Review 2.2, 2.4 - 2.6

1) $f(x) = 2\sqrt{x} + 1$

$= 2x^{\frac{1}{2}} + 1$

$f'(x) = \frac{1}{2} \cdot 2x^{-\frac{1}{2}} + 0$

$= x^{-\frac{1}{2}}$

$\boxed{f'(x) = \frac{1}{\sqrt{x}}}$

2) $f(x) = \frac{2}{3x^2}$

$= \frac{2}{3} x^{-2}$

$f'(x) = -2 \cdot \frac{2}{3} x^{-3}$

$= \frac{-4}{3} x^{-3}$

$\boxed{f'(x) = \frac{-4}{3x^3}}$

3) $f(x) = 2x^{-3} + 4 - x^{\frac{1}{2}}$

$f'(x) = -6x^{-4} + 0 - \frac{1}{2} x^{-\frac{1}{2}}$

$\boxed{f'(x) = \frac{-6}{x^4} - \frac{1}{2\sqrt{x}}}$

4) $f(x) = \frac{6x-5}{x^2+1}$

$f'(x) = \frac{(x^2+1) \cdot 6 - (6x-5)(2x)}{(x^2+1)^2}$

$= \frac{6x^2 + 6 - 12x^2 + 10x}{(x^2+1)^2}$

$\boxed{f'(x) = \frac{-6x^2 + 10x + 6}{(x^2+1)^2}}$

5) $f(x) = x(1-4x^2)^2$

$f'(x) = x\left[2(\cancel{1-4x^2})(-8x)\right] + (1-4x^2)^2 \cdot 1$

$= (1-4x^2)\left[-16x^2 + 1 - 4x^2\right]$

$= (1-4x^2)(-20x^2+1)$

$\boxed{f'(x) = (1-4x^2)(1-20x^2)}$

6) $f(x) = (3x^2+7)(x^2-2x)$

$f'(x) = (3x^2+7)(2x-2) + (x^2-2x)(6x)$

$= 6x^3 - 6x^2 + 14x - 14 + 6x^3 - 12x^2$

$\boxed{f'(x) = 12x^3 - 18x^2 + 14x - 14}$

7) $f(x) = \left(x^2 + \frac{1}{x}\right)^5$

$= (x^2 + x^{-1})^5$

$f'(x) = 5(x^2 + x^{-1})^4 (2x - x^{-2})$

$\boxed{f'(x) = 5\left(x^2 + \frac{1}{x}\right)^4 \left(2x - \frac{1}{x^2}\right)}$

8) $f(x) = (6x - x^3)^2$

$\boxed{f'(x) = 2(6x - x^3)(6 - 3x^2)}$

9) $f(x) = \dfrac{1}{x^2+3x-1}$

$f(x) = (x^2+3x-1)^{-1}$

$f'(x) = -1\,(x^2+3x-1)^{-2}(2x+3)$

$= \dfrac{-(2x+3)}{(x^2+3x-1)^2}$

11) $f(x) = \dfrac{1}{(x^2-3x)^2}$

$= (x^2-3x)^{-2}$

$f'(x) = -2(x^2-3x)^{-3}(2x-3)$

$\boxed{f'(x) = \dfrac{-2(2x-3)}{(x^2-3x)^3}}$

13) $f(x) = x\left(1-\dfrac{2}{x+1}\right)$

$= x - \dfrac{2x}{x+1}$

$f'(x) = 1 - \dfrac{(x+1)2-(2x)(1)}{(x+1)^2}$

$= 1 - \dfrac{\cancel{2x}+2-\cancel{2x}}{(x+1)^2}$

$\boxed{f'(x) = 1 - \dfrac{2}{(x+1)^2}}$

10) $f(x) = x\,(2x+3)^{\frac{1}{2}}$

$f'(x) = x\left[\cancel{\tfrac{1}{2}}(2x+3)^{-\frac{1}{2}}\cancel{(2)}\right] + (2x+3)^{\frac{1}{2}}$

