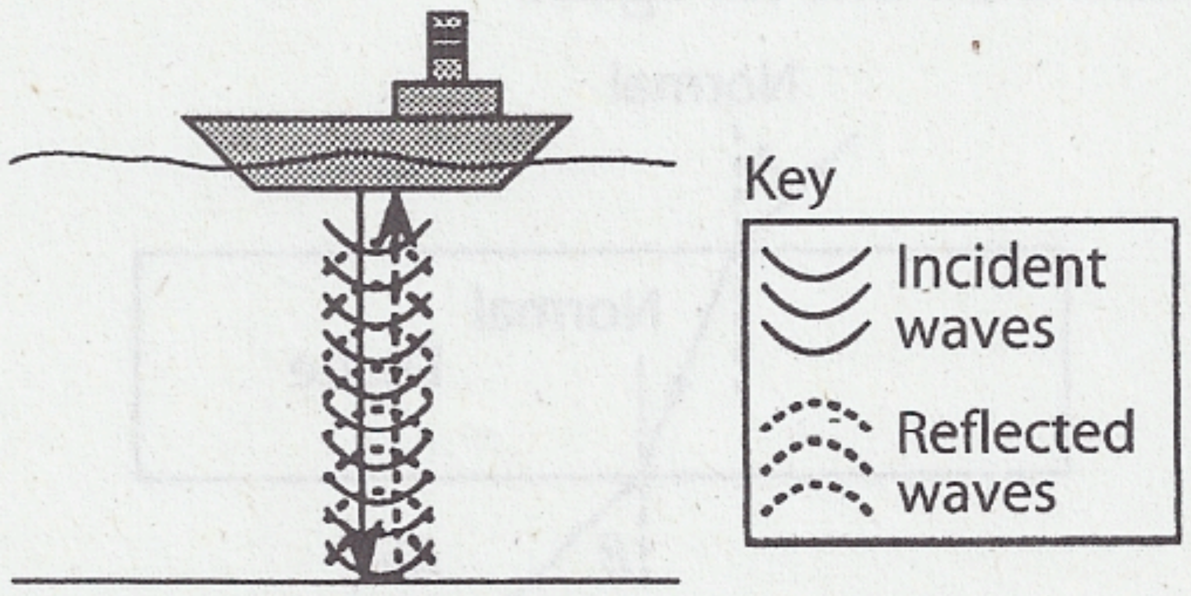
71 Calculate the angle of refraction of the light ray in the flint glass. [2]

72 Using a protractor and straightedge, draw the refracted ray on the diagram. [1]

73 What happens to the light from the incident ray that is *not* refracted or absorbed? [1]

Base your answers to questions 74 through 76 on the information and diagram below.

The sonar of a stationary ship sends a signal with a frequency of 5.0×10^3 hertz down through water. The speed of the signal is 1.5×10^3 meters per second. The echo from the bottom is detected 4.0 seconds later.



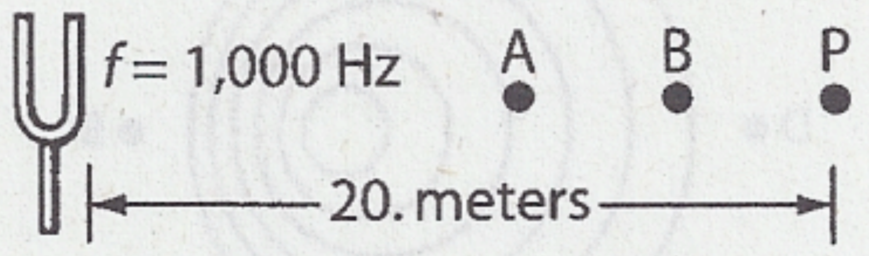
74 Calculate the wavelength of the sonar wave. [2]

75 Calculate the depth of the water under the ship. [2]

76 The echo is an example of which wave phenomenon? [1]

Base your answers to questions 77 through 80 on the information and diagram below.

A vibrating 1000-hertz tuning fork produces sound waves that travel at 340 meters per second in air. Points A and B are some distance from the tuning fork. Point P is 20. meters from the tuning fork.



