

$$(33) \int \frac{x}{3x^2-1} dx$$

$$\int \frac{1}{3x^2-1} x dx$$

$$u = 3x^2-1 \quad du = 6x dx$$

$$\int \frac{1}{u} \frac{1}{6} du$$

$$\frac{1}{6} \int \frac{1}{u} du$$

$$\frac{1}{6} \ln|u| + C$$

$$\frac{1}{6} \ln|3x^2-1| + C$$

$$(35) \int \frac{x^2}{(x^3-4)^2} dx$$

$$\int \frac{1}{(x^3-4)^2} \cdot x^2 dx$$

$$u = (x^3-4) \quad du = 3x^2 dx$$

$$\int \frac{1}{u^2} \left(\frac{1}{3} du\right)$$

$$\frac{1}{3} \int u^{-2} du$$

$$\frac{1}{3} \frac{u^{-2+1}}{-2+1} + C$$

$$\frac{1}{3} \frac{u^{-1}}{-1} + C$$

$$\frac{-1}{3u} + C = \frac{-1}{3(x^3-4)} + C$$

$$(34) \int \frac{4x}{\sqrt{1-3x^2}} dx$$

$$\int \frac{1}{(1-3x^2)^{\frac{1}{2}}} 4x dx$$

$$u = 1-3x^2 \quad du = -6x dx$$

$$4 \int \frac{1}{(1-3x^2)^{\frac{1}{2}}} x dx$$

$$4 \int u^{-\frac{1}{2}} \left(-\frac{1}{6} du\right)$$

$$-\frac{4}{6} \int u^{-\frac{1}{2}} du$$

$$-\frac{2}{3} \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

$$-\frac{2}{3} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$-\frac{2}{3} \cdot \frac{2}{1} u^{\frac{1}{2}} + C$$