$= (2x+3)^{-\frac{1}{2}}\left[x + 2x+3\right]$

$= (2x+3)^{-\frac{1}{2}}(3x+3)$

$= \dfrac{3x+3}{(2x+3)^{\frac{1}{2}}}$

$\boxed{f'(x) = \dfrac{3(x+1)}{\sqrt{2x+3}}}$

12) $h(x) = -3\sqrt[4]{2-9x}$

$= -3\,(2-9x)^{\frac{1}{4}}$

$= -\dfrac{3}{4}(2-9x)^{-\frac{3}{4}}(-9)$

$= \dfrac{27}{4}(2-9x)^{-\frac{3}{4}}$

$\boxed{h'(x) = \dfrac{27}{4(2-9x)^{\frac{3}{4}}}}$

14) $g(x) = \dfrac{x^2-2x+5}{\sqrt{x}}$

$= \dfrac{x^2}{x^{\frac{1}{2}}} - \dfrac{2x}{x^{\frac{1}{2}}} + \dfrac{5}{x^{\frac{1}{2}}}$

$f(x) = x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + 5x^{-\frac{1}{2}}$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}} - x^{-\frac{1}{2}} - \frac{5}{2}x^{-\frac{3}{2}}$

$\boxed{f'(x) = \dfrac{3}{2}\sqrt{x} - \dfrac{1}{\sqrt{x}} - \dfrac{5}{2x^{\frac{3}{2}}}}$

(15) $g(x) = \sqrt[3]{x}\,(x+1)$

$= x^{\frac{1}{3}}(x+1)$

$g(x) = x^{\frac{4}{3}} + x^{\frac{1}{3}}$

$g'(x) = \frac{4}{3}x^{\frac{1}{3}} + \frac{1}{3}x^{-\frac{2}{3}}$

$\boxed{g'(x) = \frac{4}{3}\sqrt[3]{x} + \frac{1}{3x^{\frac{2}{3}}}}$

(17) $d(x) = (x^2+4)^{\frac{2}{3}}$

$d'(x) = \frac{2}{3}(x^2+4)^{-\frac{1}{3}}(2x)$

$\boxed{d'(x) = \frac{4x}{3\sqrt[3]{x^2+4}}}$

(16) $f(x) = (3x^3+4x)(x-5)(x+1)$

$= (3x^3+4x)(x^2-4x-5)$

$f'(x) = (3x^3+4x)(2x-4) + (x^2-4x-5)(9x^2+4)$

$= 6x^4 - 12x^3 + 8x^2 - 16x + 9x^4 + 4x^2 - 36x^3 - 16x - 45x^2 - 20$

$\boxed{f'(x) = 15x^4 - 48x^3 - 33x^2 - 32x - 20}$

(18) $f(x) = \left(\frac{3x-1}{x^2+3}\right)^2$

$f'(x) = 2\left(\frac{3x-1}{x^2+3}\right)\left(\frac{(x^2+3)\cdot 3 - (3x-1)(2x)}{(x^2+3)^2}\right)$

$= 2\left(\frac{3x-1}{x^2+3}\right)\frac{3x^2+9-6x^2+2x}{(x^2+3)^2}$

$\boxed{f'(x) = \frac{2(3x-1)(-3x^2+2x+9)}{(x^2+3)^3}}$

(19) $f(x) = x^2\sqrt{1-x^2}$

$f(x) = x^2(1-x^2)^{\frac{1}{2}}$

$f'(x) = x^2\left[\cancel{\tfrac{1}{2}}(1-x^2)^{-\frac{1}{2}}(-\cancel{2}x)\right] + (1-x^2)^{\frac{1}{2}}(2x)$