77 Calculate the time required for a sound wave to travel from the tuning fork to point P. [2]

78 Calculate the wavelength of the sound waves produced by the tuning fork. [2]

79 If the waves are in phase at point A and B, what is the minimum distance separating points A and B in terms of λ? [1]

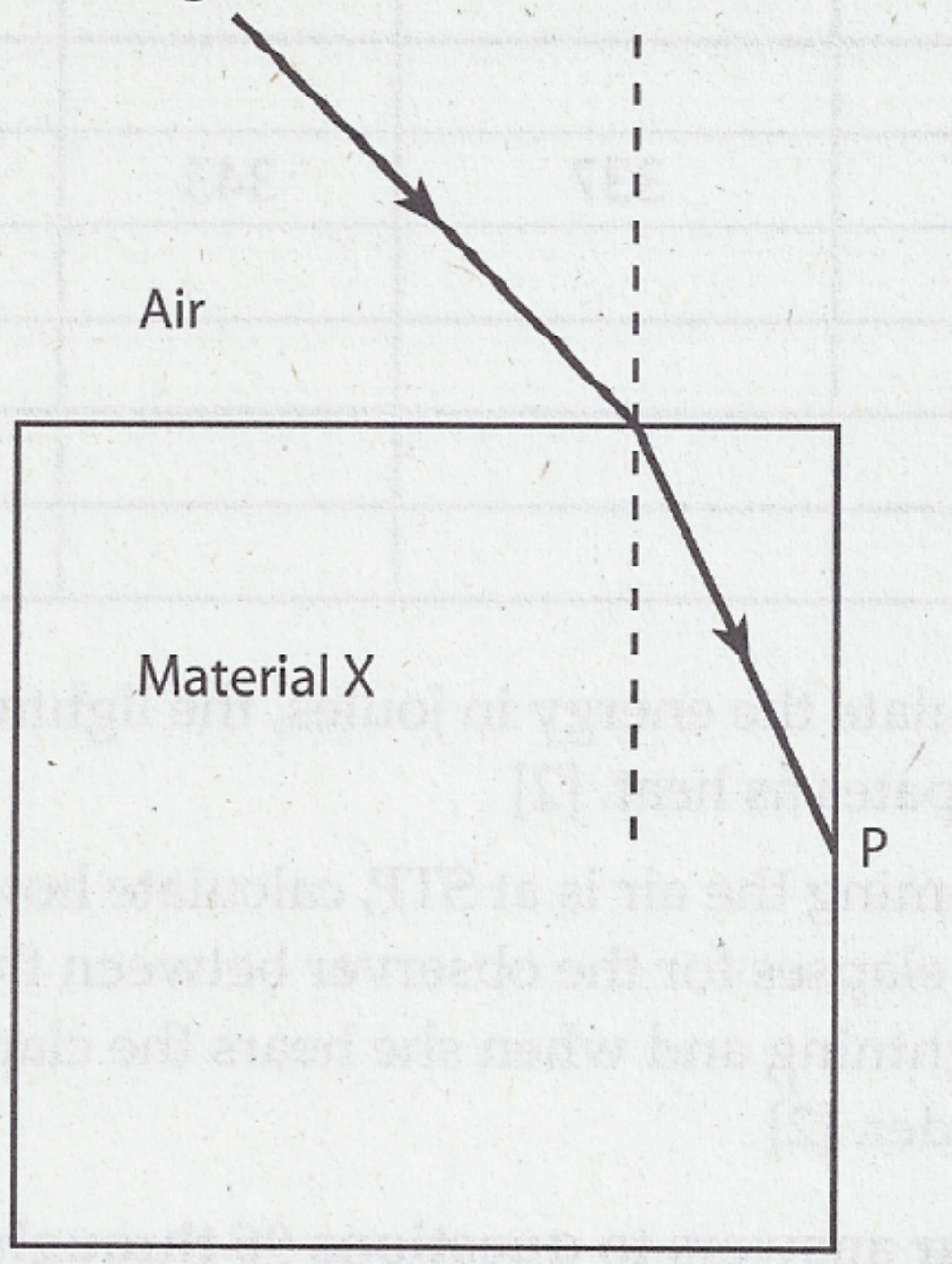
80 If the vibrating tuning fork is accelerated toward point P, what happens to the frequency of the sound observed at P? [1]

81 A sound wave has a wavelength of 5.5 meters as it travels through air at STP. Determine the wavelength of this sound in a medium where its speed is 1324 meters per second. [1]

82 Explain why, when a rapidly moving fire engine is coming toward you, the pitch of its siren sounds higher than it does when the fire engine is at rest. [1]

Base your answers to questions 83 and 84 on the information and diagram below.

A ray of light passes from air into a block of transparent material X as shown.



83 Measure the angles of incidence and refraction to the nearest degree for this light ray at the air into material *X* boundary and write your answers in the appropriate places on the diagram. [2]

84 The refracted light ray is reflected from the material *X*–air boundary at point *P*. Using a protractor and straightedge, on the diagram draw the reflected ray from point *P*. [1]

Part C

Base your answers to questions 85 through 92 on the following information, diagram, and data table.

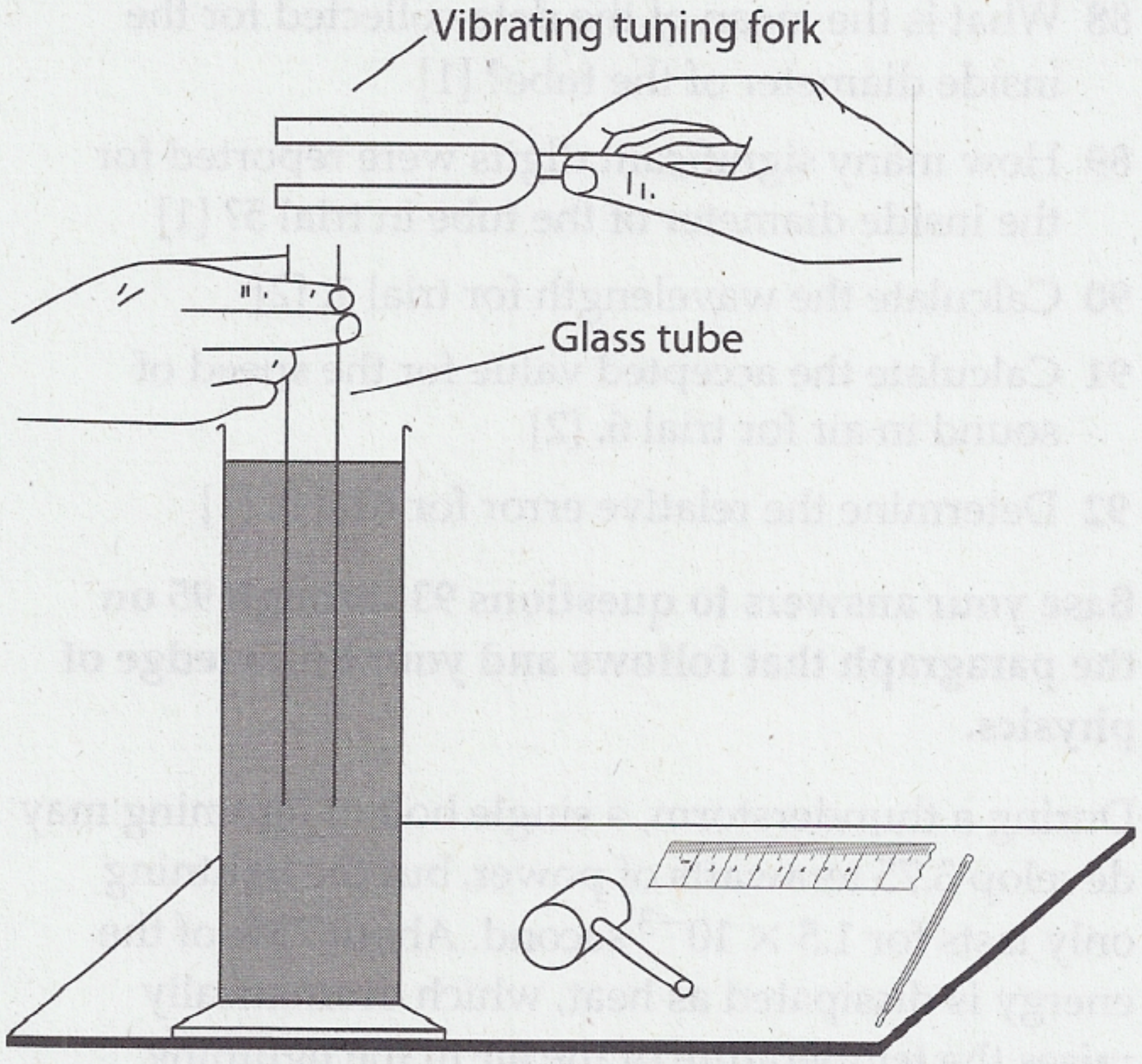
Seven pairs of students performed an experiment to determine the speed of sound in air in the classroom. The apparatus consisted of a tall cylinder nearly filled with water, a hollow glass tube, a 30-centimeter ruler, a Celsius thermometer, a tuning fork marked 512 hertz, and a rubber mallet. The glass tube was held vertically in the cylinder of water. After striking the tuning fork with the mallet, it was held over the open end of the tube as shown.

