(1) $f(x) = \dfrac{x-1}{x+2}$

$f'(x) = \dfrac{(x+2)-(x-1)}{(x+2)^2}$

$f'(x) = \dfrac{3}{(x+2)^2}$

$0 = \dfrac{3}{(x+2)^2}$

$f'(x)$ + ND +

$f(x)$ inc -2 inc

Inc: $(-\infty,-2)$ + $(-2,\infty)$

no local extrema

(2) $f(x) = \dfrac{x+2}{x-3}$

$f'(x) = \dfrac{(x-3)-(x+2)}{(x-3)^2}$

$f'(x) = \dfrac{-5}{(x-3)^2}$

$0 = \dfrac{-5}{(x-3)^2}$

$x = 3$

$f'(x)$ − ND −

$f(x)$ dec 3 dec

Dec: $(-\infty, 3)$ + $(3,\infty)$

no local extrema

(3) $f(x) = x + \dfrac{4}{x}$

$f'(x) = 1 - \dfrac{4}{x^2}$

$0 = \dfrac{x^2-4}{x^2} \quad \dfrac{x^2}{x^2} \quad -\dfrac{4}{x^2}$

$x^2-4=0$ $\qquad x^2=0$

$x^2=\sqrt{4}$ $\qquad x=0$

$x=\pm 2$

critical values

$x=\pm 2$

$f'(x)$ + 0 − ND − 0 +

$f(x)$ -2 0 2

$\boxed{\begin{array}{l}\text{inc: } (-\infty,-2) + (2,\infty)\\ \text{dec: } (-2,0) + (0,2)\\ \text{local max at } x=-2\\ \text{local min at } x=2\end{array}}$

(4) $f(x) = \dfrac{9}{x} + x$

$f'(x) = -\dfrac{9}{x^2} + 1$

$= \dfrac{x^2}{x^2} - \dfrac{9}{x^2}$

$0 = \dfrac{x^2-9}{x^2}$

$x^2-9=0$ $\qquad x=\pm 3$

$x=0$

$f'(x)$ + 0 − ND − 0 +

$f(x)$ -3 0 3

$\boxed{\begin{array}{ll}\text{inc: } (-\infty,-3) + (3,\infty) & \\ \text{dec: } (-3,0) + (0,3) & \\ \text{local max at } x=-3 & \text{local min } x=3\end{array}}$

(5) $f(x) = 1 + \frac{1}{x} + \frac{1}{x^2}$

$f'(x) = -\frac{1}{x^2} - \frac{2}{x^3}$

$-\frac{x}{x^3} - \frac{2}{x^3} = \frac{-x-2}{x^3}$

$0 = \frac{-x-2}{x^3}$

$-x-2=0$ $\quad$ $x^3=0$

$x=-2$ $\quad$ $x=0$

$f'(x)$ $-$ 0 $+$ ND $-$

-2 $\quad$ 0

$f(x)$

$\boxed{\begin{array}{l} \text{inc: } (-2,0) \\ \text{dec: } (-\infty,-2) + (0,\infty) \\ \text{local min at } x=-2 \end{array}}$