$= x(1-x^2)^{-\frac{1}{2}}\left[-x^2 + 2(1-x^2)\right]$

$= x(1-x^2)^{-\frac{1}{2}}(-x^2+2-2x^2)$

$= x(1-x^2)^{-\frac{1}{2}}(2-3x^2)$

$$\boxed{f'(x) = \frac{x(2-3x^2)}{\sqrt{1-x^2}}}$$

(20) $f(x) = (x^2-1)^2$ $(-2, 9)$ $m = -24$ $\boxed{y-9 = -24(x+2)}$

$f'(x) = 2(x^2-1)2x$

$f'(x) = 4x(x^2-1)$

$f'(-2) = 4(-2)\left[(-2)^2-1\right]$

$= -8(4-1)$

$= -8(3)$

$m = -24$

(21) $h(x) = \dfrac{2x+1}{x-1}$ $(2,5)$ $m = -3$ $\boxed{y-5 = -3(x-2)}$

$h'(x) = \dfrac{(x-1)2 - (2x+1)\cdot 1}{(x-1)^2}$

$= \dfrac{\cancel{2x} - 2 - \cancel{2x} - 1}{(x-1)^2}$

$f'(x) = \dfrac{-3}{(x-1)^2}$

$f'(2) = \dfrac{-3}{(2-1)^2}$

$m = -3$

(22) $f(x) = \dfrac{36}{(3-x)^2}$ $(0,4)$ $m = \dfrac{8}{3}$

$f(x) = 36(3-x)^{-2}$

$f'(x) = -72(3-x)^{-3}(-1)$ $y-4 = \dfrac{8}{3}x$

$\boxed{y = \dfrac{8}{3}x + 4}$

$= \dfrac{72}{(3-x)^3}$

$= \dfrac{72}{(3-0)^3}$

$m = \dfrac{72}{27}$

23) $h(x) = (x^2-9)\sqrt{x+2}$ $(-1,-8)$ $m=-6$ $\boxed{y+8=-6(x+1)}$

$= (x^2-9)(x+2)^{\frac{1}{2}}$

$h'(x) = (x^2-9)\left[\frac{1}{2}(x+2)^{-\frac{1}{2}}\right] + (x+2)^{\frac{1}{2}}(2x)$

$= (x+2)^{-\frac{1}{2}}\left[\frac{1}{2}(x^2-9) + (x+2)2x\right]$

$h'(-1) = \dfrac{\frac{1}{2}(-8) + (-1+2)(-2)}{\sqrt{1}}$

$= \dfrac{-4+(-2)}{1}$

$m = -6$

24) $f(x) = \dfrac{x+1}{(2x-3)^{\frac{1}{2}}}$ $(2,3)$ $m = -2$

$\boxed{y-3=-2(x-2)}$

$f'(x) = \dfrac{(2x-3)^{\frac{1}{2}}\cdot 1 - (x+1)\left[\frac{1}{\cancel{2}}(2x-3)^{-\frac{1}{2}}\cdot\cancel{2}\right]}{2x-3}$

$= \dfrac{(2x-3)^{-\frac{1}{2}}\left[2x-3-(x+1)\right]}{(2x-3)}$

$= \dfrac{2x-3-x-1}{(2x-3)^{\frac{3}{2}}}$

$= \dfrac{x-4}{(2x-3)^{\frac{3}{2}}}$

$f'(2) = \dfrac{2-4}{1^{\frac{3}{2}}}$

$= \dfrac{-2}{1}$

$= -2$

(25) $S(x) = \dfrac{2x}{\sqrt{x+1}}$ (3, 3) $m = \frac{5}{8}$ $\boxed{y-3 = \frac{5}{8}(x-3)}$

$S(x) = \dfrac{2x}{(x+1)^{\frac{1}{2}}}$

$S'(x) = \dfrac{(x+1)^{\frac{1}{2}} \cdot 2 - \cancel{2}x\left[\cancel{\tfrac{1}{2}}(x+1)^{-\frac{1}{2}}\right]}{(x+1)}$