Keeping the vibrating fork just above the edge of the tube, the glass tube was slowly moved up and down in the water until the position was located where the sound was loudest. The length of the air column in the glass tube at this point was measured and recorded. The inside diameter of the tube and the temperature of the air inside the tube were also measured and recorded in the incomplete data table that follows. Each pair of students used the same tuning fork and all the data was collected within a 15-minute time interval.

Students were instructed to use the formula $\lambda = 4\ell + 1.6d$ to calculate the wavelength λ of the sound wave that was produced in the air column by the tuning fork. They were also told to use the formula

$$v = 331\sqrt{1 + \frac{T_C}{273}}$$

to determine the accepted value for v, the speed of sound in air in meters per second at a particular Celsius temperature T_C.



Data Table

Trial	Length ℓ of air column (m)	Inside diameter d of tube (m)	Wavelength (m)	Frequency (Hz)	Temperature (°C)	Experimental speed of sound (m/s)	Accepted speed of sound, v (m/s)	Relative error (%)
1	0.163	0.032		512	21.5			
2	0.149	0.039		512	21.5			
3	0.150	0.037	0.659	512	20.5	337	343	
4	0.149	0.037		512	21.5			
5	0.152	0.040		512	21.8			
6	0.159	0.038	0.697	512	21.5			
7	0.152	0.040		512	21.8			

85 What type of wave was produced by the vibrating tuning fork? [1]

86 The loudest sound was produced when the natural frequency of the air in the column was the same as that of the vibrating tuning fork. What is the name of this wave phenomenon? [1]

87 What is the range of data collected for the length of the air column? [1]

88 What is the mean of the data collected for the inside diameter of the tube? [1]

89 How many significant digits were reported for the inside diameter of the tube in trial 5? [1]

90 Calculate the wavelength for trial 1. [2]

91 Calculate the accepted value for the speed of sound in air for trial 6. [2]

92 Determine the relative error for trial 3. [1]

Base your answers to questions 93 through 95 on the paragraph that follows and your knowledge of physics.

During a thunderstorm, a single bolt of lightning may develop 3.75 terawatts of power, but the lightning only lasts for 1.5×10^{-3} second. About 75% of the energy is dissipated as heat, which dramatically raises the temperature of the air in the lightning channel, causing the air to expand quickly. The movement creates sound waves that can be heard as thunder for distances up to 30. kilometers. An observer located 30. kilometers from the lightning strike sees the flash of lightning before hearing the clap of thunder.

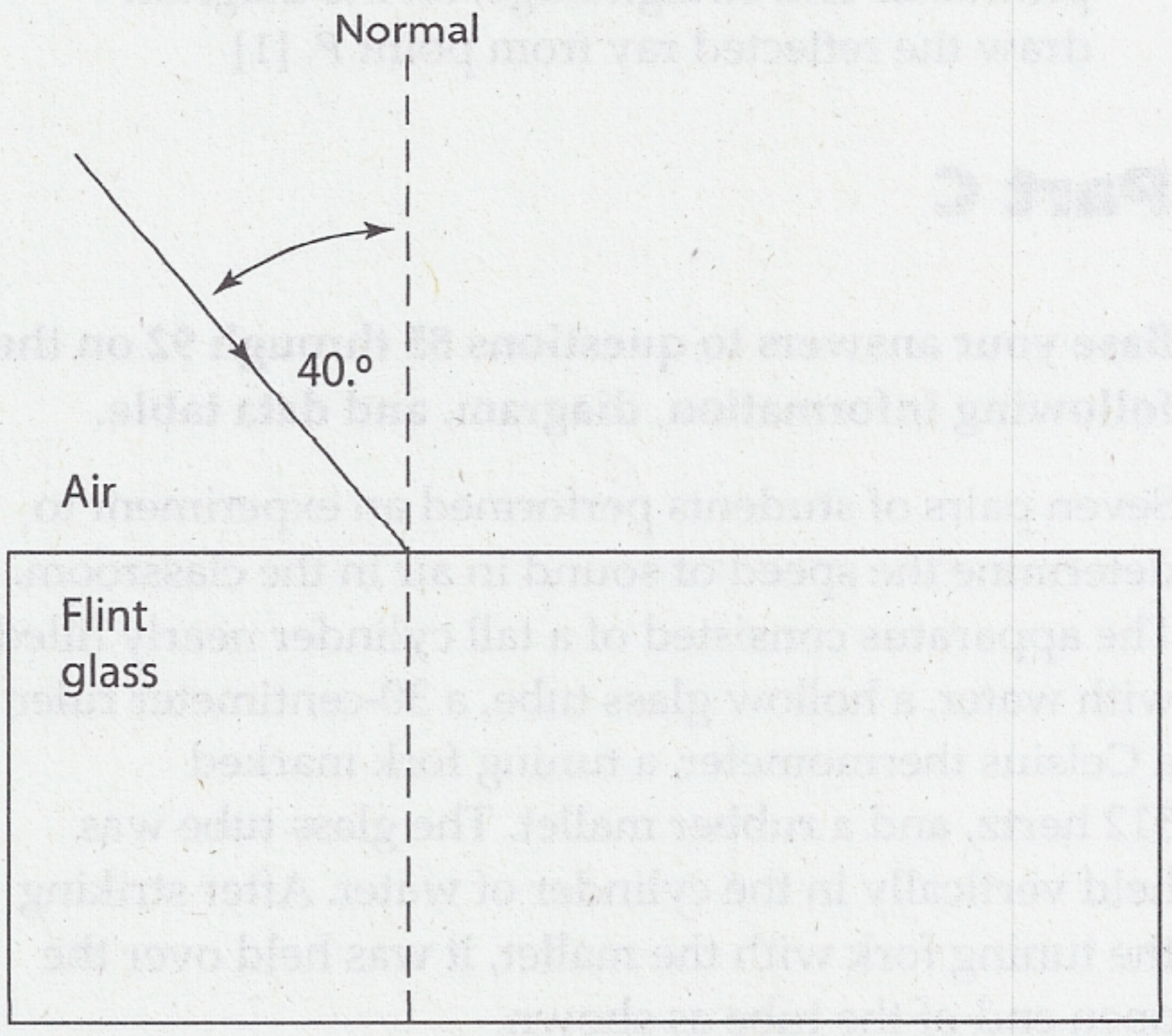
93 Express in scientific notation the power developed by the lightning bolt in watts. [1]

