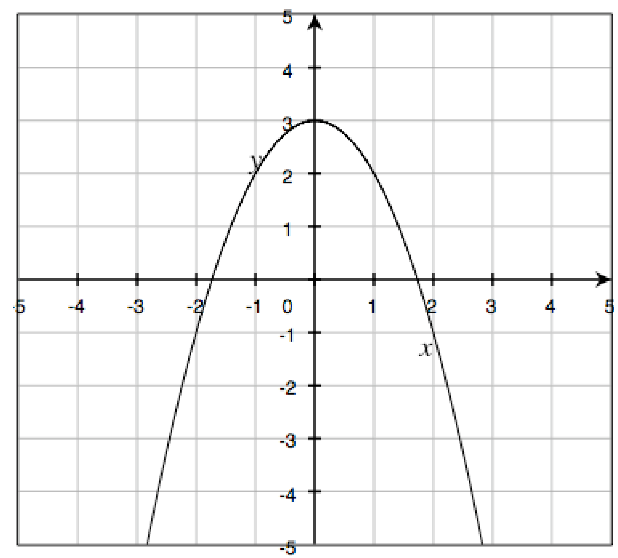
Limits

In Calculus, the LIMIT is the intended height of the function.

What are the limits for the following function?

$$f(x) = -x^2 + 3$$



1) $\lim_{x \to 2} f(x) =$

2) $\lim_{x \to -1} f(x) =$

3) $\lim_{x \to 0} f(x) =$

Definition of a Limit

If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, then

$$\lim_{x \to c} f(x) = L$$

Which is read as "the limit of $f(x)$ as x approaches c is L."

Limit of a polynomial Function

If p is a polynomial function and c is any real number, then

$$\lim_{x \to c} p(x) = p(c)$$

Find the limit:

1) $\lim_{x \to 1} (2x^2 - x + 4)$

2) $\lim_{x \to 2} (x^2 + 2x - 3)$

When the function is not a Polynomial Function

Find the limit: $$\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3}$$

Direct substitution fails because both the numerator and the denominator are zero when $x = -3$. There is a hole in the graph at $x = -3$. However, if we want the limit at any other value, direct substitution will work.

So how do we find the limit in this case?

$$\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3}$$

1) Factor
2) Cancel
3) Direct Substitution

1) $\lim_{x \to -3} \frac{(x+3)(x-2)}{x+3}$

2) $\lim_{x \to -3} (x-2)$

3) $\lim_{x \to -3} (-3-2)$

4) -5

Find the limit:

$$\lim_{x \to 3} \frac{x^2 + x - 6}{x + 3}$$

1) $$\lim_{x \to -1} \frac{x^2 + x - 6}{x + 3}$$

2) $$\lim_{x \to 0} \frac{x^2 + x - 6}{x + 3}$$

Find the limit:

1) $\lim_{x \to 3} \frac{x^2 + x - 12}{x - 3}$

2) $\lim_{x \to 2} \frac{x - 2}{x^2 - 4x + 4}$

3) $\lim_{x \to 2} \frac{x^2 + x - 6}{x + 3}$

4) $\lim_{x \to 0} \frac{x - 2}{x^2 - 4x + 4}$

What about this function? Does direct substitution work?

$$\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x}$$

Another method to try is using the conjugate. The following expressions are conjugates:

$$x+y,\ x-y;\ \sqrt{x}+3,\ \sqrt{x}-3$$

What is the conjugate of the numerator?