$= \dfrac{(x+1)^{-\frac{1}{2}}\left[2(x+1) - x\right]}{x+1}$

$= \dfrac{2x+2-x}{(x+1)^{\frac{3}{2}}}$

$S'(x) = \dfrac{x+2}{(x+1)^{\frac{3}{2}}}$

$S'(3) = \dfrac{3+2}{(3+1)^{\frac{3}{2}}}$

$= \dfrac{5}{4^{\frac{3}{2}}}$

$m = \dfrac{5}{8}$

(26) $g(x) = 3x^2 + 2x + 5$

$g''(2)$

$g'(x) = 6x + 2$

$g''(x) = 6$

$\boxed{g''(2) = 6}$

(27) $c(x) = \sqrt{4-x}$

$c(x) = (4-x)^{\frac{1}{2}}$

$c'(x) = \frac{1}{2}(4-x)^{-\frac{1}{2}}(-1)$

$c''(x) = -\frac{1}{4}(4-x)^{-\frac{3}{2}}(-1)$

$c'''(x) = \frac{3}{8}(4-x)^{-\frac{5}{2}}(-1)$

$c'''(-5) = -\frac{3}{8}(4+5)^{-\frac{5}{2}}$

$= \frac{-3}{8(\sqrt{9})^5}$

$= \frac{-\cancel{3}}{8 \cdot 3^{\cancel{5}4}}$

$= \frac{-1}{8 \cdot 81}$

$\boxed{c'''(-5) = \frac{-1}{648}}$

(28) $f(x) = \dfrac{3}{16x^2}$

$f(x) = \dfrac{3}{16} x^{-2}$

$f'(x) = \cancel{-2} \dfrac{3}{\cancel{16}_{8}} x^{-3}$

$f'(x) = -\dfrac{3}{8} x^{-3}$

$f''(x) = -3\left(-\dfrac{3}{8}\right) x^{-4}$

$f''(x) = \dfrac{9}{8} x^{-4}$

$f'''(x) = -4\left(\dfrac{9}{8}\right) x^{-5}$

$= \dfrac{-9}{2} x^{-5}$

$\boxed{f'''(x) = \dfrac{-9}{2x^5}}$

(29) $w(x) = \dfrac{3}{4x^2}$

$= \dfrac{3}{4} x^{-2}$

$w'(x) = -\cancel{2} \dfrac{3}{\cancel{4}_{2}} x^{-3}$

$w'(x) = \dfrac{-3}{2} x^{-3}$

$w''(x) = \dfrac{9}{2} x^{-4}$

$w'''(x) = -4^{2} \dfrac{9}{\cancel{2}} x^{-5}$

$= -18 x^{-5}$

$\boxed{w'''(x) = \dfrac{-18}{x^5}}$

(30) $f(x) = 5x(x+4)^3$

$f'(x) = 5x\left[3(\cancel{x+4})^2\right] + (x+4)^3 \cdot 5$

$= 5(x+4)^2\left[3x + x+4\right]$

$= 5(x+4)^2(4x+4)$

$= 20(x+4)^2(x+1)$

$f''(x) = 20\ (x+4)^2 \cdot 1 + (x+1)\left[2(x+4)\right]$

$= 20\ (x+4)\left[(x+4) + 2(x+1)\right]$

$= 20\ (x+4)(3x+6)$

$= 60(x+4)(x+2) \longrightarrow f'''(x) = 60\ (\cancel{x}+4)\cdot 1 + (\cancel{x}+2)\cdot 1$

$= 60(x+4+x+2)$

$= 60(2x+6)$

$= 60 \cdot 2(x+3)$

$= \boxed{120(x+3)}$

(31) $f(x) = x^3 - 9x^2 + 27x - 27$

$f'(x) = 3x^2 - 18x + 27$

$f''(x) = 6x - 18$

$0 = 6x - 18$

$6x = 18$

$\boxed{x = 3}$

(33) $f(x) = \dfrac{x}{x^2+3}$

$f'(x) = \dfrac{(x^2+3)\cdot 1 - x(2x)}{(x^2+3)^2}$

$= \dfrac{x^2+3-2x^2}{(x^2+3)^2}$

$f'(x) = \dfrac{-x^2+3}{(x^2+3)^2}$

$f''(x) = \dfrac{(x^2+3)^2(-2x) - (-x^2+3)\boxed{2(x^2+3)(2x)}}{(x^2+3)^4}$

$= \dfrac{2x(x^2+3)[-(x^2+3) - 2(-x^2+3)]}{(x^2+3)^4}$

$= \dfrac{2x(x^2+3)(-x^2-3+2x^2-6)}{(\quad)^4}$

$f''(x) = \dfrac{2x\cancel{(x^2+3)}(x^2-9)}{(x^2+3)^{\cancel{4}3}} = \dfrac{2x(x-3)(x+3)}{(x^2+3)^3} \rightarrow$ (33) $\boxed{x=0 \quad x = \pm 3}$