94 Calculate the energy in joules, the lightning bolt dissipates as heat. [2]

95 Assuming the air is at STP, calculate how much time elapses for the observer between the flash of lightning and when she hears the clap of thunder. [2]

Base your answers to questions 96 through 99 on the information and diagram below.

A ray of monochromatic light ($f = 5.09 \times 10^{14}$ Hz) is traveling in air. The ray is incident on the surface of a block of flint glass at an angle of 40.°, as shown. Part of the light is reflected at the air-glass interface and part is refracted in the glass.



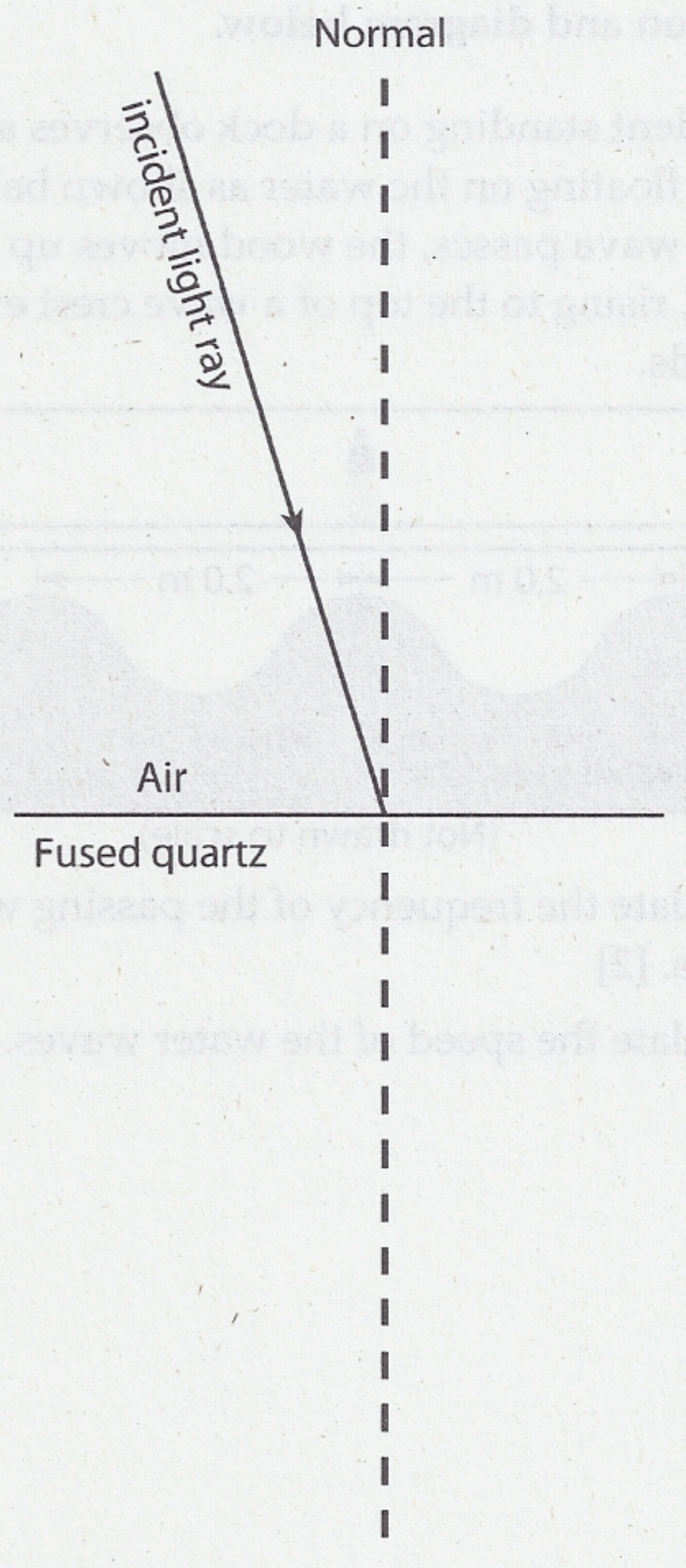
96 On the diagram, draw the reflected ray and label the angle of reflection with its measure in degrees. [2]