$$\boxed{-\frac{4}{3} \sqrt{1-3x^2} + C}$$

$$(36) \int (x-1) e^{x^2-2x} dx$$

$$u = x^2-2x \quad du = (2x-2) dx$$
$$du = 2(x-1) dx$$

$$\int e^u \frac{1}{2} du$$

$$\frac{1}{2} \int e^u du$$

$$\frac{1}{2} e^u + C$$

$$\boxed{\frac{1}{2} e^{x^2-2x} + C}$$

$$\textcircled{29} \int \frac{e^{-\frac{1}{x}}}{x^2} dx$$

$$\int e^{-\frac{1}{x}} \frac{1}{x^2} dx$$

$$u = -\frac{1}{x} \quad du = \frac{1}{x^2} dx$$

$$\int e^u du$$

$$\boxed{e^{-\frac{1}{x}} + C}$$

$$\textcircled{31} \int \frac{\ln(5x)}{x} dx$$

$$\int \ln(5x) \cdot \frac{1}{x} dx$$

$$u = \ln 5x \quad du = \frac{1}{x} dx$$

$$\int u du$$

$$\frac{u^2}{2} + C$$

$$\frac{1}{2} [\ln(5x)]^2 + C$$

$$\textcircled{30} \int \frac{(\ln u)^3}{u} du$$

$$\int (\ln u)^3 \frac{1}{u} du$$

$$u = \ln u \quad du = \frac{1}{u} du$$

$$\int u^3 du$$

$$\frac{1}{4} u^4 + C$$

$$\boxed{\frac{1}{4} (\ln u)^4 + C}$$

$$\textcircled{32} \int \frac{(\ln x)^2}{2x} dx$$

$$\int (\ln x)^2 \frac{1}{2} \frac{1}{x} dx$$

$$u = \ln x \quad du = \frac{1}{x} dx$$

$$\frac{1}{2} \int u^2 du$$

$$\frac{1}{2} \frac{u^3}{3} + C$$

$$\frac{1}{6} u^3 + C$$

$$\boxed{\frac{1}{6} (\ln x)^3 + C}$$

$$(25) \int \frac{e^{2x}}{1+e^{2x}} dx$$

$$u = 1+e^{2x} \quad du = 2e^{2x} dx$$

$$\int \frac{1}{1+e^{2x}} e^{2x} dx$$

$$\int \frac{1}{u} \frac{1}{2} du$$

$$\frac{1}{2} \int \frac{1}{u} du$$

$$\frac{1}{2} \ln|u| + C$$

$$\boxed{\frac{1}{2} \ln|1+e^{2x}| + C}$$

$$(27) \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

$$u = \sqrt{x} \quad du = \frac{1}{2\sqrt{x}} dx$$

$$\int e^{\sqrt{x}} \frac{1}{\sqrt{x}} dx \quad dx = 2du$$

$$\int e^u 2du$$

$$2 \int e^u du$$

$$2e^u + C$$

$$\boxed{2e^{\sqrt{x}} + C}$$

$$(26) \int \frac{x}{3x^2-1} dx$$

$$\int \frac{1}{3x^2-1} x dx$$

$$u = 3x^2-1 \quad du = 6x dx$$

$$\int \frac{1}{u} \left(\frac{1}{6} du\right)$$

$$x dx = \frac{1}{6} du$$

$$\frac{1}{6} \int \frac{1}{u} du$$

$$\frac{1}{6} \ln|u| + C$$

$$\boxed{\frac{1}{6} \ln|3x^2-1| + C}$$

$$(28) \int e^{2x} (e^{2x} + 1)^3 dx$$

$$u = e^{2x} + 1 \quad du = 2e^{2x} dx$$

$$\int (e^{2x} + 1)^3 e^{2x} dx$$

$$\int u^3 \frac{1}{2} du$$

$$\frac{1}{2} \int u^3 du$$

$$\frac{1}{2} \frac{u^4}{4} + C$$

$$\frac{1}{8} u^4 + C$$

$$\boxed{\frac{1}{8} (e^{2x} + 1)^4 + C}$$

$$(15) \int \left( \frac{2}{\sqrt[3]{x}} - \sqrt[3]{x^2} \right) dx$$

$$\int (2x^{-\frac{1}{3}} - x^{\frac{2}{3}}) dx$$

$$\int 2x^{-\frac{1}{3}} - \int x^{\frac{2}{3}}$$

$$\frac{2}{2} \frac{x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} - \frac{3}{\frac{5}{3}}$$

$$3x^{\frac{2}{3}} - \frac{3}{5}x^{\frac{5}{3}} + C$$

$$(17) \int \left( \frac{e^x - 3x^2}{2} \right) dx$$

$$\int \frac{e^x}{2} - \int \frac{3x^2}{2} dx$$

$$\frac{1}{2}e^x - \frac{3}{2} \cdot \frac{x^3}{3}$$

$$\frac{1}{2}e^x - \frac{1}{2}x^3 + C$$

$$(19) \int (3x^{-2} - x^{-1}) dx$$

$$\int 3x^{-2} - \int x^{-1}$$

$$\frac{3x^{-1}}{-1} - \ln x + C$$

$$-\frac{3}{x} - \ln x + C$$

$$(16) \int \left( \frac{e^x - 3x}{4} \right) dx$$

$$\int \frac{1}{4}e^x - \int \frac{3}{4}x$$

$$\frac{1}{4}e^x - \frac{3x^2}{4 \cdot 2} + C$$

$$\frac{1}{4}e^x - \frac{3}{8}x^2 + C$$

$$(18) \int (2z^{-3} + z^{-2} + z^{-1}) dz$$

$$\frac{2z^{-2}}{-2} + \frac{z^{-1}}{-1} + \ln z$$

$$-\frac{1}{z^2} - \frac{1}{z} + \ln z + C$$

$$(20) \int \frac{1}{x-5} dx = \ln|x-5| + C$$

$$(21) \int \frac{1}{x+1} dx = \ln|x+1| + C$$

$$(22) \int \frac{4}{x} dx = 4 \ln|x| + C$$

$$(23) \int 2e^x dx = 2e^x + C$$

$$(24) \int (e^x + x) dx = \int e^x + \int x$$

$$e^x + \frac{1}{2}x^2 + C$$

$$\textcircled{1} \int 7 dx = \boxed{7x + C}$$

$$\textcircled{2} \int x^6 dx = \boxed{\frac{1}{7} x^7 + C}$$

5.3

$$\textcircled{3} \int 8t^3 dt = \frac{8t^4}{4} \\ = \boxed{2t^4 + C}$$

$$\textcircled{4} \int (2u+1) du = \int 2u + \int 1 du \\ = \frac{2u^2}{2} + u + C \\ = \boxed{u^2 + u + C}$$

$$\textcircled{5} \int (5^4 - 85^5) ds$$

$$\int 5^4 - \int 85^5 \\ \frac{5^5}{5} - \frac{85^6}{6} + C$$

$$\boxed{\frac{1}{5} 5^5 - \frac{4}{3} 5^6 + C}$$

$$\textcircled{6} \int 3e^t dt \\ \boxed{3e^t + C}$$

$$\textcircled{8} \int 3 \cdot \frac{1}{s} ds \\ \boxed{3 \ln s + C}$$

$$\textcircled{7} \int 2z^{-1} dz \\ \int 2 \cdot \frac{1}{z} dz \\ \boxed{2 \ln z + C}$$

$$\textcircled{9} \int (24 - 6x) dx$$

$$\int 24 - 6 \int x$$

$$24x - \frac{6x^2}{2}$$

$$\boxed{24x - 3x^2 + C}$$

$$\textcircled{10} \int (e^x + 3) dx$$

$$\int e^x + \int 3$$

$$\boxed{e^x + 3x + C}$$

$$\textcircled{11} \int (5x^{-1} + 1) dx$$

$$\int 5 \cdot \frac{1}{x} + \int 1$$

$$\boxed{5 \ln x + x + C}$$

$$\textcircled{12} \int \left( \frac{4}{v} + \frac{v}{4} \right) dv$$

$$4 \ln v + \frac{1}{4} \cdot \frac{v^2}{2}$$

$$\boxed{4 \ln v + \frac{1}{8} v^2 + C}$$

$$\textcircled{13} \int \frac{1}{2u^5} du$$

$$\int \frac{1}{2} u^{-5} du$$

$$\frac{1}{2} \cdot \frac{u^{-4}}{-4}$$

$$\boxed{\frac{-1}{8u^4} + C}$$

$$\textcircled{14} \int \left( 10x^4 - \frac{8}{x^5} - 2 \right) dx$$

$$\frac{10x^5}{5} - 8 \frac{x^{-4}}{-4} - 2x$$

$$\boxed{2x^5 + \frac{2}{x^4} - 2x + C}$$