(6) $f(x) = 3 - \frac{4}{x} - \frac{2}{x^2}$

$= \frac{4}{x^2} + \frac{4}{x^3}$

$0 = \frac{4x+4}{x^3}$

$4x+4=0$ $\quad$ $x^3=0$

$x=-1$ $\quad$ $x=0$

$f'(x)$ $+$ 0 $-$ ND $+$

$f(x)$ inc -1 dec 0 inc

inc: $(-\infty,-1) + (0,\infty)$

dec: $(-1,0)$

local max at $x=-1$

(7) $f(x) = \frac{x^2}{x-2}$

$f'(x) = \frac{(x-2)2x - x^2(1)}{(x-2)^2}$

$= \frac{2x^2-4x-x^2}{(x-2)^2}$

$= \frac{x^2-4x}{(x-2)^2}$

$x^2-4x=0$ $\quad$ $x-2=0$

$x(x-4)=0$

$f'(x)$ $+$ 0 $-$ ND $-$ 0 $+$

$f(x)$ inc 0 2 4

inc: $(-\infty,0) + (4,\infty)$

dec: $(0,2) + (2,4)$

local max at $x=0$ $\quad$ local min at $x=4$

$$f'(x) = \frac{(x+1)(2x) - x^2}{(x+1)^2}$$

$$= \frac{2x^2+2x-x^2}{(x+1)^2}$$

$$0 = \frac{x^2+2x}{(x+1)^2}$$

$f'(x)$ + 0 − ND − 0 +

−2 −1 0

inc: $(-\infty,-2) + (0,\infty)$

dec: $(-2,-1) + (-1,0)$

local max at $x = -2$

local min at $x = 0$

(9) $f(x) = x^4(x-6)^2$

$$f'(x) = x^4 \cdot 2(x-6) + (x-6)^2 \cdot 4x^3$$

$$= 2x^4(x-6) + 4x^3(x-6)^2$$

$$= 2x^3(x-6)\left[x + 2(x-6)\right]$$

$$= 2x^3(x-6)(x+2x-12)$$

$$= 2x^3(x-6)(3x-12)$$

$x^3 = 0$ $x-6=0$ $x-4=0$

$x=0$ $x=6$ $x=4$

$f'(x)$ − 0 + 0 − 0 +

$f(x)$ dec 0 inc 4 dec 6 inc

inc: $(0,4) + (6,\infty)$

dec: $(-\infty,0) + (4,6)$

local max at $x=4$

local min at $x=0$ $x=6$

(10) $f(x) = x^3(x-5)^2$

$$f'(x) = x^3 2(x-5) + (x-5)^2 \cdot 3x^2$$

$$f'(x) = 2x^3(x-5) + 3x^2(x-5)^2$$

$$f'(x) = x^2(x-5)\left[2x + 3(x-5)\right]$$

$$= x^2(x-5)(2x+3x-15)$$

$$0 = x^2(x-5)(5x-15)$$

$x=0$ $x=5$ $x=3$

$f'(x)$ + 0 + 0 − 0 +

$f(x)$ dec 0 inc 3 dec 5 inc

inc: $(-\infty,0)$ $(0,3)$ $(5,\infty)$

dec: $(3,5)$

local max at $x=3$

local min at $x = $

(11) $f(x) = 3(x-2)^{\frac{2}{3}} + 4$

$f'(x) = 2(x-2)^{-\frac{1}{3}}$

$f'(x) = \dfrac{2}{(x-2)^{\frac{1}{3}}}$

(12) $f(x) = 6(4-x)^{\frac{2}{3}} + 4$

* $f'x = 4(4-x)^{-\frac{1}{3}}$

$f'(x) = \dfrac{4}{(4-x)^{\frac{1}{3}}}$

$0 = x-2 \qquad x=2$

$f'(x)$ − ND +

$f(x)$ dec 2 inc

inc: $(2, \infty)$

dec: $(-\infty, 2)$

local min at $x = 2$

$0 = 4-x \qquad x=4$

$f'(x)$ + ND −

$f(x)$ inc 4 dec

inc: (

(13) $f(x) = \dfrac{x^2}{x^2+4}$

$$f'(x) = \frac{(x^2+4)2x - x^2(2x)}{(x^2+4)^2}$$

$$= \frac{\cancel{2x^3} + 8x - \cancel{2x^3}}{(x^2+4)^2}$$

$$f'(x) = \frac{8x}{(x^2+4)^2}$$

$$0 = \frac{8x}{(x^2+4)^2}$$

$x = 0$

$f'(x)$	$-$		$+$
$f(x)$	dec	0	inc

$\boxed{\text{inc: } (0, \infty) \quad \text{dec: } (-\infty, 0) \quad \text{local min at } x = 0}$

(14) $f(x) = \dfrac{x}{x+1}$ $x \neq -1$

$$f'(x) = \frac{(x+1)\cdot 1 - x(1)}{(x+1)^2}$$

$$= \frac{\cancel{x} + 1 - \cancel{x}}{(x+1)^2}$$

$$0 = \frac{1}{(x+1)^2}$$

$x = -1$

		asy	
$f'(x)$	$+$		$+$
$f(x)$	inc	-1	inc

inc: $(-\infty, -1)$ $(-1, \infty)$

no local max or min

(15) $f(x) = \sqrt{4 - x^2}$

$= (4 - x^2)^{\frac{1}{2}}$

$$f'(x) = \frac{1}{\cancel{2}}(4 - x^2)^{-\frac{1}{2}}(-\cancel{2}x)$$

$$0 = \frac{-x}{(4 - x^2)^{\frac{1}{2}}}$$

$x = 0$ $x = \pm 2$

$f'(x)$	no	0	$+$	0	$-$	0	no
$f(x)$	graph	-2	inc	0	dec	2	graph

$\boxed{\text{inc: } (-2, 0) \quad \text{dec: } (0, 2) \quad \text{Rel max at } x = 0}$

(18) $f(x) = x^2\sqrt{9-x^2}$

$= x^2(9-x^2)^{\frac{1}{2}}$

$f'(x) = x^2\left[\cancel{\tfrac{1}{2}}(9-x^2)^{-\frac{1}{2}}(-\cancel{2}x)\right] + (9-x^2)^{\frac{1}{2}}(2x)$

$= x(9-x^2)^{-\frac{1}{2}}\left[-x^2 + 2(9-x^2)\right]$

$= x(9-x^2)^{-\frac{1}{2}}\left[-x^2 + 18 - 2x^2\right]$

$= \dfrac{x\left[-3x^2+18\right]}{(9-x^2)^{\frac{1}{2}}}$ $\quad x=0 \quad$ $3x^2 = 18$ $\quad x = \pm 3$