97 Calculate the angle of refraction in the flint glass to the nearest degree. [2]

98 On the diagram, draw the refracted ray. Label it "refracted ray." [1]

99 Calculate the wavelength of the light ray in flint glass. [2]

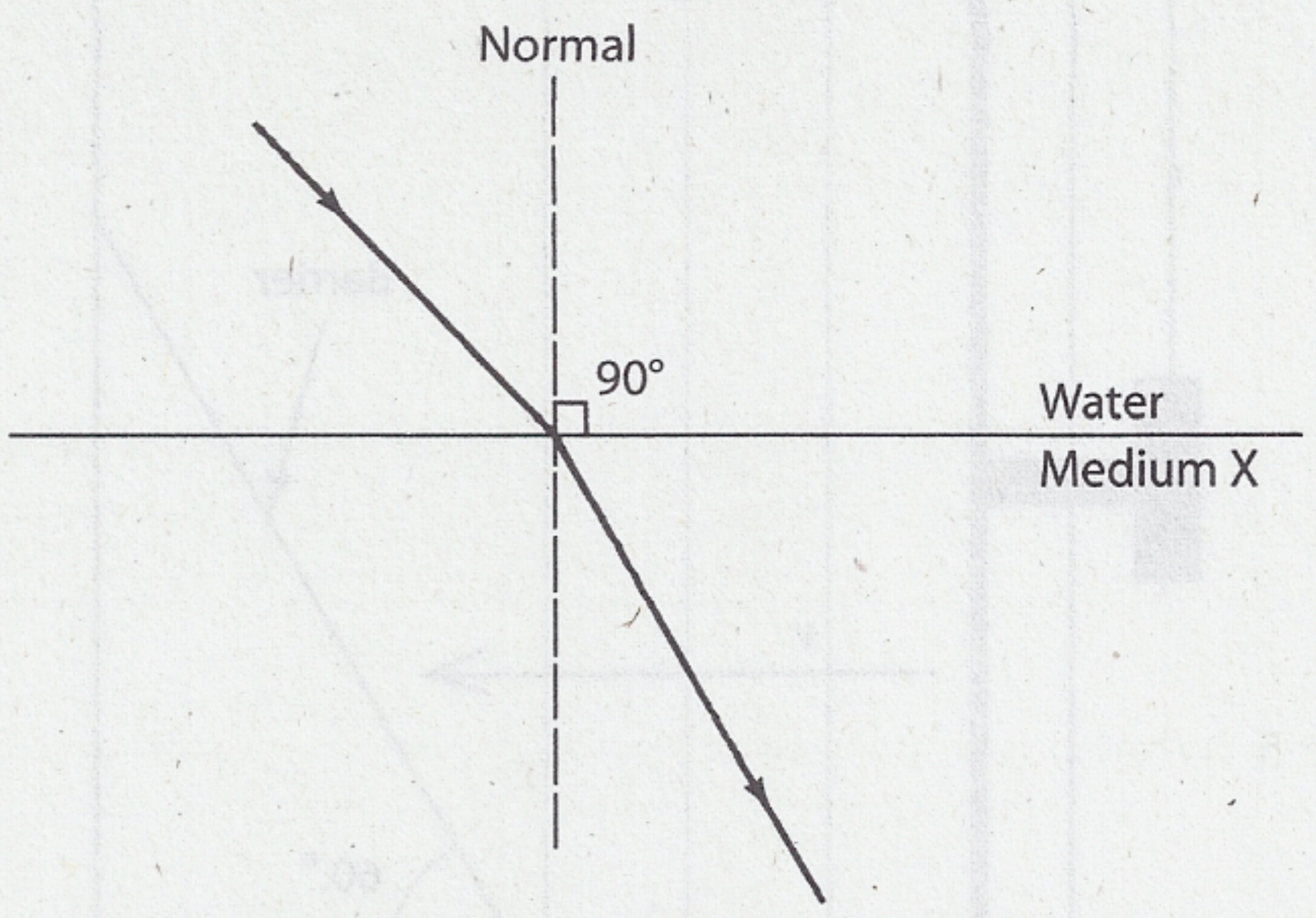
Base your answers to questions 100 and 101 on the diagram below, which shows a light ray ($f = 5.09 \times 10^{14}$ Hz) in air, incident on a boundary with fused quartz. At the boundary, part of the light is refracted and part of the light is reflected.



100 Calculate the angle of refraction of the incident light ray. [2]

101 Using a protractor and straightedge, construct the refracted light ray in the fused quartz on the diagram. [1]

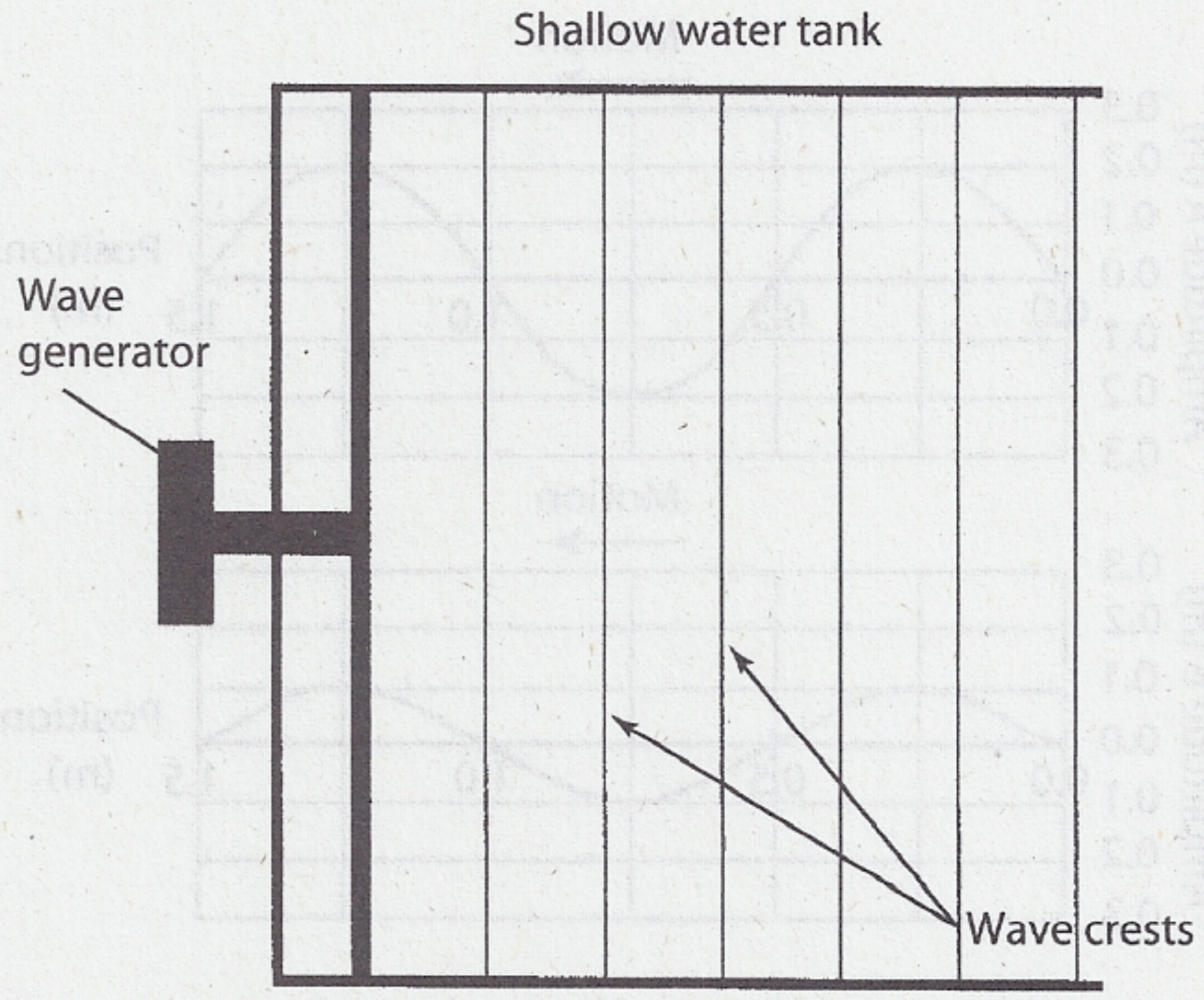
102 A ray of monochromatic light ($f = 5.09 \times 10^{14}$ Hz) is incident upon an interface of water and an unknown medium, X. The ray is refracted in medium X as shown in the diagram below.