(32) $f(x) = x(x^2-1)^{\frac{1}{2}}$

$f'(x) = x\left[\frac{1}{\cancel{2}}(x^2-1)^{-\frac{1}{2}}(\cancel{2}x)\right] + (x^2-1)^{\frac{1}{2}}\cdot 1$

$= (x^2-1)^{-\frac{1}{2}}\left[x^2 + x^2 - 1\right]$

$= \dfrac{2x^2-1}{(x^2-1)^{\frac{1}{2}}}$

$f''(x) = \dfrac{(x^2-1)^{\frac{1}{2}}4x - (2x^2-1)\left[\frac{1}{\cancel{2}}(x^2-1)^{-\frac{1}{2}}\cancel{2}x\right]}{(x^2-1)}$

$= \dfrac{x(x^2-1)^{-\frac{1}{2}}\left[(x^2-1)4 - (2x^2-1)\right]}{(x^2-1)}$

$= \dfrac{x(x^2-1)^{-\frac{1}{2}}(4x^2-4-2x^2+1)}{(x^2-1)}$

$f''(x) = \dfrac{x(2x^2-3)}{(x^2-1)^{\frac{3}{2}}}$

$0 = \dfrac{x(2x^2-3)}{(x^2-1)^{\frac{3}{2}}}$

$x = 0 \qquad 2x^2 - 3 = 0$

$2x^2 = 3$

$x^2 = \frac{3}{2}$

$\boxed{\cancel{x=0} \qquad x = \pm\sqrt{\frac{3}{2}}}$

(34) $f(x) = (x+3)(x-4)(x+5)$

$f'(x) = (x+3)(x-4) + (x+3)(x+5) + (x-4)(x+5)$

$= x^2 - x - 12 + x^2 + 8x + 15 + x^2 + x - 20$

$f'(x) = 3x^2 + 8x - 17$

$f''(x) = 6x + 8$

$0 = 6x + 8$

$\frac{6x}{6} = \frac{-8}{6}$

$\boxed{x = \frac{-4}{3}}$

35 $f(x) = 3(2-x^2)^3$

$f'(x) = 9(2-x^2)^2(-2x)$

$f'(x) = -18x(2-x^2)^2$

$f''(x) = -18x\left[2\cancel{(2-x^2)^1}(-2x) + (2-x^2)^2(-18)\right.$

$= -18(2-x^2)\left[-4x^2 + 2 - x^2\right]$

$f''(x) = -18(2-x^2)(2-5x^2)$

$0 = -18(2-x^2)(2-5x^2)$

$2-x^2=0 \qquad 2-5x^2=0$

$x^2 = 2 \qquad 5x^2 = 2$

$\boxed{x = \pm\sqrt{2}} \qquad x^2 = \frac{2}{5}$

$\boxed{x = \pm\sqrt{\frac{2}{5}}}$

36 $f(x) = \dfrac{x+1}{x-1}$

$f'(x) = \dfrac{(x-1)\cdot 1 - (x+1)\cdot 1}{(x-1)^2}$

$= \dfrac{\cancel{x}-1-\cancel{x}-1}{(x-1)^2}$

$f'(x) = \dfrac{-2}{(x-1)^2}$

$f'(x) = -2(x-1)^{-2}$

$f''(x) = 4(x-1)^{-3}\cdot 1$

$0 = \dfrac{4}{(x-1)^3}$

$\boxed{\text{Not possible}}$