The conjugate is........... $\sqrt{x+1}+1$

$$\frac{\sqrt{x+1}-1}{x} = \left(\frac{\sqrt{x+1}-1}{x}\right)\left(\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right)$$

$$= \left(\frac{(x+1)-1}{x(\sqrt{x+1}+1)}\right)$$

$$= \left(\frac{(x+1)-1}{x(\sqrt{x+1}+1)}\right)$$

$$= \left(\frac{1}{\sqrt{x+1}+1)}\right), x \neq 0$$

Now you can use substitution

$$\lim_{x \to 0}\left(\frac{\sqrt{x+1}-1}{x}\right) = \lim_{x \to 0}\frac{1}{\sqrt{x+1}+1}$$

$$= \frac{1}{1+1}$$

$$= \frac{1}{2}$$

One sided limits

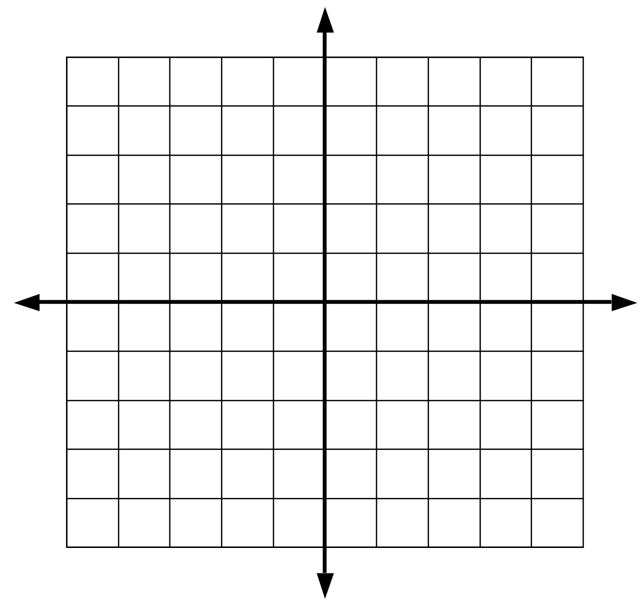
Find the limit: $f(x)=\begin{cases} x+1 & if\ x<0 \\ x^2 & if\ x\geq 0 \end{cases}$

$$\lim_{x\to 0} f(x) = ?$$

You must now find the right-handed and the left-handed limit.

$$\lim_{x\to 0^{-1}} (x+1)$$

$$\lim_{x\to 0^{+}} x^2$$



In order for the limit to exist, the right and left hand limits must be the same.

$$f(x)=\begin{cases} x+1 & \text{if } x<0 \\ x^2 & \text{if } x\geq 0 \end{cases}$$

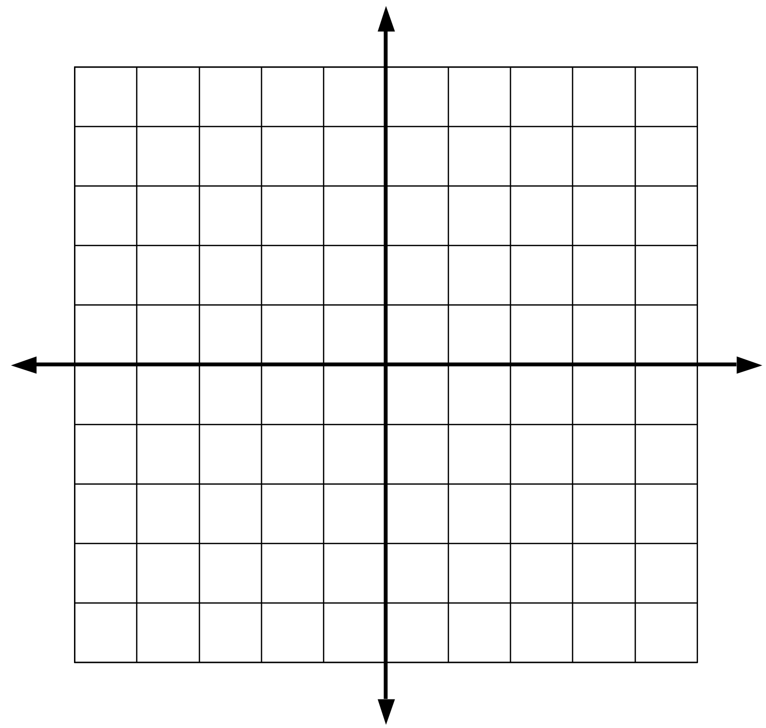
$$\lim_{x\to 0} f(x) = ?$$

Since the right-hand limit is _____ and the left-hand limit is _____, the limit does not exist.