Calculate the speed of light in medium X. [4]

Base your answers to questions 103 through 105 on the following information and diagram.

The diagram represents a wave generator having a constant frequency of 12 hertz and producing parallel wave fronts in a shallow tank of water. The velocity of the wave is v.

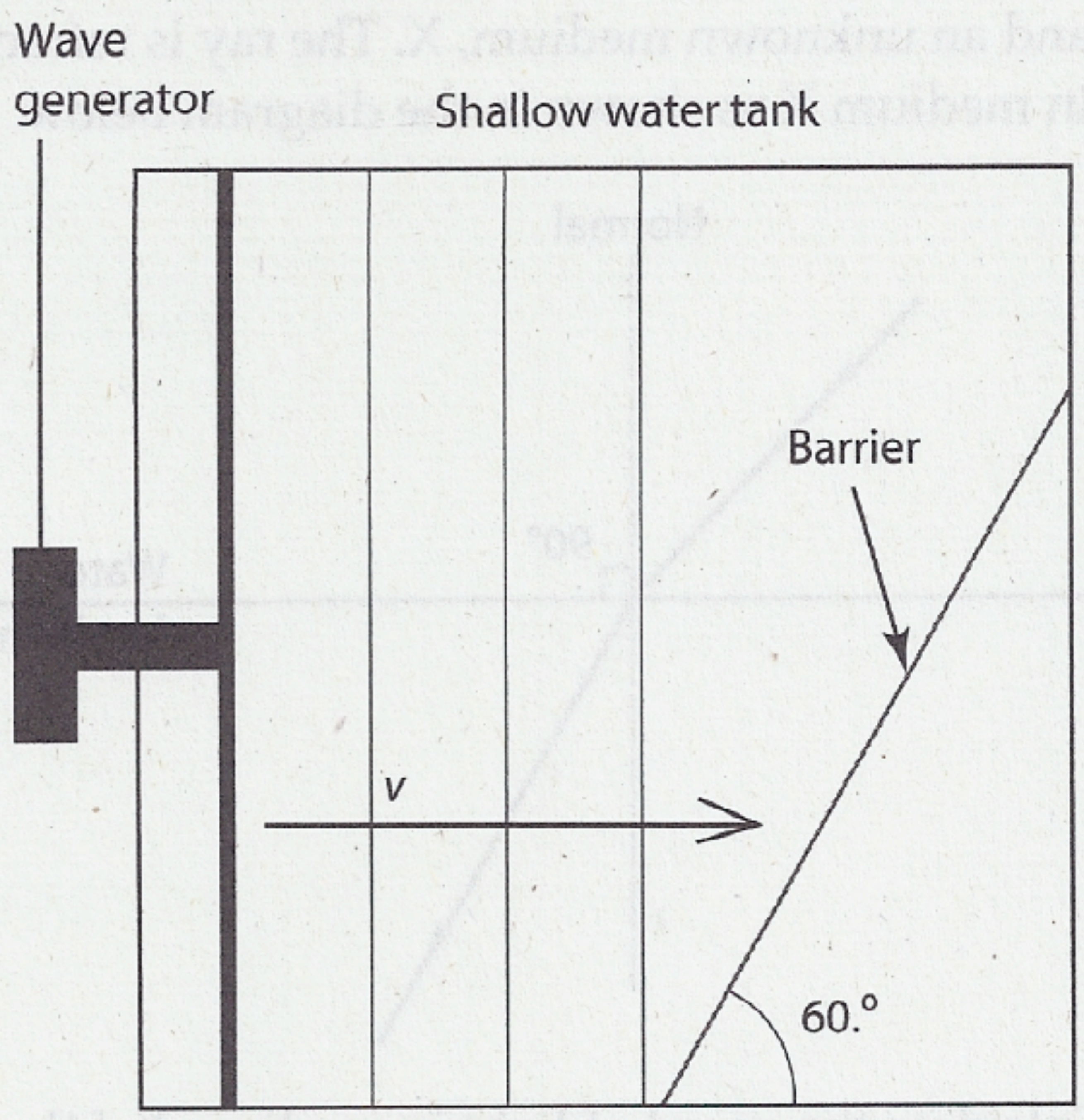


103 Determine the period of the waves. [1]