$x^2 = 6$

$x = \pm\sqrt{6}$

no graph, und, +, −, +, −, und, no graph

−3 inc $-\sqrt{6}$ dec 0 inc $\sqrt{6}$ dec 3

inc: $(-3, -\sqrt{6})$ $(0, \sqrt{6})$ $\quad$ Rel max at $x = -\sqrt{6}$, $x = \sqrt{6}$

dec: $(-\sqrt{6}, 0)$ $(\sqrt{6}, 3)$ $\quad$ Rel mini at $x = 0$

(19) $f(x) = \dfrac{x+1}{x-1}$ $\quad$ $f'(x) = \dfrac{(x-1)\cdot 1 - (x+1)\cdot 1}{(x-1)^2}$

$x \neq 1$

$= \dfrac{\cancel{x} - 1 - \cancel{x} - 1}{(x-1)^2}$

$= \dfrac{-2}{(x-1)^2}$

−, −

dec 1 dec

dec: $(-\infty, 1)$ $(1, \infty)$

$1+x$

$$f'(x) = \frac{(1+x^2)2 - 2x(2x)}{(1+x^2)^2}$$

$$= \frac{2+2x^2-4x^2}{(1+x^2)^2}$$

$$= \frac{2-2x^2}{(1+x^2)^2}$$

$$= \frac{2(1-x^2)}{(1+x^2)^2}$$

$$0 = 1-x^2$$

$$x = \pm 1$$

$f'(x)$ − 0 + 0 −

$f(x)$ dec −1 inc 1 dec

$\boxed{\begin{array}{l} \text{dec: } (-\infty,-1)\ (1,\infty) \\ \text{inc: } (-1,1) \\ \text{local min at } x=-1 \\ \text{local max at } x=1 \end{array}}$

(21) $f(x) = x^3(x-3)^2$

$$f'(x) = x^3\left[2(x-3)\right] + (x-3)^2(3x^2)$$

$$= x^2(x-3)\left[2x + 3(x-3)\right]$$

$$= x^2(x-3)\left[2x+3x-9\right]$$

$$= x^2(x-3)(5x-9)$$

$x=0 \quad x=3 \quad x=\frac{9}{5}$

$f'(x)$ + + − +

$f(x)$ inc 0 $\frac{9}{5}$ 3

$\boxed{\begin{array}{l} \text{inc: } (-\infty,\frac{9}{5}),\ (3,\infty) \\ \text{dec: } (\frac{9}{5},3) \\ \text{local max at } \frac{9}{5} \\ \text{local min at } 3 \end{array}}$

(24) $f(x) = \dfrac{x^2-2x+4}{x-2}$ VA: $x=2$

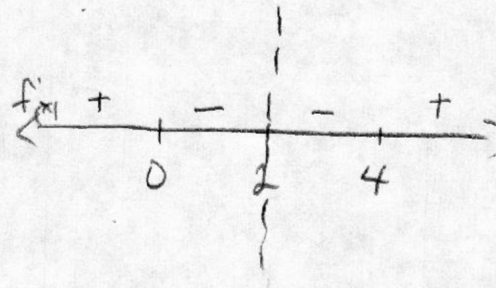
$$f'(x) = \frac{\overset{2(x-2)(x-1)}{(x-2)(2x-2)} - (x^2-2x+4)\cdot 1}{(x-2)^2}$$

$$= \frac{\overset{2(x^2-3x+2)}{2x^2-6x+4} - x^2+2x-4}{(x-2)^2}$$

$$0 = \frac{x^2-4x}{(x-2)^2} = \frac{x(x-4)}{(x-2)^2}$$

$0 = x(x-4)$ $\quad x-2=0$

$x=0 \quad x=4 \quad\quad x=2$



inc: $(-\infty, 0)$ $(4, \infty)$

dec: $(0, 2)$ $(2, 4)$

local max at $x=0$

local min at $x=4$

(25) $f(x) = \dfrac{2(x^2-9)}{x^2-4}$ VA: $x=\pm 2$

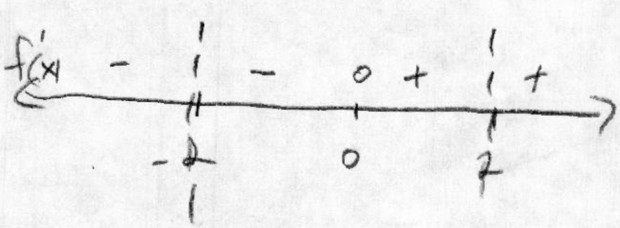
$$f'(x) = \frac{2(x^2-4)2x - 2(x^2-9)2x}{(x^2-4)^2}$$

$$= \frac{\cancel{4x^3} - 16x - \cancel{4x^3} + 36x}{(x^2-4)^2}$$

$$0 = \frac{20x}{(x^2-4)^2}$$

$20x = 0 \quad\quad x^2-4=0$

$x=0 \quad\quad x=\pm 2$



dec: $(-\infty, -2)$ $(-2, 0)$

inc: $(0, 2)$ $(2, \infty)$

local min at $x=0$