DNE

Find the limit:

$$f(x)=\begin{cases}-x^2+1 & if\ x<-1\\ \sqrt{x+1} & if\ x\geq -1\end{cases}$$



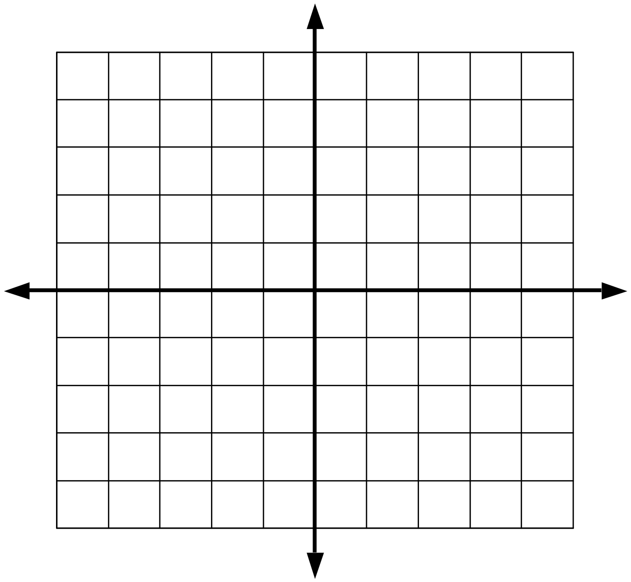
Graph

$$\lim_{x\to -1^{-1}} f(x)=$$

$$\lim_{x\to 1^{+!}} f(x)=$$

Find the limit:

$$f(x) = \begin{cases} x^2 + 1 & if\ x < 0 \\ 2x + 1 & if\ x > 0 \end{cases}$$

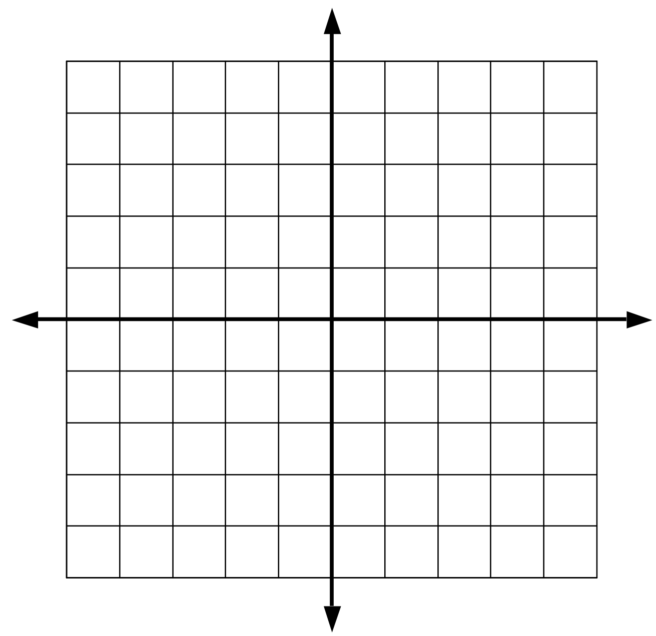


Remember, the limit is the intended height of the function

Another way a limit fails to exist: If f(x) increase or decreases without bound as x approaches c

Example: $\lim\limits_{x \to 2} \dfrac{3}{x-2}$

Graph



$$\lim_{x \to 2^{-1}} \frac{3}{x-2} =$$

$$\lim_{x \to 2^{+1}} \frac{3}{x-2} =$$

Find the limit if it exists:

1) $\lim_{x \to -2} \frac{5}{x+2}$

2) $\lim_{x \to 3} \frac{5}{x+2}$

3) $\lim_{x \to -4} \frac{5}{x+2}$

4) $\lim_{x \to 6} \frac{5}{x+2}$