104 Using a ruler, measure the wavelength of the waves to the nearest tenth of a centimeter. [1]

105 Calculate the speed of the waves in the tank. [2]

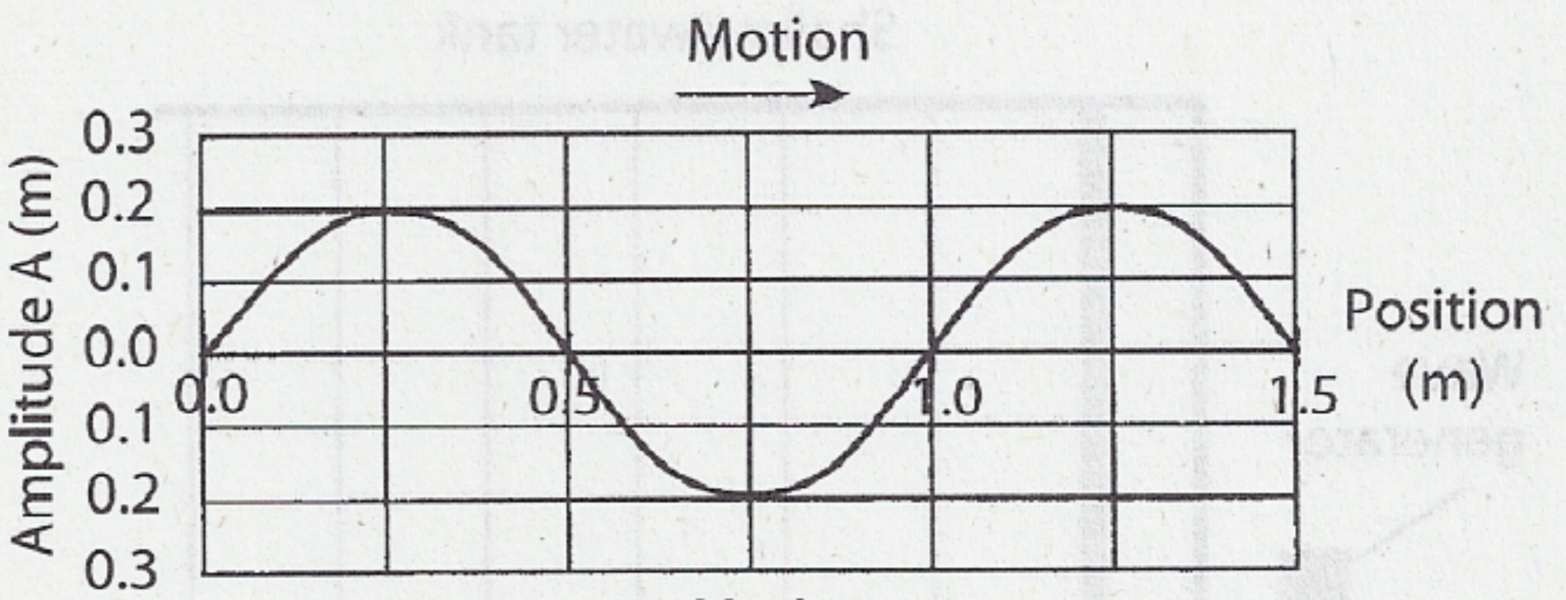
106 A barrier is placed in the tank as shown in the following diagram.

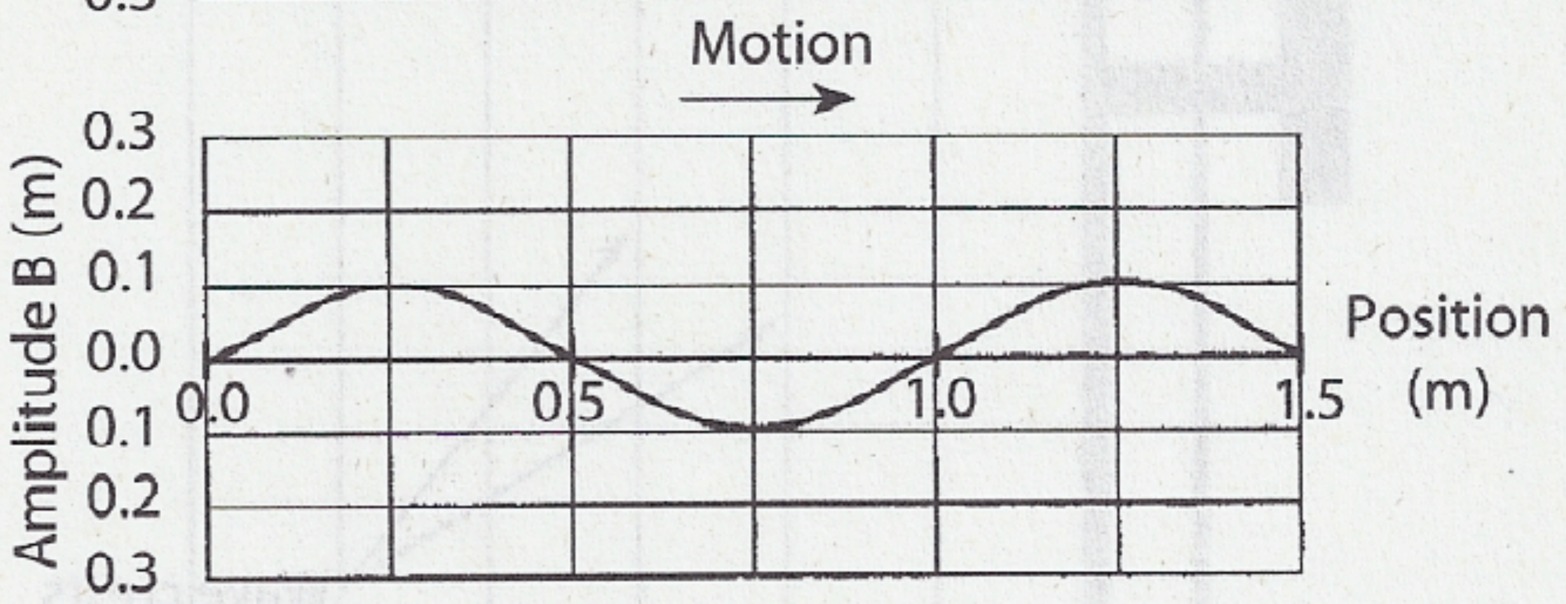


Use a protractor and a straight edge to construct an arrow to represent the direction of the velocity of the reflected waves. [1]

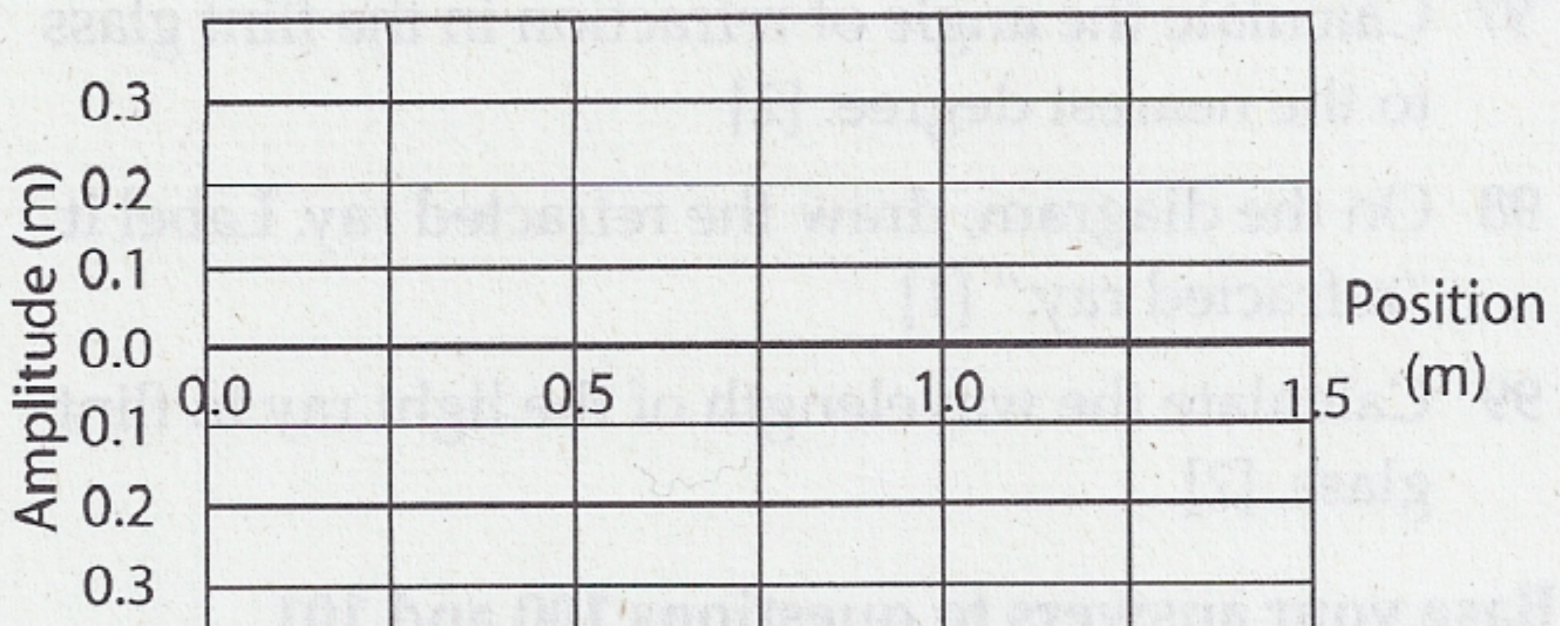
Base your answers to questions 107 through 109 on the information and diagram below.

Two waves, A and B, travel in the same direction in the same medium at the same time.





107 On the grid below draw the resultant wave produced by the superposition of waves A and B. [1]

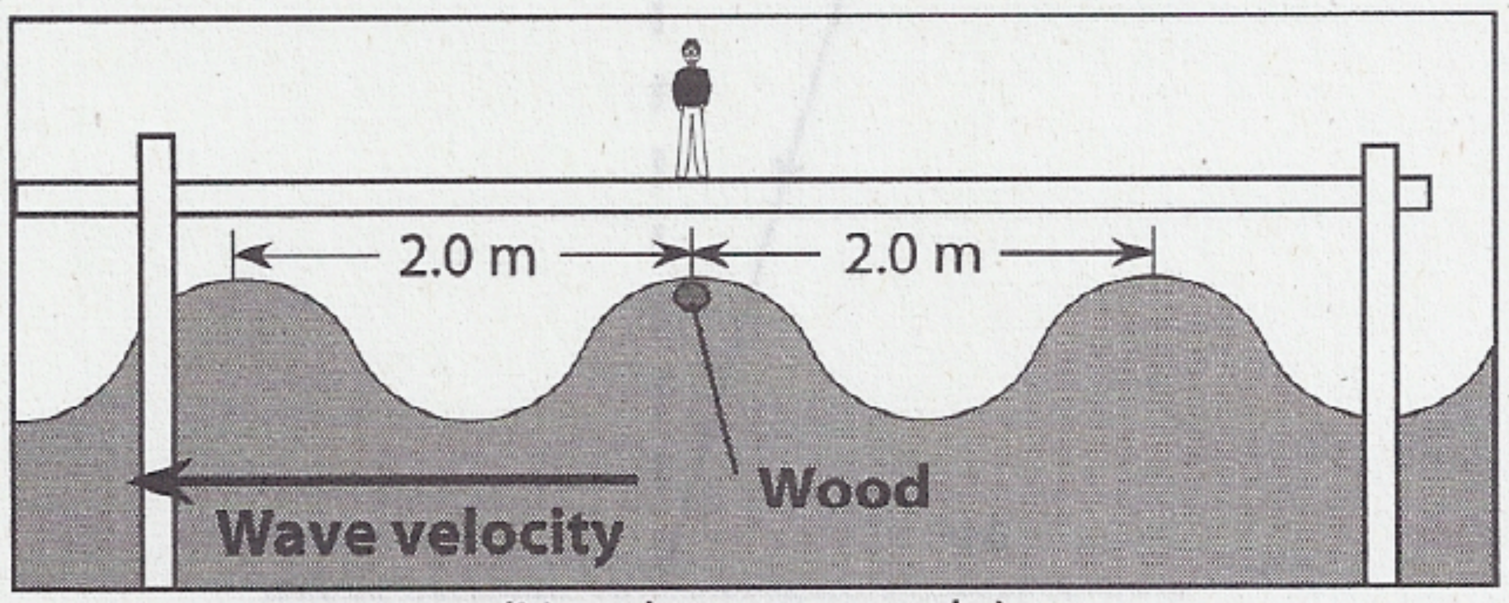


108 What is the amplitude of the resultant wave? [1]

109 What is the wavelength of the resultant wave? [1]

Base your answers to questions 110 and 111 on the information and diagram below.

A student standing on a dock observes a piece of wood floating on the water as shown below. As a water wave passes, the wood moves up and down, rising to the top of a wave crest every 5.0 seconds.



(Not drawn to scale)

110 Calculate the frequency of the passing water waves. [2]

111 Calculate the speed of the water waves. [2]